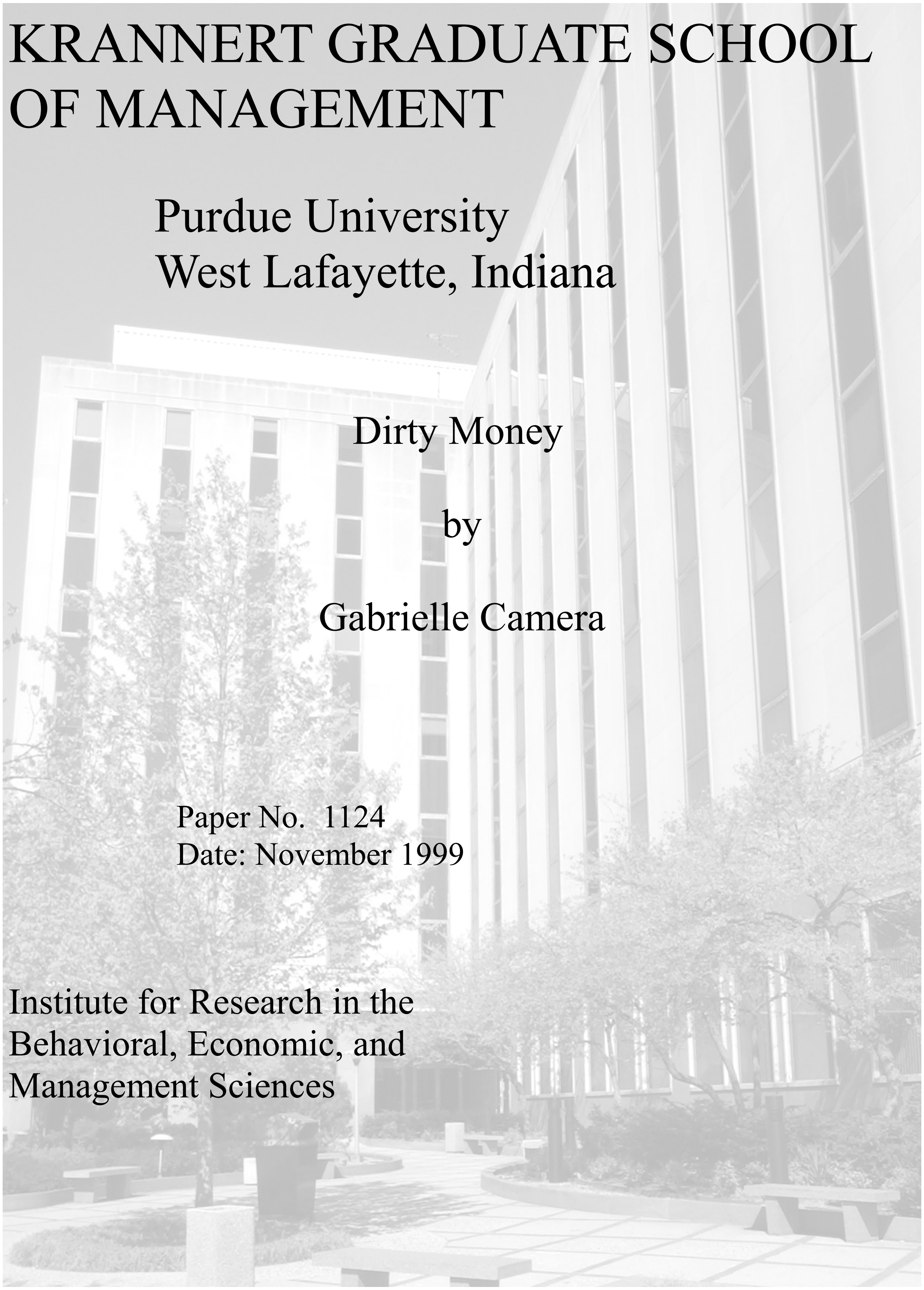

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Dirty Money

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ABSTRACT

An inter-governmental body is encouraging the replacement of currency with the objective of discouraging illegal economic activities. This policy is analyzed in a search-theoretic monetary economy with government enforcement. Individuals choose legal or illegal production and settle trades via quid-pro-quo or monetary exchange. Stationary equilibria exist where legal and illegal production coexist and money is used for purchases of both types of commodities. Larger money stocks increment the extent of illegal production on some regions of the parameter space. The findings are robust to the introduction of a costly trade technology facilitating the sale of legal output and requiring no monetary settlement.
Keywords: Search, Money, Illegal Activities (JEL C78, D83, E40)

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"Countries should further encourage in general the development of modern and secure techniques of money management [...] as a means to encourage the replacement of cash transfers"

Recommendation 24, FATF (1997).

I. Introduction

The Financial Action Task Force on Money Laundering (FATF) is an OECD-based inter-governmental body, comprising twenty-six industrialized countries, whose purpose is the development and promotion of policies to combat money laundering. Recommendation 24 aims to prevent such proceeds from being utilized in future criminal activities and from affecting legitimate economic activity. Until now, however, little has been done to construct a model capable to rationalize such a policy. While many have attempted to empirically characterize the link between underground, official economy and currency supply, a convincing theoretical approach has been missing. I attempt to fill this gap by developing a general equilibrium model capable of characterizing the link between currency and relative extent of illegitimate activities in an economy where individuals can choose among alternative productions and methods of exchange.

FATF's "Recommendation 24" appears to be a direct consequence of the commonly held belief that the underground economy almost totally uses currency for transactions.[1] While cash is believed to be the most used payment system in modern economies, there is consensus among researchers that much of the demand for currency is not associated with domestic or legal transactions. Some ascribe it mostly to the presence of large internal hoards or foreign balances, while some others point to the use of cash balances for transactions taking place in the underground economy. As an example, while recent surveys

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[1] The *underground economy* may be broadly defined as all unreported economic activities that would be taxable if reported. The focus of this paper is fraudulent economic activities, such as the ones stemming from enterprises breaking the law or engaging in plain criminal activities. A sociologist would refer to them as nonconformist or expropriative strategies that violate a specific set of normative rules, such as criminal law. *Money laundering* is the

have estimated that 4 out of 5 transactions in the US are roughly carried out with cash, Sumner (1990) has estimated that in the post-war period less than one half of the demand for currency appears to be associated with legal transactions. Others have pointed out this "special role" of currency. Goodhart (1989) for instance has noted that "much of such demand [for cash] probably relates to nefarious activities, the 'black economy', gambling, prostitution and drugs, where anonymity of currency is prized; indeed a large portion of outstanding notes is never caught in surveys". Emmons (1996) has noted that "Members of these groups [participants in the underground economy] rely to a great degree (or exclusively) on cash for making payments [...]". More recently Rogoff (1998) has indicated that probably well over 50% of the currency supply of OECD countries is held in the domestic OECD underground economy, and has suggested that the European Central Bank should consider policies that discourage the underground use of currency (such as the removal of large denomination notes from circulation).[2]

The close link between currency usage and illegal activities may be explained by the benefits such activities obtain from the anonymity guaranteed by currency transactions. Holders of cash can transfer possession of it very easily and with a high degree of anonymity since records generated by non-cash means of payment are a potentially dangerous incriminating feature. The need for anonymity is not only reflected in the use of currency, but also explains why such activities appear to be taking place outside organized markets (which makes them relatively more difficult to accomplish as compared to reported activities).

In this study I let ex-ante identical individuals be free to choose the productive activity they

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processing of criminal proceeds in order to disguise their illegal origin.

[2]Rogoff contains an extensive survey of empirical research on foreign and underground demand for currency. He also provides a simple transactions-based model of money demand to characterize the interplay between currency demand and tax evasion. His model however does not appear to provide a sufficiently rigorous theoretical approach to the study of the importance of the use of currency in the underground economy. Two are the reasons: it takes money as a primitive (thus individuals make no choice between alternative methods of exchange) and does not capture the interaction between the underground and the official economy.

prefer, legal or illegal, and then attempt a trade. Transactions can take place via quid-pro-quo or monetary exchange within the framework of a search theoretic economy where absence of records of past transactions, impediments to coalition formation, and government enforcement generate exchange frictions. To do so, I adopt a version of the Kiyotaki and Wright (1993) random matching model for its engaging ability to allow the study of endogenously generated transaction patterns and valuation of objects in environments characterized by trade frictions. Since illegal activities take place in a similar environment, such a model naturally lends itself to the investigation of how government policy and different transaction technologies may affect the extent of illegal activities. While both money and transaction patterns are not taken as primitives, both the existence of production-specific costs and trade frictions are. These will provide motivation for trade and exchange, and will generate sufficient incentives needed to support equilibria where both types of production coexist.

More precisely, I assume there is a cost advantage in the production of illegal output. This helps reproducing one of the distinctive features of illegal activities, i.e. their very high margins of return. Because of their distinctive "nonconformist" or "deviant" nature, I also embed as a primitive the sociological concept of *negative frequency dependence* of payoffs from illegal trade. This takes the form of a negative participation externality affecting the above mentioned cost advantage. Furthermore, I account for government policy by introducing government agents trading alongside private agents. Since I intend to abstract from normative analysis, I assume the government has a vested interested in reducing the size of the underground economy, thus government agents will follow an exogenously specified set of actions targeted at confiscating illegal commodities.[3] Another important primitive is the standard

[3] That is, I won't attempt to argue in favor of (or against) the claim that the existence of a large underground economy is not necessarily a bad thing. In fact, while some have argued that underground activities provide large seigniorage revenues and may stimulate an economy burdened by heavy tax rates and regulations (if most of the income earned in the process is spent in the official economy), such activities may also be a substantial source of negative externalities. Notice also that the cost specification adopted is reflected in the observation that the participants in criminal activities suffer from a negative feedback effect due—for instance—to increased competition for markets and customers, or disputes among individuals involved (see Cohen and Machalek, 1988, or

restriction on coalition formation, which makes it infinitely costly to form trade groups larger than two individuals. This will give scope to money. However, I will also consider a case where legal producers can access a costly trading technology which facilitates their sales and requires no monetary settlement. One may consider this innovative trading technology as a proxy for the existence of a competing frictionless legal transaction system which is not currency-based.[4] This will allow me to study the robustness of equilibria with illegitimate production, when legal producers can better organize their trade and need not resort to monetary settlements.

Stationary economies with different initial amounts of currency are examined. Initially, I look at economies without the costly trading technology. Different types of monetary equilibria may exist, with legal, illegal activities, or both. Under fairly broad conditions equilibria exist where high money stocks are associated to an increased (and endogenously determined) extent of illegitimate activities. Existence of these equilibria is robust to introduction of the competing transaction technology. Here too there are regions of the parameter space on which a reduction of the initial money supply can lessen the extent of illegal production. The intuition for the result is simple. Recall that in this class of models money is valued for its ability to lessen trade frictions and increase the frequency of consumption. The sale of illegal output guarantees higher payoffs but it is also more difficult to accomplish, and this may substantially diminish the frequency of consumption of an illegal producer. A greater money stock may thus prove very beneficial to illegal trade (more so than to legal ones), and hence provide incentives to engage in such activities.

To the extent that illegal activities generate negative external effects, the analysis suggests one

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Vila and Cohen, 1993 for a discussion of negative frequency dependencies). Cost differentials in production and government enforcement are also desirable due to evidence that illegal activities require a less costly acquisition of skills, deliver higher temporary payoffs, and are often not tolerated. Grossman and Shapiro's (1988) study of the trade in counterfeit products, for instance, assumes cost differentials (counterfeiters have a comparative advantage in the production of low quality goods) and a risk of confiscation of illegitimate product by an atomless government.

[4] While I could model more explicitly the existence of financial intermediaries providing costly matching services

more reason to the proponents of the view that the social costs of a country's payment system can be reduced by promoting a shift to electronics.[5] While the modeling choice adopted hinders the analysis of other interesting issues (such as the choice among alternative substitutes for currency, issues of optimal taxation, or maximization of seigniorage revenue, to cite a few) the framework is simple enough to allow a clear exposition of the mechanics of the links between currency and underground activities.

II. Environment

Time is discrete with consumption and production occurring at integer dates, while the horizon is infinite. The environment is similar to Kiyotaki and Wright (1993).

II.a Preferences, Endowments and Production

There is a [0,1] continuum of infinitely lived agents, and a [0,1] continuum of different indivisible goods whose production may be legitimate or illegitimate. Preferences are heterogeneous and agents do not consume their own production. Let $x \in (0,1]$ be the probability that any agent would want to consume a good, and, conditional on this, she wants to consume it with probability $\tau \in (0,1]$ if the good has been illegally produced (probability one otherwise). This assumptions motivate the existence of gains from specialization in production, the need for trade, and preference rankings of legal over illegal goods. Consequently, with probability x^2, $(x\tau)^2$ and $x^2\tau$, there is double coincidence of wants among two traders with, respectively, legal goods, illegal goods, or a legal and an illegal good. Similarly, single coincidence of wants occurs with probabilities x and $x\tau$. Consumption generates temporary utility $u > 0$ and future utility is discounted at rate $r>0$, corresponding to a real interest rate of r.

To account for government policy, a constant fraction $\gamma \in [0,1)$ of the population is composed of individuals who work for the government, the *government agents*, and represents the importance of

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this would not add additional insight at the expense of additional complexity.

5 Humphrey et al. (1996) estimate the cost to society of making payments as lying between 2% and 3% of GNP annually, while Humphrey and Berger (1990) suggest that debit cards and Automated Clearing House payments are

government in the economy; $1\text{-}\gamma$ are *private agents.*[6] Since the government enforces a policy against illegal production (see below), I will refer to both γ and τ as the additional frictions of illegal trade. An exogenously determined fraction $m\in(0,1)$ of individuals is initially randomly endowed with one unit of indivisible fiat money, where indivisibility is adopted for tractability. Private agents who are not endowed with money can choose to acquire a *legal* or an *illegal* production opportunity. Government agents only adopt a legal production opportunity. Let $\{g,b\}$ be the set of production opportunities (g is legal, b is illegal), and k one of its elements.

The acquisition of a legal production opportunity generates disutility $c\in(0,u)$. Let $\lambda\in[0,1]$ denote the fraction of private producers who have a legal good, which in equilibrium is a function of the strategy profile of individuals (more in section III). The acquisition of production opportunity b generates disutility defined by the continuously differentiable, non-increasing and convex function $c_b(\lambda):[0,1]\rightarrow[0,c]$. This specification is chosen because it is simple enough to make the ensuing analysis especially intuitive while still being capable to generate incentives for illegal activities to arise endogenously. It is also able to capture two important traits of illegitimate transactions: their cost differential with respect to legitimate transactions, and the concept of *negative frequency dependence* of payoffs derived by expropriative or deviant strategies, often encountered in sociological models of criminal or illegal behavior. Furthermore, the chosen cost formulation is also able rule out corner outcomes where all production is illegal even in the presence of only minimal government enforcement or marginally stronger preferences for legal output. This is a desirable feature of a model economy given that illegal behavior is by definition "nonconformist".

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the least expensive relative to all other methods, except cash.

6 This is a standard mean of introducing government policy in search environments. For instance, in Aiyagari, Wallace and Wright (1996) government agents support an equilibrium with coexisting interest bearing securities and money. In Green and Weber (1996) government agents are automata confiscating counterfeit money. In Li and Wright (1997) government agents follow prescribed rules for acceptance of different objects. Aiyagari and Wallace (1997) include government agents who do not consume or produce to examine policy. Soller and Waller's (1998) government agents enforce currency restrictions.

As soon as agent *j* acquires a legal (illegal) production opportunity, she uses it once to costlessly produce *one* unit of the storable indivisible legal (illegal) commodity *j*. For simplicity I let consumption be required in order to acquire a new production opportunity. Storage is costless and limited to one object, hence no production can take place if carrying money, and vice versa. This allows identification of private traders according to their inventory: a money trader, or buyer, has money, a legal (illegal) producer, or seller, has a legal (illegal) production opportunity. Both inventory and type of production are observable but the trading history is private information so that—due to the large population—the possibility of writing contractual arrangements for intertemporal trade is neglected.

II.b. Matching technologies and Exchange Protocols

Two different matching technologies are present. A costless random matching technology matches traders in pairs according to a known stochastic process, while a *costly* matching technology provides deterministic matches to legal producers. Traders who search meet pairwise and at random according to a Poisson process with constant arrival rate α (i.e. there is a CRS meeting technology). Conditional on an encounter, the probabilities of being matched to a money trader or a producer (legal or illegal) are a function of the strategies adopted by all others. Search takes place during t while exchange and consumption occur at the beginning of t+1. A successful trade results if mutually agreeable. Quid-pro-quo trades require double coincidence, but single coincidence is sufficient for a monetary trade. Due to the indivisibility of the inventory prices are fixed at one.

By using the costly matching technology legal producers can have their trade process greatly facilitated. For a fee, an atomless "broker" collects information on production and preferences of traders, matching them deterministically during t. At the beginning of t+1 production, exchange, consumption, payment of the fee and acquisition of a new legal production opportunity occur. The per capita cost is incurred simultaneously to consumption, and generates disutility denoted by $\phi \in [0, u\text{-}c]$. This is intended to approximate the existence of a market which—as compared to random search—is better organized and does not require cash settlements. Observe also that the formulation chosen bounds the gains from costly trade at zero when participation in it is null.

In this environment trade is necessary for consumption, double coincidence of wants is essential for exchange and consumption to occur, but if money is valued single coincidence is sufficient. The model is maintained tractable by limiting the dimensionality of the state space.

III. Symmetric Stationary Equilibria

Consider equilibria where money is valued, strategies and distribution of objects are time invariant, and identical types act alike. I adopt the simplifying assumption that money is used to purchase goods but not another unit of money (a small transaction cost would guarantee it), and consider cases where individuals adopt pure strategies in the acceptance of objects.[7]

III.a Strategies

Government agents act as private agents in matches with legal producers or money traders, but follow a predetermined rule when an illegal producer is encountered: the government agent seizes (and disposes of) his output.

Private individuals face a set of possible actions contingent on the state: which production opportunity to acquire (the agent has no inventory), whether to search or use intermediation (the agent is holding a legal good), and whether to exchange one's inventory for the trading partner's (bilateral match). The *strategy profile* of an individual is a mapping from the state into probabilities, i.e. it is a set of rules specifying which actions to take contingent on the state. The *state* for an individual is fully identified by

[7] Generalizing the model to encompass mixed best responses in the acceptance of objects, and adding a transaction cost (to rule out swaps of goods which are not to be consumed) would not add qualitatively new insight at the cost of added complexity for the analysis. Exchange of goods which are not subsequently consumed is not considered since it is inconsequential for the analysis.

her trade status (with or without a trade partner), her position in the match (buyer or seller), her inventory and her partner's. Let $s \in [0,1]$ define the probability that an average private trader with no inventory chooses to acquire a legal production opportunity. Let $e \in [0,1]$ define the probability that an average legal producer chooses costly trade. Then denote the strategy profile by $\sigma=\{s,e\}$, on which individuals have symmetric beliefs. An individual's best response, when he takes as given the strategy profile of the others, is $\sigma'=\{s', e'\}$.

III.b Distribution of Money and Goods

I next discuss the stationary distribution of money and goods which, in turn, have implications for the structure of the random matching process. Consider first the distribution of monetary assets and let m_p and m_g respectively define the fractions of private and government agents with currency, where

$$(1-\gamma)m_p+\gamma m_g=m \tag{1a}$$

Given the description of the individual's actions, the flow of private commodity traders into private money traders occurs at rate

$$\alpha x(1-\gamma)(1-m_p)[\lambda(1-e)m+(1-\lambda)(1-\gamma)m_p\tau]$$

The arrival rate of a single coincidence match is αx and $(1-\gamma)(1-m_p)$ is the proportion of private producers. A legal producer acquires money after trading with any money trader, $\lambda(1-e)m$. An illegal producer acquires money only after encountering private money traders who like illegal commodities, $(1-\lambda)(1-\gamma)m_p\tau$. The outflow of private money traders (into private producers) occurs at rate

$$\alpha x(1-\gamma)m_pA_1$$

where A_1 denotes the sum of conditional probabilities for all matches with producers (private and government) where private money holders want to trade. For a given set of parameters, in the steady state m_p is a strategy-dependent quantity defined by

$$m_p=m_p(\sigma) \tag{1b}$$

where $m_p(\sigma):[0,1]^2\rightarrow[0,m]$ is a continuously differentiable single-valued function, decreasing in e, increasing in λ and invariant to s.[8]

Next, consider the distribution of goods across private producers. Recall that individuals using the organized market enter and exit it with a legal production opportunity. Consequently, consideration of flows into and out of the proportion of legal producers (generated by search trade) is sufficient to track the distribution of goods. Outflows are due to legal traders either acquiring money or bartering and then choosing illegal production. Inflows are due to all those traders who where not legal producers and, after a transaction, have chosen to acquire a legal production: private money traders who have bought, and illegal sellers who have bartered or have lost their inventory.

Let A_3 denote the sum of conditional probabilities of all matches with private producers and government agents where illegal producers transfer their inventory. The inflow of private agents into legal producers occurs at rate

$$\alpha(1-\gamma)s[xm_pA_1+(1-m_p)(1-\lambda)A_3].$$

The term $\alpha(1-\gamma)s$ identifies the rate at which the two subsets of private agents become legal producers. The first subset of private agents who may flow into legal traders is private money traders in a single coincidence match, m_px. They may trade with both government or private producers, A_1. The second subset of private agents is composed by illegal producers, $(1-m_p)(1-\lambda)$. They may be caught by the government (and stripped of their good), or be in a double coincidence match with private producers, A_3. The outflow of legal producers (into illegal producers or private money traders) is

$$\alpha x(1-\gamma)(1-m_p)\lambda(1-e)[m+xA_1(1-s)].$$

For a given set of parameters the stationary distribution of production opportunities satisfies

$$\lambda=\lambda(\sigma) \tag{1c}$$

The function $\lambda(\sigma):[0,1]^2\rightarrow[0,1]$ is a continuously differentiable function where $\lambda(1,e)=1$ and $\lambda(0,e)=0$.

[8] All algebraic derivations and proofs are in Appendix. There I also show that m_p is decreasing in γ.

Additionally $\lambda(\sigma)$ is monotonically increasing in s, for a fixed e and a given parameterization.[9]

In a steady state equilibrium, m_p, m_g and λ must satisfy (1a)-(1c). Different strategies deliver different equilibria which can be completely characterized by the set of actions of the representative agent σ, the fraction of money traders and legal producers, $\{m_p, m_g, \lambda\}$, and the lifetime expected utilities of individuals (considered below).

III.c Value Functions

In what follows I restrict attention to private individuals' actions since government agents act similarly, and follow predetermined actions when encountering illegal producers. In period t an individual may be in three different states depending on her past trade. She may be a producer randomly searching or costly trading, or she may be a money trader. Consider an individual in search after a random match has taken place. As a producer she chooses whether to sell for money (single coincidence match with a money trader) or barter (double coincidence match with another producer). As a money trader she chooses whether to buy. Next period's trading technology and type of production is chosen at the end of t. At the beginning of t+1 the following takes place: exchange, consumption, acquisition of production opportunities, production and sanctioning of illegal producers (if they had met a government agent during t). A producer who has chosen costly trade at t, is deterministically matched during t and has only to choose next period's trading technology. Consumption and payment of the per-capita fee occur simultaneously, at the beginning of t+1.

Let V_g, V_b and V_m be the stationary value functions of respectively, an individual holding a legal, illegal good, and money. Let $\Pi_{k,m}$ and Π_k denote the maximal net payoff to a producer k who, respectively, sells for money and barters. Let Π_m be the maximal net payoff to a money trader and Π

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[9] In Appendix I show that since $\lambda(\sigma)$ is a single-valued function it is also invertible and the stationarity condition can be rearranged as $s=s(\lambda,e)$, also a single-valued function increasing in λ.

denote the maximal expected lifetime utility to an individual without inventory. In a symmetric steady state individuals choose their strategy profile to maximize their lifetime utility, taking everybody else's strategies as given. The value functions may be represented by the standard flow-return version of the Bellman equation

$$rV_g=\alpha mx\Pi_{g,m}+\alpha A_1x^2\Pi_g \tag{2a}$$

$$rV_b=\alpha(1-\gamma)m_p x\tau\Pi_{b,m}+\alpha(1-\gamma)(1-m_p)A_2x^2\tau\Pi_b+\alpha\gamma(\Pi\text{-}V_b) \tag{2b}$$

$$rV_m=\alpha A_1x\Pi_m \tag{3}$$

Expression (2a) shows that a legal producer searching meets someone randomly at rate α. With probability mx she meets a money trader (private or government) willing to trade, and nets $\Pi_{g,m}$. She may also meet a producer with whom she would like to trade (legal, illegal or government agent) with probabilities respectively given by $(1\text{-}\gamma)(1\text{-}m_p)\lambda(1\text{-}e)x^2$, $(1\text{-}\gamma)(1\text{-}m_p)(1\text{-}\lambda)x^2\tau$ and $\gamma(1\text{-}m_g)x^2$ (which sum up to A_1x^2), netting Π_g. Expressions (2b) and (3) have a similar interpretation.

Let O be the lifetime expected utility to a legal producer who has chosen the costly trade technology. In the steady state, his expected lifetime utility satisfies

$$rO=(u\text{-}c)1_{\{e\neq 0\}}\text{-}\phi+max\{V_g\text{-}O,0\} \tag{4}$$

where $1_{\{e\neq 0\}}$ is an indicator function taking the value 1 if e is non-zero. Expression (4) indicates that the costly trade technology is capable of providing a deterministic match only if there is a positive measure of individuals adopting that technology. In this case the individual exchanges her good for another netting $u\text{-}\phi\text{-}c$, because of the costs generated by the fee and acquisition of a new legal production opportunity.

III.d Best Response Correspondences

From the discussion above in a steady state

$$\Pi_{k,m}=V_m\text{-}V_k \tag{5a}$$

$$\Pi_k=\Pi+u\text{-}V_k \tag{5b}$$

$$\Pi_m=\Pi+u-V_m \tag{5c}$$

$$\Pi=\max_{s'}\{s'[\max\{V_g,O\}-c]+(1-s')[V_b-c_b(\lambda)]\} \tag{5d}$$

for $k\in\{g,b\}$. In an equilibrium where symmetric Nash strategies are adopted, the representative individual chooses $\sigma'=\{s',e'\}$ taking as given both σ and the associated stationary distribution of money and goods implied by (1b)-(1c). In addition, define

$$\Delta(\sigma)=V_b-c_b(\lambda)-max\{V_g,O\}+c$$

as a measure of the relative gain from acquiring an illegal (vs. legal) production opportunity, and let

$$K(\sigma)=V_g-O$$

denote the gain from search trade relative to costly trade (omitting the argument σ where no confusion arises). When, following a successful exchange, a trader is left with no inventory, she chooses the production opportunity providing her with the highest lifetime utility. Her best response correspondence is

$$s'\begin{cases}=0 \;\; if\;\Delta(\sigma)>0\\ \in[0,1] \;\; if\;\Delta(\sigma)=0\\ =1 \;\; if\;\Delta(\sigma)<0\end{cases} \tag{6a}$$

Since I am considering pure strategies in the acceptance of objects, an individual will exchange her inventory only if that makes her strictly better off. As a seller, she accepts money in exchange for her good $k\in\{g,b\}$ if she is strictly better off

$$V_m-V_k>0 \tag{6b}$$

As a buyer, she offers her money for a good she likes if she is strictly better off

$$\Pi+u>V_m \tag{6c}$$

and she would offer her good k in a quid-pro-quo exchange also if that makes her strictly better off

$$\Pi+u>V_k \tag{6d}$$

Contingent on being a legal producer, her decision to trade in the organized market depends on the relative benefit it would provide her with. If trade by means of costly matching increases (decreases) her lifetime utility with respect to random matching trade, then she will adopt (not adopt) the costly technology. When the two alternatives generate similar lifetime utilities she will randomize. Her equilibrium choice is then identified by the best response correspondence

$$e' \begin{cases} = 0 \;\; if \; K(\sigma) > 0 \\ \in [0,1] \;\; if \; K(\sigma) = 0 \\ = 1 \;\; if \; K(\sigma) < 0 \end{cases} \tag{6e}$$

Finally, symmetry requires

$$\sigma' = \sigma \tag{6f}$$

and since individuals do not participate in trading activities if their expected lifetime utility is negative, $V_k > 0$ (for at least one k) is needed in an equilibrium with production.

III.e Definition of Equilibrium

A *symmetric stationary monetary equilibrium* is a set of non-negative value functions $V=\{V_g,V_b,V_m,O\}$, a strategy profile σ, a proportion of legal producers λ, and a proportion of money traders $\{m_p,m_g\}$ such that: (i) individuals maximize their expected lifetime utilities and act symmetrically, i.e. for a given distribution of money and goods $\{m_p,m_g,\lambda\}$, then V and σ must satisfy (2a)-(4) and (6a)-(6f); (ii) given value functions and symmetric strategies $\{V, \sigma\}$, the distribution of money and goods, $\{m_p,m_g,\lambda\}$, satisfies the stationarity conditions (1a)-(1c).

To summarize, in a symmetric stationary monetary equilibrium, taking as given the strategy profile of the others, σ, and the stock of currency, m, the fraction of private traders engaging in legal production, λ, and the distribution of money, $\{m_g,m_p\}$, are endogenously determined. Outcomes are fully described by the combinations of the elements of the strategy profile σ.

IV. Stationary Equilibria: A Benchmark

A benchmark model is now provided where, with no loss in generality, the arrival rate of a match is normalized to one (α=1) and u-c=1. For simplicity I consider the linear cost function $c_b(\lambda) = c(1-\lambda)$, so that $c_b(\lambda)\to c$ as $\lambda\to 0$. This specification assures that a quid-pro-quo exchange always takes place contingent on double coincidence. Because the objective of the paper is to study possible links between currency and extent of illegitimate activities, attention is focused on equilibria where monetary transactions involve both legal and illegal output.[10] I refer to these outcomes as *interior*, as opposed to the *corner* solutions where only legal or illegal output is produced.

Definition. An *interior* outcome is such that $s,\lambda\in(0,1)$. A *corner* outcome is such that $\lambda=s\in\{0,1\}$. An interior outcome with costly trade is such that $e\in(0,1)$.

The requisite for coexistence of the two types of production is satisfied whenever traders are indifferent between production of either type, and this requires the lifetime utility generated by illegal production, V_b, to be lower (than V_g). This is due to the cost structure which, when the outcome is interior, implies uniformly lower disutility from acquisition of an illegal production opportunity, and so generates higher temporary payoffs from sales of illegal goods.

Since illicit transactions are subject to more intense trade frictions due to both government enforcement and preferences over commodity types, for a given strategy profile all value functions are increasing in τ while V_b is decreasing in γ. Recall also that money is valued for its ability to ameliorate trade frictions arising from randomness and absence of double coincidence in matches. If money is accepted in exchange for legal output then it must be also accepted by illegal sellers. The following lemma summarizes all of these features.

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[10] That is I do not study cases where some currency is accepted in underground transactions but is not a legal tender, as for instance is the case for the stock of US dollars or OECD currencies circulating in Russia.

Lemma 1. An interior outcome where money buys both legal and illegal goods exists if $V_g \geq O > V_b > 0$. In this equilibrium illegal producers' payoffs are the largest, $\Pi_{b,m} > \Pi_{g,m}$ and $\Pi_b > \Pi_g$. V_b is decreasing while V_g, and V_m are non-decreasing in γ. All value functions are increasing in τ.

Lemma 1 implies that any characterization of the interaction between money and illegal activities must focus on interior equilibria where money is used to buy both types of goods. Since legal sellers sell for money only in instances when they are randomly matched, I will next focus attention on those interior monetary equilibria where costly trade arrangements are not strictly preferred ($e<1$). To characterize them, I first conjecture a strategy profile capable of generating them, i.e. I consider a σ such that $\Delta=0$ and $K \geq 0$. Then I derive the distribution of goods and money, $\{\lambda(\sigma), m_p(\sigma)\}$, and solve for the value functions. Finally, I provide conditions under which the conjectured strategy is an equilibrium.

The first step in this process is showing that money is valued in equilibrium for all σ which satisfy the requisites for an interior solution. This is done in the next lemma and confirms that in a rational expectations equilibrium where individuals believe money is accepted with certainty, buying and selling for money is optimal for all traders.

Lemma 2. If σ satisfies $\Delta(\sigma)=0$ and $K(\sigma) \geq 0$ then $\Pi_m > 0$ and $\Pi_{g,m} > 0$.

The proof works off the fact that an interior outcome implies equality of expected lifetime utilities, net of acquisition costs. Since no transaction or production cost are incurred from a sale and time is discounted, any legal seller in a single coincidence match will be willing to accept money, if she rationally believes that everyone else will. And since illegal sellers have an even lower acquisition cost, they must also be certainly willing to accept currency.[11]

Now I turn to examination of the conditions under which the conjectured strategy profile is an equilibrium, by defining regions of the parameter space which support it. To simplify the analysis, first I look at the case where both types of producers face the same trade technology, and must use either quid-pro-quo or monetary exchanges. Then I examine the robustness of equilibria when legal producers can access the improved trading technology, i.e. can sell/buy output via costly matching and avoid the use of currency altogether.

IV.a All individuals face a similar trading technology

Let the disutility from entrance in the organized market be fixed at $\phi=u-c$, so that in equilibrium (4) and (6e) imply $e=0$. The relevant component of the strategy profile is s. First I show there are equilibria where legal and illegal production coexist, and provide sufficient conditions in the form of regions of the parameter space which admit interior outcomes.

Proposition 1. *There are interior monetary equilibria if* $\gamma\in(0,\gamma_{H1}]$, *where* $\gamma_{H1}=1-\dfrac{1}{\tau(1+c)}$. *The equilibria may be multiple.*

In an interior outcome legal and illegal producers sell for money or transact directly by bartering commodities. Not surprisingly, this requires illegal trade to be subject to more severe trade impediments than legal trade (both $\tau=1$ and $\gamma=0$ do not support it). However, such trade impediments cannot be exceedingly high. In particular, the risk of confiscation must be moderate in order for the average individual to be indifferent between the two production types. The frequency of illegal sales must also be

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[11] It is also obvious that this model admits both a non-monetary and mixed monetary equilibria, which are not

acceptably high: this explains why only a sufficiently large τ insures that the set $(0, \gamma_{H1}]$ be non-empty. Furthermore, since $\Delta(\sigma)$ is non-linear in λ, more than a single s may solve Δ=0. This is a source of potential multiplicity of interior equilibria, (since $\lambda(\sigma)$ is single valued it implies a unique s and vice versa). When the requisites on γ and τ are not met, the model may admit corner outcomes.

<u>**Corollary to Proposition 1**</u>. *There exist corner outcomes with only illegal (legal) producers if there are no (large) illegal trade frictions.*

Clearly corner solutions where all individuals choose to produce legal output are always possible for extreme degrees of illegal trade impediments (high γ and low τ). When the expected frequency of sales of illegal output is too low (compared to the legal alternative), engaging in illegal production is a dominated choice ($\Delta(\sigma)<0$ for all $s<1$). It is also obvious that when the extent of frictions is independent of the type of object traded ($\gamma=0$ and $\tau=1$), a corner outcome results with only illegal production taking place ($s=0$). This is due to that production's more favorable cost structure. However, the participation externality affecting acquisition costs of illegal commodities rules out the existence of a similar corner solution in a neighborhood of $\gamma=0$ and $\tau=1$. This is occurs because full participation in illegal trade generates acquisition costs similar to legal activities ($c_b(\lambda)\to c$ as $\lambda\to 0$), hence strict preference for illegal production cannot be supported as an equilibrium for, say, even marginally small government enforcement (see figure 1).[12]

Now I show that on some regions of the parameter space, a larger initial money stock generates incentives for individuals to engage in illegal production.

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examined here.

[12] Suppose we let $\gamma>0$ small and $\tau<1$ large and conjecture a case where all traders choose illegal production. Since

Proposition 2. *Let γ be in a neighborhood of zero. λ is decreasing in the initial currency stock if x or m are sufficiently large. Conversely, there exists an optimal m that maximizes λ.*

The proposition gives support to the FATF's Recommendation 24, and suggests that policies directed at restricting the currency supply may be beneficial in diminishing the extent of illegal economic activities. The result is due to several effects which help diminish the more severe trade impediments faced by an illegal seller. First, all else equal, added currency helps an illegal producer to move out of his (illegal) inventory position more quickly thus standing a lower chance of future punishment (in matches with government agents). Second, for a given set of value functions, more frequent monetary trades generate larger benefits for illegal producers since their net payoffs are higher (they "prize" money more than legal traders). Furthermore, increased currency stocks generate "crowding out" of production which has a negative impact on lifetime utilities because of the lower frequency of encounters with producers. This negative effect encourages production of those goods providing the highest payoff form trade (i.e. illegal goods) and is more pronounced at higher (lower) levels of liquidity (trade frictions). All these factors tend to increase the appeal of illegal production in those instances where trade frictions are not extensive and the exchange process benefits *the least* from increases in money. That is, instances where the random matching process is moderately affected by double coincidence problems and where exchange is facilitated by largely available currency (see figure 2). However, when currency is scarce the opposite may occur and the proportion of legal producers may be positively correlated to the quantity of money. This, in turn, may generate a hump-shaped response of s (and hence λ) to different currency stocks.

Figure 3a (drawn for the baseline u=3.1, r=0.01, τ=0.35, γ=0.01, x=0.25) contains an illustration. The change in the steady state value function brought by an increase in the initial level of money stock is

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$\Delta(\sigma)<0$ for s=0 this cannot be a symmetric equilibrium because s'=1 from (6a).

the largest for illegal producers (DV_b vs. DV_g in the figure) which results in a decreasing proportion of legal traders (s and λ are both decreasing). Recall that $\Delta = V_b - c_b(\lambda) - V_g + c = 0$ in an interior equilibrium, and $c_b(\lambda)$ is decreasing in λ. Therefore, to restore an indifference balance affected by changes in m, λ must fall when V_g increases by less (or decreases by more) than V_b since this increases the cost $c_b(\lambda)$. The reverse result may take place in the presence of limited currency and more substantive search frictions (Figure 3b for x=0.125). In this case the benefits generated by increased liquidity are initially more pronounced for legal producers (for m<0.07). This results in a positive correlation between m and λ, for low levels of initial currency stock, and a negative correlation otherwise ($m \geq 0.07$).[13]

IV.b Legal traders can access an improved trading technology

Now I consider $\phi < u - c$ in order to address the robustness of interior equilibria to the introduction of an improved trading technology for sales of legal output. Below I provide sufficient conditions identifying regions of the parameter space where coexistence of both types of production is consistent with adoption of the costly trade technology by some legal producers.

Proposition 3. *There exist interior monetary equilibria with costly trade if* $\gamma \in (0, \gamma_H]$ *and* $\phi \in (\phi_L, \phi_H)$, $\gamma_H = \min\{\gamma_{H1}, \gamma_{H2}\}$, $\gamma_{H2} = \dfrac{m(1-m)}{r + x(1-m)} - m$. *Equilibria may be multiple.*

The proof works off the fact that the representative individual must be indifferent not only between the two types of production, but also between the two matching technologies, i.e. between sustaining a cost to receive a deterministic match or freely attempt to sell her output to randomly

[13] Two separate effects influence λ. One is the indirect effect due to the variation in the strategy s induced by a change in m. The other is the direct effect of changes in m, since λ must satisfy the steady state law of motion (1c'), specified in the appendix. They need not have similar signs and this explains why λ may be decreasing even if s is

encountered partners. Two different requisites must be met, the first of which involves the cost ϕ which must be moderate but not too low (otherwise all legal producers would avoid monetary exchange). In particular, since less producers are randomly met when more legal sellers choose costly trade, then V_g is decreasing in e, for a given s, while O is invariant to it. In order to find an $e\in(0,1)$ leaving a legal producer indifferent between the two trading technologies, then both V_g-O=K>0 as $e\rightarrow 0$ and K<0 as $e\rightarrow 1$ are needed. The first inequality is guaranteed by ϕ large enough ($\phi>\phi_L$), so that if in the limit all legal producers engage in random search, then the best response for any trader is to choose search as well (e'=0). But ϕ cannot be too high otherwise no legal producer would choose costly trade, when encounters with legal sellers are extremely rare ($e\rightarrow 1$). That is K<0 as $e\rightarrow 1$, guaranteed by $\phi<\phi_H$ (the definitions of ϕ_H and ϕ_L are in the proof).[14]

The second ingredient is the degree of additional trade impediment sustained by illegal sellers. As before, there must be some degree of government enforcement ($\gamma\neq 0$), but this cannot be too severe ($\gamma<\gamma_H$). But since now legal producers have availability of an additional trading technology, extra conditions (spelled out in the proof) are needed to assure the existence of such a γ. In particular the discount factor must be moderate (r<1-x) since only if traders are not too impatient would they consider random trade (and hence possibly illegal production). Also, individuals would search only if the frequency of matches with sellers is sufficiently high, whereas they would all switch to legal production (and costly trade) otherwise. This is guaranteed by the existence of a moderate money supply since money in this environment "crowds-out" production ($m<m_H$).[15] These two last conditions guarantee the

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increasing in m (up to m=0.22 in figure 3b).

[14] Notice that since for a given s $K(\sigma)$ is strictly decreasing in e, any equilibrium e^* would be inherently "unstable". For instance, shuffling some legal producers into search would induce all the remaining ones to choose e'=0 (since K>0 to the left of e^*). However, since s is not a constant, there may be instances in which such an experiment would modify the distribution of commodities in such a way that V_g falls (K<0 to the left of e^*) thus making the equilibrium "stable".

[15] While it is possible to consider environments where multiple objects can be stored by any individual trader, this would expand the state space tremendously and render the model substantially (and unnecessarily) more complex.

non-emptiness of the set of feasible γ. Finally, as in the case without costly trade, corner solutions are possible.[16] Figure 1 contrasts the two cases (with and without costly trade) by showing regions of the parameters (γ,τ,m) where existence of an interior solution is guaranteed. For instance, a parameterization lying in the region identified by C is sufficient to guarantee interior solutions with either organized trade or without (depending on ϕ), for some s. A corner solution with only illegal production occurs at A, similar outcomes are possible in the area marked by D, while the opposite corner outcome may occur at B.[17]

Finally, a version of proposition 2 can be proven for the case where $e \neq 0$.

__Proposition 4__. λ is decreasing in m whenever e is increasing in the initial money stock.

This result can be explained as follows. First, the strategy e must be an equilibrium, which requires $K(\sigma)=V_g-O=0$, i.e. the lifetime utility for a legal producer must be held constant ($V_g=O$). Changes in the money stock which affect V_g are compensated by "appropriate" changes in the strategy e. This can be accomplished since, all else equal, V_g falls when e increases (it lowers the fraction of legal producers searching for trade matches, and hence the frequency of consumption). Therefore e rises whenever V_g benefits from an increase in m. Second, the strategy s must also be an equilibrium, which requires $\Delta(\sigma)=V_b-c_b(\lambda)-V_g+c=0$. When a higher money stock causes V_g to rise, then V_b will also rise. And since legal sellers' lifetime utility is kept constant the rise in V_b must be contrasted by larger acquisition costs, $c_b(\lambda)$. Consequently λ must fall in m.

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[16] The proof follows from the one of the corollary of proposition 1, recalling that when $\lambda=0$ then $e=0$. Notice that the corner outcome where no costly trade occurs may arise even if the alternative matching technology is costless. As long as no one uses it there are no incentives for any one trader to do so (a coordination failure).

[17] Notice that since the propositions provide only sufficient (but not necessary) conditions, it is not necessarily the case that only corner outcomes with $s=1$ occur in D. Indeed, interior outcomes may occur on some neighborhood of the lower bound of the set D.

Finally, notice that proposition 4 suggests that λ may be an ever-decreasing function of the stock of money. It is a well known result that sellers benefit from increases in the stock of currency when m is small (hence λ decreases as m rises). If for large money supplies increased liquidity lowers V_g more than V_b (a "crowding out" effect induced by the inventory restriction) then λ must once again decrease.

V. A Dynamic Experiment.

The preceding section has shown that, under certain conditions, higher currency levels are associated with steady state equilibria with a larger fraction of illegitimate production. Can we characterize the equilibrium path following a one-time reduction of the amount of currency circulating? What are the strategies adopted along the transition to the new steady state? These questions are here considered in the context of a numerical example for the benchmark case where e=0. Letting t'=t+1, the laws of motion for m_p and λ are

$$(1-\gamma)m_{p,t'}=(1-\gamma)m_{p,t}+\alpha(1-\gamma)(1-m_{p,t})x[\lambda_t m+(1-\lambda_t)(1-\gamma)m_{p,t}\tau]-\alpha(1-\gamma)m_{p,t}xA_{1,t} \tag{1b'}$$

and

$$(1-\gamma)(1-m_{pt'})\lambda_{t'}=(1-\gamma)(1-m_{p,t})\lambda_t+\alpha(1-\gamma)[(1-m_{p,t})(1-\lambda_t)A_{3,t}+m_{p,t}xA_{1,t}]s_t$$
$$-\alpha(1-\gamma)(1-m_{p,t})\lambda_t[(1-s_t)x^2A_{1,t}+mx] \tag{1c'}$$

respectively for private money traders and private legal producers. The Bellman equations describing the value functions of traders in different states are

$$(1+r)V_{g,t}=\alpha mx\Pi_{g,m,t'}+\alpha A_{1,t}x^2\Pi_{g,t'}+V_{g,t'} \tag{2a''}$$

$$(1+r)V_{b,t}=\alpha(1-\gamma)m_{p,t}x\tau\Pi_{b,m,t'}+\alpha(1-\gamma)(1-m_{p,t})A_{2,t}x^2\tau\Pi_{b,t'}+\alpha\gamma(\Pi_{t'}-V_{b,t'})+V_{b,t'} \tag{2b''}$$

$$(1+r)V_{m,t}=\alpha A_{1,t}x\Pi_{m,t'}+V_{m,t'} \tag{3''}$$

I choose m and the other parameters so that a stationary equilibrium with $s\in(0,1)$ exists and use (1a)-(1c) and (2a)-(3) to derive the steady state vector $\{V, m_p, \lambda, s\}$. I then conjecture an equilibrium sequence $\{s_t\}_{t\in\{1,\dots,T\}}$, and find a sequence $\{m_{p,t}, \lambda_t\}\rightarrow\{m_p, \lambda\}$ for the given $\{s_t\}$. To do so, I use (1b')-(1c') backwards to compute the $\{m_{p,t}, \lambda_t\}_{t\in\{1,\dots,T\}}$ sequence for given sequence $\{s_t\}$. I assume the economy is at the long run equilibrium, at T, i.e. $\{V_t, m_{p,t}, \lambda_t, s_t\}_{t=T}=\{V, m_p, \lambda, s\}$. Then I take the sequences $\{s_t\}$, $\{m_{p,t}, \lambda_t\}$ and use (2a")-(3") to work backward the sequence $\{V_t\}_{t\in\{1,\ldots,T\}}$. To obtain V_t, given V_{t+1}, I make sure that the conjectured s_t is an equilibrium by checking (6a). That is if s_t=0 (1) then it must be that Δ_t>0 (<0).

An illustration is contained in figures 3a-3b where T=32, m=0.1 and the other parameters are as in the previous examples. The economy is initially characterized by a large extent of illegal activities but the stock of currency is not large enough to support this outcome as a steady state equilibrium. Along the path to the steady state, individuals without an inventory always choose to acquire a legal production opportunity: s_t=1, since Δ_t<0 along the path. This strategy is implemented until the distribution of production opportunities and money is such that there is indifference between acquiring any of the two production opportunities, so that the economy settles at a steady state where Δ=0, λ= 0.610 and s= 0.680.

VI. Conclusions

I have developed a simple general equilibrium model capable of characterizing the links between availability of currency and extent of illegitimate economic activities. This was motivated by the desire of providing a rationale for a policy recommendation explicitly directed at encouraging the replacement of cash transfers, and set forth by an OECD-based organization, the FATF.

In the search-theoretic model proposed, individuals can engage in illegitimate and legitimate production and trade via quid-pro-quo or monetary exchange, while government agents sanction illegal producers with an exogenously set frequency. Different stationary equilibria exist depending on the severity of impediments encountered in attempting to trade illegitimate output, which I have embedded as primitives of the structure of preferences and institutional enforcement. Depending on the strength of such impediments corner solutions (with only one type of production) may arise. More interestingly, I have shown the existence of "interior" monetary equilibria where legitimate and illegitimate activities coexist with valued money, on some regions of the parameter space. In this latter case, the distribution of production is susceptible to changes in the initial stock of currency. In particular, increasing the latter may prove detrimental to the composition of output in that it may enlarge the fraction of illegal output produced. This is more likely to occur in environments where trade impediments are not too pervasive, which, in this model, are taken to be a sufficiently large currency supply, and a limited double coincidence problem. In such an instance, greater liquidity may well provide increased incentives to engage in the higher-return illegal production because it lessens its specific trade frictions. These results are robust to the introduction of an improved trading technology allowing legal producers to trade on a more organized market with a costly payment system requiring no cash settlements.

If the reduction of illegitimate economic activities is the goal, then the analysis carried out provides a rationale for the policy recommendation concerning currency expressed both by an OECD-based organization and several economists. Would the society benefit from such a policy? One way to answer this question is to examine how utility varies with different initial quantities of money by considering some measure of weighted lifetime utility of private traders in the economy. For instance $W=m_p V_m+(1-m_p)\lambda V_g+(1-m_p)(1-\lambda)V_b$ may be one such criterion, in which case standard arguments suggest that there may exist an interior quantity of money that maximizes W, depending on the severity of the double coincidence of wants problem. Availability of a medium of exchange increases the frequency of consumption only when double coincidence matches are infrequent, while the crowding out of production caused by a larger supply of the medium of exchange is detrimental to both buyers and sellers (see Kiyotaki and Wright, 1993). If this is paired to low trade frictions (high x) an increase in m will also lower the equilibrium proportion of legal producers, and this produces additional negative effects on both illegal production costs and on the frequency of trade (since $\tau<1$). However, this study has shown that when trade impediments are substantive, there may be an interior level of currency maximizing the extent of legal production. In such a case stripping the economy of *all* the currency would not be welfare maximizing since not only it would depress *all* trading activities, but it would also increase the incentives to engage in illegal ones.

While a more complete characterization of the equilibria is warranted (in particular with respect to consideration of fiscal policy and dynamic analysis), this study is a first step in the direction of providing theoretical support to the common belief of an existing link between a country's unreported economic activities and the organization of its payment systems.

Appendix

Derivation of (1b) and characterization of m_p

Let A_1=$\gamma(1\text{-}m_g)$+$(1\text{-}\gamma)(1\text{-}m_p)\lambda(1\text{-}e)$+$(1\text{-}\gamma)(1\text{-}m_p)(1\text{-}\lambda)\tau$, that is the sum of all conditional probabilities for matches with producers (private and government) where private money holders trade. In the steady state, inflows and outflows of private money traders need to be balanced, or

$$(1\text{-}m_p)[\lambda(1\text{-}e)m+(1\text{-}\lambda)(1\text{-}\gamma)m_p\tau]=m_p[\gamma(1\text{-}m_g)+(1\text{-}\gamma)(1\text{-}m_p)\lambda(1\text{-}e)+(1\text{-}\gamma)(1\text{-}m_p)(1\text{-}\lambda)\tau]$$

which, using (1a) can be rewritten as a quadratic equation in m_p

$$m_p^2a_1+m_pa_2+a_3=0$$

where a_1=-$(1\text{-}\gamma)[1\text{-}\lambda(1\text{-}e)]$, a_2=$(m\text{-}\gamma)[1\text{-}\lambda(1\text{-}e)]\text{-}\lambda(1\text{-}e)$, a_3=$\lambda m(1\text{-}e)$. The equation has two real roots and, for $a_1 \neq 0$, m_p is uniquely defined by the larger of the roots

$$m_p=\frac{-a_2-\sqrt{a_2^2-4a_1a_3}}{2a_1}\equiv m_p(\sigma) \qquad \text{(1b')}$$

since $a_1<0$ while the square root term is always positive and larger than a_2, thus the only non-negative solution has a negative sign in front of the square root.

From (1b') m_p=0 for e=1. Observe that a maximum for m_p can be obtained by choosing e=0 for all λ (since by so doing the denominator is minimized and the numerator is maximized) i.e.

$$m_p|_{e=0}=\frac{(m-\gamma)(1-\lambda)-\lambda+\sqrt{[(m-\gamma)(1-\lambda)-\lambda]^2+4(1-\gamma)(1-\lambda)\lambda m}}{2(1-\gamma)(1-\lambda)}$$

A maximum for the square root term is $[(m\text{-}\gamma)(1\text{-}\lambda)+\lambda]$ when $m>\gamma$, hence an upper bound for the right hand side of the inequality above is $\frac{m-\gamma}{1-\gamma}\leq m$ for $m>\gamma$. Also, for $m<\gamma$ a maximum for the square root term is $\frac{\lambda m}{\lambda+(\gamma-m)(1-\lambda)} \leq m$.[18] Hence m_p is bounded by m above and 0 below.

Denote by a prime (') the partial derivative with respect to e. Then m_p is decreasing in e if

$$\frac{\partial m_p}{\partial e} \propto -a'_2 - \left(\sqrt{a_2^2-4a_1a_3}\right)^{-1}(a_2a'_2-2a'_1a_3-2a_1a'_3)-2a'_1m_p<0$$

where $a'_1=-(1-\gamma)\lambda<0$, $a'_2=(m-\gamma)\lambda+\lambda>0$ and $a'_3=-\lambda m<0$. Notice that $(a_2a'_2-2a'_1a_3-2a_1a'_3)>0$. Also $-2a'_1m_p-a'_2=(1-\gamma)(2m_p-1)-m<0\ \forall\gamma,m$. This is seen by substituting $\sup(m_p)=m$ and observing that if the LHS of the inequality is increasing (decreasing) in m, then for $m=1$ ($m=0$) the inequality still holds. Hence m_p is monotonically decreasing in e.

From (1b') observe also that, for all e, as $\gamma\to 0$ then $m_p\to m$ and m_p is decreasing in γ, i.e.

$$\frac{\partial m_p}{\partial \gamma} \propto -1-(a_2+2a_3)/\sqrt{a_2^2-4a_1a_3}+2m_p<0$$

since $a_2+2a_3>\sqrt{a_2^2-4a_1a_3}$ (because $|a_1|<1$) implies $-1-(a_2+2a_3)/\sqrt{a_2^2-4a_1a_3}<-2$, while $2m_p<2$.

Finally, I show that m_p is increasing in λ. Dividing both the numerator and the denominator of (1b') by $1-\lambda(1-e)$, taking the partial with respect to λ and rearranging

$$\frac{\partial m_p}{\partial \lambda} \propto -1+\frac{1-\lambda(1-e)}{\sqrt{a_2^2-4a_1a_3}}[\lambda/(1-\lambda(1-e))+(\gamma+m-2\gamma m)]$$

which reaches a minimum at $e=0$. A maximum for $(a_2^2-4a_1a_3)/(1-\lambda)^2$ (i.e. a minimum for the ratio involving the square root term) is $[\lambda/(1-\lambda)+(\gamma+m-2\gamma m)]^2$ (since $|m-\gamma| < |\gamma+m-2\gamma m|$). Also, $\gamma+m-2\gamma m>0$ for all γ and m.[19] Thus the second term on the RHS of $\frac{\partial m_p}{\partial \lambda}$ is larger than one, hence $\frac{\partial m_p}{\partial \lambda}>0\ \forall e$. ■

Derivation of (1c) and characterization of λ

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[18] This is obtained by taking a linear Taylor expansion of the square root term, treating $4(1-\gamma)(1-\lambda)\lambda m$ as a variable and expanding around $4(1-\gamma)(1-\lambda)\lambda m=0$.

Let A_3=γ+$(1-\gamma)(1-m_p)\lambda(1-e)x^2\tau+(1-\gamma)(1-m_p)(1-\lambda)x^2\tau^2$ denote the sum of conditional probabilities for matches with private producers and government agents where transactions with (or sanctioning of) illegal traders occur. Also, define $A_2=\lambda(1-e)+(1-\lambda)\tau$, so that $A_1=\gamma(1-m_g)+(1-\gamma)(1-m_p)A_2$, and $A_3=\gamma+\tau x^2(1-\gamma)(1-m_p)A_2$. Using (1a) then $A_1=1-m-(1-\gamma)(1-m_p)(1-A_2)\in(0,1)$, and $A_1=(1-m)A_2$ if $\gamma=0$.

The stationary distribution of production opportunities satisfies

$$s[(1-m_p)(1-\lambda)A_3+xm_pA_1]=x(1-m_p)\lambda(1-e)[xA_1(1-s)+m] \tag{1c'}$$

which for convenience can be rearranged as

$$F(\sigma,\lambda)=sf_1-(1-s)f_2-f_3=0 \tag{1c''}$$

where f_1, f_2, $f_3\geq 0$ are continuously differentiable functions of (λ,e), easily obtained from (1c'). I next show that $F(\sigma,\lambda)$ defines an implicit function $\lambda=\lambda(\sigma)$, such that $\lambda(\sigma)$: $[0,1]^2\rightarrow[0,1]$, monotonically increasing in s.

First I show that $F(\sigma,\lambda)=0$ is satisfied by $s=\lambda=0$ and $s=\lambda=1$, $\forall e$. Clearly for $s=0$ (1c') is satisfied only by $\lambda=0$. When $s=1$ (1c') is satisfied only by $\lambda=1$. This can be confirmed by substituting $s=\lambda=1$ and observing that at those values (1c') can be rearranged to yield exactly $m_p^2a_1+m_pa_2+a_3=0$, which is satisfied in the steady state only when $m_p=m_p(\sigma)$.

Observe also that $s\geq 0$ $\forall\lambda,e$ since by rearranging $F(\sigma,\lambda)=0$ as

$$s=\frac{f_3+f_2}{f_1+f_2}=\frac{(1-m_p)\lambda(1-e)xm+(1-m_p)\lambda(1-e)x^2A_1}{xA_1m_p+(1-m_p)(1-\lambda)A_3+(1-m_p)\lambda(1-e)x^2A_1}\equiv s(\lambda,e)$$

it can be seen that the right hand side is clearly non-negative $\forall e,\lambda$.

Next fix e, m, x, γ, and τ. I show that $F(\sigma,\lambda)=0$ defines an implicit function $\lambda=\lambda(\sigma)$. Let F_i and $f_{j,i}$ denote the partial derivatives of (1c'') with respect to the variable $i\in\{s,\lambda\}$ for $j\in\{1,2,3\}$. Clearly $F_s>0$ $\forall\lambda,s$. $F_\lambda\neq 0$ is now shown by means of a contradiction. Suppose $F_\lambda=0$, then there must exists a unique

[19] It's clear if $m<1/2$. If $m>1/2$ then the inequality requires $\gamma<m/(2m-1)$, but $m/(2m-1)<1$ since $m<1$.

$s'>0$ such that $s'=\dfrac{f_{3,\lambda}+f_{2,\lambda}}{f_{1,\lambda}+f_{2,\lambda}}$ (because $F_\lambda=0$) and $s'=s(\lambda,e)=\dfrac{f_3+f_2}{f_1+f_2}$ (because $F(\sigma,\lambda)=0$). But $f_{2,\lambda}\neq f_{3,\lambda}$ hence $F_\lambda\neq 0$ $\forall\sigma,\lambda$. Then, by the implicit function theorem $F(\sigma,\lambda)=0$ defines an implicit function $\lambda\text{-}\lambda(\sigma)=0$ for $\sigma\in[0,1]^2$.

Finally I show that $F_\lambda<0$ $\forall\sigma,\lambda$. From (1c") $\dfrac{ds}{d\lambda}=-\dfrac{F_\lambda}{F_s}>0$ because $s=0$ for $\lambda=0$, $s>0$ $\forall e,\lambda\neq 0$, $s=1$ for $\lambda=1$ and $F_\lambda\neq 0$ $\forall\sigma,\lambda$. Consequently $\dfrac{d\lambda}{ds}>0$ $\forall e$. Thus for all $e\in[0,1]$ the steady state law of motion (1c) defines a function $\lambda(\sigma):[0,1]^2\rightarrow[0,1]$ monotonically increasing in s, and a function $s(\lambda,e)$: $[0,1]^2\rightarrow[0,1]$ monotonically increasing in λ.■

Proof of Lemma 1.

In a stationary monetary equilibrium with interior solution σ, $s\in(0,1)$ and thus $\lambda\in(0,1)$ by (1c), and $c>c_b(\lambda)$. Since the interior s requires $\max\{V_g,O\}-c=V_b-c_b(\lambda)=\Pi$, from (6a), $V_g>V_b>0$ is necessary. This also implies $\Pi_{b,m}>\Pi_{g,m}$, and $\Pi_b>\Pi_g$, from (5a)-(5b). If $\Pi_g>0$ and $\Pi_{g,m}>0$, legal traders sell for both money and goods, and so do illegal producers. Since (5c) implies that a money trader's gain is independent of the type of output bought, if money traders buy legal goods they also buy illegal ones.

If $O>V_g$ then (6e) implies $e=1$, which rules out monetary legal transactions, hence $O\leq V_g$ is required for money to circulate across to the illegal sector.

Finally for a given σ, as γ rises illegal producers are subject to less frequent matches involving trade and more frequent matches involving penalties (see (2b). By observing (2a) and (3), an increase in γ has a non-negative effect on the frequency of consumption of legal and money traders, since $\tau\leq 1$. Hence V_b is decreasing in γ and V_g, V_m are non-decreasing in γ. Since larger τ implies higher likelihood of exchange, V_g, V_b, and V_m are all increasing in τ.■

Proof of Lemma 2.

Conjecture σ such that Δ=0 and $K \geq 0$. Using (2a)-(5d), and u-$c_b(\lambda)$=1+$c\lambda$, the value functions are

$$rV_g=x^2B_0 \tag{2a'}$$

$$rV_b=(1-\gamma)x^2\tau(1+c\lambda)B_1-\gamma B_2 \tag{2b'}$$

$$V_m-V_k=[xA_1(u-c_k(\lambda))-rV_k]/(r+xA_1) \tag{3'}$$

$$O=\frac{(u-c)1_{e\neq 0}-\phi+V_g}{1+r} \tag{4'}$$

where k={g,b}, $c_g(\lambda)=c$, so that $O=(u-c-\phi)r^{-1}$ for V_g-O=0 and $e\neq 0$, and $\frac{-\phi}{1+r}+\frac{x^2B_0}{r(1+r)}$ for V_g-O>0 and e=0.

Here $B_0=A_1(r+m+xA_1)/(r+mx+xA_1)$, $B_1=[m_pA_1+(1-m_p)A_2(r+xA_1)]/[r+(1-\gamma)x\tau m_p+xA_1]$, $B_2=c(1-\lambda)(r+xA_1)\}/(r+(1-\gamma)x\tau m_p+xA_1)$.

The equilibrium is monetary if Π_m>0 (money traders buy) and $\Pi_{k,m}$>0 (all producers sell for money). From (3') $u-c>V_m-V_g$ and $u-c_b(\lambda)>V_m-V_b$ hence Π_m>0. By Lemma 1 a maximum for V_k is V_g, so use (2a'); observe that $rV_g=x^2B_0<xA_1$ so that, using (3'), $V_m-V_k>0$ for all k. Consequently both (6b) and (6c) are satisfied.■

Lemma A.1

There is a $\lambda\in(0,1)$ such that Δ=0 if the following sufficient conditions hold:

a) $0<\gamma\leq\gamma_{H1}=1-[\tau(1+c)]^{-1}$, when e=0

c) $0<\gamma\leq\gamma_H=\min\{\gamma_{H1},\gamma_{H2}=\frac{m(1-m)}{r+x(1-m)}-m\}$, when $e\in(0,1)$.

Proof of Lemma A.1

Conjecture the existence of σ such that $\lambda\in(0,1)$ and $\Delta=0\leq K$. I provide sufficient conditions supporting the conjecture.

First, $\Delta \rightarrow \Delta_0 < 0$ as $\lambda \rightarrow 0$. Since V_b is decreasing in γ for λ given (by Lemma 1), let $\gamma=0$ in order to obtain a maximum for V_b. As $\lambda \rightarrow 0$ then $m_p \rightarrow (m-\gamma)/(1-\gamma)=m$ (since $\gamma=0$), $A_2 \rightarrow \tau$, $A_1 \rightarrow (1-m)\tau$, $c_b(0) \rightarrow c$, and using (2a')-(2b')

$$V_b \rightarrow V_g \tau(r+mx+xA_1)/(r+mx\tau+xA_1) < V_g.$$

Consequently as $\lambda \rightarrow 0$ then $\Delta \rightarrow V_b - V_g < 0$ $\forall \gamma$, e, and $\tau \neq 1$, while $\Delta \rightarrow V_b - V_g = 0$ for $\tau=1$ and $\gamma=0$.

Second, I provide conditions guaranteeing that $\Delta \rightarrow \Delta_1 > 0$ as $\lambda \rightarrow 1$.

As $\lambda \rightarrow 1$ then $A_1 \rightarrow A'_1 = 1-m-(1-m_p)(1-\gamma)e$, $A_2 \rightarrow 1-e$, $c_b(\lambda) \rightarrow 0$, and from (2a')-(2b')

$$rV_g \rightarrow x^2 B'_0$$

$$rV_b \rightarrow x^2(1-\gamma)\tau(1+c)B'_1$$

where $B'_1=[m_p A'_1+(1-m_p)(1-e)[r+xA'_1]\}/[r+(1-\gamma)\tau x m_p+xA'_1]$ and $B'_0=A'_1[r+m+xA'_1]/(r+mx+xA'_1)$. Notice that $(1-\gamma)\tau(1+c)\geq 1$ is satisfied by $\gamma \leq \gamma_{H1} = 1-[\tau(1+c)]^{-1}$. Since γ must be positive then the interval $(0,\gamma_{H1})$ is non-empty only if $\tau \geq \tau_L = (1+c)^{-1}$. It is easy to see that $B'_1 \geq B'_0$ when $e=0$ (since $m_p \rightarrow m$ as $\lambda \rightarrow 1$). When $e \in (0,1)$, $B'_1 \geq B'_0$ whenever the numerator of B'_1 is larger than the numerator of B'_0. Let $A'_1=1-m$ (its maximum) and $m_p=0$ (its minimum). Then a sufficient condition for $B'_1 \geq B'_0$ is $0<\gamma\leq\gamma_{H2}=\dfrac{m(1-m)}{r+x(1-m)}-m$, where $(0,\gamma_{H2})$ is non-empty if $0<m<m_H=1-r/(1-x)$ and $r<1-x$. Let $\gamma_H=\min\{\gamma_{H1},\gamma_{H2}\}$, then $0<\gamma\leq\gamma_H$ is sufficient for $\Delta \rightarrow \Delta_1 = V_b - V_g + c > 0$ as $\lambda \rightarrow 1$.

Let $0<\gamma\leq\gamma_H$, where $(0,\gamma_H)$ is non-empty, and e be such that $K\geq 0$. Since Δ is continuous in λ, $\Delta \rightarrow \Delta_0 < 0$ as $\lambda \rightarrow 0$, and $\Delta \rightarrow \Delta_1 > 0$ as $\lambda \rightarrow 1$, the intermediate value theorem guarantees the existence of a $\lambda^* \in (0,1)$ such that $\Delta=0$. Then use (1c) to find $\sigma^*=(s^*,e)$ where $s^*=s(\lambda^*,e)$ and $\lambda^*=\lambda(s^*,e)$.■

<u>Lemma A.2</u>

Let $\phi \in (\phi_L, \phi_H)$. There exists a unique $e^* \in (0,1)$ such that $K=0$.

Proof of Lemma A.2

Conjecture the existence of σ such that $\Delta=0=K$. In what follows I provide sufficient conditions to support the conjecture. Since $V_g=O$ under the conjecture, (2a') and (4') imply $K=[x^2B_0-(u-c-\phi)]r^{-1}=0$ or $K=0 \leftrightarrow \phi=u-c-x^2B_0$.

Denote with a prime (') the partial derivative with respect to e. Since $m'_p<0$ (from Lemma 2) and $A'_2<0$, then $A'_1<0$. Hence

$$\frac{\partial B_0}{\partial e} \propto A'_1[1+xA_1\left(\frac{1}{r+m+xA_1}-\frac{1}{r+mx+xA_1}\right)]<0$$

since $xA_1|[(r+m+xA_1)^{-1}-(r+mx+xA_1)^{-1}]|<1$ (the difference of two terms smaller than one is less than one). Let $B_H=\lim_{e\to0}B_0$, $\lim_{e\to1}B_0=B_L$, and notice that since V_g can never exceed $(u-c)/r$, then $u-c=1>x^2B_H>x^2B_L\geq0$. Additionally, $\lim_{m\to0}B_0>\lim_{m\to1}B_0=0\ \forall e$. This is so because as $m\to1$ then $m_p\to1$ so $A_1\to0$ and $B_0\to0\ \forall e$. As $m\to0$ then $m_g,m_p\to0$ so that $A_1\to A=1-(1-\gamma)(1-A_2)=1-(1-\gamma)[1-\lambda(1-e)-(1-\lambda)\tau]>0$, hence $B_0\to A\ \forall e$. Since $B_0>0\ \forall m$, then B_0 must also be decreasing in m in a left neighborhood of $m=1$.

Next, if $\phi>u-c-x^2B_H=\phi_L$ then $\lim_{e\to0}K>0$ and if $\phi<u-c-x^2B_L=\phi_H$ then $\lim_{e\to1}K<0$. From the discussion above $0<\phi_L\leq\phi_H\leq u-c$, and ϕ_L, $\phi_H\to(u-c)$ as $m\to1$. The set (ϕ_L,ϕ_H) is non-empty as long as $m<1$. Let $\phi\in(\phi_L,\phi_H)$ and $m<1$. By the intermediate value theorem, there exists an $e^*\in(0,1)$ such that $K=0$. Since B_0 is monotonically decreasing in e, then e^* is also unique. Finally, since B_0 is falling in e and in m around $m=1$, then $de^*/dm<0$ for m sufficiently large.■

Proof of Proposition 1.

Let $e=0$ and choose τ such that $(0,\gamma_{H1})$ is non-empty. Let $0<\gamma\leq\gamma_{H1}$. From lemma A.1 there exists $\sigma^*=(s^*,0)$ where $s^*=s(\lambda^*,0)$ such that $\Delta=0$. Let $\sigma'=\sigma^*$, then lemma 2 assures σ^* is a symmetric stationary monetary equilibrium.■

Proof of Corollary to Proposition 1.

In the following I show that (a) If $\gamma\neq 0$ or $\tau\neq 1$ then s=0 is not an equilibrium; (b) If γ=0, τ=1 and $\phi=u-c$, the only equilibrium is s=0 and e=0; (c) If $\gamma<1$ and τ is sufficiently small the equilibrium is s=1 $\forall e$.

To prove (a) let $\gamma\neq 0$ or $\tau\neq 1$, and s=0, hence λ=0. From the proof of Lemma A.1, this implies $\Delta<0$ so that s'=1. Consequently s=0 is not an equilibrium.

To prove (b) let γ=0, τ=1, $\phi= u-c$, s=0, hence $\lambda=0=e$. From the proof of Lemma A.1, this implies Δ=0 so that $s'=0=s$ and $e'=0=e$ is the only equilibrium.

To prove (c) let $\gamma<1$, τ=0, s=1, hence λ=1. From the proof of Lemma A.1, this implies $\Delta<0$ so that $s'=1=s$ is an equilibrium. Since V_b is decreasing in γ, and continuous in γ and τ, one can find an $\varepsilon>0$ and $\gamma(\varepsilon)<\gamma$, such that $\Delta<0$ if $\tau=\varepsilon$, and $s=1=\lambda$, in which case $s'=1=s$ is an equilibrium. ■

Proof of Proposition 2.

Let γ=0 then $m_p=m$, $A_1=(1-m)A_2$, $A_2=\lambda(1-e)+(1-\lambda)\tau$. In an interior equilibrium $\Delta=x^2\tau B_1/r-x^2B_0/r+c\lambda(1+x^2\tau B_1/r)=0$ which, after can be rearranged as

$$\Delta=c\lambda(B_4+\tau)+\tau-B_3$$

where $B_3=\dfrac{r+xm\tau+xA_1}{r+xm+xA_1}$ and $B_4=\dfrac{r(r+xm\tau+xA_1)}{x^2A_1(r+m+xA_1)}$. Since $c\lambda=(B_3-\tau)/(B_4+\tau)>0$, then $d\lambda/dm$ is proportional to $\dfrac{\partial B_3}{\partial m}(B_4+\tau)-\dfrac{\partial B_4}{\partial m}(B_3-\tau)$. Observe that when e=0 (a constant) then $\partial A_1/\partial m<0$, $\dfrac{\partial B_3}{\partial m}<0$, and $B_3>\tau$, hence in equilibrium $d\lambda/dm<0$ if $\dfrac{\partial B_4}{\partial m}\geq 0$. It can be shown that

$$\frac{\partial B_4}{\partial m}\propto(r+m+xA_1)(r+x\tau)-(1-m)(1-xA_2)(r+x\tau m+xA_1) \tag{7}$$

and since $r+m+xA_1>r+x\tau m+xA_1$ then $(r+x\tau)-(1-m)(1-xA_2)\geq 0$ is sufficient to guarantee (7) is positive.

Since $A_2>\tau$ then rewrite the last inequality as $(r+x\tau)-(1-m)(1-x\tau)\geq 0$ so that $m \geq m_L=1-\frac{r+x\tau}{1-x\tau}$ is sufficient for (7) to be positive. Furthermore $m_L<1$ if $x<\frac{1-r}{2\tau}$ and $m_L\leq 0$ if $1>x\geq\frac{1-r}{2\tau}$. Consequently $d\lambda/dm<0$, $\forall m\in(0,1)$ if $1>x\geq(1-r)/2\tau$, and for $m\geq m_L$ if $x<(1-r)/2\tau$. Finally, by standard continuity arguments, there must exist a γ in a neighborhood of 0 such that $d\lambda/dm<0$. ■

Proof of Proposition 3.

Choose τ, r, m such $(0,\gamma_H)$ is non-empty, and let $0<\gamma\leq\gamma_H$. Lemma A.1 assures the existence of a $\lambda^*\in(0,1)$ such that $\Delta=0$ for $s^*=s(\lambda^*,e)$. Lemma A.2 assures the existence of $e^*\in(0,1)$ such that $K=0$ for $s=s(\lambda^*,e^*)$ if $\phi\in(\phi_L, \phi_H)$. Let $\sigma'=\sigma^*=\{s^*, e^*\}$. Lemma 2 assures σ^* is a symmetric stationary monetary equilibrium.■

Proof of Proposition 4.

Let $e\neq 0$ such that $K=0$, $s\neq 0$ such that $\Delta=0$, and suppose e is increasing in m. From the proof of proposition 2, notice that since $A_1=(1-m)[\lambda(1-e)+(1-\lambda)\tau]$ then $\partial A_1/\partial m= -A_2 - (1-m)(\partial e/\partial m)<0$ also applies to the case where $e\neq 0$. Furthermore, both $1-\frac{r+x\tau}{1-x}<m_H$ and $x>\frac{1-r}{1+\tau}$, satisfy the requisites for the non-emptiness of the set $(0,\gamma_{H2})$ provided in the proof of proposition 3, and hence support the existence of an interior equilibrium. Consequently the proof of proposition 2 applies, and $d\lambda/dm<0$. ■

References

Aiyagari, S.R. and N. Wallace, (1997), "Government Transaction Policy, the Medium of Exchange and Welfare", Journal of Economic Theory,, 74, p. 1-18

Aiyagari, S.R., N. Wallace, and R. Wright, (1996), "Coexistence of Money and Interest-Bearing Securities", Journal of Monetary Economics, 37, p. 397-419

Cohen L.E., and R. Machalek, (1988), A General Theory of Expropriative Crime: An Evolutionary Ecological Approach", *American Journal of Sociology*, Vol. 94, 3, p.465-501

Emmons, W. R., (1997), "Price Stability and the Efficiency of the Retail Payments System", *Federal Reserve Bank of St. Louis Review*, September/October, p.49-68

FATF, (1997), Annual Report 1996/97, Organization for Economic Cooperation and Development

Goodhart, C. A. E., (1989), *Money, Information and Uncertainty*, Houndmills, Basingstoke and London, MacMillan., p.381-382.

Green, E. J., and W. E. Weber, (1996), "Will The New $100 Bill Decrease Counterfeiting?", Federal Reserve Bank of Minneapolis, Working Paper 571, September.

Grossman, Gene, M., and C. Shapiro, (1988), "Counterfeit-Product Trade", *American Economic Review*, Vol.78, No.1, p.58-75

Humphrey D.B. and A. N. Berger, (1990), "Market failure and Resource Use: Economic Incentives to Use Different Payment Instruments", in D. B. Humphrey, Ed., *The U.S. Payment System: Efficiency, Risk, and the Role of the Federal Reserve*, Boston, Kluwer Academic Publishers, p.45-86

Humphrey, D. B., L. B. Pulley, and V. M. Jukka, (1996), "Cash, Paper, and Electronic Payments: A Cross-Country Analysis", *Journal of Money, Credit and Banking*, Vol. 28, N. 4, Part 2, p. 914-939

Kiyotaki, N. and R. Wright, (1993), "A Search Theoretic Approach to Monetary Economics", *American Economic Review*, Vol. 83, No.1, p.63-77.

Li, Y., and R. Wright, (1998), "Government Transaction Policy, Media of Exchange, and Prices", *Journal of Economic Theory*, forthcoming.

Rogoff K., (1998), Blessing or Curse? Foreign and underground demand for euro notes", Economic Policy, April, p.262-303

Soller, E. and C. Waller, (1998), "A Search Theoretic Model of Legal and Illegal Currency", Manuscript.

Sumner, S. B., (1990), "The Transactions and Hoarding Demand for Currency", *Quarterly Review of Economics and Business*, Vol. 30, No.1, p. 75-89

Vila, B.J., and L.E. Cohen, (1993), "Crime as Strategy: Testing an Evolutionary Ecological Theory of Expropriative Crime", *American Journal of Sociology*, Vol. 98, 4, p.873-912

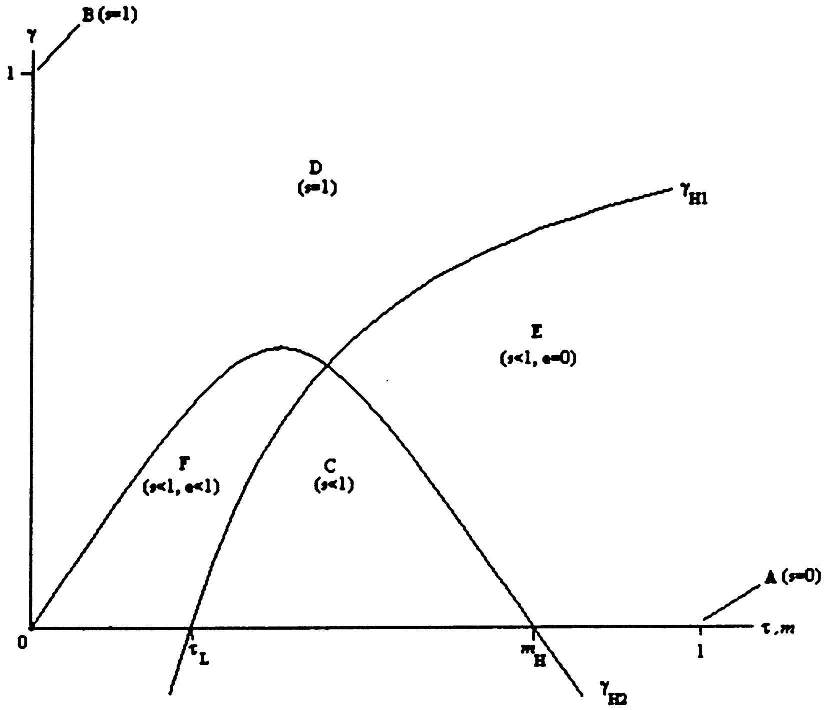

Figure 1 – Equilibria

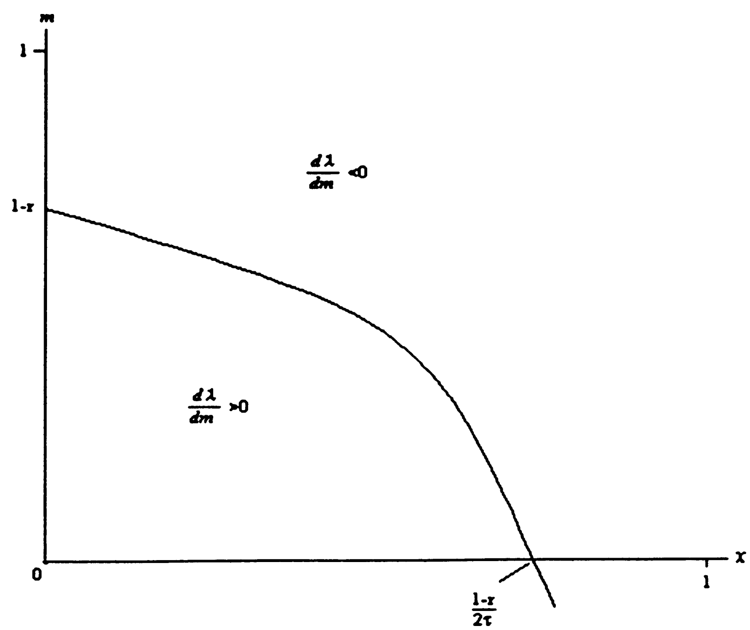

Figure 2

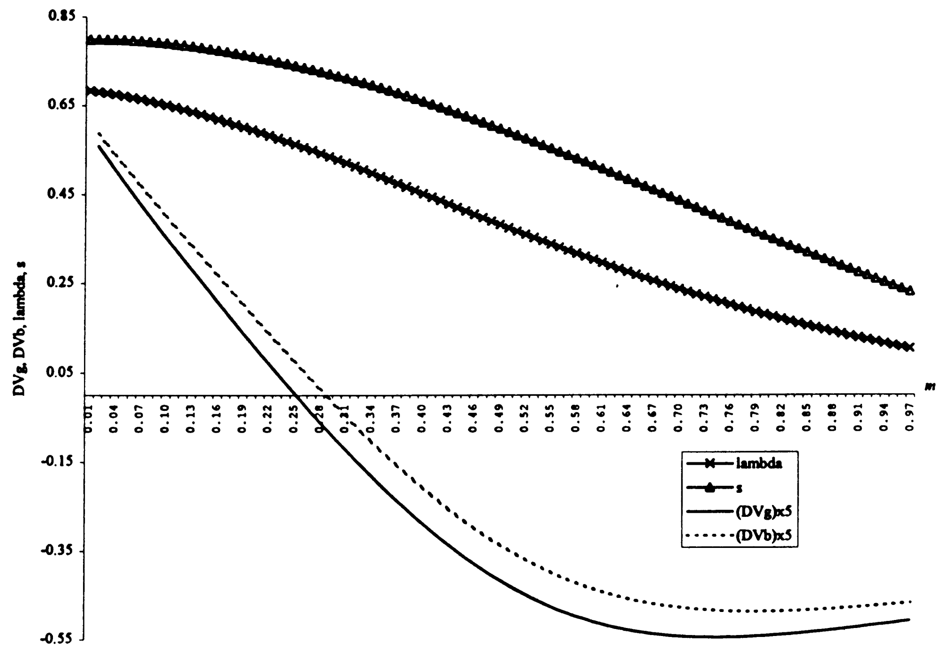

Figure 3a - λ and *s* across *m* for "small" search frictions (*x*=0.25)

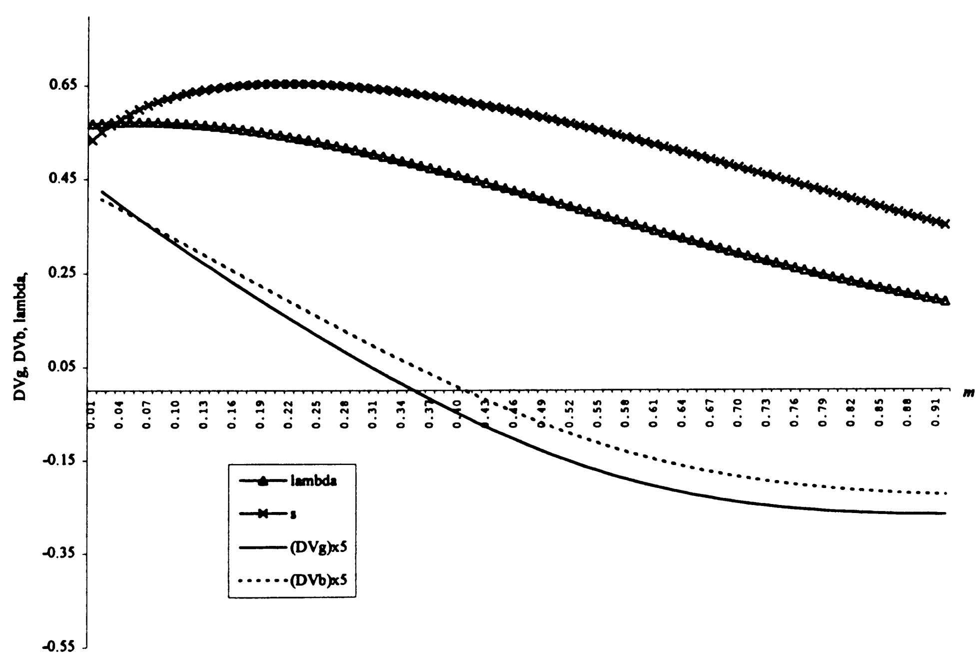

Figure 3b - λ and *s* across *m* for "larger" search frictions (*x*=0.125)

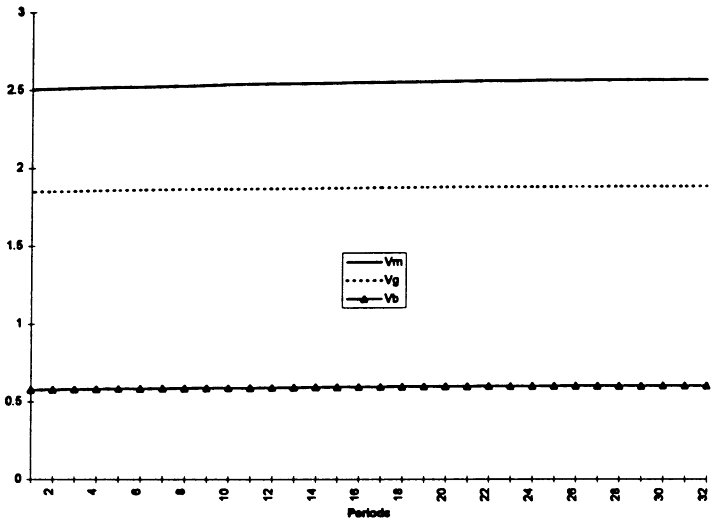

Figure 4a - Value functions in a dynamic experiment (baseline parameters)

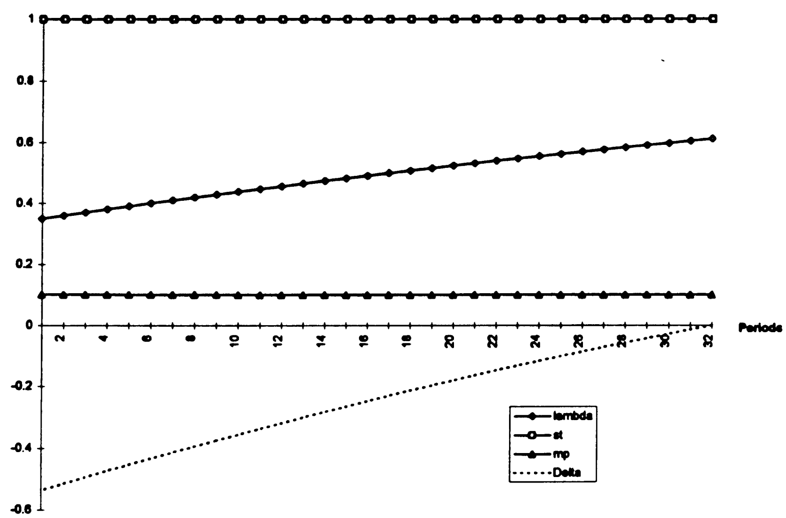

Figure 4b - Strategies and Distributions in a dynamic experiment (baseline parameters)

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