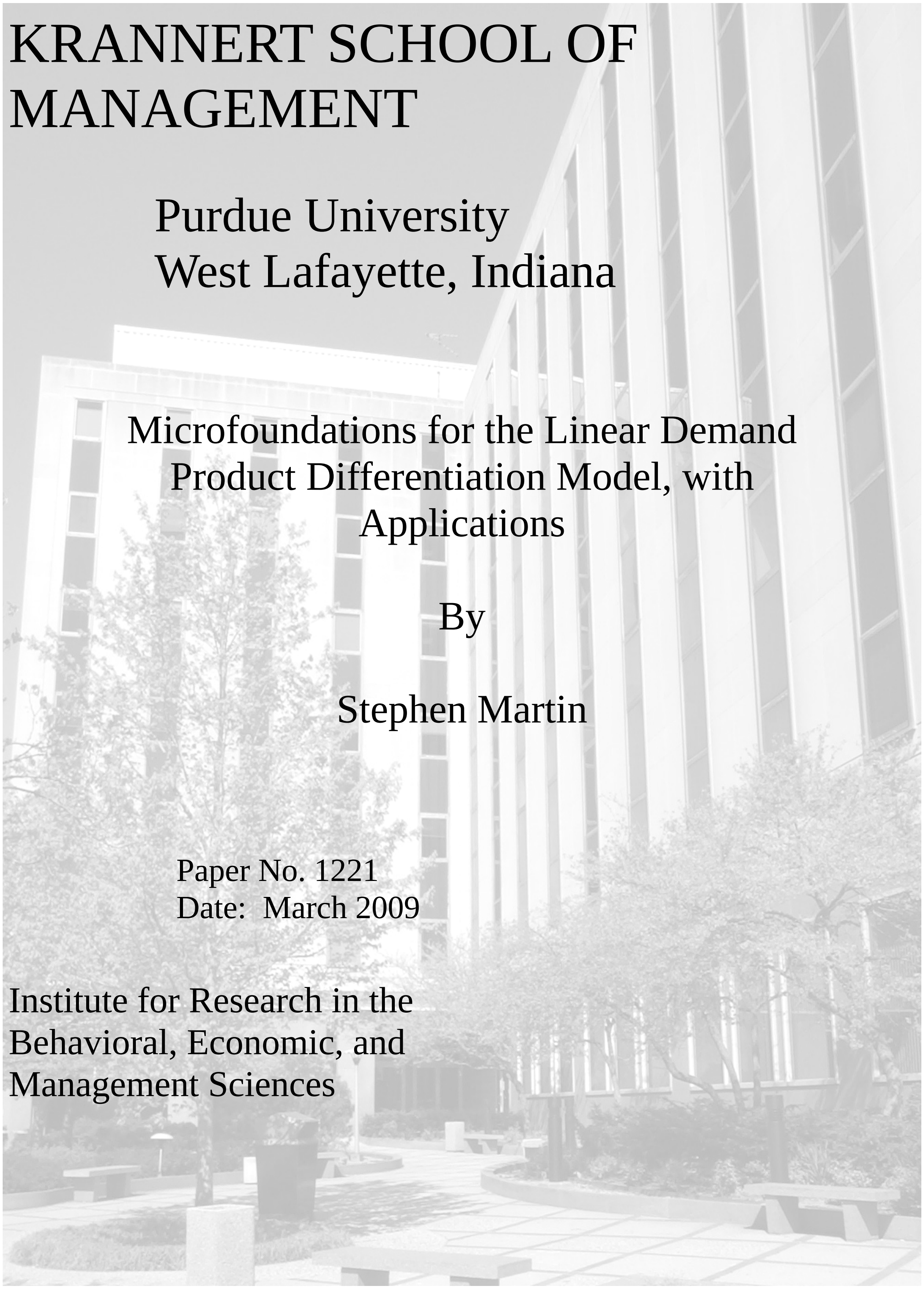

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Microfoundations for the Linear Demand Product Differentiation Model, with Applications

By

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Microfoundations for the Linear Demand Product Differentiation Model, with Applications

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Abstract

This paper shows (1) that the Spence-Dixit-Vives model of linear demand for differentiated varieties is implied if supplies of substitutes reduce individual consumers' reservation prices as indicated in the paper, (2) that for the micro-foundation-based version SDV demand and endogenous sunk costs, the equilibrium number of varieties is independent of the number of consumers in the market and the marginal cost of a variety of unit quality, and (3) that with endogenous sunk cost, if demand does not expand with the number of varieties (as in the SDV model), the equilibrium number of varieties is unchanged, but equilibrium qualities and quantities purchased are less, all else equal.

I am grateful for comments received at the Katholieke Universiteit Leuven and the European University Institute. Responsibility for errors is my own.

Contents

1 Introduction

The linear aggregate demand product differentiation specification is widely used in industrial economics. It is used by Sutton (1998 and elsewhere) in his pathbreaking analysis of market structure. It is increasingly used in the international trade literature. But it lacks micro foundations in a specification of individual demand, of the kind that has been common since Hotelling (1929).

Linear aggregate demand equations for differentiated products are typically either posited *a priori* or derived from a quadratic aggregate welfare function. The quadratic aggregate welfare function is itself either simply assumed or (less often) derived by aggregation from a population of individuals, each of whom is assumed to have a quadratic welfare function of the same type as the aggregate.

Strictly speaking, this specification has a long history in economics. Edgeworth (1897, p. 26/1925, p. 122) uses a quadratic utility function and linear inverse demand equations in discussion of complementary goods. Bowley (1924, p. 56) uses a quadratic aggregate welfare function. He does not include among its arguments a composite good with constant marginal utility; the implied demand equations are not linear. Despite these distinguished antecedents, it is to Spence (1976) and Dixit (1979) that the modern economics literature owes the linear demand for differentiated goods specification and the observation that it can be derived from a quadratic representative consumer welfare function. One might also highlight Singh and Vives (1984). Häckner (2000), who allows for many varieties and quality differences, provides what is probably the most complete generalization of Dixit (1979).[1]

Clear micro foundations have the merit of making explicit the assumptions about individual demand that stand behind a model of aggregate demand (Kirman, 1992). In this paper I outline a way to derive the quadratic aggregate welfare function from a model of individual demand,[2] and use the resulting specification to model the equilibrium number of firms in quantity-setting oligopoly with exogenous and alternatively endogenous sunk cost per

[1]For a review of the specification, see Vives (1999). Pinkse and Slade (2002, 2004) approximate a general individual indirect utility function by a quadratic functional form that aggregates to a demand equation that is linear in prices. Foster *et al.* (2008) use a quadratic aggregate welfare function that allows for a continuum of varieties.

[2]That is, from a model of individual demand that is not the aggregate relationship writ small.

variety.

In Section 2 I review the derivation from micro foundations of a linear aggregate demand function for the case of a homogeneous product. In Section 3 I show how to generalize this derivation from micro foundations to obtain linear aggregate demand functions for varieties of a differentiated product class. In Section 4 I relate Cournot oligopoly equilibrium, considering alternatively the cases of exogenous and endogenous sunk costs, to parameters describing individual demand characteristics. I examine two demand specifications, that of Spence-Dixit-Vives and a modification that corresponds, in a sense made precise below, to that of Shubik and Levitan (1980). The nature of the demand-parameter, equilibrium number of varieties relationship differs fundamentally between the exogenous and endogenous sunk cost cases. Section 5 concludes. Terse derivations of results are in the Appendix; a detailed Appendix with full proofs is available on request from the author.

2 Linear Demand, Homogeneous Product

In preparation for what follows, I review the standard derivation of a linear market demand curve for a homogeneous product from the micro behavior of a continuous mass of N consumers, each consumer purchasing either μ or zero units of the good.[3] Let x denote a consumer's reservation price for the good, suppose that x is uniformly distributed on the interval $0 \leq x \leq \rho$,[4] and refer to a consumer who obtains utility x from consuming one unit of the good as "the consumer at x." Then if a consumer at x purchases one unit of the product at price p, that consumer obtains net utility $x - p$. If all consumers have sufficient income to purchase the product, should they choose to do so, the quantity demanded at price p is

$$Q\left(p\right) = \frac{\mu N}{\rho}\int_{p}^{\rho} dx = \mu N - \frac{\mu N}{\rho} p \tag{1}$$

for $0 \leq p \leq \rho$ and 0 for $p \geq \rho$. μN is the quantity demanded at price 0. At price p, the $\frac{N}{\rho}p$ consumers with reservation prices between zero and p do not

[3]This aspect of the specification is discussed in Section 3.3.2.

[4]The assumption that reservation prices are uniformly distributed recommends itself for the simplicity of the results it brings. See Schmalensee (1984) for an early analysis of a normal distribution of reservation prices.

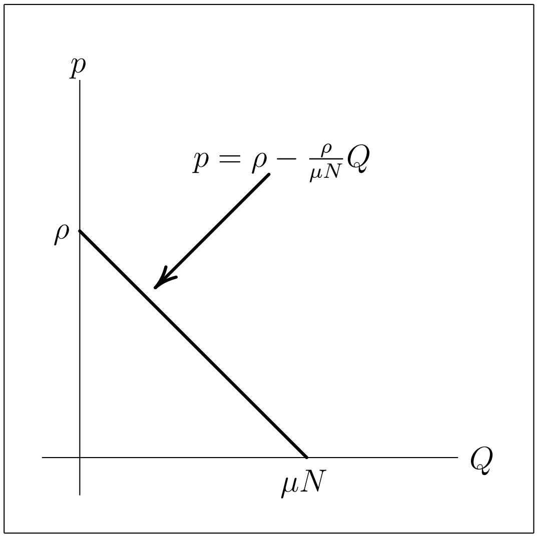

Figure 1: Linear market demand curve, homogeneous product.

purchase the good, and the remaining $N-\frac{N}{\rho}p$ consumers purchase μ units each.

From (1) one obtains the inverse demand equation

$$p=\rho-\frac{\rho}{\mu N}Q, \tag{2a}$$

the graph of which is shown in Figure 1. The slope coefficient, $\rho/\mu N$, is the inverse of the uniform density of quantity demanded by reservation prices.

The inverse demand equation can alternatively be derived by maximizing an aggregate or representative consumer welfare function that is quadratic in Q and linear in a constant marginal utility Hicksian "all other goods" or "money," M, the price of which is normalized to be 1,

$$U\left(Q\right)=M+\rho Q-\frac{1}{2}\frac{\rho}{\mu N}Q^2, \tag{3}$$

subject to the budget constraint

$$pQ+M=Y, \tag{4}$$

taking the solution with $M>0$.

3 Linear Demand, Horizontal Product Differentiation

In Section 3.1, limiting myself for simplicity to the case of two varieties, I give a standard form of linear inverse demand equations with differentiated varieties and show how it can be derived from a modified version of the individual behavior that underlies equation (1). In Section 3.3 I introduce asymmetries in the maximum reservation price and quantity purchased per consumer.

3.1 Symmetric Duopoly

Generalizing the notation of the previous section, for the duopoly case, linear inverse demand equations for horizontally differentiated goods are

$$p_1 = \rho - \frac{\rho}{\mu N}\left(q_1 + \sigma q_2\right), \tag{5}$$

$$p_2 = \rho - \frac{\rho}{\mu N}\left(\sigma q_1 + q_2\right). \tag{6}$$

Here ρ, N, and μ are as in the homogeneous product case. At the aggregate level, it is customary to interpret σ as a product differentiation parameter. An alternative aggregate interpretation, and a micro interpretation, will be presented below.

If $\sigma = 1$, the varieties are perfect substitutes; if $\sigma = 0$, the varieties are independent in demand; and if $\sigma < 0$, the varieties are demand complements. It is natural to use such a specification to model quantity-setting oligopoly.[5]

Spence (1976, footnote 6) notes that total surplus for linear aggregate

[5] For equivalent specifications that are in some ways better suited for models of price-setting oligopoly, see Shubik and Levitan (1980, p. 69), Vives (1985).

inverse demand equations of the form (5), (6) is[6]

$$T = \rho\left(q_1 + q_2\right) - \frac{1}{2}\frac{\rho}{\mu N}\left(q_1^2 + 2\sigma q_1 q_2 + q_2^2\right). \tag{8}$$

Then as is well known, and by analogy with the homogeneous product case, the aggregate inverse demand equations (5) and (6) can be derived by maximizing the aggregate welfare function

$$U = M + \rho\left(q_1 + q_2\right) - \frac{1}{2}\frac{\rho}{\mu N}\left(q_1^2 + 2\sigma q_1 q_2 + q_2^2\right) \tag{9}$$

subject to the budget constraint

$$Y = M + p_1 q_1 + p_2 q_2. \tag{10a}$$

3.2 Micro Foundations

The aggregate inverse demand equations (5) and (6) can alternatively be derived from individual behavior as follows. Let there be N consumers potentially in the market for varieties 1 and 2. Consumers' maximum reservation prices are uniformly distributed, with density N/ρ, along the interval $0 \leq x \leq \rho$. Now assume that if q_j units of good j are on the market, the reservation price for good i of a consumer at x is $x - \sigma\frac{\rho}{\mu N}q_j$, provided the latter is nonnegative, and zero otherwise. Intuitively, if apples are plentiful, consumers are willing to pay less for oranges, all else equal, and vice versa. Then the quantity demanded of variety i is

$$q_i = \frac{\mu N}{\rho}\int_{p_i}^{\rho - \sigma\frac{\rho}{\mu N}q_j} dx = \mu N - \sigma q_j - \frac{\mu N}{\rho}p_i. \tag{11}$$

[6] He also remarks, in his footnote 6 (simplifying to the case of two varieties and using the notation of the present paper) that total surplus is

$$T = \int_0^{q_1} p\left(s_1, 0\right) ds_1 + \int_0^{q_2} p\left(q_1, s_2\right) ds_2 =$$

$$\int_0^{q_1}\left(\rho - \frac{\rho}{\mu N}s_1\right) ds_1 + \int_0^{q_2}\left(\rho - 2\sigma\frac{\rho}{\mu N}q_1 - \frac{a}{\mu N}s_2\right) ds_2, \tag{7}$$

so that for a class of differentiated goods, aggregate gross welfare is not the sum of the areas under the demand curves, but rather the sum under the areas under curves derived from the demand curves by suitably adjusting price-axis intercepts.

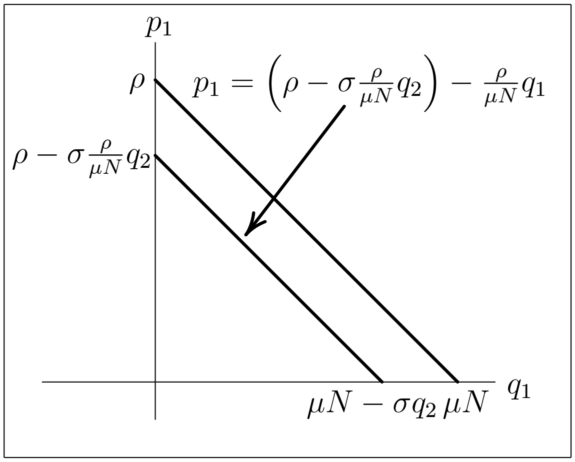

Figure 2: Linear market demand curve, variety 1, horizontal product differentiation.

This leads to (5) and (6); the demand curve for variety 1 is illustrated in Figure 2. At the micro level, σ indicates the extent to which the availability of substitute varieties lowers a consumer's reservation price.

3.3 Asymmetries

In the previous section, all varieties have the same maximum reservation price ρ, and each consumer purchases the same quantity (μ) of every variety, if the variety is purchased at all. For many purposes (modelling vertical product differentiation, for example, or examining the impact of changes in market size on the equilibrium number of firms), it is useful to extend the model to permit differences across varieties in these aspects of consumer demand.

3.3.1 Different reservation prices

We begin with different maximum reservation prices for different varieties, keeping other aspects of demand the same for all varieties. Limiting ourselves for expositional purposes to the duopoly case, if the maximum reservation price is the same for both varieties, demand for variety i is given by (11). If there are different maximum reservation prices for different varieties, a general formulation for the demand for variety i is

$$q_i = \frac{\mu N}{\rho_i} \int_{p_i}^{\rho_i - \sigma \frac{f(\rho_1, \rho_2)}{\mu N} q_j} dx, \tag{12}$$

for some function $f(\rho_1, \rho_2)$. f should satisfy $f(\rho, \rho) = \rho$, so (12) reduces to (11) if $\rho_1 = \rho_2$. Since nothing fundamental changes if varieties are renumbered, f should also be symmetric. With these considerations in mind, and for simplicity, let

$$f(\rho_1, \rho_2) = \sqrt{\rho_1 \rho_2}. \tag{13}$$

Then the quantities demanded, inverse demand equations, and aggregate welfare functions are

$$q_i = \frac{\mu N}{\rho_i} \int_{p_i}^{\rho_i - \sigma \frac{\sqrt{\rho_1 \rho_2}}{\mu N} q_j} dx = \frac{\mu N}{\rho_i} \left(\rho_i - \sigma \frac{\sqrt{\rho_1 \rho_2}}{\mu N} q_j - p_i \right), \tag{14}$$

$$p_i = \rho_i - \frac{1}{\mu N} \left(\rho_i q_i + \sigma \sqrt{\rho_1 \rho_2} q_j \right) \tag{15}$$

and

$$U = M + \rho_1 q_1 + \rho_2 q_2 - \frac{1}{2} \frac{1}{\mu N} \left(\rho_1 q_1^2 + 2\sigma \sqrt{\rho_1 \rho_2} q_1 q_2 + \rho_2 q_2^2 \right), \tag{16}$$

respectively.

3.3.2 Different quantities per consumer

At this point a word on the role of the parameter μ is in order. It would be usual, and in discussion of the homogeneous-product case would raise no difficulty of interpretation, to assume that there are N potential consumers, each of whom takes either one or zero units of a good. Then one way in which market size can change is a change in N. In the specification used here, the quantity that would be demanded if price were zero is μN, and it is a change in μN that is one kind of change in market size. The micro foundation for the linear product differentiation model proposed in equation (11) makes consumer reservation prices for a variety a declining function of the quantities of substitute varieties. Although the mathematical formulation does not require it, a natural economic interpretation is that it is one and the same N consumers who are potential purchasers of all varieties. The parameter μ (which generalizes to μ_i in the asymmetric case) makes it possible to model changes in market size in the sense of changing the maximum possible quantity demanded, holding N constant.[7]

If the maximum individual quantity purchased of different varieties may differ, write in place of (11)

$$q_i = \frac{\mu_i N}{\rho} \int_{p_i}^{\rho - \sigma \frac{\rho}{g(\mu_1, \mu_2) N} q_j} dx = \frac{\mu_i N}{\rho} \left[\rho - \sigma \frac{\rho}{g(\mu_1, \mu_2) N} q_j - p_i \right]. \tag{17}$$

By the arguments made about $f(\rho_1, \rho_2)$ in the previous section, $g(\mu_1, \mu_2)$ should be symmetric and satisfy $g(\mu, \mu) = \mu$. Then let

$$g(\mu_1, \mu_2) = \sqrt{\mu_1 \mu_2}, \tag{18}$$

which leads to demand and inverse demand equations

$$q_i = \frac{\mu_i N}{\rho} \int_{p_i}^{\rho - \sigma \frac{\rho}{\sqrt{\mu_1 \mu_2} N} q_j} dx = \frac{\mu_i N}{\rho} \left(\rho - \sigma \frac{\rho}{\sqrt{\mu_1 \mu_2} N} q_j - p_i \right) \tag{19}$$

and

$$p_i = \rho - \sigma \frac{\rho}{\sqrt{\mu_1 \mu_2} N} q_j - \frac{\rho}{\mu_i N} q_i, \tag{20}$$

respectively. The corresponding aggregate welfare function is

$$(q_1, q_2) = M + \rho (q_1 + q_2) - \frac{1}{2} \frac{\rho}{N} \left[\frac{q_1^2}{\mu_1} + 2\sigma \frac{q_1 q_2}{\sqrt{\mu_1 \mu_2}} + \frac{q_2^2}{\mu_2} \right]. \tag{21}$$

3.3.3 An Encompassing Specification

A specification of demand for variety i that combines the two previous cases is

$$q_i = \frac{\mu_i N}{\rho_i} \int_{p_i}^{\rho_i - \sigma \sqrt{\frac{\rho_1}{\mu_1} \frac{\rho_2}{\mu_2} \frac{q_j}{N}}} dx = \frac{\mu_i N}{\rho_i} \left(\rho_i - \sigma \sqrt{\frac{\rho_1}{\mu_1} \frac{\rho_2}{\mu_2} \frac{q_j}{N}} - p_i \right), \tag{22}$$

[7] All results presented here for differences in μ across varieties are replicated if μ is set equal to 1 and it is differences in N that are considered. Further, having worked out formulation considered in the paper, it is straightforward to generalize the model so that total demand for (say) variety 1 is the sum of demand by one group of consumers for whom other varieties are substitutes and another group of consumers who purchase, if at all, only variety 1.

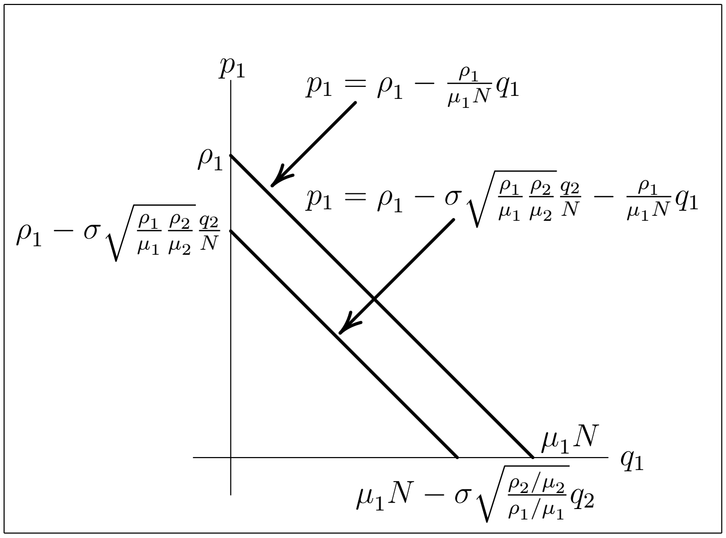

Figure 3: Linear market demand curve, variety 1, asymmetric maximum reservation prices, asymmetric quantities demanded per consumer.

from which follow inverse demand and quadratic aggregate welfare equations

$$p_i = \rho_i - \frac{1}{N}\left(\frac{\rho_i}{\mu_i} q_i + \sigma\sqrt{\frac{\rho_1}{\mu_1}\frac{\rho_2}{\mu_2}} q_j\right) \tag{23}$$

and

$$U(q_1, q_2) = M + \rho_1 q_1 + \rho_2 q_2 - \frac{1}{2}\frac{1}{N}\left[\frac{\rho_1}{\mu_1} q_1^2 + 2\sigma\sqrt{\frac{\rho_1}{\mu_1}\frac{\rho_2}{\mu_2}} q_1 q_2 + \frac{\rho_2}{\mu_2} q_2^2\right], \tag{24}$$

respectively.

The demand curve for variety 1 is drawn in Figure 3. ρ_1 is the maximum reservation price if the substitute good is not available, μ_1 the quantity demanded per consumer, and $\mu_1 N$ the maximum quantity demanded. For reservation price parameter σ, with quantity q_2 of the substitute variety available, the reservation price of a consumer at x falls to $x-\ \sigma\sqrt{\frac{\rho_1}{\mu_1}\frac{\rho_2}{\mu_2}\frac{q_2}{N}}$ or to zero, whichever is greater.

In what follows I will work with generalizations of the encompassing specification from duopoly to oligopoly.

4 Market Size and the Equilibrium Number of Firms

Here I examine the relation between micro demand parameters and the equilibrium number of firms in quantity-setting oligopoly. I first consider the n-firm generalization of the variety of the duopoly specification (23), then a modification in which (in a certain sense) market size is constant as the number of varieties changes. In symmetric oligopoly with exogenous sunk cost per variety, the equilibrium number of firms is related to a measure of market size that emerges from the analysis, a measure that depends on ρ, μ, and marginal cost. If variety-specific values of ρ and μ are endogenized, along the lines of Sutton (1998), the equilibrium number of firms depends on cost parameters and σ. For reasons of tractability, I consider cases in which each firm produces one variety.[8]

4.1 Exogenous sunk cost

Writing in place of (11)

$$q_i = \frac{\mu N}{\rho} \int_{p_i}^{\rho - \sigma \frac{\rho}{\mu N} Q_{-i}} dx = \mu N - \sigma Q_{-i} - \frac{\mu N}{\rho} p_i, i = 1, ..., n, \tag{25}$$

where $Q_{-i} = \sum_{j \neq i} q_j$ is the combined output of all varieties except that of variety i, one obtains an n-firm version of the symmetric duopoly inverse demand equations (5)/(6):

$$p_i = \rho - \frac{\rho}{\mu N} \left(q_i + \sigma Q_{-i} \right). \tag{26}$$

Let F be fixed and sunk cost per variety and constant marginal cost per

[8]Extensions of the models developed here to the case of multiple varieties per firm lack analytical solutions.

unit ρc per unit.[9] Firm's i's single-period payoff and objective function, is

$$\pi_i = \left[(1-c)\,\rho - \frac{\rho}{\mu N}\left(q_i + \sigma Q_{-i}\right)\right] q_1 - F. \tag{27}$$

The first-order condition,[10] which can be written

$$2q_i + \sigma Q_{-i} \equiv (1-c)\,\mu N, \tag{28}$$

implies that firm i's equilibrium payoff is

$$\pi_i^* = \frac{\rho}{\mu N}\left(q_1^*\right)^2 - F, \tag{29}$$

with an asterisk denoting an equilibrium value.

From (28), symmetric equilibrium output per firm is

$$q^* = \frac{1-c}{2+(n-1)\,\sigma}\mu N, \tag{30}$$

making equilibrium profit per firm

$$\pi^* = \frac{2}{\left[2+(n-1)\,\sigma\right]^2}\left[\frac{1}{2}\left(1-c\right)^2\rho\mu N\right] - F = \frac{2S}{\left[2+(n-1)\,\sigma\right]^2} - F. \tag{31}$$

Here

$$S = \frac{1}{2}\left(1-c\right)^2\rho\mu N \tag{32}$$

is a natural measure of market size. It is consumer surplus in a market with inverse demand equation $p = \rho - \frac{\rho}{\mu N}q$ if price equals marginal cost, or equivalently the profit of a single supplier that can costlessly engage in first-degree price discrimination.

If the number of firms adjusts so profit per firm equals zero, we obtain[11]

[9] In this section, writing constant marginal cost as ρc is merely a normalization. When we move on to the case of endogenous sunk cost, we interpret ρ as a measure of quality, and the assumption that the marginal cost of a variety of quality ρ is ρc is the assumption that marginal cost is proportional to quality. One might expect constant marginal cost to increase more than proportionately to quality, and the specification employed here is chosen on the usual ground that it permits analytic solutions.

[10] The second-order condition is met.

[11] This ignores the fact that the number of firms must be an integer. See Amir and Lambson (2000, 2007) for consideration of this point.

Result 1: With exogenous sunk cost F per variety, constant marginal cost ρc per unit, and aggregate demand (26), the long-run Cournot oligopoly equilibrium number of firms is

$$n^* = 1 + \frac{1}{\sigma}\left[(1-c)\sqrt{\frac{\rho\mu N}{F}} - 2\right] = 1 + \frac{1}{\sigma}\left(\sqrt{2\frac{S}{F}} - 2\right), \quad (33)$$

for S given by (32).

The condition for it to be profitable for at least one firm to supply the market is $S \geq 2F$. The individual demand characteristics ρ and μ affect equilibrium profit (and therefore the long-run equilibrium number of firms) only via their effects on S. S rises as ρ and μN rise, and as c falls, as therefore does n^*. n^* rises as σ and F fall. All these results are what one would expect with exogenous sunk costs.

4.2 Endogenous sunk cost

In this section, we consider a two-stage game. In the second stage, firms compete as quantity-setting oligopolists, with the maximum reservation price ρ_i and quantity demanded per consumer μ_i of each variety given. In the first stage, there is simultaneous entry of the zero-profit number of firms. Each firm noncooperatively makes fixed and sunk investments that determine the ρ and μ values of its variety. When it makes these investments, the firm anticipates the consequences of its choices for its second stage payoff.

4.2.1 Demand, asymmetric Cournot oligopoly

If varieties differ in their demand characteristics, (25) and (26) generalize to

$$q_i = \frac{\mu_i N}{\rho_i}\int_{p_i}^{\rho_i - \frac{\sigma}{N}\sqrt{\frac{\rho_i}{\mu_i}}\sum_{j\neq i}^{n}\sqrt{\frac{\rho_j}{\mu_j}}q_j} dx = \frac{\mu_i N}{\rho_i}\left(\rho_i - \frac{\sigma}{N}\sqrt{\frac{\rho_i}{\mu_i}}\sum_{j\neq i}^{n}\sqrt{\frac{\rho_j}{\mu_j}}q_j - p_i\right) \quad (34)$$

and

$$p_i = \rho_i - \frac{1}{N}\left(\sigma\sqrt{\frac{\rho_i}{\mu_i}}\sum_{j\neq i}^{n}\sqrt{\frac{\rho_j}{\mu_j}}q_j + \frac{\rho_i}{\mu_i}q_i\right), \quad (35)$$

respectively.[12]

4.2.2 Second-stage equilibrium

Firm i's second period objective function — which is gross of sunk costs incurred in the first period — is

$$\pi_i^{II} = \left[\rho_i\left(1-c\right) - \frac{1}{N}\left(\sigma\sqrt{\frac{\rho_i}{\mu_i}}\sum_{j\neq i}^{n}\sqrt{\frac{\rho_j}{\mu_j}}q_j + \frac{\rho_i}{\mu_i}q_i\right)\right]q_i \tag{36}$$

The first-order condition to maximize (36) implies that firm i's equilibrium second-stage payoff is

$$\pi_i^{II*} = \frac{\rho_i}{\mu_i N}\left(q_i^*\right)^2. \tag{37}$$

It is shown in the Appendix that

Result 2: For single product firms and aggregate inverse demand equations of the form (35), variety i's equilibrium output is

$$q_i^* = \frac{\left(1-c\right)N}{\left(2-\sigma\right)\left[2+\left(n-1\right)\sigma\right]}\left\{\left[2+\left(n-2\right)\sigma\right]\mu_i - \sigma\sqrt{\frac{\mu_i}{\rho_i}}\sum_{j\neq i}^{n}\left(\sqrt{\rho_j\mu_j}\right)\right\}. \tag{38}$$

4.2.3 First-stage equilibrium

Let ε denote the fixed and sunk cost of designing a variety with $\rho = 1$, and η the fixed and sunk cost of marketing a variety with $\mu = 1$. Suppose further that the fixed cost of quality rises exponentially at rate $\beta > 2$, and the fixed cost of increasing quantity purchased rises exponentially at rate $\gamma > 2$.[13] Fixed and sunk cost per variety, now endogenous, is then

$$\varepsilon\rho_i^{\beta} + \eta\mu_i^{\gamma}. \tag{39}$$

[12] These inverse demand equations are valid provided all quantities demanded are nonnegative, and we limit our attention to such cases. The condition for a variety to have nonnegative quantity demanded, in Cournot equilibrium, depends on the market sizes (in the sense of (32)) of all varieties and on σ.

[13] This is the formulation Sutton (1998, p. 59) uses for a firm with a single choice variable. As Sutton explains in the context of his model, the assumption that the exponents are greater than 2 ensures that fixed cost rises (here, with ρ, μ) at least as rapidly as profit.

A firm can increase maximum reservation prices for its variety, given the quantity an individual consumer will purchase if the consumer purchases at all. A firm can increase the quantity a consumer with a given maximum reservation price will buy, if the consumer buys at all, given the consumer's maximum reservation price. By way of interpretation, one can think of $\varepsilon\rho_i^\beta$ as product design (or R&D) costs that determine the quality of the variety, while $\eta\mu_i^\gamma$ is marketing (or advertising) costs that affect purchase amounts, given quality. Firms face increasing costs of product design and of marketing.[14]

Firm i's first period payoff is

$$\pi_i^I = \frac{\rho_i}{\mu_i N}\left(q_i^*\right)^2 - \varepsilon\rho_i^\beta - \eta\mu_i^\gamma, \tag{40}$$

with q_i^* given by (38).

It is shown in the Appendix that

Theorem 3: (a) for given n, equilibrium ρ, μ, and market size $S = \frac{1}{2}\left(1-c\right)^2 \rho\mu N$ satisfy

$$\rho^{\frac{(\gamma-1)(\beta-1)-1}{\gamma}} = N\frac{\left(1-c\right)^2}{2-\sigma}\frac{2+\left(n-2\right)\sigma}{\left[2+\left(n-1\right)\sigma\right]^2}\left(\varepsilon\beta\right)^{-\left(1-\frac{1}{\gamma}\right)}\left(\eta\gamma\right)^{-\frac{1}{\gamma}} \tag{41}$$

$$\mu^{\frac{(\gamma-1)(\beta-1)-1}{\beta}} = N\frac{\left(1-c\right)^2}{2-\sigma}\frac{2+\left(n-2\right)\sigma}{\left[2+\left(n-1\right)\sigma\right]^2}\left(\varepsilon\beta\right)^{-\frac{1}{\beta}}\left(\eta\gamma\right)^{-\left(1-\frac{1}{\beta}\right)} \tag{42}$$

and

$$\left(2S\right)^{(\gamma-1)(\beta-1)-1} = \left[\left(1-c\right)^2 N\right]^{\beta\gamma}\left\{\frac{1}{2-\sigma}\frac{2+\left(n-2\right)\sigma}{\left[2+\left(n-1\right)\sigma\right]^2}\right\}^{\beta+\gamma}\left(\varepsilon\beta\right)^{-\gamma}\left(\eta\gamma\right)^{-\beta} \tag{43}$$

respectively;

(b) the equilibrium number of varieties is

$$n^* = 1 + \frac{2-\sigma}{\sigma}\frac{\left(\beta-1\right)\left(\gamma-1\right)-1}{\beta+\gamma} > 1; \tag{44}$$

equilibrium values of ρ, μ, and and S satisfy

$$\rho^{\frac{(\gamma-1)(\beta-1)-1}{\gamma}} = \left(1-c\right)^2 N\frac{\beta\gamma\left(\beta+\gamma\right)}{\left[\left(2-\sigma\right)\beta\gamma+\sigma\left(\beta+\gamma\right)\right]^2}\left(\varepsilon\beta\right)^{-\left(1-\frac{1}{\gamma}\right)}\left(\eta\gamma\right)^{-\frac{1}{\gamma}} \tag{45}$$

[14] In a more general formulation, the two types of costs might be related.

$$\mu^{\frac{(\gamma-1)(\beta-1)-1}{\beta}} = (1-c)^2 N \frac{\beta\gamma(\beta+\gamma)}{\left[(2-\sigma)\beta\gamma+\sigma(\beta+\gamma)\right]^2} (\varepsilon\beta)^{-\frac{1}{\beta}} (\eta\gamma)^{-\left(1-\frac{1}{\beta}\right)} \tag{46}$$

and

$$(2S)^{(\gamma-1)(\beta-1)-1} =$$

$$\left[(1-c)^2 N\right]^{\beta\gamma} \left\{\frac{\beta\gamma(\beta+\gamma)}{\left[(2-\sigma)\beta\gamma+\sigma(\beta+\gamma)\right]^2}\right\}^{\beta+\gamma} (\varepsilon\beta)^{-\gamma} (\eta\gamma)^{-\beta}, \tag{47}$$

respectively.

Sutton (1998, p. 46) illustrates his general analysis of the determinants of market structure with an example in which every consumer in a market has a quadratic utility function. In this framework, markets grow larger as the number of consumers increases. As the number of consumers increases, variety-specific aggregate demand curves rotate in a counterclockwise direction around the maximum reservation price that is common to all consumers. In this framework, Sutton looks for (Section 1.2) and finds the result that concentration is bounded away from zero as market size increases.

Here consumers differ in reservation prices, and varieties differ potentially both in terms of the maximum reservation price and the quantity a consumer will buy, if the consumer buys at all (demand characteristics that depend on firms' sunk investments). β and ε, γ and η are parameters that determine the cost to a firm of getting a higher ρ or a larger μ. In the context of the present model, it is N and c that parametrically affect market size.[15] The specific manifestation of Sutton's lower bound on concentration that appears as (44) is that the equilibrium number of firms is invariant to changes in N and c.

From (44), the equilibrium number of firms falls as reservation prices are more sensitive to the availability of substitutes (higher σ)

$$\frac{\partial}{\partial\sigma}\left(\frac{2-\sigma}{\sigma}\right) = -\frac{2}{\sigma^2} < 0, \tag{48}$$

and rises the more rapidly sunk costs rise with ρ (higher β):

$$\frac{\partial}{\partial\beta}\left(\frac{\beta\gamma-\beta-\gamma}{\beta+\gamma}\right) = \left(\frac{\gamma}{\beta+\gamma}\right)^2 > 0 \tag{49}$$

[15] But, as regards N, see footnote 7.

or with μ (higher γ):

$$\frac{\partial}{\partial\gamma}\left(\frac{\beta\gamma-\beta-\gamma}{\beta+\gamma}\right)=\left(\frac{\beta}{\beta+\gamma}\right)^2>0. \tag{50}$$

4.3 "Constant Market Size"

There is a sense[16] in which the Spence-Dixit-Vives specification implies that market size increases with the number of varieties. To see this, consider the total number of units demanded, as implied by (26), if all firms set the same price. Since the quantity demanded of a single variety at equal prices is

$$q=\frac{1}{1+(n-1)\,\sigma}\frac{\rho-p}{\rho}\mu N, \tag{51}$$

the total number of units demanded[17] is

$$nq=\frac{n}{1+(n-1)\,\sigma}\frac{\rho-p}{\rho}\mu N, \tag{52}$$

from which

$$\lim_{n\to\infty}\frac{nq}{\frac{\rho-p}{\rho}\mu N}=\lim_{n\to\infty}\frac{n}{1+(n-1)\,\sigma}=\lim_{n\to\infty}\frac{1}{\frac{1-\sigma}{n}+\sigma}=\frac{1}{\sigma}>1 \tag{53}$$

(for $0<\sigma<1$).

This leads to the alternative interpretation of σ that is referred to at the start of Section 3.1: in the Spence-Dixit-Vives specification, the inverse of σ measures the expandability of the market as the number of varieties increases, all else equal.

4.3.1 Shubik-Levitan

For the Spence-Dixit-Vives specification, the total number of units demanded at a common price increases as the number of varieties increases. This market expansion effect is not inherent in linear aggregate demand models of product differentiation. Shubik and Levitan (1980) put forward an alternative aggregate linear demand, differentiated product specification in which

[16] As I have earlier (Martin, 1985) observed.

[17] This sum should be interpreted in the sense n_1 apples plus n_2 oranges add to n_1+n_2 pieces of fruit.

total demand at identical prices is constant with respect to changes in the number of varieties.

If there are n varieties, the Shubik-Levitan demand equation for variety i is

$$q_i = \frac{1}{n}\left[\alpha - \beta p_i - \beta\gamma(p_i - \overline{p})\right]. \tag{54}$$

The corresponding inverse demand equation is

$$p_i = \frac{\alpha}{\beta} - \frac{1}{\beta}\frac{n+\gamma}{1+\gamma}\left(q_i + \frac{\gamma}{n+\gamma}Q_{-i}\right). \tag{55}$$

Comparing (26) and (55), for a given number of varieties, the SDV and SL specifications are equivalent for[18]

$$\alpha = \frac{n}{1+(n-1)\,\sigma}\mu N \qquad \beta = \frac{\alpha}{\rho} \tag{56}$$

$$\gamma = \frac{n\sigma}{1-\sigma}. \tag{57}$$

In the Shubik-Levitan specification, it is γ that indicates the extent of product differentiation. From (54), "varieties" are completely independent in demand if $\gamma = 0$. From (57), $\gamma = 0$ corresponds to $\sigma = 0$ in the SDV specification. Also from (54), varieties approach perfect substitutability in the SL specification as $\gamma \to \infty$,[19] and this corresponds to $\sigma = 1$ in the SDV specification.

For present purposes, however, it suffices to note that if all varieties charge the same price p (which is a characteristic of symmetric equilibrium), the quantity demanded (q) will be the same for all varieties, and (54) simplifies to

$$nq = \alpha - \beta p, \tag{59}$$

the right-hand side of which is independent of n. In the Shubik-Levitan model, in contrast to the SDV specification, for a common price, the total number of units demanded is independent of the number of varieties.

[18] Derivations are straightforward and available on request from the author.

[19] Alternatively, this may be seen by rewriting (54) as

$$p_i - \overline{p} = \frac{1}{\beta\gamma}\left[\alpha - \beta p_i - nq_i\right]. \tag{58}$$

As $\gamma \to \infty$, the right-hand side goes to zero; as varieties approach perfect substitutability, there is less and less leeway for the price of any variety to deviate from the industry average price.

4.3.2 Scaled-demand SDV

From (52), in the Spence-Dixit-Vives specification, the total number of units demanded, at a common price, is proportional to $\frac{n}{1+(n-1)\sigma}$. A constant-market-size version of the SDV specification can be obtained by scaling demand equations by

$$\frac{1+(n-1)\,\sigma}{n} = 1 - \left(1 - \frac{1}{n}\right)(1-\sigma) < 1. \tag{60}$$

Linear demand equations that exhibit constant market size in this sense can be obtained from microfoundations by assuming that a single consumer's quantity demanded of a variety, if the consumer purchases at all, is $\left[1 - \left(1 - \frac{1}{n}\right)(1-\sigma)\right]\mu < \mu$. The aggregate demand equation for variety i (for the symmetric parameter case) becomes,

$$q_i = \frac{1+(n-1)\,\sigma}{n}\frac{\mu N}{\rho}\int_{p_i}^{\rho-\sigma\frac{n}{1+(n-1)\sigma}\frac{\rho}{\mu N}Q^s_{-1}} dx, \tag{61}$$

leading to inverse demand equations of the form[20]

$$p_i = \rho - \frac{n}{1+(n-1)\,\sigma}\frac{\rho}{\mu N}\left(q^s_i + \sigma Q^s_{-i}\right). \tag{63}$$

4.3.3 Exogenous sunk cost

If fixed cost is exogenously determined and demand is scaled as indicated above, firm i's objective function is

$$\pi^s_i = \left[(1-c)\,\rho - \frac{n}{1+(n-1)\,\sigma}\frac{\rho}{\mu N}\left(q^s_i + \sigma Q^s_{-i}\right)\right] q^s_i - F. \tag{64}$$

The first-order condition to maximize (64), which can be written

$$2q^s_i + \sigma Q^s_{-i} \equiv \frac{1+(n-1)\,\sigma}{n}(1-c)\,\mu N, \tag{65}$$

[20] The demand equation for variety i is

$$q^s_i = \frac{1}{n}\frac{\mu N}{\rho}\left\{\rho - \frac{[1+(n-2)\,\sigma]\,p_i - \sigma\sum_{j\neq i}^{n} p_j}{1-\sigma}\right\}. \tag{62}$$

If all varieties charge a common price p, this reduces to $q^s_i = \frac{\mu N}{n}\frac{\rho - p}{\rho}$; the total number of units demanded at a common price, $nq^s_i = \mu N\frac{\rho-p}{\rho}$, is independent of n.

implies that firm's i's payoff for its best-response output is

$$\pi_i^s = \frac{n}{1+(n-1)\,\sigma}\frac{\rho}{\mu N}\left(q_i^s\right)^2 - F. \tag{66}$$

If the number of firms n^s takes the value that makes symmetric equilibrium profit equal to zero, n^s is determined by the cubic equation

$$\left[1+(n^s-1)\,\sigma\right]\frac{2S}{F} = n^s\left[2+(n^s-1)\,\sigma\right]^2. \tag{67}$$

Analysis of the analytic solution to (67) is uninformative. A simple argument, however, verifies the expected result, that $n^s < n^*$ (where n^* is given by Result 1). For a given number of firms,

$$\pi^* - \pi^s = \frac{(n-1)\,(1-\sigma)}{n}\frac{2S}{\left[2+(n-1)\,\sigma\right]^2} > 0. \tag{68}$$

Hence for the value of n that makes $\pi^s = 0$, firms in an otherwise identical market with unscaled demand earn positive profit.

4.3.4 Endogenous sunk cost

Second-stage equilibrium Model endogenous sunk cost above. If demand equations are scaled so the total number of units demanded at a constant price is independent of the number of varieties, firm i's second period objective function for the endogenous sunk-cost case is

$$\pi_i^{II} = \left\{\rho_i\,(1-c) - \frac{n}{1+(n-1)\,\sigma}\frac{1}{N}\left[\frac{\rho_i}{\mu_i}q_i^s + \sigma\sqrt{\frac{\rho_i}{\mu_i}}\sum_{j\neq i}^{n}\sqrt{\frac{\rho_j}{\mu_j}}q_j^s\right]\right\}q_i^s \tag{69}$$

The first-order condition to maximize (69), which can be written

$$(2-\sigma)\sqrt{\frac{\rho_i}{\mu_i}}q_i^s + \sigma\sum_{j=1}^{n}\sqrt{\frac{\rho_j}{\mu_j}}q_j^s = \frac{1+(n-1)\,\sigma}{n}(1-c)\,N\sqrt{\mu_i\rho_i} \tag{70}$$

implies that firm i's equilibrium second-stage payoff is

$$\pi_i^{II} = \frac{n}{1+(n-1)\,\sigma}\frac{1}{N}\frac{\rho_i}{\mu_i}\left(q_i^s\right)^2. \tag{71}$$

Equilibrium second-stage outputs are given by[21]

Result 4: For single product firms and aggregate inverse demand equations scaled as indicated above, variety i's equilibrium output is

$$q_i^s = \frac{1+(n-1)\sigma}{2+(n-1)\sigma}\frac{N}{n}\frac{1-c}{2-\sigma}\left\{[2+(n-2)\sigma]\mu_i - \sigma\sqrt{\frac{\mu_i}{\rho_i}}\sum_{j\neq i}^{n}\left(\sqrt{\mu_j\rho_j}\right)\right\}. \tag{73}$$

First-stage equilibrium It is shown in the Appendix that

Theorem 5: For single product firms and aggregate inverse demand equations scaled as indicated above,

(a) the equilibrium number of varieties is the same as for the Spence-Dixit-Vives specification;

(b) equilibrium values of ρ^s and μ^s satisfy

$$\left(\frac{\rho^s}{\rho}\right)^{\frac{(\gamma-1)(\beta-1)-1}{\gamma}} = \left(\frac{\mu^s}{\mu}\right)^{\frac{(\gamma-1)(\beta-1)-1}{\beta}} = 1-\left(1-\frac{1}{n}\right)(1-\sigma) < 1, \tag{74}$$

where ρ and μ are given by Theorem 3;

(c) and market size $S^s = \frac{1}{2}(1-c)^2\rho^s\mu^s N$ satisfies

$$\frac{S^s}{S} = \frac{\rho^s\mu^s}{\rho\mu} = \left[1-\left(1-\frac{1}{n}\right)(1-\sigma)\right]^{\frac{\beta+\gamma}{(\gamma-1)(\beta-1)-1}} < 1, \tag{75}$$

where S is given by Theorem 3.

It is for this reason that the heading of this section ("Constant Market Size") is in quotation marks. With endogenous sunk costs, market size is

[21] The system of first-order equations is

$$diag\sqrt{\frac{\rho_i}{\mu_i}}\left[(2-\sigma)I_n + \sigma J_n J_n'\right] diag\sqrt{\frac{\rho_j}{\mu_j}}\,\mathrm{col}_n\left(q_i^s\right) = \frac{1+(n-1)\sigma}{n}N(1-c)\,\mathrm{col}\,\rho_i. \tag{72}$$

Solution gives (73). Details are contained in an Appendix that is available on request from the author.

endogenous. By investing in product design and advertising, firms influence reservation prices and quantities demanded, and this holds whether or not the total number of units demanded, at a common price, varies with the number of varieties or not. With exogenous sunk cost, if the total number of units demanded at a common price is independent of the number of varieties, the result is a smaller equilibrium number of varieties, relative to the Spence-Dixit-Vives specification. With endogenous sunk cost, the long-run equilibrium number of varieties is the same with either specification, but equilibrium market size, maximum reservation prices, and quantities demanded per consumer are smaller for demand scaled in the Shubik-Levitan manner.

5 Conclusion

This paper makes three contributions. One is to show that the Spence-Dixit-Vives model is implied if supplies of substitutes reduce individual consumers' reservation prices as indicated in Section 3.2. The second is to show that for the micro-foundation-based version SDV demand explored here and endogenous sunk costs, the equilibrium number of varieties is independent of the number of consumers in the market and the marginal cost of a variety of unit quality. Instead, the equilibrium number of varieties depends on the way endogenous sunk costs vary with quality and individual quantity demanded, and with the way reservation prices are affected by the availability of substitute varieties. The third is to show that with endogenous sunk cost, if (in contrast to the SDV model) demand does not expand with the number of varieties, the equilibrium number of varieties is unchanged from the SDV model, but equilibrium qualities and quantities purchased are less, compared with the expandable demand case.

6 Appendix

6.1 Derivation of Result 2

The first-order condition to maximize firm i's second-stage payoff, (36) can be written

$$(2-\sigma)\frac{\rho_i}{\mu_i}q_i + \sigma\sqrt{\frac{\rho_i}{\mu_i}}\sum_{j=1}^{n}\sqrt{\frac{\rho_j}{\mu_j}}q_j \equiv (1-c)\,N\rho_i. \tag{76}$$

The system of first-order equations for all varieties is

$$diag_n\left(\sqrt{\frac{\rho_i}{\mu_i}}\right)\left[(2-\sigma)\,I_n + \sigma diag_n\left(\sqrt{\frac{\rho_i}{\mu_i}}\right)J_nJ_n'\right]diag_n\left(\sqrt{\frac{\rho_i}{\mu_i}}\right)\mathrm{col}_n\left(q_i\right) = (1-c)\,N\rho, \tag{77}$$

where J_n is a column vector of 1s. Taking advantage of the known inverse of the second matrix on the left,[22] equilibrium outputs satisfy

$$\mathrm{col}_n\left(q\right) = \left[\mathrm{col}_n\left(\mu_i\right) - \frac{\sigma}{2+(n-1)\,\sigma}\left(\sum_i \rho_i\mu_i\right)\mathrm{col}_n\left(\sqrt{\frac{\mu_i}{\rho_i}}\right)\right]\frac{1-c}{2-\sigma}N, \tag{79}$$

and for a single variety, this gives (38).

6.2 Derivation of Theorem 3

For notational compactness, write

$$K = \frac{1}{N}\left[\frac{(1-c)\,N}{(2-\sigma)\left[2+(n-1)\,\sigma\right]}\right]^2. \tag{80}$$

Then firm 1's first-stage payoff function is

$$\pi_1^I = K\frac{\rho_1}{\mu_1}\left[\left[2+(n-2)\,\sigma\right]\mu_1 - \sigma\sqrt{\frac{\mu_1}{\rho_1}}\sum_{i=2}^{n}\left(\sqrt{\rho_i\mu_i}\right)\right]^2 - \varepsilon\rho_1^{\beta} - \eta\mu_1^{\gamma}. \tag{81}$$

[22]
$$\left[(2-\sigma)\,I_n + \sigma J_nJ_n'\right]^{-1} = \frac{1}{2-\sigma}\left[I_n - \frac{\sigma}{2+(n-1)\,\sigma}J_nJ_n'\right]. \tag{78}$$

It is useful to note the partial derivatives

$$\frac{\partial}{\partial \rho_1}\left\{\rho_1\left[\left[2+(n-2)\,\sigma\right]\mu_1-\sigma\sqrt{\frac{\mu_1}{\rho_1}}\sum_{i=2}^{n}\left(\sqrt{\rho_i\mu_i}\right)\right]^2\right\}= \tag{82}$$

$$\left[2+(n-2)\,\sigma\right]\mu_1\left\{\left[2+(n-2)\,\sigma\right]\mu_1-\sigma\sqrt{\frac{\mu_1}{\rho_1}}\sum_{i=2}^{n}\left(\sqrt{\rho_i\mu_i}\right)\right\}$$

and

$$\frac{\partial}{\partial \mu_1}\left\{\frac{1}{\mu_1}\left[\left[2+(n-2)\,\sigma\right]\mu_1-\sigma\sqrt{\frac{\mu_1}{\rho_1}}\sum_{i=2}^{n}\left(\sqrt{\rho_i\mu_i}\right)\right]^2\right\}= \tag{83}$$

$$\frac{2+(n-2)\,\sigma}{\mu_1}\left\{\left[2+(n-2)\,\sigma\right]\mu_1-\sigma\sqrt{\frac{\mu_1}{\rho_1}}\sum_{i=2}^{n}\left(\sqrt{\rho_i\mu_i}\right)\right\}.$$

Using (82) and (83), the first-order conditions with respect to ρ_1 and $\mu_1$can be written

$$K\left[2+(n-2)\,\sigma\right]\left\{\left[2+(n-2)\,\sigma\right]\mu_1\rho_1-\sigma\sqrt{\mu_1\rho_1}\sum_{i=2}^{n}\left(\sqrt{\rho_i\mu_i}\right)\right\}\equiv\beta\varepsilon\rho_1^{\beta}. \tag{84}$$

and

$$K\rho_1\left[2+(n-2)\,\sigma\right]\left\{\left[2+(n-2)\,\sigma\right]\mu_1-\sigma\sqrt{\frac{\mu_1}{\rho_1}}\sum_{i=2}^{n}\left(\sqrt{\rho_i\mu_i}\right)\right\}\equiv\gamma\eta\mu_1^{\gamma}, \tag{85}$$

respectively.

In symmetric equilibrium, (84) and (85) imply

$$\varepsilon\rho^{\beta}=K\,(2-\sigma)\left[2+(n-2)\,\sigma\right]\frac{\mu\rho}{\beta} \tag{86}$$

and

$$\eta\mu^{\gamma}=K\,(2-\sigma)\left[2+(n-2)\,\sigma\right]\frac{\mu\rho}{\gamma}. \tag{87}$$

These can be solved for (41) and (42), which in turn imply (43).

Symmetric equilibrium profit per firm is

$$\pi=K\rho\mu\,(2-\sigma)^2-\varepsilon\rho^{\beta}-\eta\mu^{\gamma}. \tag{88}$$

Substituting (86) and (87) in (88) gives

$$\pi = K\rho\mu\,(2-\sigma)\left\{2-\sigma-[2+(n-2)\,\sigma]\left(\frac{1}{\beta}+\frac{1}{\gamma}\right)\right\}. \tag{89}$$

from which (44) follows. Other statements in the theorem follow substituting this expression for n^* in expressions in intermediate parts of the derivation, and simplifying the resulting expressions.

6.3 Derivation of Theorem 5

For notational compactness, write

$$K^s = \frac{1+(n-1)\,\sigma}{n}\left[\frac{1}{2+(n-1)\,\sigma}\right]^2 N\left(\frac{1-c}{2-\sigma}\right)^2. \tag{90}$$

Then firm 1's objective function has the form (81), substituting K^s for K. That the equilibrium number of firms is the same as implied by Theorem 3 follows from this. The equilibrium values of ρ^s, μ^s, S^s also follow; details are in an Appendix available on request from the author.

7 References

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