

The background of the entire page is a grayscale photograph of a modern, multi-story building with a glass facade. In the foreground, there is a paved courtyard area with several trees and benches. The text is overlaid on this image.

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Search, Dealers, and the Terms of Trade

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## Search, Dealers, and the Terms of Trade<sup>(\*)</sup>

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### Abstract

I illustrate a search theoretic environment that allows endogenous determination of number of trade facilitators and markup charged on mediated sales. Developed around Kiyotaki and Wright's (1989) exchange economy, the study relaxes the assumptions of exogenous prices and distribution of specialized consumption-production activities. There is a unique equilibrium where dealers endogenously arise, buying the lowest storage-cost good at a discount, reselling it at a premium. The resulting markup responds in predictable ways to extent of trading frictions, storage cost, and distribution of specialty production. There is scope for price dispersion in that mediated transactions occur at unequal terms of trade for different agents, even if storage cost and time discounting vanish. Due to a trading externality generated by indirect exchange, absence or choice of pricing mechanism has implications for existence and efficiency of the transaction arrangement.

Key Words: Search, Intermediation, Prices

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## 1. Introduction

In a world burdened by absence of double coincidence of wants, there is little scope for intermediation or terms of trade dispersion when homogenous agents can trade multilaterally and simultaneously in a centralized market. Specialized preferences and production would not hinder the satisfaction of agents' effective-demand (as in Clower, 1965) nor the triumph of the law of one price, at each date. This is less obvious, however, in a more realistic world of decentralized, multiple bilateral exchanges where transaction dates are random.

While a large body of literature has shown why intrinsically useless fiat money may arise as a better way to organize exchange (see Wallace, 1998), trade facilitators may also emerge as a natural response to exchange frictions, and in this paper I endogenize a prototypical trade intermediary institution.<sup>1</sup> Basic economic intuition tells us that incentives for mediated exchange would probably exist if impatient buyers are willing to pay a premium, and sellers offer discounts, to someone capable of quickly satisfying their effective demand. One basic economic question is how this “bid-ask” spread, or markup, would respond to extent of trading frictions, intermediation costs, and the composition of the demand and supply for the different goods. Another question is whether the equilibrium can be efficient, in the sense that the decentralized allocation corresponds to the solution of a planner's problem who, taking as given the trading arrangement, maximizes social welfare by imposing the terms of trade.

In this study I illustrate a search theoretic environment that allows endogenous determination of the number of trade facilitators and the markup they charge on their intermediated sales. The exercise intends to show how the equilibrium terms of trade offered by dealers respond to extent of trading frictions, intermediation cost, and distribution of specialty production. Second, it investigates whether there is scope for dispersion in the terms of trade, in that different agents trade at different prices with the dealer. Third, because of the trading externality generated by intermediation, I investigate whether the existence of a non-degenerated distribution of terms of trade is necessarily inefficient. I extend and complement work on random matching models of

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<sup>1</sup> The role and scope of intermediation in bilateral search markets has been the focus of a number of studies where price formation is not endogenous. Among them, Rubinstein and Wolinsky (1987), and Cosimano (1996) examine endogenous determination and extent of intermediaries for a non-monetary economy, and Bose and Pingle (1995) for a monetary economy. In Bhattacharya and Haggerty (1989) endogenous intermediaries and producers coexist in the presence of a positive trading externality. Yavas (1994) shows that if search is sufficiently expensive costly middlemen may improve welfare, and so does Li (1998, 1999) where middlemen allow traders to overcome information frictions. More recently Shevchenko (1999) studies the optimal inventory of middlemen (i.e. quantity and type of goods stored) in a non-monetary

decentralized exchange, by pointing at the implications that both the absence, but also the choice of mechanisms for endogenous price determination may have on existence and efficiency of the equilibrium transaction arrangement.

The study essentially develops along the lines of Kiyotaki and Wright's (1989) commodity money model. This is a natural starting point because the model's market frictions, stemming from bilateral random matches and specialized consumption/production, make trade essential for consumption, and intermediation necessary for the acquisition of some goods: certain agents choose to undertake the role of dealers, costly storing a commodity they don't consume to resell it to others. I amend the model by relaxing the two key assumptions of terms of trade exogenously fixed at par, and exogenously uniform distribution of agents who specialize in the different consumption-production activities.<sup>2</sup> That is to say, I endogenize both prices and the number of agents who produce and supply the different commodities.

The analysis focuses on one natural transaction pattern, which Kiyotaki and Wright labeled "fundamental". In this outcome some individuals choose to satisfy their effective demand by engaging in a sequence of indirect trades involving only the lowest storage cost good, which thus becomes a "commodity money". There are several reasons for focusing on this equilibrium. First, this directs attention to the endogenous determination of equilibrium terms of trade, the agents' choice of specialty production, and extent of intermediation, rather than the many different transaction patterns shown to exist in this class of models.<sup>3</sup> Second, the fundamental pattern of transactions is often thought to be the most "natural" in environments where decentralized trade requires costly storage of goods. This has been suggested by both simulation of random-matching economies, and experimental work. For instance, Marimon et Al. (1990) have shown that in a prototypical Kiyotaki and Wright exchange economy with artificially intelligent agents, trading and consumption patterns converge to the stationary equilibrium where the good with lowest storage cost emerges as the medium of exchange. By placing human subjects in an experimental environment resembling the Kiyotaki and Wright model, Brown (1996) and Duffy and Ochs (1998) have also found that the observed trading pattern generally favors the indirect exchange of

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Kiyotaki and Wright economy where middlemen are generalists in consumption.

<sup>2</sup> Equilibrium price formation with divisible non-storable goods and indivisible money has been discussed by Shi (1995) and Trejos and Wright (1995).

<sup>3</sup> The multitude of equilibria that can be obtained in this class of models is well illustrated by Aiyagari and Wallace (1991) and Kehoe et Al. (1993) who generalize the original Kiyotaki and Wright model to include, alternatively,  $N$  commodities, and either stationary or dynamic mixed strategies.

the good with lowest storage cost, even when it is not an equilibrium behavior as predicted by the theory.

Finally, by focusing on a fundamental equilibrium I can point to the implication that the absence of a mechanism for endogenous price determination has on the occurrence of some transaction patterns. I exemplify it by resolving an issue raised by Wright (1995), namely that the cheapest-to-store good won't have a chance to become the unique medium of exchange when agents choose their specialty production but the terms of trade are exogenously fixed at par. In fact, by considering arbitrary endogenous price formation rules, I demonstrate the existence of a continuum of terms of trades consistent with this fundamental trading pattern.

I conduct most of the analysis by adopting a simple pricing rule following Diamond (1984). Namely, the terms of trade in each match are negotiated via a bilateral bargaining process that is always successful and that divides evenly the net utility gain from completing the transaction. Adoption of this simple protocol keeps the model tractable and has the virtue of engendering clarity in both derivation, characterization of equilibria, and particularly the analysis of dispersion of terms of trade. The latter is an important element of the analysis. As recognized by Stigler long ago (1961), if buyers must search for sellers and time is discounted, unfavorable offers may be accepted today even if better offers can be encountered tomorrow. As the frequency of transaction becomes large, however, it is reasonable to expect that price dispersion should vanish.<sup>4</sup> This study shows that when prices guarantee equal trade surplus to both the middleman and her customer, dealers arise endogenously only if they can obtain discounts from producers and, sometimes, charge a premium to buyers of the intermediated good. In other words, not only in equilibrium there is a wedge between "bids" and "offers", but this wedge does not generally vanish as intermediation cost or rate of time discounting approach zero. Furthermore, there is scope for terms of trade dispersion whereby the relative amounts of consumption commodities delivered by the dealer to different types of agents--buyers or sellers of the intermediated good--are also dissimilar.

To summarize the other main results, I show that in equilibrium participation in intermediation and terms of trade are fully flexible and respond in predictable ways to extent of trading frictions, intermediation cost, and individuals' choice of specialty production. As trading

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<sup>4</sup> Camera and Corbae (1999) study endogenous price dispersion in a random-matching monetary economy and show that price dispersion disappears as search frictions vanish, given that buyers (who have money) can extract the entire surplus from sellers (who don't).

frictions or intermediation cost grow, the number of dealers falls, the markup they charge grows, and dealers ask for larger discounts to producers of the intermediated commodity. By considering arbitrary pricing mechanisms I show the existence of an equilibrium in which terms of trade are identical across the economy, whereby exchange of any good occurs at par, and where the distribution of the consumption-production types becomes uniform (as in Kiyotaki and Wright, 1989) as the intermediation cost vanishes. This is only one of a continuum of price vectors capable of supporting the fundamental trading pattern and, because of a trading externality generated by indirect exchange, is not the most efficient.

What follows contains a description of the environment (section 2), a discussion of the existence of the equilibrium (section 3), a characterization (section 4), a discussion of existence and welfare under different pricing rules (section 5).

## 2. Environment

The model is a version of the random matching model that Kiyotaki and Wright (1989) refer to as model A. To summarize it, time is discrete and continues forever. There is a constant population comprised of a continuum of ex-ante identical infinitely-lived individuals of measure one. They can all produce a non-storable unit of an autarkic good, good 0, whose consumption generates net utility  $a \geq 0$ . At the beginning of their life, they are given the choice to dispose of their autarkic production opportunity in order to acquire a market production opportunity  $i \in \mathbb{N} \equiv \{1, 2, 3\}$ . By doing so the individual chooses to specialize in the costly production of some non-negative quantity of good  $i$ , that she cannot consume, and whose production requires consumption of some amount of market good  $i-1$ . At any point in time market production opportunities can be freely disposed in favor of reverting back to autarkic production.

Because of the link between production and consumption, let producers of  $i+1$  be identified as individuals of type  $i$ . Consumption of  $q_i$  units of good  $i$  generates instantaneous utility  $u(q_i) > 0$  if the individual is of type  $i$  (zero otherwise), where  $u(q)$  is strictly increasing, weakly concave, continuously differentiable, and  $u(0) = 0$ .<sup>5</sup> Future utility is discounted at rate  $\beta \in (0, 1)$ . Production of  $q_{i+1}$  generates disutility  $\gamma q_{i+1}$ ,  $\gamma > 0$ , and let  $q = \{q_1, q_2, q_3\}$  denote the production vector in the

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<sup>5</sup> Strict concavity is not needed because of the storage upper bound. As it will be clear later, this helps bounding *all* production so individuals never produce infinite amounts.

economy. Additionally, any individual has the ability to store any good  $i$  in unit amounts, from one date to the next. This activity, however, generates disutility  $c_i$  per period, where  $0 < c_1 \leq c_2 \leq c_3$ . Furthermore, storage and market production are mutually exclusive, and once good  $i$  is stored it must be traded as an indivisible unit.

Agents are spatially separated, and autarkic producers stay put at their location where they cannot search nor be met by others. Market producers, however, search for trade partners and at each date are paired randomly and anonymously with another agent. While the type of trade partner (objects stored and preferences) and the actions taken in a match are common knowledge, trading histories of past matches are private information. Furthermore, agents cannot commit themselves to future actions.

Once in a match, two producers can trade contingent on the existence of double coincidence of wants, in which case both simultaneously produce and consume at the beginning of the period following the encounter. Given  $q$ , let  $U_i = u(q_i) - \gamma q_{i+1}$  denote the net instantaneous utility derived by  $i$ , and assume that  $u(q) - \gamma q$  is positive and increasing on  $q \in (0, 1]$ . Because of her ability to store commodities, note that agent  $i$  can also choose to produce in exchange for a unit of a commodity she does not consume.

Since some of these features may seem extreme, I briefly explain their rationale before moving further into the analysis. First, production and preferences' specification is a simple way to assume that technologies and tastes are specialized. This feature, jointly with random and anonymous pairings is a simple yet effective way to motivate the existence of potential gains from trade from specializing in the production of some market good. Second, the storage upper bound and indivisibility limits the state space and keeps the model tractable, thus avoiding the complications generated by tracking a very complex distribution of multiple inventories.<sup>6</sup> Third, because of absence of double coincidence in matches between producers, private information on trade histories, and absence of commitment, trade needs to be facilitated by an intermediary whereby at least one commodity serves as a medium of exchange and store of value. Trade intermediation, however, is a costly activity because of the existence of storage costs, and time discounting. This has implications for the choice to become a middleman, and what terms of trade to offer. The virtue of these assumptions, I think, is that they make the model sufficiently

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<sup>6</sup> Shevchenko (1999) analyzes exactly this issue by relaxing the inventory assumption and studying the possible distribution of inventories.

simple, and the results clear and easy to derive, yet without sacrificing the rigorousness of the analysis.<sup>7</sup>

### 3. Stationary Symmetric Fundamental Equilibrium

I focus on the existence of outcomes with no autarkic production, and where individuals always accept the lowest storage-cost good.<sup>8</sup> The focus is on symmetric rational expectations equilibria that are stationary, and where individuals adopt symmetric Nash strategies taking prices and strategies of others as given. The amount of goods to be exchanged in each match is determined via bargaining. Specifically, most of the analysis is conducted assuming that the terms of trade are negotiated via a bilateral bargaining process which is always successful and that divides evenly the net utility gain from completing the transaction. This pricing rule is based on the correct evaluation of the gains from trade in each possible match, is adopted for tractability, and may be thought of as implementing a solution to a more structured game in which two individuals engaged in bargaining may meet (and deal with) other traders continuously, during the bargaining process.<sup>9</sup> In equilibrium production and trading decisions are individually optimal, given the correctly perceived strategies of others and distribution of objects, and are also time-invariant and identical for individuals of identical type.

I proceed by conjecturing an equilibrium in which all commodities in  $N$  are produced, and individuals play pure fundamental trading strategies for some given production vector  $q$ . Then I show that  $q$  is unique and provide conditions for existence of the equilibrium. Finally I characterize the outcome in terms of the endogenous distribution of consumption-production types and objects, value function (defined over type and holdings), trading strategies, and prices. Each of these elements is discussed separately below.

#### 3.1 The initial choice of productive activity

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<sup>7</sup> Neil Wallace (1997) cleverly describes both the virtues and drawbacks of these and other standard assumptions of prototypical random matching models.

<sup>8</sup> While equilibria could exist where some choose autarky, this is beyond the scope of the present exercise. The reader can consult Baye and Cosimano's (1987) study of Nash equilibria when players choose the sides of a matching process and costly participation in economic activity. Shi (1997) also studies a random matching model of money with explicit market participation choices.

<sup>9</sup> See for instance Diamond (1984), and Luce and Raiffa (1957).



In the initial period,  $t=0$ , individuals simultaneously choose a production opportunity, taking as given the actions of others. Each market production opportunity has a chance to be chosen by someone if it is weakly preferred to autarky and to the remaining others. Because of the link between production and types, I can interpret the choice of production as a choice over types. Let  $s'(i)$  denote the probability that, at the beginning of life, the average individual chooses to produce good  $i=0,1,..3$ , given the choices  $\{s(i)\}_{i=0,1,..3}$  of all others. Then for any two production choices  $j \neq i$

$$s'(i) \begin{cases} = 1 & \text{if } i \succ j \ \forall j \\ \in [0,1] & \text{if } i \sim j \ \forall j \\ = 0 & \text{if } i \prec j \ \text{for some } j \end{cases} \quad (1)$$

where  $\succ$  represents strict preference and  $\sim$  weak preference over types of production. Thus  $s'(i)$  is the individual's Nash best response to the choices of all others.

Define a symmetric stationary *search* equilibrium as an outcome where there is no autarkic production, i.e.  $s(i)=s'(i)=0$ . In such an equilibrium market production is strictly preferred to autarky, hence there must be a positive demand for each market commodity. This necessary feature implies that individuals must be indifferent among any market production.<sup>10</sup> For this reason in a search equilibrium  $\sum_{i \in N} s(i) = 1$ , and  $s(i) \neq \{0,1\}$ , so that I let  $s = \{s(1), s(2), s(3)\}$  denote the vector of production choices of all others.

Let  $p_i$  denote the proportion of the population who has chosen to produce market commodity  $i+1$ . This is equivalent to saying that  $p_i$  is the proportion of the population who has chosen to become a trader of type  $i \in N$ , or who is, for short, of type  $i$ . Because of the discussion above, in a search equilibrium  $p = \{p_1, p_2, p_3\}$  must be a vector with elements laying strictly in the unit interval.

Due to the existence of storage costs, it is easy to see why an individual  $i$  would never prefer to produce and store her own production of good  $i+1$ . Furthermore, because of time discounting, she would consume her preferred commodity  $i$  as soon as she could obtain it.<sup>11</sup> She could,

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<sup>10</sup> Assume  $s(i)=0$  for some market good  $i$  and  $s(j)>0$  for  $j \neq i$ . Given the specification of preferences, production technology, and matching, then autarky would be preferred to market production  $i+1$ , since good  $i$  could never be consumed. A similar conclusion holds if  $s(i)=1$  for some  $i$ .

<sup>11</sup> Conditions assuring that this is the case will be provided in what follows.

however, decide to store commodity  $i+2$  which she is incapable of producing, and that generates no consumption utility to her, if this allows her to obtain her preferred commodity in some future match. For this reason let  $p_{ij}$  denote the proportion of individuals of type  $i$  who owns a market production opportunity (if  $j=i+1$ ), or, alternatively, the proportion of individuals of type  $i$  who are storing *one* unit of good  $j$  (if  $j=i+2$ ). Then

$$\sum_{i \in \mathbf{N}} p_i = \sum_{j \in \mathbf{N}} p_{ij} = 1. \quad (2)$$

Because good  $i$  is never stored by type  $i$ , at each point in time she can be either a *producer*, of good  $i+1$ , or a middleman or *dealer* storing one good  $i+2$ . Using (1) and (2), it follows that in a symmetric search equilibrium the stationary distribution of types must be such that for all  $i$ ,

$$s_i = p_i \in (0,1), \quad (3)$$

that is the elements of  $p$  are such that in equilibrium ex-ante identical individuals are indifferent between the choice of any of the three market production activities.

To determine the endogenous distribution of types, let  $V_{ij}$  denote the lifetime utility of  $i$  when she stores one unit of good  $j$  ( $j=i+2$ ) or when she is a producer of that good ( $j=i+1$ ), and let  $V_a$  be the autarkic lifetime utility. Additionally, let  $E(V_i) \equiv \sum_{j \in \mathbf{N}} p_{ij} V_{ij}$  denote the expected lifetime utility of an individual of type  $i$ , *unconditional* on her current inventory. It may also be interpreted as the average utility for an agent of type  $i$ , a function of the endogenous distribution of inventories  $\{p_{ij}\}$ .

I say that an agent is ex-ante indifferent between the choice of becoming a type  $i$  or  $h$  when the two expected lifetime utilities, unconditional on inventory, are identical but larger than the value attached to autarkic production.<sup>12</sup> It follows that when the individual takes as given the terms of trade, the endogenous distribution of types implied by the initial choice of productive activity, and the endogenous distribution of objects implied by the trading strategies of all others, in a stationary symmetric search equilibrium agents must be indifferent across production types,

$$E(V_i) = E(V_h) \quad (4)$$

for all  $i, h \in \mathbf{N}$ , and must also strictly prefer market production to autarky,

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<sup>12</sup> While other measures are possible (for instance measuring expected utility conditional on type *and* current inventory) this measure is easy to work with, and it has been previously proposed (Wright, 1995).

$$E(V_i) > V_a. \tag{5}$$

### 3.2 Trading Strategies

This section focuses on the endogenous choice of whether to provide intermediation services, or not. That is, the choice of  $i$  when she is a producer, and she is matched to an individual who offers her a commodity that  $i$  can neither consume nor produce (good  $i+2$ ).

Recall that storage is limited to one unit, and that in equilibrium  $i$  always accepts to trade for good  $i$ . Recall also that when  $i$  can produce good  $i+1$ , she has no advantage in acquiring it from someone else. Thus, as a producer,  $i$ 's relevant choice is whether she would be willing to trade some of her output in exchange for one unit of a good she cannot consume or produce. She decides to do so while taking the possible gains from trade as given. That is to say, prior to entering the bargaining process, her choice of accepting or refusing to trade is based on the (correct) forecast of the resulting terms of trade. Contingent on having decided to acquire one unit of good  $i+2$ , then  $i$  will bargain over the terms of trade. This latter process defines the quantity  $q_{i+1}$  she will produce, and its equilibrium outcome is described later, in section 3.5.

Let  $\pi_i \in \{0,1\}$  denote the *trading strategy* of an individual of type  $i$ , when she takes as given the trading strategy of all others, summarized by  $\pi = \{\pi_1, \pi_2, \pi_3\}$ , the terms of trade, summarized by  $q$ , and the distribution of objects and types,  $p$  and  $\{p_{i,j}\}$ . The strategy  $\pi_i$  defines the probability that a type  $i$  accepts one unit of good  $i+2$  (modulo 3) in exchange for an amount  $q_{i+1}$  of her production. Note that the definition of  $\pi_i$  implies that  $i$  plays a pure strategy, and never mixes. This is without loss in generality, in equilibrium, because of the structure of the matching process and the perfect divisibility of the quantity of goods that  $i$  can offer. In equilibrium, agent  $i$ 's counterparty (agent  $j$ ) must want good  $i+1$  in order to consume it because: (i) if  $j$  could produce  $i+1$  then she would not accept it, hence  $j \neq i$ , and (ii) if she could not produce it and did not want to consume it, then it must be that  $j = i+2$ , but this can't be either since I am considering the case where  $j$  offers good  $i+2$ . Note that  $j$  must have a strict positive surplus from receiving the equilibrium quantity  $q_{i+1}$ . However, suppose agent  $i$  is indifferent between abstaining from trade or not, and so chooses a mixed strategy  $\pi_i < 1$ . Agent  $j$  would strictly prefer to marginally reduce her request of  $q_{i+1}$ , so that agent  $i$  strictly prefers trade.

It follows that in a *fundamental* strategy equilibrium types  $i=1,3$  only accept their respective

consumption commodities, while type 2 always accepts both his consumption commodity, but *also* one unit of commodity one. Hence in a symmetric equilibrium where the fundamental trading strategy is played

$$\pi = \pi' = \pi^* \equiv \{0, 1, 0\}. \quad (6)$$

### 3.3 Distribution of types and objects.

I now consider the distribution of objects across types, associated with  $p$  and  $\pi$ . Following the discussion in the previous section, in a search equilibrium individuals consume their consumption commodity which implies  $p_{i,i} = 0 \forall i \in N$ . When the fundamental strategy defined in (6) is an equilibrium, then  $p_{1,3} = p_{3,2} = 0$  since types 1 and 3 never accept and store a good they don't consume. Using (2) it follows

$$p_{1,2} = p_{3,1} = 1. \quad (7)$$

Type 2, however, accepts good one, so sometimes may store it, while sometimes holds his production opportunity. This implies that  $p_{2,3}$  and  $p_{2,1}$  should generally both be positive and must satisfy the two laws of motion

$$p'_{2,3} = p_{2,3} + p_{2,1} p_1 p_{1,2} - p_{2,3} p_3 p_{3,1} \quad (8)$$

$$p'_{2,1} = p_{2,1} - p_{2,1} p_1 p_{1,2} + p_{2,3} p_3 p_{3,1} \quad (9)$$

where a prime denotes next period's value. The second term on the right hand side of (8) reflects how frequently a type 2 switches from his role as a middleman (storing good 1) to being a producer of good 3. This switch occurs whenever, as a middleman, he transfers the good in his storage to a type 1, an event that occurs with probability  $p_1 p_{1,2}$ . The third term refers to instances in which a type 2 switches from being a producer to being a middleman: with probability  $p_3 p_{3,1}$  he sells some of his output to a type 3 and acquires one unit of good 1. In the steady state (9) is redundant because of (2), and the stationary proportions are

$$p_{2,1} = \frac{p_3}{p_1 + p_3} \quad \text{and} \quad p_{2,3} = \frac{p_1}{p_1 + p_3}. \quad (10)$$

### 3.4 Value functions

Agents choose their strategies with the objective of maximizing the expected discounted

lifetime utility derived from consumption. For clarification purposes I briefly discuss an encounter between  $i$  and  $j \in \{i-1, i+1\}$ , the only type of match that may lead to exchange. With probability  $p_j p_{j,h}$  type  $i$  meets a type  $j$  offering good  $h$ . Individual  $i$  might buy if  $h \in \{i, i+2\}$ , and if a purchase occurs  $i$  consumes it if  $h=i$ , otherwise she stores it. When  $i$  decides to make the purchase, she bargains over the terms of trade, a process that results in her acquisition of  $q_h$  units of good  $h$ , in exchange for either for  $q_{i+1}$  units of her output (if she is a producer), or her entire inventory (if she is a dealer). The vector  $q$  summarizes the terms of trade in all matches, taken as given by the individual.

Next, for a given  $q$ , suppose that a fundamental trading strategy is played, i.e.  $\pi = \pi^*$ , and that autarkic production does not take place, i.e. (2)-(3) and (10) hold. If such an equilibrium exists (which I verify in the following subsections) the steady state value function for an agent of type  $i$  who is currently able to offer some quantity of good  $j$ , must satisfy

$$V_{1,2} = \beta \{ p_1 V_{1,2} + p_2 [ p_{2,1} \max \{ U_1 + V_{1,2}, V_{1,2} \} + p_{2,3} \max_{\pi'_1} \{ \pi'_1 (V_{1,3} - \gamma q_2) + (1 - \pi'_1) V_{1,2} \} ] + p_3 V_{1,2} \} \quad (11)$$

$$V_{2,1} = -c_1 + \beta \{ p_1 \max \{ u(q_2) + V_{2,3}, V_{2,1} \} + p_2 V_{2,1} + p_3 V_{2,1} \} \quad (12)$$

$$V_{2,3} = \beta \{ p_1 V_{2,3} + p_2 V_{2,3} + p_3 \max_{\pi'_2} [ \pi'_2 (V_{2,1} - \gamma q_3) + (1 - \pi'_2) V_{2,3} ] \} \quad (13)$$

$$V_{3,1} = \beta \{ p_1 \max_{\pi'_3} \{ \pi'_3 (V_{3,2} - \gamma q_1) + (1 - \pi'_3) V_{3,1} \} + p_2 [ p_{2,3} \max \{ U_3 + V_{3,1}, V_{3,1} \} + p_{2,1} V_{3,1} ] + p_3 V_{3,1} \}. \quad (14)$$

Equation (11) describes the expected flow return to a type 1 capable of producing good 2. With probability  $p_1$  he meets a type 1 and no trade occurs since, given the conjectured trading strategy, none of the parties has something it would lead to a mutually beneficial exchange. With probability  $p_2 p_{2,1}$  he meets an intermediary (type 2) storing one unit of good 1. Therefore, if trade occurs it implies consumption of  $q_1=1$  units of the good, and production of  $q_2$ , generating net utility  $U_1$  and continuation utility  $V_{1,2}$ . With probability  $p_2 p_{2,3}$  agent 1 meets a type 2 able to produce good 3. Agent 1 has the option of accepting one unit of good 3 and store it ( $\pi'_1=1$ ), or

not ( $\pi'_1=0$ ).<sup>13</sup> Given the proposed trading strategy, with probability  $p_3$  no exchange can occur: a type 3 is met but, according to distribution implied by the proposed trading strategy, he is not willing to store anything that can be offered by a type 1. The other expressions have a similar interpretation.

Taking a closer look at expressions (12)-(13) one sees why the terms of trade in matches with the middleman need not be equal to one. The dealer suffers from both direct and indirect costs of keeping an inventory. First she must expend  $c_1$  resources per period, to keep the intermediated commodity in inventory. The severity of these costs is a function of the extent of the random demand for the intermediated good, that is  $p_1$ . The less frequently she is able to turn around the intermediated good, however, the longer she will have to keep it in storage. An especially impatient dealer will feel the bite of the inventory costs much more. These losses can be made up in two different ways. The dealer can ask for discounts to the producer of the intermediated commodity, type 3, by offering him  $q_3 < q_1 = 1$ . She can also charge a premium to the consumer, type 1, by asking her to deliver  $q_2 > q_1 = 1$ . How much more she can “ask” to consumers, and how much less she can “bid” to producers, in equilibrium, is the focus of the next section.

### 3.5 Equilibrium Strategies and Prices.

Under the proposed trading strategy  $\pi^*$ , the maximization on the right hand side of the functional equations (11)-(14) implies a set of conditions that the terms of trade specified by  $q$  must satisfy, in order for the exchanges considered to be mutually beneficial.

I say that, given the strategy  $\pi^*$ ,  $q$  is incentive *feasible* when it specifies terms of trade which, in equilibrium, are never rejected by the buyer of the consumption good. Specifically,  $q$  is feasible if individuals  $i=1,3$  agree to produce  $q_{i+1}$  in order to receive an amount  $q_i$  of their specialty consumption, that is their net gain from any exchange is strictly positive,  $U_i > 0$  (which implies  $V_{1,2}, V_{3,1} > 0$ ). Additionally, the dealer, individual  $i=2$ , must also strictly benefit from delivering her inventory  $q_1=1$  in exchange for  $q_2$ . These three requirements amount to

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<sup>13</sup> Notice that out of equilibrium actions must take into account the terms of trade implied by the proposed equilibrium vector  $q$ . Hence if individual 1 were to consider the out-of-equilibrium action of buying (and then storing) one unit of good 3 from a type2, she would rationally expect to be required to produce  $q_2$ .

$$q_2 < \frac{u(q_1)}{\gamma} \quad (15)$$

$$q_3 > u^{-1}(\gamma q_1) \quad (16)$$

$$u(q_2) + V_{2,3} - V_{2,1} > 0. \quad (17)$$

Now consider  $i$ 's trading strategy, i.e. she is a potential buyer of a commodity that is neither her consumption nor production good. The following conditions guarantee that choosing  $\pi' = \pi^*$  is individually optimal, given the terms of trade implied by  $q$ , and that all others choose  $\pi^*$ ,

$$\pi'_1 = 0 \quad \text{if} \quad V_{1,2} > V_{1,3} - \gamma q_2 \quad (18)$$

$$\pi'_2 = 1 \quad \text{if} \quad V_{2,3} < V_{2,1} - \gamma q_3 \quad (19)$$

$$\pi'_3 = 0 \quad \text{if} \quad V_{3,1} > V_{3,2} - \gamma q_1. \quad (20)$$

Next, I discuss  $i$ 's decision to produce a quantity  $Q_{i+1}$  in a match with  $i-1$  or  $i+1$ , when she takes as given the terms of trade in all other matches,  $q$ , and when she takes as given the proposed strategy,  $\pi^*$ . Individuals bargain only if by doing so they can obtain a positive gain from trade that, in a rational expectations equilibrium, is always correctly evaluated for each possible match. When  $\pi^*$  is an equilibrium, trade will be beneficial only in matches between the dealer and type 1 or type 3.

Recall that, because of the storage assumptions, no middleman would request more than one unit to store. Thus, when a match comprises types 2 and 3, bargaining involves determination of a quantity  $Q_3$  to be produced by agent 2 in exchange for  $Q_1=1$ . When a match involves types 1 and 2, trade will take place only if agent 2 is storing good 1. Because the dealer has to offer her entire inventory, bargaining involves determination of a quantity  $Q_2$  to be produced by 1 in exchange for the stored unit of good 1.

I do not model explicitly the bilateral bargaining process, instead I consider a bargaining solution that is the result of an always successful process and where the negotiated quantity divides the surplus equally between the two parties.<sup>14</sup> That is, given  $Q_1=1$ , the quantities  $Q_2$  and  $Q_3$  must solve

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<sup>14</sup> This is known as the Raiffa bargaining solution. Even though taking this approach leaves the nature of the bargaining game unresolved, it is well known that well-defined strategic bargaining games exist that deliver an identical outcome (see Osborne and Rubinstein, 1994).

$$u(Q_2) + V_{2,3} - V_{2,1} = u(Q_1) - \gamma Q_2 \quad (21)$$

$$V_{2,1} - \gamma Q_3 - V_{2,3} = u(Q_3) - \gamma Q_1, \quad (22)$$

and must satisfy  $U_1, U_3 > 0$ . The left hand side of equation (21) shows the surplus from trade to the middleman in a match with a type 1. The transaction provides her with temporary utility  $u(Q_2)$ . She also obtains net continuation utility  $V_{2,3} - V_{2,1}$ , because of the change in state due to her disposing of good 1 and becoming a producer of good 3. Her trade partner, individual of type 1, consumes and produces enjoying net momentary utility  $U_1$ , and zero net continuation utility since no change in state occurs. A similar description applies to expression (22).

In a symmetric equilibrium  $q_1 = Q_1 = 1$ ,  $q_2 = Q_2$  and  $q_3 = Q_3$  must satisfy (21)-(22) and (15)-(16). Since  $q_1 = 1$ , then  $q_2$  defines the real price offered by the dealer (the “ask” price) and  $q_3$  the real price paid by the dealer (the “bid” price) to, respectively, consumers and producers of the intermediated commodity. Thus  $q_2/q_3 - 1$  can be taken to measure the real markup, as the percentage increment over the acquisition price. Furthermore,  $q_2$  and  $1/q_3$  represent the terms of trade at which agents 1 and 3, respectively, trade for their consumption good, or the real price of their purchases from the dealer.

### 3.6 Definition of Symmetric Stationary Equilibrium

A *rational expectations symmetric stationary equilibrium* is a set of value functions  $\{V_{ij}\}$ , strategies  $\{s', \pi'\}$ , a distribution of types and objects,  $p$  and  $\{p_{ij}\}$ , and terms of trade  $Q$ , such that:

- (i) individuals maximize their expected lifetime utilities, that is  $\{V_{ij}\}$  satisfy (11)-(14) and  $\{s', \pi'\}$  satisfy (3)-(6) and (18)-(20), given  $p$ ,  $\{p_{ij}\}$  and  $q$ ;
- (ii) given  $s$ ,  $\pi$ , and  $\{V_{ij}\}$ , the stationarity conditions for the distribution of types and inventory holdings are satisfied, i.e.  $p$ ,  $\{p_{ij}\}$  satisfy (2), (7) and (10);
- (iii)  $Q$  is feasible, consistent with storing restrictions and equal sharing of trade surplus, and  $q = Q$ , i.e. it satisfies (15)-(17), (21)-(22), and  $q_1 = 1$ .

## **4. Existence and Characterization**

In order to prove the existence of the conjectured equilibrium I need to check that, given  $s$ ,  $\pi$



and  $q$ , (i) no individual prefers autarky, (ii) no one deviates from the conjectured trading strategy, and (iii) there exists at least one vector  $Q$  consistent with the split-the-surplus rule. Requirement (i) is easily satisfied for  $\alpha$  sufficiently small, which I henceforth assume, while (iii) is satisfied if there exists at least a pair  $\{Q_2, Q_3\}$  capable of equating the trade surplus of the dealer to her partner's. Since in equilibrium the value functions depend on  $q$ , then the pair  $\{Q_2, Q_3\}$  must also be a fixed point of the map defined by (21)-(22), i.e.  $\{Q_2, Q_3\} \equiv \{q_2, q_3\}$ . I consider these two issues later, and first deal with requirement (ii).

As seen from the discussion in section 3.5, the trading strategy  $\pi^*$  is individually optimal if the following three actions are *never* chosen in equilibrium: a type 1 accepts and stores a unit of good 3, a type 3 accepts and stores a unit of good 2, and a type 2 rejects a unit of good 1. The value functions associated to contingencies where types 1 and 3 have followed out of equilibrium actions, are respectively given by

$$V_{1,3} = \{-c_3 + \beta p_3 [u(q_1) + V_{1,2}]\} / [1 - \beta(p_1 + p_2)], \quad (23)$$

when individual of type 1 has accepted (and now stores) one unit of commodity 3, and

$$V_{3,2} = \{-c_2 + \beta p_2 p_{2,3} [u(q_3) + V_{3,1}]\} / [1 - \beta(p_1 + p_2 p_{2,1} + p_3)], \quad (24)$$

when individual 3 has accepted (and now stores) one unit of commodity 2. Note that in determining  $V_{1,3}$  and  $V_{3,2}$ , which can only be observed in out of equilibrium nodes, prices must still be specified by the equilibrium  $q$ . That is,  $q_1=1$  is the amount produced by a type 3 when type 1 sells him the stored unit of good 3. Similarly  $q_3$  is the amount produced by type 2 when type 3 supplies to her his unit inventory of good 2.

The following provides sufficient conditions for  $\pi' = \pi^*$  to be individually optimal, given feasible prices and distributions.

#### Lemma 1

Let  $q$  satisfy (15)-(16),  $\pi = \pi^*$ , and  $p$  satisfy (1)-(5). Then  $q$  is feasible and  $\pi' = \{0, 1, 0\}$  is individually optimal if

$$c_3 > \beta(p_3 - p_2 p_{2,1}) U_1 - \gamma q_2 (1 - \beta) \quad (25)$$

$$c_1 < \beta p_1 u(q_2) - \gamma q_3 [1 - \beta(1 - p_1)] \quad (26)$$

Proof. See Appendix.

This lemma provides conditions under which  $\pi' = \pi = \pi^*$  is an optimal symmetric trading

strategy: some individuals take on the role of middlemen only if the storage cost isn't too large. The inequalities can be interpreted as saying that the cost of starting the activity of middleman must be smaller than the present value of the expected stream of benefits. Notice that there are two cost components. A direct component due to the disutility generated by the specific intermediation activity ( $c_1$ ), and an indirect component due to the discounting of the utility of future consumption, influenced not only by search frictions (reflected in  $\beta$ ), but also the endogenous distribution of the demand-supply of the different commodities ( $p$ ).

By rearranging expression (26), for instance, one sees that the cost suffered by the dealer in acquiring one unit of good 1,  $\gamma q_3$ , must be less than the present value of the net expected utility derived by trading that good forever. This latter component is the utility of consuming and producing from tomorrow on,  $\beta p_1 [u(q_2) - \gamma q_3] / (1 - \beta)$ , minus the storage cost from today on,  $c_1 / (1 - \beta)$ . Thus, all else equal, there are two requirements for dealers of good 1 to endogenously arise. First, storing goods must be sufficiently inexpensive ( $c_1$  low). Second, individuals must be sufficiently patient ( $\beta$  high) and the sale frequency of the intermediated good must be sufficiently high ( $p_1$  high). Absent an adequate demand for the intermediated commodity, the dealer would suffer both from severe inventory costs, but also from a limited frequency of consumption. Here I note (and later prove) the existence of a "trade-off" between cost and discounting requirements. For instance, should storage become more expensive, the dealer would still find it worthwhile to supply commodity 1 if the number of agents consuming it increased as well.

Next, using the stationary distribution of objects characterized by (7) and (10), and the functional equations (11)-(14), I provide conditions for the existence of a unique symmetric mixed strategy  $s$ , given a feasible  $q$  and  $\pi = \pi^*$ . Obviously, by doing so I also pin down a unique distribution  $p$ .

### Lemma 2

Let  $\pi = \pi^*$ ,  $q$  be feasible, and  $a > 0$  arbitrarily small. There is a unique optimal  $s'$  satisfying (1) and (3), such that  $s' = s = p^*$ , where  $p^*$  satisfies (2), (4)-(5), and

$$p_1^* = \frac{\beta U_1 + c_1}{\beta(U_1 + U_2 + U_3)} \quad (27)$$

$$p_3^* = p_1^* \frac{U_3}{U_1}. \quad (28)$$

Proof: See appendix.

Recall that the symmetric equilibrium studied rests on the assumption of rational expectations: individuals have identical beliefs about which object will serve as a medium of exchange, and who will store it (the strategy  $\pi$ ), when prices are taken as given. In equilibrium these beliefs are consistent with the trading pattern conjectured. Thus, in general, proving the existence of an equilibrium amounts to (i) invoke the rational expectations assumption, (ii) studying the individually optimal strategy  $\pi'$ , when  $\pi$  and  $q$  are taken as given, and then (iii) impose symmetry and find all the fixed points of a resulting best response correspondence, say  $\pi'=\pi'(\pi)$ . Because in this study I have relaxed the assumption of exogenous distribution of types, I have one more best response correspondence to study,  $s'=s'(s)$ . Thus I must consider the beliefs about the distribution of types,  $p$ , when both trading strategies and prices are taken as given. In equilibrium these beliefs must be consistent with the equilibrium choice of productive activity. Furthermore, since I am considering equilibria with no autarky, all commodities must be produced. That is I must find only those fixed points of  $s'$  that are in  $(0,1)$ . An equilibrium must then have the property that, ex-ante, every individual symmetrically selects to be type  $i$  with probability  $p_i^*$ , which is identical to the equilibrium proportion of type  $i$  agents.

Notice from lemma 2 that when indirect trade of the cheapest-to-store good is an equilibrium, the individuals' choice of consumption-production type is a function of three crucial elements: the absolute cost of storing the medium of exchange, the prices paid and offered by the dealer, and the discount factor. Because the equilibrium terms of trade are taken as given when choosing the type,  $p_1^*$  is an increasing function of  $c_1$ , hence  $p_3^*$  increases and  $p_2^*$  decreases in  $c_1$ . The intuition is simple. In equilibrium as the storage cost grows the return from intermediation falls, ceteris paribus. Indifference across types can be restored if agents believe (correctly so) that as dealers they now also face an increased frequency of matches with consumers of the intermediated good. This offsets the length of storage and increases dealers' frequency of consumption, since only type 1 agents can satisfy the effective demand of dealers. Similar considerations apply to a decrease in the discount factor,  $\beta$ , or a decrease in the sale price,  $q_2$ : higher impatience and lower consumption both make less attractive becoming an intermediary, ceteris paribus. Interestingly, Lemma 2 demonstrates that if all trades were to occur at par, i.e.  $q_i=1$  for all  $i$ , then the distribution of consumption-production types would become uniform (as in Kiyotaki and Wright, 1989) as the intermediation cost vanishes. Whether such degenerated terms of trade can be an equilibrium under the assumed pricing rule, is what I consider next.

I now discuss the existence of a vector  $Q=q=q^*$  that (i) satisfies the pricing rule defined by the equilibrium conditions (21)-(22), (ii) is consistent with the ex-ante choice  $s=p^*$ , and (iii) ensures that only those trades that are implied by the proposed fundamental strategy  $\pi=\pi^*$  are mutually advantageous and so are undertaken. I proceed as follows. First, conjecturing the existence of an equilibrium with  $s=p^*$ ,  $\pi=\pi^*$ , and feasible  $q$ , I show the existence and uniqueness of a vector  $Q=q=q^*$  and then characterize it. Then I provide a sufficient condition for the existence of the fundamental equilibrium under  $p^*$  and  $q^*$ .

Lemma 3

Let  $\pi=\pi^*$ ,  $p=p^*$ , and  $q$  feasible. There exists a unique feasible equilibrium vector  $Q=q=q^*$  which satisfies the price rules (21)-(22), and that has the following properties: (i)  $q_2^*/q_3^* > 1$ , increasing in  $c_1$  and decreasing in  $\beta$ , (ii)  $q_2^*$  and  $q_3^*$  are decreasing in  $c_1$  and increasing in  $\beta$ , and (iii)  $q_3^* < 1$ .

Proof. In Appendix.

Interestingly in the equilibrium considered there is always a positive spread between “bid” and “ask” price of the intermediated good. This markup responds to trade frictions, intermediation costs, and distribution of agents who specialize in different consumption-production activities, as illustrated in Figures 1 and 2.<sup>15</sup> All else equal, a lower storage cost spurs the provision of intermediary services ( $p_2$  rises) because of a greater return from being a dealer. This positive effect is reflected in a decreased markup on sales of the intermediated commodity, i.e.  $q_2/q_3 - 1$  falls. As the markup falls, however, the terms of trade worsen for consumers of the intermediated good, both in absolute terms (since  $q_2$  increases) but also relative to the terms of trade faced by producers (because  $q_3$  increases). A lower storage cost translates into higher surplus to the dealer in matches with producers of good 1 who, therefore, are offered better terms of trade ( $q_3$  rises). This provides a greater incentive to undertake production of good 1 ( $p_3$  rises), which is also furthered by the expectation of more frequent encounters with dealers. It follows that less agents will select to become type 1 ( $p_1$  falls). This is reflected in dealers’ increased

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<sup>15</sup> All figures are for the parameterization  $u(q)=2q^{0.5}$ ,  $\gamma=1$ ,  $c_1=c_3=0.5$ , and  $\beta=0.95$ . Feasibility requires  $q_3 > q^L=0.25$  and  $q_2 < q^H=2$ .

length of storage ( $p_{2,1}$  increases relative to  $p_{2,3}$ ) and less frequent consumption matches, a negative occurrence which dealers counteract by increasing the price of good 1.

To summarize, as the storage cost falls the extent of intermediation and the return to being a dealer both increase, so that the markup falls (but does not reach zero). The result extends to a decrease in search frictions ( $\beta$  rises). Furthermore, the terms of trade at which different types acquire their specialty consumption from the dealer (by selling to him some quantity of their specialty output) are generally dissimilar, unless  $q_3=1/q_2<1$ . In Figure 1, types 1 and 3 always acquire their consumption good at dissimilar terms of trade ( $q_2$  and  $1/q_3$  always differ). In other words, the equilibrium is characterized by terms of trade dispersion, generally.

Because of their effect on the return to being a dealer, not all costs and discount factors support a fundamental transaction pattern. I next provide a sufficient condition, in terms of the parameters of the model, for the existence of a unique equilibrium where the fundamental transaction pattern is optimal.

Proposition 1

If  $c_1 < c_1(\beta, \gamma) \equiv \beta U_1 - 2\gamma q_3(1-\beta)$  and  $\alpha > 0$  arbitrarily small, there exists a unique search equilibrium where the terms of trade are specified by  $q^*$ , the trading strategy by  $\pi^*$ , and the distribution of types by  $p^*$ . The set  $(0, c_1(\beta, \gamma))$  is non-empty if  $\gamma$  is sufficiently small.

Proof. In Appendix.

To summarize, a unique fundamental equilibrium exists when the distribution of production-consumption types and terms of trade are endogenized. However, because individuals must be ex-ante indifferent between types, disutility from storage can't be excessive. It is the inability of bargaining over better terms of trade, reselling the intermediated commodity at a premium, that rules out a fundamental transaction pattern as an equilibrium in similar random matching models where the choice of specialty production is endogenized (as illustrated by Wright, 1995). In other words, the model points to the importance that the absence of a mechanism for endogenous price determination has on the occurrence of some equilibria and, in particular, a "natural" transaction pattern.

Because the equilibrium described is characterized by the presence of a strictly positive markup, one natural question is whether, under the simple pricing rule adopted, the difference between "ask" and "bid" price vanishes as time discounting and inventory costs become

negligible. I find that this is not the case, unless the price mechanism adopted assigns unequal gains from trade to some of the partners in some matches.

#### Corollary to Proposition 1

Let  $\pi = \pi^*$ . The markup  $q_2/q_3 - 1$  remains positive as search frictions go to zero, even if  $c_1$  is negligible. An equilibrium where all goods trade at par and the distribution of types is approximately uniform exists only if individuals are sufficiently patient, storage costs are low, and trade surpluses are unequal in some match.

Proof: See appendix.

I illustrate this result by considering the most stringent case, when commodity 1 can be costlessly stored. Assume that all goods trade at par ( $q_i = 1 \forall i$ ), that the fundamental trade pattern is an equilibrium, and the trade surpluses are equal in all matches (implying  $U_i = U \forall i$ ). Since the return from choosing a consumption-production activity  $i$  depends on the frequency of trade it guarantees (recall we are assuming trade surpluses are equal in *all* matches), then ex-ante indifference over types exists if each  $i$  trades with identical frequency. However, this can't possibly occur because types 1 and 3 would trade less frequently than the dealer. Since  $U_i = U$  implies  $p_i = 1/3$  (from lemma 2), then  $p_{2,1} = p_{2,3} = 1/2$  and the dealer would spend half of her life storing the intermediated good, and trading with probability 1/3 in each period. The two remaining types, however, would trade less frequently. Once a dealer is met, with probability 1/3, trade would occur only half of the times, which generates disparity of their ex-ante lifetime utilities, relative to a dealer. This implies that the fundamental pattern of trade where goods are traded at par cannot be an equilibrium under a pricing rule that splits the trading surplus equally in every match.

Thus, if every match generates equal surpluses, then the frequency of trade must adjust to support ex-ante indifference. Under the conjectured transaction pattern, this can only be achieved by an increase in the number of dealers. The latter occurs endogenously if the necessary economic incentives are in place, in that dealers can charge a sufficient premium over the acquisition price of the intermediated good.

### **5. Fixed Prices and Welfare**

Since bargaining theory is capable of supporting many equilibria, not only the ones where

prices generate equal surplus from trade, in this section I demonstrate the existence of a continuum of arbitrary prices capable of supporting the strategy  $\pi^*$ , and its associated distribution  $p^*$ . I also show that the result concerning the existence of a wedge between the sale and acquisition price of the intermediated good is not a construct of the simple pricing rule considered.

To do so I first set  $a > 0$  arbitrarily small (to rule out autarky), fix  $q_1 = 1$  because of storage bounds, and let  $p = p^*$  and  $\pi = \pi^*$ . Then I search over all arbitrary  $\{q_2, q_3\}$  pairs that are feasible (i.e.  $q_2 < q^H$  and  $q_3 > q^L$ ) and are consistent with the fundamental transaction pattern (i.e. satisfy (25)-(26)).

Substituting  $p = p^*$  in (25)-(26) I obtain

$$f(q_2, q_3) \equiv c_3 - U_3 \left[ \frac{2(\beta U_1 + c_1)}{U_1 + U_2 + U_3} - \frac{\beta U_1}{U_1 + U_3} \right] + \gamma q_2 (1 - \beta) \quad (29)$$

$$h(q_2, q_3) \equiv \frac{\beta U_1 U_2 - c_1 (U_1 + U_3)}{U_1 + U_2 + U_3} - \gamma q_3 (1 - \beta), \quad (30)$$

hence any pricing rule whereby  $\{q_2, q_3\}$  satisfies  $f(q_2, q_3) > 0$  and  $h(q_2, q_3) > 0$ , also satisfies (25)-(26). The existence of a continuum of terms of trade  $q$  slightly departing from the split-the-surplus rule, and supporting a fundamental equilibrium, is a simple extension of proposition 1. By continuity, if there exists a feasible  $q$  such that (25)-(26) are satisfied, for some  $c_1$ , then there is a continuum of vectors  $q$  in a neighborhood of  $q^*$ , capable of supporting the equilibrium conjectured. The following proposition goes a step further and considers existence of equilibria for pricing rules that may depart sharply from the split-the-surplus assumption. It does so by showing that the fundamental pattern of exchange can be an equilibrium even if, under some arbitrary pricing mechanism, the dealer were to offer significantly worse terms of trade to the supplier of the intermediated commodity.

Proposition 2.

If  $\{p^*, \pi^*\}$  is an equilibrium for  $q = q^*$ , then there exists a continuum of terms of trade that supports the equilibrium  $\{p^*, \pi^*\}$ , and such that  $q_2$  is only marginally different from  $q_2^*$ , but  $q_3$  can be substantially smaller than  $q_3^*$ .

Proof: In appendix.

I conclude that the adoption of the simple pricing rule is only one of many that can generate equilibria where prices are fully flexible, the distribution of types of traders is endogenous, and exchange of only the low storage-cost commodity is intermediated. There is a large set of arbitrary pricing mechanisms which, although altering the distribution of surpluses between the matched parties, supports the existence of a fundamental equilibrium where dealers charge a positive markup. Figure 3 ( $\beta=0.8$  and  $c_1=0.01$ ) provides an illustration. There is a large set  $\{q_2, q_3\}$  supporting  $p^*$  and  $\pi^*$ , represented by the intersection of the areas lying below each curve. It includes point A, where all goods are traded at par ( $q_i=1 \forall i$ ), and point B, where prices satisfy the split-the-surplus rule ( $q_2^*=1.005, q_3^*=0.54$ ).

What bargaining protocol can generate these arbitrary terms of trade? One may consider  $q$  as the solution to a Nash bargaining problem with non-zero threat points and bargaining weights  $\theta=\{\theta_1, \theta_3\}$ , where  $\theta_i \in (0,1)$  is the bargaining power of type 2 in the match with  $i=1,3$  ( $1-\theta_i$  the other's bargaining weight).<sup>16</sup> That is taking as given  $q$  and  $\theta$ ,  $Q_2$  and  $Q_3$  solve respectively

$$\begin{aligned} \max_{Q_2} [V_{2,3} + u(Q_2) - V_{2,1}]^{\theta_1} [u(1) - \gamma Q_2]^{(1-\theta_1)} \\ \max_{Q_3} [V_{2,1} - \gamma Q_3 - V_{2,3}]^{(1-\theta_3)} [u(Q_3) - \gamma]^{\theta_3} \end{aligned}$$

subject to feasibility constraints and (26), all necessary requirements since trade surpluses must be positive.<sup>17</sup> The pair  $\{Q_2, Q_3\}$  is an equilibrium when, given  $q$ , it is a solution to the first order conditions

$$\begin{aligned} u(Q_2) + V_{2,3} - V_{2,1} &= [u(1) - \gamma Q_2] \frac{\theta_1 u'(Q_2)}{(1-\theta_1)\gamma} \\ V_{2,1} - \gamma Q_3 - V_{2,3} &= [u(Q_3) - \gamma] \frac{(1-\theta_3)\gamma}{\theta_3 u'(Q_3)}, \end{aligned}$$

while satisfying the inequalities (29)-(30), feasibility, and  $Q=q$ . In this case I can express both existence conditions and equilibrium price vector as functions of the bargaining weights,

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<sup>16</sup> If  $\theta_i=1/2$  the Nash cooperative solution is the outcome of a non-cooperative strategic bargaining game where individuals are randomly chosen to make an offer (initially or following a rejection), can walk away from a match, can meet other partners during the bargaining process, and must wait a negligible time interval between subsequent bargaining rounds (see Trejos and Wright, 1995).

<sup>17</sup> Feasibility implies that (15)-(16) are satisfied, hence types 1 and 3 have strictly positive surpluses. Equation (26) is an explicit form for (19), which guarantees a positive surplus to type 2 in the match with 3.



$c_1 < c_1(\theta)$ ,  $c_3 > c_3(\theta)$ , and  $q = q(\theta)$ . For instance, if  $u(q) = uq$  the simple pricing rule adopted is equivalent to setting  $\theta_i = \theta = \gamma / (u + \gamma)$ , and to setting  $\theta_1 [1 + u'(q_2^*) / \gamma] = \theta_3 [1 + \gamma / u'(q_3^*)] = 1$ , in the presence of risk-aversion. While lemma 3 and continuity indicate that the subset on which  $\theta_i$  may take values is non-empty, it must also be a proper subset of the unit interval. Not all bargaining weights support feasibility and indifference over  $i$ 's. For instance,  $\theta_i = 0, 1$  (take-it-or-leave-it offers) cannot support a non-autarkic outcome since surpluses from trade must be bounded away from zero. In other words, the choice of pricing mechanism has implications for the existence of the equilibrium.

Since many different prices are capable of supporting a fundamental equilibrium in which dealers arise endogenously, I next study the existence of a set of efficient prices in the sense that the resulting allocation corresponds to the solution of a planner's problem who, taking as given the trading arrangement, chooses the quantities to be exchanged to maximize social welfare. One natural question is whether the efficient allocation can be supported as an equilibrium where pricing decisions are decentralized, and whether the outcome with trades occurring at par is the most preferred. In other words, can an economy where middlemen arise endogenously attain an efficient allocation, and if trading at par can be supported as a fundamental equilibrium, does it maximize social utility?

To address this issue consider the welfare measure  $W = \sum_{i=1}^3 p_i E[V_i]$ , i.e. the ex-ante expected utility of an agent. Since the corollary to Proposition 1 has shown that unit trades are an equilibrium when good 1 is sufficiently cheap to store, I focus on this latter parameterization.

Proposition 3

Let  $c_1 > 0$  small. There is a unique welfare-maximizing feasible  $q$ , such that  $\pi = \pi^*$ ,  $p = p^*$ , and  $q_i \neq 1$  for some  $i$ .

Proof: In appendix.

The rationale for the inefficiency of the outcome where all trades occur at par, is due to the positive trading externality generated by the existence of dealers. The planner's objective coincides with maximization of the ex-ante lifetime utility of a representative agent, and since

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It also implies (17) in equilibrium, which guarantees a positive surplus to type 2 in the match with 1.

lifetime utility increases in the frequency of consumption, this is the variable of interest to the planner. In this decentralized trading environment dealers facilitate consumption by engaging in indirect trade with both buyers and producers of good 1. They also economize on societal storage costs, since they are the only ones to store goods and hold the cheapest possible inventory.<sup>18</sup> Excessive intermediation, however, generates high costs and diminishes the frequency of consumption of some type of individuals. Thus there is an optimal proportion of dealers, bounded away from both one and zero, and such that it exhausts all net marginal benefits generated by a further increase.

Because the planner takes as given the trading arrangement, and dealers arise endogenously, he can't just impose a distribution of consumption-production types. However, he can bypass the boundaries imposed by the pricing rule, and thus can provide the right economic incentives so that a desirable number agents chooses to undertake the role of trading intermediary. The proposition shows that if the storage cost is small these incentives generally require the presence of a positive bid-ask spread on all mediated exchanges. That is, the planner internalizes the positive externalities created by dealers, rewarding them with some extra consumption.

The efficient terms of trade are indicated by point C in Figures 3 and 4, and are unique because of concavity of  $W$  in  $q_2$  and  $q_3$ . Note that the externality generated by intermediation is sufficiently large to warrant existence of about twice as many dealers, relative to any other type of agent ( $p_2=.481$ ). This allows both types 1 and 3 to consume often. Because the distribution of types is nearly uniform when  $c_1$  is small and goods are exchanged at par, however, mediated sales of good 1 must occur at a substantial markup in order to generate the necessary economic incentives for intermediation ( $q_2=1.236$ ,  $q_3=.769$ ). Clearly, there is a tradeoff between extent of intermediation and welfare. An even larger number of dealers could be induced by mandating an even larger markup, but this may worsen the terms of trade for consumers or producers of good 1, negatively affecting their lifetime utilities and, ultimately, welfare. This tradeoff is demonstrated by comparing the efficient outcome to the equilibrium where prices satisfy the

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<sup>18</sup> Possibly, storage of other commodities by different individuals might be beneficial for the society if it increases the frequency of exchange to such an extent that the disutility engendered by extra storage costs is smaller than the benefit derived from more frequent consumption. Although an interesting question in its own right, it is beyond the scope of the present exercise. Thus, I am not asking whether the fundamental trading pattern is the best way to organize exchange, or if there is a better transaction pattern. The latter is equivalent to solving the problem of a planner who chooses not only prices but also the trading arrangement, to maximize social welfare.

split-the-surplus rule. In the latter case the markup is larger since  $q_2=1.005$  and  $q_3=.54$ , implying a markup of about 86% (relative to 60% in the former). This induces a larger extent of intermediation ( $p_2=.493$ ) but lower welfare, as evident from Figures 3 and 4.

## 6. Concluding Remarks

In this study I have illustrated a decentralized trading environment that allows endogenous determination of the number of trade facilitators and the markup they charge on intermediated sales. The starting point of the analysis is the search theoretic commodity-money model of Kiyotaki and Wright (1989), of which I have relaxed the two key assumptions of exogenous terms of trade and distribution of agents who specialize in different consumption-production activities, endogenizing them both.

By focusing on the natural transaction pattern where only exchange of the cheapest-to-store good is mediated, I have demonstrated the existence of a unique equilibrium where agents split the trade surplus, dealers arise endogenously and generally charge a markup on their sales of the intermediated good. In equilibrium, participation in intermediation and the markup's size are fully flexible and respond in predictable ways to extent of trading frictions, intermediation cost, and distribution of specialty production. Specifically, as trading frictions or storage cost grow, the number of dealers falls, and the markup they charge increases. Furthermore, there is scope for dispersion in the terms of trade, in that different agents acquire their consumption commodity at different prices, from the dealer. Interestingly, this disparity does not disappear even if the intermediation cost and time discounting vanish. By considering arbitrary pricing rules, I have also found that this same transaction pattern is supported by a continuum of price vectors, some of which may be thought as being traced by varying the weights of the participants in a generalized Nash bargaining process. One of these equilibria is such that all goods exchange at par as in Kiyotaki and Wright (1989), but is socially inefficient.

This study also contributes to further our understanding of search theoretic models of exchange. First, it has pointed at the implication that the absence of a mechanism for endogenous price determination has on the occurrence of some equilibrium transaction patterns. This is exemplified by an issue raised by Wright (1995), namely that the cheapest-to-store good won't have a chance to become the unique medium of exchange when agents choose their specialty production but the terms of trade are exogenously fixed at par. In fact, I have proved the

existence of a continuum of terms of trades consistent with this fundamental trading pattern. Second, it has demonstrated that because the equilibrium distribution of consumption-production types, terms of trade, and transaction arrangements are intertwined, the choice of the pricing mechanism has also implications for the efficiency of equilibria. Since intermediation can provide a positive trading externality, by increasing the consumption frequency, societal gains can be generated by choosing a suitable pricing mechanism.

## Appendix

### Reduced forms of Value Functions

In equilibrium  $\pi'=\pi=\pi^*$ , and  $s'=s=p\in(0,1)$ , and rearrange the Bellman equations (11)-(14) as

$$(1-\beta)V_{1,2}=\beta p_2 p_{2,1} U_1 \quad (\text{A.1})$$

$$(1-\beta)V_{2,1}=-c_1+\beta p_1[u(q_2)+V_{2,3}-V_{2,1}] \quad (\text{A.2})$$

$$(1-\beta)V_{2,3}=\beta p_3[V_{2,1}-\gamma q_3-V_{2,3}] \quad (\text{A.3})$$

$$(1-\beta)V_{3,1}=\beta p_2 p_{2,3} U_3. \quad (\text{A.4})$$

Notice that, contrary to Kiyotaki-Wright (89) or Wright (95), *no storage cost* component appears in the value functions of types 1 and 3 (since they only produce their commodity). Furthermore there is a *production cost* component in  $V_{2,3}$  but *not* in  $V_{2,1}$  since he produces only after having met an individual who can produce the medium of exchange. From these I derive the reduced forms

$$V_{2,1}=\frac{-\gamma q_3 \beta^2 p_1 p_3 + [1-\beta(p_1+p_2)][\beta p_1 u(q_2)-c_1]}{(1-\beta)(1-\beta p_2)}$$

$$V_{2,3}=\beta p_3 \frac{-\gamma q_3 [1-\beta(p_2+p_3)] + \beta p_1 u(q_2) - c_1}{(1-\beta)(1-\beta p_2)}.$$

### Proof of Lemma 1

Assume  $\pi=\pi^*$ ,  $q$  satisfying (15)-(16), and  $p$  satisfying (1)-(5).

a)  $\pi'_1=\pi_1=0$ . Using (A.1) and (23),  $(V_{1,2}-V_{1,3})[1-\beta(p_1+p_2)]=c_3-\beta[p_3 u(q_1)-p_2 p_{2,1} U_1]$ . Since  $\pi'_1=0$  is a best response to  $\pi^*$  when (18) holds, it requires  $V_{1,2}-V_{1,3}+\gamma q_2>0$ . Use the definition of

$$U_1 \text{ and rearrange to get } \frac{c_3 + \beta U_1 (p_2 p_{2,1} - p_3) + \gamma q_2 (1-\beta)}{1-\beta(p_1+p_2)} > 0.$$

b)  $\pi'_3=\pi_3=0$ . Using (A.4) and (24),  $(V_{3,1}-V_{3,2})[1-\beta(p_1+p_2 p_{2,1}+p_3)]=c_2-\beta p_2 p_{2,3}[u(q_3)-U_3]$ . Since  $\pi'_3=0$  is a best response to  $\pi^*$  when (20) holds, it requires  $V_{3,1}-V_{3,2}+\gamma q_1>0$ . Use the definition of

$$U_3 \text{ and rearrange to get } \frac{c_2 + \gamma q_1 (1-\beta)}{1-\beta(p_1+p_2 p_{2,1}+p_3)} > 0, \text{ always satisfied.}$$

c)  $\pi'_2=\pi_2=1$ . Using (A.2)-(A.3),  $(V_{2,1}-V_{2,3})(1-\beta p_2)=\beta[p_1 u(q_2)+p_3 \gamma q_3]-c_1$ . Since  $\pi'_2=1$  is a best response to  $\pi^*$  when (19) holds, it requires  $V_{2,1}-V_{2,3}-\gamma q_3>0$ . Hence

$$\frac{\beta p_1 U_2 - c_1 - \gamma q_3 (1 - \beta)}{1 - \beta p_2} > 0.$$

Finally I show that when  $\pi' = \pi = \pi^*$ , if an equilibrium  $q$  exists it is feasible as long as it satisfies (15) and (16). Feasibility implies that in equilibrium consumers buy their consumption good at the specified terms of trade. When (15)-(16) hold types 1 and 3 want buy their consumption from type 2. Since  $\pi_3 = \pi_3^* = 0$  implies that 2 wants to sell to 3, then I have only to show that 2 wants also to trade with type 1. The latter occurs if (17) holds, i.e. when  $V_{2,1} - V_{2,3} - u(q_2) < 0$ , rearranged as  $\frac{-\beta p_3 U_2 - u(q_2)(1 - \beta) - c_1}{1 - \beta p_2} < 0$ , seen to be always satisfied since  $U_2 > 0$  must hold when the *optimal* trading strategy is  $\pi'_2 = \pi_2^* = 1$  (a contradiction otherwise). Hence (15) and (16) are sufficient for feasibility, under the conjectured fundamental trading strategy ■

### Proof of Lemma 2

I first provide conditions such that if  $q$  satisfies (15)-(16) (i.e. it is feasible),  $\{p_{ij}\}$  satisfy (2),  $\pi = \pi^*$ , and (25)-(26) hold (i.e.  $\pi' = \pi^*$  is a best response) then an individual is ex-ante indifferent among choosing any of the three alternative types of specialized production. This allows me to find the endogenous proportions of producers  $s = p$  as a function of parameters of the economy, trading strategies, and prices. Finally I provide a condition assuring that no one prefers autarkic consumption, that is  $s(0) = 0$ .

I first show conditions for  $s'(i) = s(i) \notin \{0, 1\}$  for all  $i \in N$ . This requires (4) to hold where

$$E[V_1] = p_{1,2} V_{1,2} + p_{1,3} V_{1,3} = V_{1,2} \text{ (since } p_{1,3} = 0 \text{ in the equilibrium conjectured)}$$

$$E[V_2] = p_{2,1} V_{2,1} + p_{2,3} V_{2,3} = \frac{p_3 V_{2,1} + p_1 V_{2,3}}{p_1 + p_3} \text{ (using (5) for the equilibrium } p_{2,1} \text{ and } p_{2,3})$$

$$E[V_3] = p_{3,1} V_{3,1} + p_{3,2} V_{3,2} = V_{3,1} \text{ (since } p_{3,2} = 0 \text{ in the equilibrium conjectured)}$$

When  $1 > p_2 > 0$ , the requisite of indifference among the three productive activities generates two conditions

$$V_{1,2} - V_{3,1} = 0 \tag{A.5}$$

$$V_{1,2} - \frac{p_3 V_{2,1} + p_1 V_{2,3}}{p_1 + p_3} = 0. \tag{A.6}$$

Consider (A.5). Using (A.1) and (A.2)

$$p_3 = p_1 \frac{U_3}{U_1} > 0 \quad (\text{A.7})$$

satisfies  $V_{1,2} - V_{3,1} = 0$ , since  $U_3 > 0$  by (16). Consider (A.6). Use (12)-(13), (A.1)-(A.2) and the definition of  $U_2$ . Algebraic manipulations lead to

$$\beta(1-p_1-p_3)U_1 = \beta p_1 U_2 - c_1 \quad (\text{A.8})$$

Substituting (A.7) in (A.8) I obtain the with unique solution

$$p_1^* = \frac{\beta U_1 + c_1}{\beta(U_1 + U_2 + U_3)} \in (0,1) \quad (\text{A.9})$$

since  $U_1 > 0$  by (15), and  $c_1 < \beta U_2$  when  $\pi = \pi^*$  (by Lemma 1).

I now show conditions assuring that  $p_2^* \in (0,1)$ , by giving conditions guaranteeing  $1 > p_1^* + p_3^* > 0$ . Consider  $p_2^* > 0$ , implied by  $1 > p_1^* + p_3^*$ . Using (A.7) and (A.9) rearrange  $p_1^* + p_3^* < 1$  as  $\frac{\beta U_1 + c_1}{\beta(U_1 + U_2 + U_3)} < \frac{U_1}{U_1 + U_3}$ , implying  $c_1 < \frac{\beta U_1 U_2}{U_1 + U_3}$ . Since (26) is assumed to be holding (the trading strategy is assumed to be optimal), I substitute  $p_1^*$  in it and rearrange it as

$$c_1 < \frac{\beta U_1 U_2}{U_1 + U_3} - \gamma q_3 (1 - \beta) \frac{U_1 + U_2 + U_3}{U_1 + U_3}. \quad (\text{A.10})$$

Its right hand side is strictly less than  $\frac{\beta U_1 U_2}{U_1 + U_3}$ ,  $\forall q_3 > 0$ . Hence  $p_2^* > 0$ . It is obvious that  $p_2^* < 1$  since  $p_2^* = 1 - (p_1^* + p_3^*) > 0$  and both  $p_1^*$  and  $p_3^*$  are positive. Hence if (15)-(16) and (26) hold then  $p_i^* \in (0,1) \forall i$ .

I now show when individuals strictly prefer search to autarky, i.e. give conditions for  $s'(0) = s(0) = 0$ . This requires (5) to hold, given  $q$ ,  $\pi$ , and  $p$ . Using  $V_a = a/(1-\beta)$  (consumption of the home produced good yields utility  $a$  forever after), and the reduced form for  $V_{1,2}$  from (A.1) then

$$a < \beta p_2 \frac{p_3}{1-p_2} U_1 \text{ satisfied by } a > 0 \text{ arbitrarily small.} \blacksquare$$

Proof of Lemma 3.

The proof is organized as follows. Fix the strategy  $\pi=\pi^*$ , and the distribution  $p=p^*$  that depends on a feasible  $q$  (all taken as given in a match).  $Q_2$  and  $Q_3$  denote the quantities to be determined in the two matches, according to the specified pricing rule. Surpluses from trade are equal when (21)-(22) hold, and are positive since  $q$  is assumed to be feasible (i.e. (15)-(16) hold). I first show that there exists a unique value  $z^*=Q_2-Q_3>0$  that satisfies (21) and (22), defining a unique and feasible locus of pairs  $\{Q_2, Q_3\}$ . I then use (21), (22) and (26) to show that there is a unique pair  $\{Q_2, Q_3\}=\{q_2, q_3\}$ .

To start, I show that if  $\{Q_2, Q_3\}$  satisfies (15)-(16), then it generates positive surplus to both parties. For  $Q_2 < q^H = \frac{u(1)}{\gamma}$  and  $Q_3 > q^L = u^{-1}(\gamma)$  (where  $q^H > q^L$  because of the assumptions on preferences) both (15) and (16) hold, hence (21) and (22) imply positive surpluses, if an equilibrium  $Q=q$  exists. Hence in what follows I consider  $0 < Q_2 < q^H$  and  $Q_3 > q^L$ .

In equilibrium I can rearrange  $V_{2,1}$  and  $V_{2,3}$  from (A.2)-(A.3) using the definition of  $p_3^*$  and equality of surpluses,

$$(1-\beta)V_{2,1} = -c_1 + \beta p_1^* U_1 \quad (\text{A.11})$$

$$(1-\beta)V_{2,3} = \beta p_1^* U_3^2 / U_1. \quad (\text{A.12})$$

Let  $\Delta = V_{2,1} - V_{2,3}$ . Using (21) and (22),

$$u(Q_2) - \Delta = U_1 \quad (\text{A.13})$$

$$\Delta - \gamma Q_3 = U_3 \quad (\text{A.14})$$

which in equilibrium jointly imply

$$U_1 + U_3 = U_2. \quad (\text{A.15})$$

Notice that (A.15) implies  $u(Q_1) - \gamma Q_1 = u(Q_2) + \gamma Q_2 - [u(Q_3) + \gamma Q_3]$ , thus in equilibrium  $Q_2 > Q_3$  since  $u(Q_1) - \gamma Q_1 > 0$  (by assumption). Substituting  $q_1 = Q_1 = 1$  (A.15) becomes

$$u(Q_2) - u(Q_3) + \gamma Q_2 - \gamma Q_3 = u(1) - \gamma. \quad (\text{A.16})$$

Since  $Q_2 > Q_3$  let  $z = Q_2 - Q_3 \in (0, q^H - q^L)$ , where  $Q_i$  is feasible and  $z = q^H - q^L$  is the maximum when  $Q_2 = q^H$  and  $Q_3 = q^L$ . Denote the left hand side of (A.16) by the continuous real valued function  $g(z): (0, q^H - q^L) \rightarrow \mathbb{R}$ . Notice that  $g(z)$  is strictly increasing in  $z$ ,  $g(0) = 0 < u(1) - \gamma$ , and



$g(q^H - q^L) > u(1) - \gamma$  (because  $u(1) - \gamma q^H = 0$  and  $u(q^L) - \gamma = 0$ ). By the intermediate value theorem there exists a unique  $z^* = (Q_2 - Q_3) \in (0, q^H - q^L)$  such that  $g(z^*) = u(1) - \gamma$ . That is (A.15) is satisfied by a unique  $z^*$  defining the locus of pairs  $\{Q_2, Q_3\}$  with feasible elements that satisfy  $Q_2 \in (z^* + q^L, q^H)$  and  $Q_3 \in (q^L, q^H - z^*)$ .

To show that there is a unique equilibrium pair  $\{Q_2^*, Q_3^*\} = \{q_2^*, q_3^*\} \in (z^* + q^L, q^H) \times (q^L, q^H - z^*)$ , I thus let  $Q_2 \in (z^* + q^L, q^H)$  and  $Q_3 \in (q^L, q^H - z^*)$  and  $Q_2 - Q_3 = z^*$ . Now conjecturing an equilibrium impose  $Q = q$ , and then can use (A.11)-(A.12),  $\beta p_1^* = \frac{\beta U_1 + c_1}{2U_2}$ , and (A.15) to obtain

$$(1 - \beta)\Delta = -c_1 + (\beta U_1 + c_1)(1 - U_3/U_1)/2. \quad (\text{A.17})$$

Denote the second term on its right hand side by the continuous and real valued function  $f(Q_2, Q_3)$ . From (A.13)

$$\Delta = u(Q_2) - U_1 = u(Q_2) + \gamma Q_2 - u(1) \quad (\text{A.18})$$

when the split-the-surplus pricing rule is adopted. Hence in equilibrium  $\{Q_2, Q_3\}$  must satisfy

$$(1 - \beta)\Delta - f(Q_2, Q_3) = -c_1 \quad (\text{A.19})$$

where  $\Delta$  is given by (A.18).

I next show that  $\max\{(1 - \beta)\Delta - f(Q_2, Q_3)\} > -c_1$  and  $\min\{(1 - \beta)\Delta - f(Q_2, Q_3)\} < -c_1$ . Recall that in the conjectured equilibrium  $Q_3 = Q_2 - z^*$ , hence  $Q_3$  is increasing in  $Q_2$  (and vice versa). It is easy to see that  $\Delta$  is increasing in  $Q_2$ , and it can be easily verified that  $f(Q_2, Q_3)$  is decreasing in  $Q_2$ , since  $U_3$  rises and  $U_1$  falls in  $Q_2$ . Hence the left hand side of (A.19) is increasing in  $Q_2$ , approaching a minimum as  $Q_2 \rightarrow z^* + q^L$  (i.e. as  $Q_3 \rightarrow q^L$ ) and a maximum as  $Q_2 \rightarrow q^H$ . In particular, as  $Q_2 \rightarrow q^H$  then  $U_1 \rightarrow 0$  and so  $(1 - \beta)\Delta - f(Q_2, Q_3) \rightarrow \infty > -c_1$ . As  $Q_3 \rightarrow q^L$  then  $U_3 \rightarrow 0$  and the left hand side of (A.19) becomes  $2(1 - \beta)u(Q_2) - U_1(2 - \beta)$ , which I next show to be less than  $-c_1$ . The right hand side of (26) in equilibrium (substituting  $p^*$  and  $U_2 = U_1 + U_3$ ) is  $\beta U_1 - 2\gamma Q_3(1 - \beta)$ . Observe that when  $Q_3 = q^L$ ,  $2(1 - \beta)u(Q_2) - U_1(2 - \beta) = \beta U_1 - 2\gamma Q_3(1 - \beta)$  since it simplifies to  $U_2 = U_1$  (that holds only when  $Q_3 = q^L$ ). Hence  $(1 - \beta)\Delta - f(Q_2, Q_3) \rightarrow \beta U_1 - 2\gamma Q_3(1 - \beta) > -c_1$  as  $Q_3 \rightarrow q^L$ , whenever (26) holds (i.e. when  $\pi = \pi^*$ ). Consequently if  $\pi = \pi^*$  is the optimal trading strategy, by the intermediate value theorem there exists a unique pair  $\{Q_2^*, Q_3^*\} = \{q_2^*, q_3^*\} \in (z^* + q^L, q^H) \times (q^L, q^H - z^*)$  such that  $q_2^*$

$q_3^* = z^*$  and (21)-(22) are satisfied.

I now show that  $Q_2$  and  $Q_3$  are decreasing and  $Q_2/Q_3$  is increasing in  $c_1$ . Since in equilibrium  $f(Q_2^*, Q_3^*) + c_1 = 0$  increasing in  $Q_2$  and  $Q_3$ , then if  $c_1' > c_1$  I need  $Q_2' < Q_2^*$  and  $Q_3' < Q_3^*$  such that  $Q_2' - Q_3' = Q_2^* - Q_3^* = z^*$ . Next,  $Q_2'/Q_3'$  is an increasing function of  $c_1$  since  $Q_2' - Q_3' = Q_2^* - Q_3^*$  can be

$$\text{rearranged as } \frac{\frac{Q_2' - 1}{Q_3' - 1}}{\frac{Q_2^* - 1}{Q_3^* - 1}} = \frac{Q_2^*}{Q_3^*} > 1 \text{ only if } Q_2'/Q_3' > Q_2^*/Q_3^*.$$

I now show that *in equilibrium*  $Q_2$  and  $Q_3$  are increasing and  $Q_2/Q_3$  is decreasing in  $\beta$ , by deriving a contradiction. Call  $h(\beta)$  the left hand side of (A.17) and  $g(\beta)$  its right hand side, with (A.18) giving the equilibrium value of  $\Delta$ . Recall that  $Q_1 = 1$  is constant and since  $Q_2 - Q_3 = z^*$  in equilibrium, if  $Q_2$  and  $Q_3$  move when a parameter changes, they do so in the same direction.

Suppose that  $\frac{\partial Q_i}{\partial \beta} = 0$ . It is easy to see that  $\frac{\partial h}{\partial \beta} < 0$  and  $\frac{\partial g}{\partial \beta} > 0$ , which contradicts the existence of

the equilibrium. Suppose instead that  $\frac{\partial Q_i}{\partial \beta} < 0$ . It is easy to see that  $\frac{\partial h}{\partial \beta} < 0$  and  $\frac{\partial g}{\partial \beta} > 0$ . The latter

inequality follows from  $U_1$  decreasing in  $Q_2$  and  $U_3$  increasing in  $Q_3$  hence when  $\beta$  grows  $U_1$

increases,  $U_3$  decreases and  $U_3/U_1$  falls. But this is also a contradiction. Hence  $\frac{\partial Q_i}{\partial \beta} > 0$ .

Furthermore, since  $Q_2 > Q_3 = Q_2 - z^*$  then  $Q_2/Q_3$  must fall as  $\beta$  grows.

Now I show that *in equilibrium*  $Q_3 < 1$ , by deriving a contradiction. Since  $Q_3$  is increasing in  $c_1$ , I study the most stringent case where  $c_1 \rightarrow 0$ . Recall that  $V_{2,1} > V_{2,3}$ , which, using (A.11) and (A.12), implies  $U_1 > U_3$ , rearranged as  $u(1) + \gamma > u(Q_3) + \gamma Q_2$ . Since in equilibrium  $Q_2 > Q_3$ , if  $Q_3 > 1$  then  $Q_2 > Q_3 > 1$ , but this violates  $U_1 > U_3$ . Hence  $Q_3 < 1$  in equilibrium.

Finally, let  $q_2^* = Q_2^* < q^H$ ,  $q_3^* = Q_3^* > q^L$  and  $c_1$  satisfy (26), hence in the equilibrium conjectured there exists a unique feasible  $Q = q = q^*$  with the properties described above and that satisfies (21)-(22). ■

### Proof of Proposition 1

Recall that (1)-(5) are satisfied by  $p^*$ , and  $q^*$  satisfies (15)-(17). Also recall that  $\pi^*$  is a individual best response when (25)-(26) hold (lemma 1). I make use of the distribution of types,  $p^*$ , the equilibrium price condition of equal trade surpluses,  $q^*$ , and lemmas 1-3 to show that when  $\pi=\pi^*$ : (i) if  $a>0$  arbitrarily small, and (ii) (26) holds, then all feasibility constraints are met, and  $\pi^*$  is a best response.

When  $p=p^*$  and  $q=q^*$  I can use the definition of the elements of  $p^*$  together with (A.15) to rearrange (25) and (26) respectively as

$$c_3 - c_1 U_3 / (U_1 + U_3) > -\gamma q_2 (1 - \beta) \quad (\text{A.20})$$

$$c_1 < \beta U_1 - 2\gamma q_3 (1 - \beta) \equiv c_1(\beta, \gamma). \quad (\text{A.21})$$

It is easy to see that (A.20) is always satisfied, since  $c_3 > c_1$  by assumption. Hence when (A.21) holds lemmas 1-3 imply the existence of a unique search equilibrium with prices, distributions and strategies given by  $q^*$ ,  $p^*$ , and  $\pi^*$ .

Finally I show the non-emptiness of the set  $(0, c_1(\beta, \gamma))$ . Notice that the upper bound of this set is decreasing in  $q_3$ . When  $q_3=1$  (A.15) implies  $u(Q_2) + \gamma Q_2 = 2u(1)$ , and let  $Q_2^H$  be its solution (observe that  $1 < Q_2^H < q^H$ ). The set is non-empty if  $\gamma < \beta[(Q_2^H) - u(1)]2(1 - \beta)$ , a positive number. ■

#### Proof of Corollary to Proposition 1

Let  $\pi=\pi^*$  be the (conjectured) equilibrium trading strategy. I first show that, as  $\beta \rightarrow 1$ , an equilibrium where  $\pi=\pi^*$  and the pricing rule is split-the-surplus may exist. Then I show that  $q$  cannot be a unit vector.

Let  $\beta \rightarrow 1$ . From (A.21) the sufficient condition for existence of the equilibrium conjecture is  $c_1 < U_1$ . If the split-the-surplus pricing rule is adopted then (A.11) must hold, which is satisfied only by  $z^* = Q_2 - Q_3 > 0$ , independent of  $\beta$  (see lemma 3). Thus if  $c_1 < U_1$  then  $q_2^* - q_3^* > 0$  as  $\beta \rightarrow 1$ .

Next, I show conditions for the existence of an equilibrium without  $q_i=1$ . Under the conjecture  $q_i=1$  then  $U_i=U > 0$ ,  $p_i^* \rightarrow 1/3 \forall i$  (by lemma 2) and  $p_{2,1}^* = p_{2,3}^* = 1/2$  (from (10)). I must verify that  $\pi^*$  is a best response by checking both (25) and (26), since (21)-(22) do not necessarily hold. Equation (26) requires  $c_1 < c_1(\beta, \gamma) = \beta U/3 - \gamma(1 - \beta)$ , and since  $c_1 > 0$  then there is a feasible  $c_1$  only if  $\beta > \beta_L = 3\gamma/(3\gamma + U)$ . Equation (25) holds for  $c_3 > c_3(\beta, \gamma) = \beta U/6 - \gamma(1 - \beta)$ . For

$\beta \in (\beta_L, 1)$  an equilibrium  $\pi^*, p_i^* = 1/3$  and  $q_i = 1 \forall i$  exists if  $c_1 \in (0, c_1(\beta, \gamma))$  and  $c_3 > \min\{0, c_3(\beta, \gamma)\}$ .

Finally, I show that equality of surpluses is inconsistent with  $q_i = 1$  even in the most stringent case where  $c_1$  tends to zero. After providing conditions for the existence of an equilibrium with  $c_1 = 0$  I show that it does not support a split-the-surplus rule in each match, even if I let  $\beta \rightarrow 1$ . I then invoke continuity to argue that the result holds for  $c_1$  in a (right) neighborhood of 0.

When  $c_1 = 0$ , equation (26) is satisfied by  $\beta > \beta_L$ , and equation (25) is satisfied for all  $c_3 > 0$  if  $\beta \leq \beta_H = 6\gamma/(6\gamma + U)$ , because the right hand side of (25) is negative for  $\beta < \beta_H$ . Notice that  $0 < \beta_L < \beta_H < 1$ . Alternatively, (25) is satisfied for all  $\beta$  if  $c_3 > U/6$ . Hence an equilibrium where  $c_1 = 0$ ,  $\pi = \pi^*, p_i^* = 1/3$  and  $q_i = 1 \forall i$  exists for either  $\beta \in (\beta_L, \beta_H)$ , or for  $\beta \in (\beta_L, 1)$  and  $c_3 > U/6$ .

Notice that  $U$  is the trade surplus for individuals 1 and 3, and (using (A.1)-(A.4)) that  $E[V_1] = V_{1,2} = E[V_3] = V_{3,1} = E[V_2] = \beta U/6$ . It is easy to see that equations (21)-(22) are not satisfied in

this equilibrium. Rearranging (A.2)-(A.3)  $V_{2,1} - V_{2,3} = \frac{\beta/3}{1-\beta/3} [u(1) + \gamma]$ . This implies that for all

$\beta \in (\beta_L, \beta_H)$  surpluses are either equal in the match  $\{1,2\}$   $u(1) - \frac{\beta/3}{1-\beta/3} [u(1) + \gamma] = U$  or they are

equal in the match  $\{2,3\}$ ,  $-\gamma + \frac{\beta/3}{1-\beta/3} [u(1) + \gamma] = U$  but not in both (unless  $U=0$ , which is not).

Similar considerations can be made if  $c_3 > U/6$  and  $\beta \rightarrow 1$ . By continuity, the above argument holds for  $c_1$  in a neighborhood of 0. I conclude that price uniformity is inconsistent with equality of surpluses even in the least stringent case of absence of intermediation costs. ■

### Proof of Proposition 2

When  $p = p^*, \pi = \pi^*$  and  $q = q^*$  is an equilibrium, then both (29) and (30) hold as strict inequalities (it is a consequence of lemma 3). It is easy to verify that both  $h(Q_2, Q_3)$  and  $f(Q_2, Q_3)$  are decreasing in  $Q_3$  (recall that  $u(Q_3) - \gamma Q_3$  increases in  $Q_3$  when  $Q_3 = q_3^* < 1$ , by assumption on preferences and technology). If  $Q_2$  is kept constant at  $q_2^*$ ,  $Q_3$  may fall down to  $q^L$  without (29)-(30) ever being violated. Furthermore by continuity there exists a neighborhood of  $Q_2$ ,  $q_2^* \pm \varepsilon$  for  $\varepsilon > 0$  small, and an associated neighborhood for  $Q_3$ ,  $q_3^* + \delta_\varepsilon$  for  $\delta_\varepsilon > 0$  small, on which  $\{Q_2, Q_3\}$  can

be located without violating (29)-(30). Thus a vector  $\{Q_2, Q_3\}$  such that  $Q_2 = q_2 = q_2^* \pm \varepsilon$ ,  $Q_3 = q_3 \in (q_3^L, q_3^* + \delta_\varepsilon)$  with  $\varepsilon > 0$  and  $\delta_\varepsilon > 0$  small supports the equilibrium conjectured. ■

### Proof of Proposition 3

When  $p = p^*$  and  $\pi = \pi^*$  by (2) and (4)  $W = E[V_i] \forall i$ , hence consider  $W = E[V_1] \equiv V_{1,2}$  that in equilibrium is  $W = \frac{\beta}{1-\beta} \left[ \frac{U_1}{U_1 + U_3} - \frac{\beta U_1 + c_1}{\beta(U_1 + U_2 + U_3)} \right] U_3$ . When  $c_1 = 0$  then  $W: R_2 \rightarrow R$  is a continuous, real-valued, non-negative function on the compact set  $S = \{(q_2, q_3) \mid q_2 \in [u^1(\gamma q_3^L), q_2^H], q_3 \in [q_3^L, u(q_2^H)/\gamma]\}$ , a subset of  $R^2$  (chosen as such since  $V_{1,2}$  cannot be negative, according to (5)).  $W$  achieves a minimum,  $W = 0$ , for  $q_2 = q_2^H$ ,  $q_3 = q_3^L$ , and along the locus  $q_3 = u(q_2)/\gamma$  ( $W > 0$  otherwise). The Weierstrass Theorem guarantees the existence of an interior  $q \in S$  such that  $W(q) \geq W(q') > 0 \forall q' \in S$ . To show that the maximum is unique let  $c_1 = 0$  and take the first derivatives  $\frac{\partial W}{\partial q_i} \equiv W_i$  for  $i = 2, 3$ . They vanish if, respectively

$$W_2 \equiv u'(q_2) U_1 (U_1 + U_3)^2 + \gamma U_2 [U_1^2 - U_3 (U_2 + U_3)] = 0$$

$$W_3 \equiv u'(q_3) U_2 [U_3^2 - U_1 (U_1 + U_2)] + \gamma U_3 (U_1 + U_3)^2 = 0.$$

It is easy to show that  $W_2$  is uniformly decreasing in  $q_2$ , while  $W_3$  is uniformly increasing in  $q_3$ . By continuity a unique maximum exists for  $c_1 > 0$  small. To show that the unit vector cannot be a maximum evaluate the partials  $W_i$  at  $q_i = 1$  ( $U_i = U \forall i$ ) and notice that  $W_2 = 4u'(1) - \gamma \neq W_3 = u'(1) - 4\gamma$ . Hence if  $W_2 = 0$  then  $q_3 < 1$ , while if  $W_3 = 0$  then  $q_2 > 1$  is necessary for a maximum. ■

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Figure 1

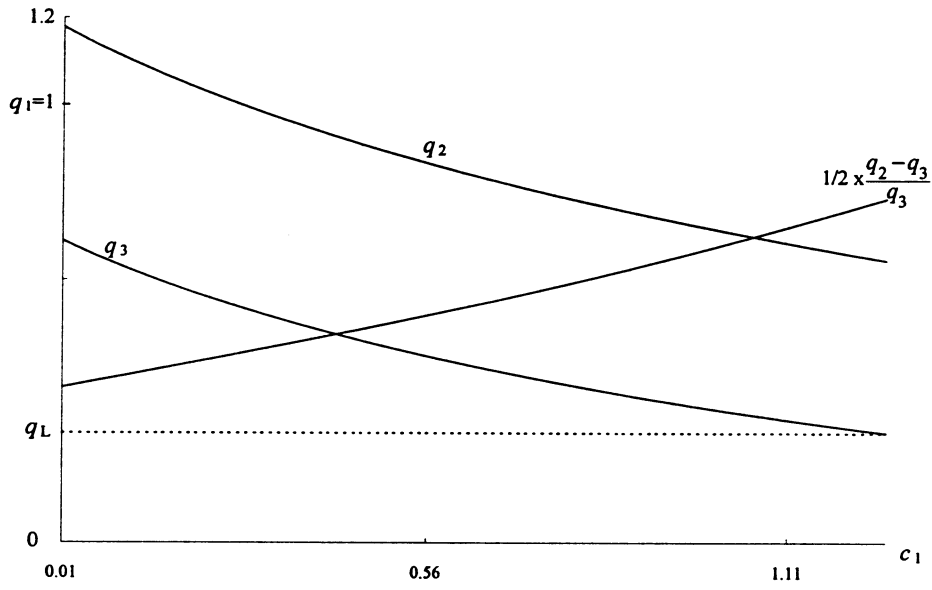


Figure 2

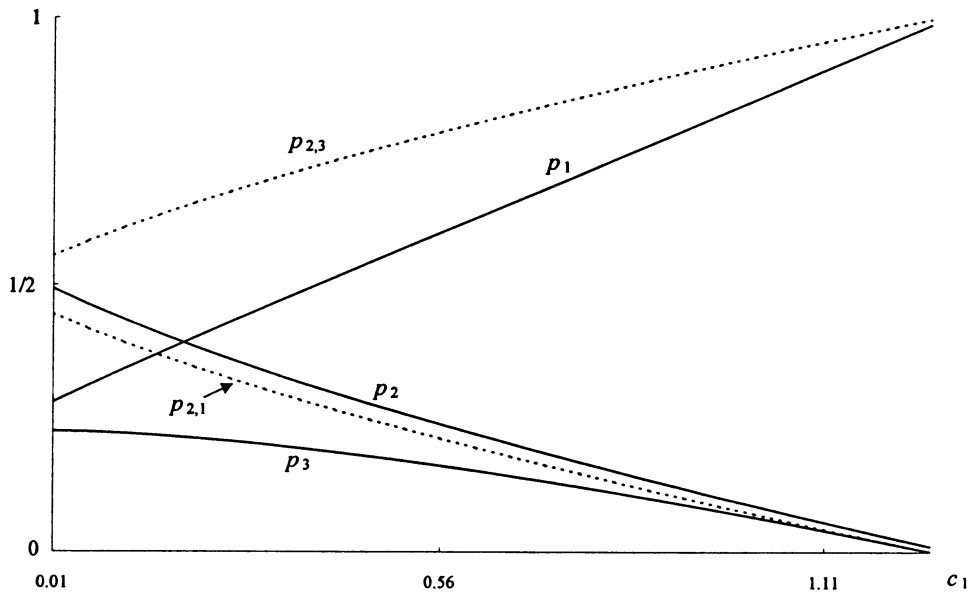


Figure 3

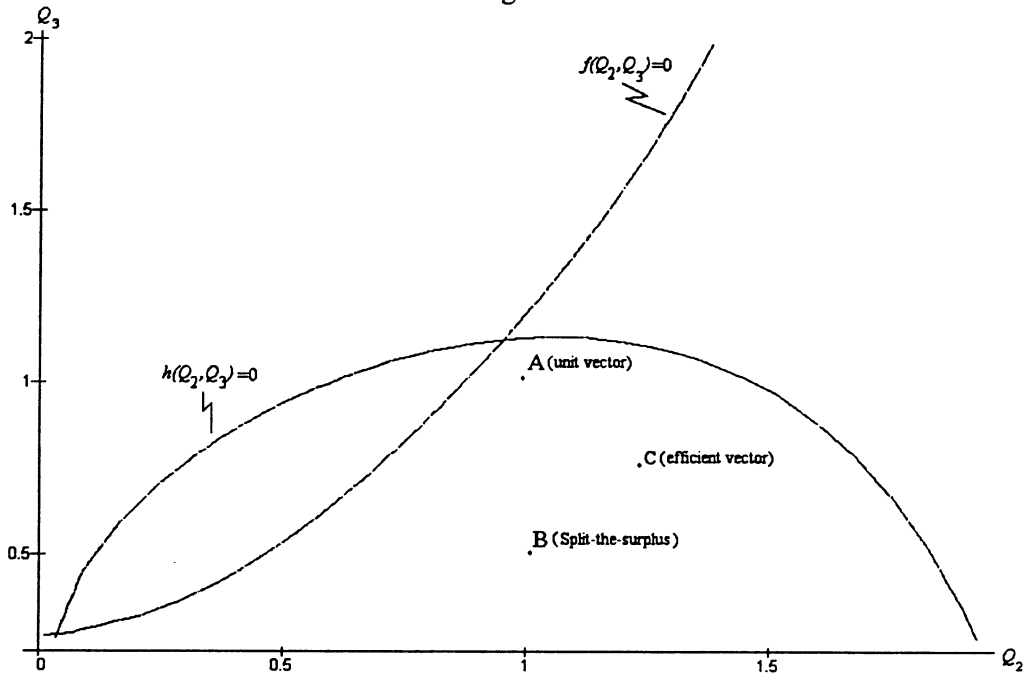
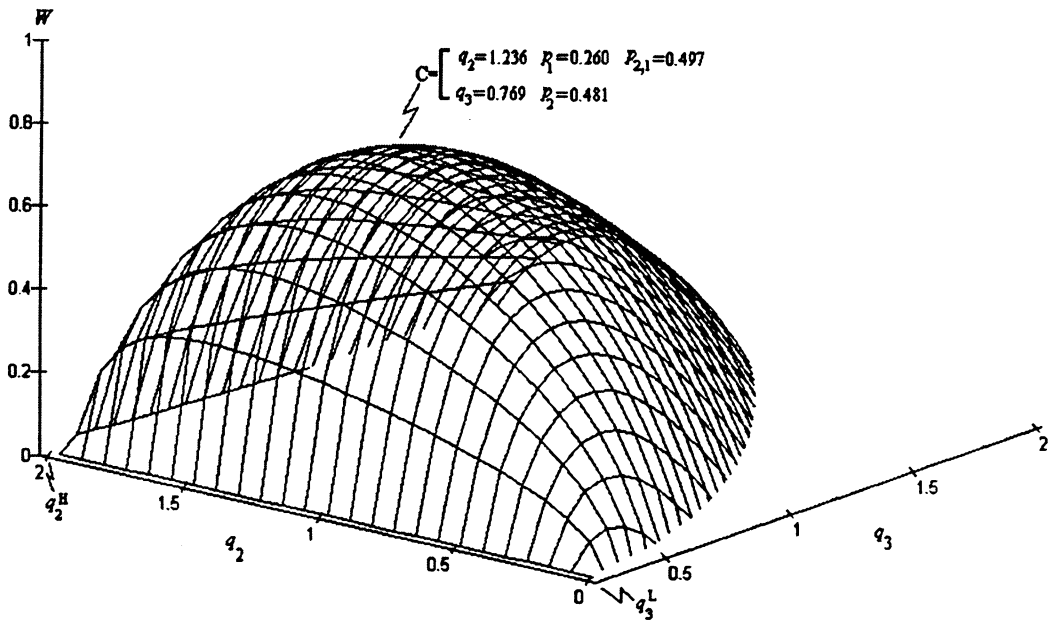


Figure 4





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