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Abstract

We examine conditions under which a platform firm can exclude rivals by bundling a product that some on one side of the market regard as essential with its platform, and pursue implications for market performance. We show that the impact of an exclusive dealing contract between the upstream firm and one of the downstream firms on market performance depends on the strength of consumer preferences for the products of the two downstream firms and the relative size of the market segment for which the complementary consumption good is essential. In some cases this may reduce the net social welfare.

Keywords: exclusion; essential components; exclusive contract; platform market.

JEL codes: L12, L13, L22, L42

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1 Introduction

We model a platform market in which a portion of agents on one side of the market require that a complementary good be supplied to be willing to transact on the platform. We show that if the fraction of such agents is sufficiently great, an exclusive dealing arrangement between one platform and the supplier of the complementary good can make rival platforms unprofitable, excluding them from the market, although they would not be excluded (and consumer welfare and net social welfare would improve, under some circumstances), absent the exclusive dealing arrangement. We also find conditions under which this exclusion can be welfare improving.

Our stylized model is exemplified by a real-world episode from the Dallas, Texas newspaper market.[1] The newspaper industry is a prototypical example of a platform market. It can be modelled as involving four sets of players: newspapers, readers, advertisers and press syndicates. Newspapers are platforms, with readers and advertisers as the two interacting groups. A newspaper commonly publishes features, articles, comics, puzzles, etc., along with news and advertisements. Newspaper employees prepare some published material; the remainder is purchased from press syndicates. Press syndicates are upstream firms that sell specialized material to newspapers; they are not news wire services. Some press syndicates specialize in the distribution of comic strips, acting as agents for cartoonists, often under exclusive territorial contracts.

On August 2,1989, the Dallas Morning News ('Morning News') signed an exclusive contract for 26 columns and comic strips provided by the Universal Press Syndicate, offerings that had until that time been available through the Dallas Times Herald ('Times Herald'). The two newspapers had competed for more than a century. Universal Syndicate conceded that it thought the move was 'predatory', but that the cancellations were required by its contract with the Morning News. The Times Herald suffered a circulation loss of 9,000 – 10,000 in weekday and 15,000 in Sunday circulation, and filed an antitrust lawsuit asking for $33 million in actual damages and up to three times of that amount in punitive damages against the Morning News and its parent company.

[1] See the Appeals Court decision in Times Herald Printing Co. *v.* A. H. Belo Corporation *et al.* (Court of Appeals of Texas, Fourteenth District) 820 S.W.2d 206; 1991 Tex. App. LEXIS 2899; 335-66 Trade Cas. (CCH) P69, 680 (1991). Also see Gelsanliter (1995).

A state judge in Texas refused to grant the Times Herald a preliminary injunction to prevent the movement of the syndicated features, on the ground that the Times Herald could be supplied with substitute syndicated features by competing syndicates. The Times Herald subsequently lost a District Court jury trial and an appeal of the District Court outcome. However, the Morning News paid $1.5 million to the Times Herald as part of an outside settlement. The Times Herald did not recapture its lost reader base and advertising revenue. The Morning News' parent corporation purchased the Times Herald on December 8, 1991 and stopped its publication the next day.[2]

A similar episode occurred in the U.S. television industry. Project Runway, a reality show based on fashion design, was shown by the Bravo Network from 2004 to 2008. On July 2006 the show's producers made an exclusive deal to move the show to Lifetime Television starting from 2009. Litigation followed, and was privately settled after Bravo Network prevailed in early stages. Bravo Network subsequently launched a competing program ("The Fashion Show"), which enjoyed about one-quarter Project Runway's number of viewers, and correspondingly less advertising revenue. The switch of Project Runway to a rival network has the potential to exclude the Bravo Network from the market.[3]

Some other examples of this setting are the "killer app" available only on a single platform; see Viecens (2009), who notes "tendency of firms which operate software platforms to integrate with so-called killer applications." An exclusive license, in this perspective, is a form of contractual integration.

We model a vertically-structured two-sided market. An upstream firm provides what for some consumers is an essential complementary consump-

[2]The rise of the internet has made print media a declining industry. Our stylized model is not meant to imply that the Morning News' exclusive arrangement with the United Press Syndicate was the unique factor responsible for the demise of the Times Herald. The general increase in concentration in the newspaper markets of US cities, and the corresponding reasons and consequences are discussed in Bucklin *et al.* (1989) and Genesove (2003). But the fact that the Times Herald's otherwise unsuccessful legal action resulted in a $1.5 million private settlement is consistent with the view that the exclusive arrangement was *one* factor in the demise of the Times Herald.

[3]See New York Supreme Court case 601011-2008, NBC Universal, Inc. and Bravo Media Llc. v. Weinstein Co. Llc., Defendant-Intervenor: Lifetime Entertainment Services; Huff, Richard "'Project Runway' quits Bravo for Lifetime," *NYDailyNews.com* 7 April 2008; Lafayette, Jon "NBCU wins round in 'Project Runway,'' *TVWeek.com*, 26 September 2008; Associated Press, "'Project Runway' is cleared for move to Lifetime from Bravo," 1 April 2009.

tion good[4] to two downstream firms that supply differentiated platforms to two sides of the final market. We show that the impact of an exclusive dealing contract between the upstream firm and one of the downstream firms on market performance depends on the strength of consumer preferences for the products of the two downstream firms and the relative size of the market segment for which the complementary consumption good is essential. We show that for strong preferences about the views expressed in a newspaper (which we model as "transportation cost" in a Hotelling framework), and a sufficiently large fraction of the population that regards comics as an essential component of a newspaper can exclude the unlicensed firm from the market. If readers' preferences are pronounced, an exclusive contract can also leave the market with a fraction of buyers who do not purchase any newspaper.[5]

In Section 2 we review the parts of the literature on two-sided markets most closely related to the present study. Section 3 contains the setup of the model, describing assumptions about readers, advertisers, and newspapers. In Section 4, we present results for the monopoly case. Section 5 contains the basic model. In Section 6 we extend the basic model of reader demand to the case in which some consumers regard syndicate-supplied material as an essential component of a newspaper and characterize equilibrium outcomes. Section 7 discusses equilibrium licensing behavior, and Section 8 examines the welfare consequences of an exclusionary exclusive license. Section 9 concludes. Proofs are given in the Appendix.

2 Related Literature

2.1 Two-sided Markets

Rochet and Tirole (2006) define a two-sided market as a special type of market in which there are two distinct user groups of a particular 'platform' (a product or a service) and the users benefit from the capacity of the platform to connect the two user groups. The platform is able to charge distinct

[4]The "essential component" aspect of features evolves through habit formation of newspaper readers. See Argentesi (2004).

[5]These results are obtained in a model of a market where there is a single supplier of an essential complementary good. Qualitatively similar results will obtain if there are multiple suppliers of vertically-differentiated varieties of the complementary good and one is perceived by consumers to be of markedly higher quality than the others. It is essential for exclusionary effect that consumers regard the complementary good as essential.

prices to the two user groups. Examples of this type of market include credit cards, travel agencies, video games, personal computer operating systems, and newspapers. There is a large theoretical literature on two-sided markets,[6] and here we limit our discussion to the parts of this literature that are directly related to our work.

There are several kinds of markets (newspapers, television channels, credit cards) in which platforms compete as oligopolists to supply the same to user groups. Rochet and Tirole (2003) introduce a general model of platform competition (closely related to the credit card market) and show how prices and end-user surpluses are determined. In a platform duopoly, end users have to decide whether to transact with only one or with both platforms. Since the decision of end users on one side of the market affects the incentives of end users on the other side of the market, end users face a tradeoff. We can use Rochet and Tirole's results to justify the assumption that if consumers "single home," reading at most one of all available newspapers, then advertisers will advertise on both newspapers.

Caillaud and Jullien (2003) deal with the chicken-egg problem[7] in an intermediate service market. They build a model of imperfect competition among intermediaries and analyze efficient allocations and pricing strategies. When users use only one of two intermediaries (the case of single homing), the efficient allocation is one where all users join the same intermediary. If users are allowed to join both intermediaries, all users are willing to join both intermediaries and the optimal pricing strategy of platforms is to charge a transaction fee rather than a registration fee.

The extensive literature that follows Rochet and Tirole (2003) analyzes different aspects of competition in platform markets. We adapt the basic model of Armstrong (2006) to analyze one type of exclusionary conduct in a two-sided market.

Armstrong (2006) develops three related models of two-sided markets: a monopoly platform, a model of single-homing and a model of competitive bottlenecks (similar to the multi-homing model of Caillaud and Jullien, 2003). In the competitive bottlenecks scenario, platforms compete for a group of single-homing users (Armstrong, 2006, p. 679):

[6] See Rochet and Tirole (2002) and Schmalensee (2002) for applications of models of two-sided markets to the credit card industry, and Rochet and Tirole (2006) and Rysman (2009) for comprehensive surveys of the literature.

[7] This is that failure to capture one side of the market results in losing the other side, regardless of the prices offered.

> Here, if it wishes to interact with an agent on the single-homing side, the multi-homing side has no choice but to deal with that agent's chosen platform. Thus, platforms have monopoly power over providing access to their single-homing customers for the multi-homing side.

Using a similar structure, Armstrong and Wright (2007) consider a model of two-sided markets where each side of the market has a different level of product differentiation. Asymmetric product differentiation, if exists, causes competitive bottlenecks in the market.

2.2 Exclusion in Two-Sided Markets

Church and Gandal (2004) argue that the direct denial of compatibility, and the restriction of the compatibility of complementary products, are exclusionary in the telecommunications industry. Nocke *et al.* (2007) show that exclusion can reduce welfare if platform effects are weak, but that if platform size is large, exclusion can improve welfare. Hagiu and Lee (2009), Weeds (2009) and Stennek (2007) discuss exclusionary effects of exclusive contracts between distributors and TV channels. They also argue that exclusive contracts may be welfare improving.

The case of an essential commodity in consumer demand is little discussed in the literature. To our knowledge, Hogendorn and Yuen (2009) is the paper most closely related to the present one. They analyze a situation in which a player in one side of the market provides an essential consumption good to the other side and, for that reason, enjoys a higher reservation price.[8]

The newspaper market is often analyzed with respect to media bias and partisanship.[9] However, some investigations suggest that it is characteristic

[8] In models of vertically-differentiated products, it is generally the case that higher-quality varieties have higher equilibrium market shares. Considering for simplicity the case of duopoly, if one variety is of drastically lower quality than another, the low-quality variety will have zero equilibrium market share. The central result of this paper is that exclusion can occur *without* such quality-difference effects. If we modify the model so that readers who regard comics as essential have higher reservation prices for a newspaper-comics bundle, the exclusionary effect of bundling would be reenforced.

[9] Ellman and Germano (2009) use a two-sided framework to analyze media bias in the newspaper industry. They show that a monopoly newspaper is prone to under-reporting news, to the detriment of its advertisers. We use the concept of media bias or partisanship as a Hotelling transportation cost to consumers.

of firms in this market to supply components to readers, along with the primary product. Dewenter (2003) shows that newspapers, among other media, can form consumer habits that translate into demand for a commodity that becomes an essential component of the media product. Argentesi (2004) shows empirically that weekly supplements (comics, puzzles, etc.) increase readership of and as a result advertisement in, newspapers. If a newspaper is denied the possibility of supplying habit-forming content, content that a portion of the population regards as essential, the newspaper will see its reader base decline.

3 Setup

The basic model is a specialized version of that of Armstrong (2006). There are two newspapers, A located at the left end and B located at the right end of a Hotelling line of length 1. Newspapers sell advertising space to advertisers and print copies of newspapers to readers. We normalize the mass of readers and the mass of advertisers to be one. n_R^i denotes the number of readers of newspaper i, and n_a^i denotes the number of firms that advertise in newspaper i.[10]

3.1 Readers

The net utility of a reader of newspaper i, before allowing for "transportation cost" t is

$$u_R^i = \alpha n_a^i - p^i. \tag{1}$$

We assume readers single-home. For a reader located at x on the Hotelling line, net utilities are

$$u_R^A - tx \tag{2}$$

from newspaper A,

$$u_R^B - t\left(1 - x\right), \tag{3}$$

from newspaper B.

[10] In what follows, unless otherwise noted, references to "newspaper i" should be understood to carry the qualification "for $i = A, B$."

Boundary readers are at a location the yields the same net utility from either newspaper, $u_R^A - tx^* = u_R^B - t(1 - x^*)$, yielding boundary location

$$x^* = \frac{1}{2} + \frac{u_R^A - u_R^B}{2t} = \frac{1}{2} + \frac{\alpha n_a^A - p^A - \left(\alpha n_a^B - p^B\right)}{2t}. \tag{4}$$

Each reader selects the newspaper that offers the greatest net utility, provided that net utility is nonnegative.

The number of readers of each newspaper are

$$n_R^A = \frac{1}{2} + \frac{\alpha\left(n_a^A - n_a^B\right) - p^A + p^B}{2t} \tag{5}$$

$$n_R^B = \frac{1}{2} + \frac{\alpha\left(n_a^B - n_a^A\right) - p^B + p^A}{2t}. \tag{6}$$

3.2 Advertisers

Let γ^i denote newspaper i's per-reader advertising rate.[11] The cost of placing an ad in newspaper i is

$$\gamma^i n_R^i. \tag{7}$$

Advertisers differ in their profit per sale, β. Following Armstrong (2006), we assume that newspapers do not observe the β of any particular advertiser, but know the distribution of β in the population of advertisers. We assume β is uniformly distributed over $0 \leq \beta \leq 1$.

It will be profitable for an advertiser to place an ad in newspaper i if the profit from placing the ad is greater than or equal to the cost of placing the ad, $\beta n_R^i \geq \gamma^i n_R^i$. The number of ads demanded from newspaper i is therefore

$$n_a^i = 1 - \gamma^i. \tag{8}$$

Substituting (8) in (5) and (6), the number of readers per newspaper become

$$n_R^A = \frac{1}{2t}\left[t + \alpha\left(\gamma^B - \gamma^A\right) - p^A + p^B\right] \tag{9}$$

and

$$n_R^B = \frac{1}{2t}\left[t + \alpha\left(\gamma^A - \gamma^B\right) - p^B + p^A\right], \tag{10}$$

[11] See Rosse (1970) for an estimation of advertising cost in newspaper and Armstrong (2006) for discussion of the case in which the price of placing an advertisement is not proportional to the number of readers.

respectively.

Advertisers with $\beta \geq \gamma^A$ make profit $\beta - \gamma^A$ on each of the n_R^A sales they make to readers of platform A. Advertisers' profits on sales to readers of platform A are[12]

$$n_R^A \int_{\beta=\gamma^A}^{\beta=1} \left(\beta - \gamma^A\right) d\beta = \frac{1}{2} n_R^A \left(1-\gamma^A\right)^2. \tag{11}$$

In the same way, profit on firms' advertisements in platform B are

$$\frac{1}{2} n_R^B \left(1-\gamma^B\right)^2. \tag{12}$$

Advertisers' total profits are

$$\frac{1}{2} n_R^A \left(1-\gamma^A\right)^2 + \frac{1}{2} n_R^B \left(1-\gamma^B\right)^2. \tag{13}$$

3.3 Platforms

Newspapers have a constant marginal cost c to produce a newspaper with n_a advertisements, and fixed cost F.[13] Firm i's payoff function is

$$\pi^i = n_R^i p^i + \gamma^i n_R^i n_a^i - c n_R^i n_a^i - F = n_R^i \left[p^i + \left(\gamma^i - c\right)\left(1-\gamma^i\right)\right] - F. \tag{14}$$

Let

$$\pi_R^i = p^i + \left(\gamma^i - c\right)\left(1-\gamma^i\right) \tag{15}$$

denote newspaper i's profit per reader — p^i on the sale of the newspaper to the reader, $\gamma^i - c$ profit per reader per advertisement placed, and $n_a^i = 1-\gamma^i$ advertisements placed.

Firm i's profit maximization problem is

$$\max_{p^i,\gamma^i} n_R^i \pi_R^i - F. \tag{16}$$

[12] (11) can more simply be derived as n_R^A times the area of a triangle with base $1-\gamma^A$, the mass of firms that advertise, and height $1-\gamma^A$, the profit of firms with the highest β.

[13] The fixed cost of gathering news to produce the first copy of the paper is typically high, the variable cost to print and sell additional copies of newspaper lower. See Rosse (1970) and Strömberg (2004) for estimation and interpretation of cost structures in the newspaper market.

Since β is uniformly distributed on $(0, 1)$, the price per reader of an advertisement cannot be greater than 1. Otherwise no advertisements would be demanded. In principle, in a platform market, the price-per-reader of an advertisement could be negative. We will assume that prices to advertisers and prices to readers are nonnegative. This gives us

$$0 \leq \gamma^i \leq 1. \tag{17}$$

A second constraint appears in the duopoly version of the model. The usual Hotelling boundary condition ensures that consumers at the boundary location get identical net utility from either newspaper. An additional requirement, if readers at the boundary location are to be served, is that this net utility be nonnegative,

$$\alpha n_a^i - p^i - tx^* \geq 0. \tag{18}$$

Substitute (4) to eliminate x^* and rearrange terms to obtain an expression for the market-coverage constraint,

$$2\alpha - t \geq \alpha \left(\gamma^A + \gamma^B\right) + p^A + p^B, \tag{19}$$

with choice variables on the right, parameters on the left.

It would be possible to analyze scenarios in which the center of the market is not served in duopoly equilibrium. But we confine our attention to the contrary case.

3.4 Syndicate and timing

We model the incentive of a syndicate to offer an exclusive license and the incentive of a newspaper to accept a license, exclusive or not, if offered. The four stages of the game are shown in Figure 1.

We model the syndicate's costs as being entirely sunk before it interacts with newspapers.[14] In stage I, the syndicate offers a license to publish the complementary material to either one (without loss of generality, to firm A) or both newspapers. In stage II, if a newspaper is offered a license, it decides

[14] It would be possible to model the syndicate's arrangements with the authors of the material it markets; this would take us far afield from our topic. We take it that whatever the arrangements of a syndicate with its own providers, their interests are served if the syndicate acts to maximize its profit, given its bargaining power vis-à-vis newspapers.

I →	II →	III →	IV
Syndicate: offer one or two licenses	Platforms: accept, reject and remain, or reject and exit; set prices	Advertisers: place ads	Readers: select newspaper, if any

Figure 1: Sequence of decisions.

among three options: accept the license, reject the license and remain in the market, or reject the license and exit the market. If a newspaper is not offered a license, it decides to remain in the market or to exit. Newspapers that remain in the market set advertising rates and newspaper prices.

4 A Monopoly Platform

Suppose there is only one platform, firm A. If firm A is a monopoly supplier, the net utility of a reader located at x is

$$u_R^A = \alpha\left(1-\gamma^A\right) - p^A - tx. \tag{20}$$

If price is low, the market is covered. If price is high, readers far from newspaper A in preference space are not served under monopoly. The number of readers is

$$n_R^A = \begin{cases} 1 & p^A \le \alpha\left(1-\gamma^A\right) - t \\ \frac{\alpha\left(1-\gamma^A\right)-p^A}{t} & p^A \ge \alpha\left(1-\gamma^A\right) - t \end{cases}. \tag{21}$$

4.1 Low p^A

In the low-price case, firm A's problem is

$$\max_{p^A,\gamma^A} (1)\left[p^A + \left(\gamma^A - c\right)\left(1-\gamma^A\right)\right] - F \tag{22}$$

such that

$$p^A \le \alpha\left(1-\gamma^A\right) - t. \tag{23}$$

As shown in the Appendix, A's problem can be analyzed formally using Lagrangian methods. But intuitively, for firm A to maximize profit in the low-price case, the constraint must be binding for the most distant reader,

$$p^A = \alpha \left(1 - \gamma^A\right) - t. \tag{24}$$

It cannot be optimal for firm A to leave the most distant consumer with any surplus.

Given (24), firm A's problem can be reformulated as

$$\max_{\gamma^A} \left(1 - \gamma^A\right) \left(\alpha + \gamma^A - c\right) - t - F. \tag{25}$$

The first-order condition to solve (25) is

$$\left(1 - \gamma^A\right) - \left(\alpha + \gamma^A - c\right) \equiv 0. \tag{26}$$

A marginal increase in γ^A reduces the number of advertisements sold, $1 - \gamma^A$. A marginal increase in γ^A increases profit per advertisement, $\alpha + \gamma^A - c$. Part of the change in profit per advertisement is the decrease in the price readers pay, (24). Part of the increase in profit per advertisement is the increase in profit from sales to advertisers, $\gamma^A - c$.

From (26),

$$\gamma^A = \frac{1}{2}\left[1 - (\alpha - c)\right] \equiv \gamma^*. \tag{27}$$

γ^* is the equilibrium price-per-reader of an advertisement not only for the case of a monopoly platform, but in all the models considered in this paper. This is the "competitive bottleneck" aspect of the basic model: depending on the details (monopoly, duopoly, essential component), a platform's equilibrium number of readers will vary. But it is a monopolist with respect to those readers' access to advertisements.

4.2 High p^A

In the high-price regime, firm A's problem is

$$\max_{p^A,\gamma^A} n_R^A \pi_R^A - F = \frac{\alpha\left(1 - \gamma^A\right) - p^A}{t}\left[p^A + \left(\gamma^A - c\right)\left(1 - \gamma^A\right)\right] - F, \tag{28}$$

such that $p^A \geq \alpha\left(1 - \gamma^A\right) - t$. In the appendix, we solve the problem without imposing the constraint, then examine conditions for the solution to satisfy

	$t \leq \frac{1}{2}z^2$	$\frac{1}{2}z^2 \leq t \leq \frac{2}{3}z^2$
Licensed	$\pi^m_{l1} = z^2 - t - F$	$\pi^m_{l2} = \frac{1}{4t}z^4 - F$
Unlicensed	$\pi^m_{nl1} = (1-\mu)(z^2 - t) - F$	$\pi^m_{nl2} = \frac{1-\mu}{4t}z^4 - F$

Table 1: Monopoly payoffs.

Licensed	$\pi^d_{ll} = \frac{1}{2}t - F$
Unlicensed	$\pi^d_{nlnl} = \frac{1-\mu}{2}t - F$

Table 2: Duopoly payoffs.

the constraint. The consistency condition for the high-price solution to be valid is that transportation cost be sufficiently great,

$$t \geq \frac{1}{2}z^2, \tag{29}$$

where we write

$$z = 1 - \gamma^* \tag{30}$$

for notational compactness.

4.3 Monopoly Payoffs

For low levels of transportation cost, $t \leq \frac{1}{2}z^2$, a monopoly supplier sets price so the market is covered, extracting all surplus from the most distant readers. For higher levels of transportation cost, the market is not covered. Row 1 of Table 1 gives the equilibrium payoff of a monopolist newspaper if all readers are in the market.

5 Newspaper Duopoly

Our duopoly results, derived in the Appendix, are valid for $t \leq \frac{2}{3}z^2$, for which values of t the market is covered in duopoly equilibrium (see equation (121)). Duopoly payoffs if all readers are in the market are given in row 1 of Table 2.

6 Essential Component

6.1 Readers' Demands

We now assume that the specification of readers' demand in the base model describes the preferences of a fraction $1-\mu$ of the population, for $0 \leq \mu \leq 1$.

Then quantities demanded of each newspaper from this part of the population are

$$(1-\mu)\left[\frac{1}{2}+\frac{\alpha\left(n_a^A-n_a^B\right)-p^A+p^B}{2t}\right] \tag{31}$$

and

$$(1-\mu)\left[\frac{1}{2}+\frac{\alpha\left(n_a^B-n_a^A\right)-p^B+p^A}{2t}\right] \tag{32}$$

from platforms A and B, respectively.

We assume that the remaining portion μ of the population will read only a newspaper that publishes comics. One can interpret μ as the portion of consumers who will read only a certain comic strip and will not accept substitutes.[15] Otherwise, the utility of this group of consumers is as above. That is, for a consumer who regards comics as an essential component of a newspaper, comics yield no utility in and of themselves, but are a prerequisite for getting utility from a newspaper. A consumer who regards comics as an essential component of a newspaper purchases a newspaper only if it contains comics and if the net utility from reader the newspaper, allowing for transportation cost, is nonnegative.

Row 2 of Table 1 gives the equilibrium payoffs of an unlicensed monopolist. Row 2 of Table 2 gives equilibrium payoffs in a market supplied by two unlicensed firms. From (15), the reduction in profit of an unlicensed firm includes lost advertising revenue, a kind of loss specific to a firm that supplies a platform market.

Taking up the case of duopoly with one firm licensed and one firm unlicensed, suppose newspaper A has a license to publish comics. The most distant reader from the "comics" group who reads newspaper A is

[15] This phenomenon occurs in other industries. For example, in the 'Project Runway"case discussed in the first section, 75% of viewers chose to leave Bravo Network to watch the "killer app" show, although Bravo telecast a substitute show ('The Fashion Show').

(a) at the right end of the line if (recall the length of the line is 1)

$$u_R^A - t\,(1) = \alpha n_a^A - p^A - t \geq 0, \tag{33}$$

or equivalently if p^A is sufficiently low,

$$p^A \leq \alpha n_a^A - t, \tag{34}$$

(b) at distance $x_\mu \leq 1$ that makes net utility zero,

$$u_R^A - tx = \alpha n_a^A - p^A - tx_\mu = 0,$$

$$x_\mu = \frac{\alpha n_a^A - p^A}{t}, \tag{35}$$

if

$$p^A > \alpha n_a^A - t. \tag{36}$$

The number of A readers from the comics group is

$$\begin{array}{cc} \mu & p^A \leq \alpha n_a^A - t \\ \mu x_\mu & p^A \geq \alpha n_a^A - t \end{array}. \tag{37}$$

Quantities demanded of the two newspapers are

$$n_R^A = (1-\mu)\left(\frac{1}{2} + \frac{u_R^A - u_R^B}{2t}\right) + \left\{ \begin{array}{cc} \mu & p^A \leq \alpha n_a^A - t \\ \mu \frac{\alpha n_a^A - p^A}{t} & p^A \geq \alpha n_a^A - t \end{array} \right. . \tag{38}$$

$$n_R^B = (1-\mu)\left(\frac{1}{2} + \frac{u_R^B - u_R^A}{2t}\right). \tag{39}$$

If the utilities are expressed in terms of prices, quantities demanded are

$$n_R^A = (1-\mu)\left[\frac{1}{2} + \frac{\alpha\left(\gamma^B - \gamma^A\right) - p^A + p^B}{2t}\right]$$

$$+ \left\{ \begin{array}{cc} \mu & p^A \leq \alpha\left(1-\gamma^A\right) - t \\ \mu \frac{\alpha\left(1-\gamma^A\right) - p^A}{t} & p^A \geq \alpha\left(1-\gamma^A\right) - t \end{array} \right. . \tag{40}$$

$$n_R^B = (1-\mu)\left[\frac{1}{2} + \frac{\alpha\left(\gamma^A - \gamma^B\right) - p^B + p^A}{2t}\right]. \tag{41}$$

Equilibrium differs depending on whether transportation cost is low ($t \leq \frac{1}{2}z^2$) or high ($t \geq \frac{1}{2}z^2$). Further, in the low-transportation cost regime, equilibrium differs depending on whether μ is less or greater than

$$\mu^* = \frac{3z^2 - 6t}{3z^2 - 2t}. \tag{42}$$

	$\mu \le \mu^*$	$\mu \ge \mu^*$
A (licensed)	$\pi_{l1}^{Ad} = \frac{(3+\mu)^2}{1-\mu}\frac{t}{18} - F$	$\pi_{l2}^{Ad} = \left[1-(1-\mu)\frac{z^2}{4t}\right](z^2-t) - F$
B (unlicensed)	$\pi_{nl1}^{Bd} = \frac{(3-\mu)^2}{1-\mu}\frac{t}{18} - F$	$\pi_{nl2}^{Bd} = \frac{1-\mu}{8t} z^4 - F$

Table 3: Essential component model payoffs, low transportation cost.

A (licensed)	$\pi_{l3}^{Ad} = \frac{1+\mu}{2t}\left[\frac{3(1-\mu)t+4\mu z^2}{3+5\mu}\right]^2 - F$
B (unlicensed)	$\pi_{nl3}^{Bd} = \frac{1-\mu}{2t}\left[\frac{(3+\mu)t+2\mu z^2}{3+5\mu}\right]^2 - F$

Table 4: Essential component model payoffs, high transportation cost.

6.2 Payoffs

Payoffs for the low-t and high-t cases are given in Tables 3 and 4, respectively. Payoffs for the case that firm B is licensed and firm A unlicensed are symmetric with the payoffs shown in the tables.

7 Exclusion

A short argument (Section 7.1) shows that an exclusive license is not exclusionary for the low-t, low-μ case. For the low-t, high-μ and high-t cases, we examine equilibrium payoffs in two cases, first that the syndicate offers a license to one firm (without loss of generality, firm A), and second that the syndicate offers a license to both firms.

7.1 Low t, low μ

Subtraction shows that the payoff of an unlicensed duopolist that competes with a licensed rival is, for the low-t, low-μ case, greater than duopoly profit if both firms are licensed,

$$\frac{(3-\mu)^2}{1-\mu}\frac{t}{18} - F - \left(\frac{1}{2}t - F\right) = \frac{t}{18}\frac{\mu(3+\mu)}{1-\mu} > 0. \tag{43}$$

Thus

$$\pi_{nl1}^{Bd} = \frac{(3-\mu)^2}{1-\mu}\frac{t}{18} - F > \frac{1}{2}t - F = \pi_{ll}^{d}. \tag{44}$$

We assume that duopoly is profitable if both firms have licenses, $\pi_{ll}^{d} > 0$. This implies $\pi_{nl1}^{Bd} > 0$. Then if firm A operates with an exclusive license, firm B will operate, profitably, without a license. An exclusive license is not exclusionary if consumers regard the two newspapers as close substitutes (low t) and few consumers regard comics as essential (low μ).

7.2 Low t, high μ and high t

Theorem 1 *In the low-t, high-μ and high-t cases, for μ sufficiently close to 1, and in the high-t case for $F \geq \frac{7}{24}z^2$, it is a subgame perfect equilibrium for the syndicate to offer an exclusive license to firm A for a license fee slightly greater than $2\pi_{ll}^{d}$, for firm A to accept the offer, and for firm B to exit the market.*

7.2.1 Payoffs

Here we present the argument for the low-t, high-μ case. Minor changes in the first part of the argument, which are given in the Appendix, lead to the same result for the high-t case.

The inequalities

$$\max\left(\pi_{nlnl}^{d}, \pi_{nl1}^{m}, \pi_{nl2}^{Bd}\right) < 0 \leq \min\left(\pi_{ll}^{d}, \pi_{l1}^{m}, \pi_{l2}^{Ad}\right) \tag{45}$$

correspond to

$$\max\left[\frac{1-\mu}{2}t, (1-\mu)\left(z^2-t\right), \frac{1-\mu}{8t}z^4\right] < F \leq$$
$$\min\left\{\frac{1}{2}t, z^2-t, \left[1-(1-\mu)\frac{z^2}{4t}\right]\left(z^2-t\right)\right\}. \tag{46}$$

As $\mu \rightarrow 1$, (46) approaches

$$\max\left(0,0,0\right) = 0 < F \leq \min\left(\frac{1}{2}t, z^2-t, z^2-t\right). \tag{47}$$

Considering the expression on the right,

$$z^2 - t - \frac{1}{2}t = \frac{3}{2}\left(\frac{2}{3}z^2 - t\right) > 0. \tag{48}$$

Hence as $\mu \to 1$ (47) reduces to

$$0 < F \leq \frac{1}{2}t, \tag{49}$$

and the assumption that licensed duopoly is profitable[16] guarantees that (49) is satisfied. Assume μ is large enough so (45) holds. Then it is profitable to be a licensed monopolist ($\pi_{l1}^m > 0$) or duopolist($\pi_{ll}^d > 0$, $\pi_{l2}^{Ad} > 0$), unprofitable to be an unlicensed monopolist ($\pi_{nl1}^m < 0$) or duopolist ($\pi_{nlnl}^d < 0$, $\pi_{nl2}^{Bd} < 0$).

7.2.2 Exclusive license

Let the syndicate offer A an exclusive contract for a license fee that leaves A a positive payoff. A's options are to reject the contract and exit the market (breaking even), refuse the contract and remain in the market, or accept the contract. If A rejects the contract and continues in the market without a license, B's options are to exit or to continue in the market. If B exits, firm A is an unlicensed monopolist, earning $\pi_{nl1}^m < 0$. If B continues in the market, both firms earn $\pi_{nlnl}^d < 0$. If A accepts the contract, B's options are to exit (breaking even) or to compete as an unlicensed duopolist (earning $\pi_{nl2}^{Bd} < 0$); firm B's payoff-maximizing choice is to exit. If firm B exits, economic profit from the operation of newspaper A is $\pi_{l1}^m > 0$. As the license fee (discussed further below) leaves A with a positive payoff, accepting the offer of a license dominates A's alternative choices. If the syndicate offers firm A an exclusive license, the equilibrium outcome is that A accepts the offer, B exits, newspaper A generates monopoly profit π_{l1}^m, and the license fee determines the division of π_{l1}^m between A and the syndicate.

7.2.3 Dual licenses

We expect that in a market with two suppliers, each would learn the terms of the license offered to the other.[17] Let the syndicate simultaneously and publicly offer licenses to A and B for a license fee that leaves each firm at least small positive payoff if both firms accept the offer of a license. If A rejects the license and exits, B earns a positive profit (approximately $\pi_{l1}^m - \pi_{ll}^d$) if it

[16] If licensed monopoly is profitable, $\pi_{l1}^m \geq 0$, and licensed duopoly not profitable, $\pi_{ll}^d < 0$, there is one newspaper in equilibrium. But exclusion is not a factor.

[17] Hart and Tirole (1990) examine the different implications of public as opposed to private vertical contracts.

accepts the license, which dominates the losses it would make as an unlicensed monopolist or breaking even if it exits the market. If A rejects the license and continues in the market, B makes a positive profit (given the symmetry of payoffs, approximately $\pi_{l2}^{Ad} - \pi_{ll}^{d}$) if it accepts the license, which dominates the losses it would make as an unlicensed duopolist or breaking even if it exits the market. If A accepts the license, and B accepts the license as well, B makes a small positive profit, which dominates the losses it would make ($\pi_{nl2}^{Bd} < 0$) competing without a license against a licensed firm A or breaking even if it exits. No matter how firm A responds to the offer of a license, firm B maximizes its payoff by accepting the offer of a license. Firm A's incentives are the same. If the syndicate offers both firms licenses on terms that leave them small positive payoffs if both accept, the equilibrium outcome is for both firms to accept the offer.

7.2.4 Syndicate's payoff and overall outcome

The economic profit generated by newspaper A as a licensed monopolist is $\pi_{l1}^{m} = z^2 - t - F$. The economic profit generated by either newspaper if both firms have licenses is $\pi_{ll}^{d} = \frac{1}{2}t - F$. Monopoly profit exceeds total duopoly profit,

$$\pi_{l1}^{m} - 2\pi_{ll}^{d} = z^2 - t - F - 2\left(\frac{1}{2}t - F\right) = 2\left(\frac{1}{2}z^2 - t\right) + F > 0. \tag{50}$$

(recall that $t \leq \frac{1}{2}z^2$ in the low-t case).

If the syndicate makes public offers of licenses to both newspapers, asking a license fee slightly less than π_{ll}^{d}, the best alternative for either newspaper is to accept the offer of a license. Neglecting the small reductions in the license fees, the syndicate's payoff would be $2\pi_{ll}^{d}$. Then if the syndicate offers an exclusive license to (say) firm A, firm A could offer to pay the syndicate a license fee slightly greater than $2\pi_{ll}^{d}$, leaving the syndicate strictly better off than if it were to license both firms. Firm A's payoff, slightly less than $\pi_{l1}^{Am} - 2\pi_{ll}^{d} > 0$, would dominate its near-zero payoff as one of two licensed duopolists.[18]

[18] The mechanism at work here is essentially the same as that underlying "pay for delay" settlements between patented and generic drug manufacturers in the pharmaceutical industry. See, for example, Hemphill (2006).

	Monopoly		Duopoly
	low-t, high-μ	high-t	
Newspapers' profit	$z^2 - t - F$	$\frac{1}{4t}z^4 - F$	$2\left(\frac{1}{2}t - F\right)$
CS	$\frac{1}{2}t$	$\frac{1}{8t}z^4$	$z^2 - \frac{5}{4}t$
Advertisers' profit	$\frac{1}{2}z^2$	$\frac{1}{4t}z^4$	$\frac{1}{2}z^2$
NSW	$\frac{3}{2}z^2 - \frac{1}{2}t - F$	$\frac{5}{8t}z^4 - F$	$\frac{3}{2}z^2 - \frac{1}{4}t - 2F$

Table 5: Consumer Surplus and Net Social Welfare.

8 Welfare Consequences

We show in the Appendix that profit, consumer surplus, and net social welfare in the various regimes are as reported in Table 5. The "newspapers' profit" given in the first row of the table is the total profit generated by the operation of active newspapers. The license fee determines the division of this surplus between newspaper and syndicate, but does not affect the amount of the surplus.

8.1 Comparison: duopoly and low-t, high-μ monopoly

Comparing duopoly and low-t, high-μ monopoly gives

$$\pi_{l1}^m - 2\pi_{ll}^d = 2\left(\frac{1}{2}z^2 - t\right) + F > 0. \tag{51}$$

$$CS^d - CS_{ltl\mu}^m = \frac{7}{4}\left(\frac{4}{7}z^2 - t\right) > 0. \tag{52}$$

Advertisers' profit is the same under both regimes, since the market is covered in both cases. Thus monopoly profit is greater than total duopoly profit, and duopoly consumer surplus is greater than monopoly consumer surplus, in the low-t, high-μ case.

Duopoly net social welfare may be greater or less than monopoly net social welfare.

$$NSW^d - NSW_{ltl\mu}^m = \frac{1}{4}t - F. \tag{53}$$

We have assumed that licensed duopoly is profitable for both firms, $\frac{1}{2}t - F \geq 0$. (53) is thus of ambiguous sign. If reader preferences are strong (large t) and fixed cost low, duopoly net social welfare exceeds monopoly net social welfare. If reader preferences are weak and fixed cost high, monopoly net social welfare (which economizes on fixed cost, relative to duopoly) exceeds duopoly net social welfare. This result is comparable with the findings of Weeds (2009) and other exclusivity analyses in TV market.

8.2 Comparison: duopoly and high-t monopoly

Monopoly profit exceeds the profit of one duopolist. For duopoly and high-t monopoly, we have

$$\pi_{l2}^{m} - 2\pi_{ll}^{d} = \frac{1}{t}\left(\frac{1}{2}z^2 - t\right)\left(\frac{1}{2}z^2 + t\right) + F. \tag{54}$$

In the high-t case $\frac{1}{2}z^2 \leq t \leq \frac{2}{3}z^2$, for which values of t the first term on the right is nonpositive. $\pi_{l2}^{m} - 2\pi_{ll}^{d} = F > 0$ for $t = \frac{1}{2}z^2$. As t rises from $\frac{1}{2}z^2$ to $\frac{2}{3}z^2$, the first term on the right falls from 0 to $-\frac{7}{24}z^2$.[19] If $F \geq \frac{7}{24}z^2$, $\pi_{l2}^{m} \geq 2\pi_{ll}^{d}$ for all values of t admissible in the high-t case. For F in the range $0 \leq F < \frac{7}{24}z^2$, $\pi_{l2}^{m} - 2\pi_{ll}^{d}$ is positive, zero, or negative as t is less than, equal to, or greater than $\frac{1}{2}F + \frac{1}{2}\sqrt{F^2 + z^4}$.

Consumer surplus,

$$CS^{d} - CSh_{ht}^{m} = \frac{5}{4}\left[\left(\frac{2}{3}z^2 - t\right) + \frac{2}{15t}z^2\left(t - \frac{1}{10}z^2\right)\right] > 0. \tag{56}$$

and advertisers' profit,

$$\pi_{Ad}^{m} - 2\pi_{Ad}^{d} = \frac{1}{2t}z^2\left(t - \frac{1}{2}z^2\right) > 0. \tag{57}$$

are both greater under duopoly than under high-t monopoly.

The difference in net social welfare,

$$NSW^{d} - NSW_{ht}^{m} = \frac{1}{4}\left(\frac{2}{3}z^2 - t\right) + \frac{4}{3t}z^2\left(t - \frac{15}{32}z^2\right) - F, \tag{58}$$

is of ambiguous sign (the first two expressions on the right are positive). It is sufficient for duopoly net social welfare to exceed monopoly net social

[19] That is,

$$\frac{\partial}{\partial t}\left(\pi_{l2}^{m} - 2\pi_{ll}^{d}\right) = -\left[1 + \frac{1}{4}\left(\frac{z^2}{t}\right)^2\right] < 0. \tag{55}$$

welfare that monopoly newspaper profit be less than duopoly newspaper profit. Generally, the right-hand side of (58) is more likely to be positive the smaller is fixed cost and the stronger[20] are reader preferences.

9 Conclusion

In this article we build a model of a two-sided market with an essential complementary commodity. A fraction of agents on one side of the market view the complementary good as essential for transacting on the platform. If a platform makes an exclusive deal with the supplier of the essential commodity, then (depending on the proportion of the agents who view the commodity as essential and the transportation cost (preferences) of agents), it is possible that the rival platform is excluded from the market. This may result in a loss in consumer surplus and net social welfare. We also show that under certain conditions (weak consumer preferences between newspapers, few consumers who regard the complementary good as essential), an exclusive deal might not result in exclusion, and increases the profit of both platforms.

The literature on one-sided markets suggests (for example, Whinston (1990)) that tying, bundling, and exclusive contracts may, but need not, have exclusionary effect. Our results extend this finding to two-sided markets. Many regional markets — regional in physical space, regional in product characteristic space — will support at most a small number of firms. In such markets, an exclusive contract for a complementary product that a sufficiently large number of consumers view as essential can make unlicensed firms unprofitable, inducing exit, reducing consumer surplus and, in some cases (strong reader preferences, low fixed cost) reducing net social welfare.

There are several possible directions in which this analysis might be extended. One would be to explicitly model multiple syndicates and competition between syndicates. It would be possible to endogenize the location of platforms in preference space, and examine the impact of interactions with the essential commodity on location choice. Finally, rather than treating the proportion of agents who view a commodity as essential as exogenous, it would be possible to model consumers habit formation through marketing efforts or investment in quality.

[20] $\frac{\partial}{\partial t}\left(NSW^d - NSW^m_{ht}\right) = -\frac{5}{2t^2}\left(\frac{1}{\sqrt{10}}t - \frac{1}{2}z^2\right)\left(\frac{1}{\sqrt{10}}t + \frac{1}{2}z^2\right) > 0$, since in the high-$t$ case $\frac{1}{\sqrt{10}}t - \frac{1}{2}z^2 \leq \left(\frac{2}{3}\frac{1}{\sqrt{10}} - \frac{1}{2}\right)z^2 \approx -0.28918z^2 < 0$.

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11 Appendix

In Section 11.1 we derive payoffs under the various market regimes considered in the paper. In Section 11.2 we derive expressions for consumer surplus and net social welfare for the licensed-monopoly and licensed-duopoly regimes. In Section 11.3 we give steps in the proof of Theorem 1 for the high-t case.

11.1 Payoffs

11.1.1 Licensed Monopoly

Suppose there is only one platform, firm A. If firm A is a monopoly supplier, its objective function is

$$n_R^A \pi_R^A - F. \tag{59}$$

Profit per reader is

$$\pi_R^A = p^A + \left(\gamma^A - c\right)\left(1 - \gamma^A\right). \tag{60}$$

The number of advertisements is

$$n_a^A = 1 - \gamma^A. \tag{61}$$

Net utility of a reader located at x is

$$u_R^A = \alpha\left(1 - \gamma^A\right) - p^A - tx. \tag{62}$$

If the firm A has a license, the number of readers is

$$n_R^A = 1 \tag{63}$$

if

$$\alpha\left(1 - \gamma^A\right) - p^A - t \geq 0 \tag{64}$$

or equivalently

$$p^A \leq \alpha\left(1 - \gamma^A\right) - t \tag{65}$$

and

$$x = \frac{\alpha\left(1 - \gamma^A\right) - p^A}{t} \tag{66}$$

if

$$p^A \geq \alpha\left(1 - \gamma^A\right) - t. \tag{67}$$

This gives firm A's licensed monopoly number of readers, (21),

$$n_R^A = \begin{cases} 1 & p^A \leq \alpha\left(1-\gamma^A\right)-t \\ \frac{\alpha\left(1-\gamma^A\right)-p^A}{t} & p^A \geq \alpha\left(1-\gamma^A\right)-t \end{cases}. \tag{68}$$

Consider the low-price and high-price regimes in turn.

$p^A \leq \alpha\left(1-\gamma^A\right)-t$ If $p^A \leq \alpha\left(1-\gamma^A\right)-t$, firm A's problem is

$$\max_{p^A,\gamma^A} (1)\left[p^A+\left(\gamma^A-c\right)\left(1-\gamma^A\right)\right]-F \text{ s.t. } p^A \leq \alpha\left(1-\gamma^A\right)-t. \tag{69}$$

Set up (69) as a constrained optimization problem. A Lagrangian is

$$\mathcal{L} = p^A+\left(\gamma^A-c\right)\left(1-\gamma^A\right)-F+\lambda\left[\alpha\left(1-\gamma^A\right)-t-p^A\right]. \tag{70}$$

Kuhn-Tucker first-order conditions are

p^A:

$$\frac{\partial\mathcal{L}}{\partial p^A} = 1-\lambda = 0. \tag{71}$$

γ^A:

$$\left(\gamma^A-c\right)(-1)+(1)\left(1-\gamma^A\right)-\lambda\alpha = 0 \tag{72}$$

Substituting $\lambda = 1$ and rearranging terms gives

$$\gamma^A = \frac{1}{2}\left[1-(\alpha-c)\right] = \gamma^*. \tag{73}$$

λ:

$$\frac{\partial\mathcal{L}}{\partial\lambda} = \alpha\left(1-\gamma^A\right)-t-p^A \geq 0 \tag{74}$$

$$\lambda\left[\alpha\left(1-\gamma^A\right)-t-p^A\right] = 0 \tag{75}$$

$$\lambda \geq 0. \tag{76}$$

Then $\lambda = 1$ implies that the constraint is binding, (writing $z = 1-\gamma^*$, (30))

$$p^A = \alpha z - t. \tag{77}$$

If it maximizes profit subject to the constraint that the market be covered, firm A sets a price that takes all surplus from the most distant readers.

Firm A's monopoly payoff in the low-price regime is

$$p^A+\left(\gamma^A-c\right)\left(1-\gamma^A\right)-F = \alpha z - t + z\left(\gamma^*-c\right)-F$$

(and using $\alpha+\gamma^*-c = 1-\gamma^* = z$)

$$= z\left(\alpha+\gamma^*-c\right)-t-F = z^2-t-F. \tag{78}$$

$p^A \geq \alpha\left(1-\gamma^A\right) - t$ If $p^A \geq \alpha\left(1-\gamma^A\right) - t$, firm A's problem is

$$\max_{p^A,\gamma^A} n_R^A \pi_R^A - F \text{ s.t. } p^A \geq \alpha\left(1-\gamma^A\right) - t. \tag{79}$$

We first work out the solution without imposing the constraint, then determine a condition under which the unconstrained solution satisfies the constraint.

First-order conditions for the unconstrained problem are

$$n_R^A \frac{\partial \pi_R^A}{\partial p^A} + \pi_R^A \frac{\partial n_R^A}{\partial p^A} = 0 \tag{80}$$

and

$$n_R^A \frac{\partial \pi_R^A}{\partial \gamma^A} + \pi_R^A \frac{\partial n_R^A}{\partial \gamma^A} = 0 \tag{81}$$

with

$$\pi_R^A = p + \left(\gamma^A - c\right)\left(1-\gamma^A\right) \tag{82}$$

(so that $\frac{\partial \pi_R^A}{\partial p} = 1$, $\frac{\partial \pi_R^A}{\partial \gamma} = 1 - 2\gamma + c$) and

$$n_R^A = \frac{\alpha\left(1-\gamma^A\right) - p^A}{t}, \tag{83}$$

(so that $\frac{\partial n_R^A}{\partial p} = -\frac{1}{t}$, $\frac{\partial n_R^A}{\partial \gamma} = -\frac{\alpha}{t}$.)

Substituting, the monopoly first-order conditions are

$$n_R^A - \frac{1}{t}\pi_R^A = 0 \tag{84}$$

$$n_R^A\left(1-2\gamma+c\right) - \frac{\alpha}{t}\pi_R^A = 0. \tag{85}$$

Substitute $\pi_R^A = t n_R^A$ from (84) into (85) to obtain

$$n_R^A\left(1-2\gamma+c-\alpha\right) = 0, \tag{86}$$

from which

$$\gamma^A = \frac{1}{2}\left[1-(\alpha-c)\right] = \gamma^*. \tag{87}$$

Substituting $\gamma^A = \gamma^*$ into (84) gives firm A's monopoly price,

$$n_R^A - \frac{1}{t}\pi_R^A = 0$$

$$\frac{\alpha z - p}{t} - \frac{1}{t}\left[p + z\left(\gamma^* - c\right)\right] = 0,$$

which yields (omitting several steps)

$$p^{AM} = \frac{1}{2} z\left(\alpha + c - \gamma^*\right). \tag{88}$$

The consistency condition for (88) to be a valid solution is that "transportation cost" be sufficiently great:

$$p^{AM} \geq \alpha z - t,$$

which leads to

$$t \geq \frac{1}{2} z^2. \tag{89}$$

Now using (83), firm A's equilibrium number of readers is

$$n_R^A = \frac{\alpha z - p^A}{t} = \frac{z^2}{2t}. \tag{90}$$

Firm A's equilibrium monopoly payoff in the high-t case is

$$n_R^A \pi_R^A - F =$$

(substituting $\pi_R^A = t n_R^A$)

$$t\left(n_R^A\right)^2 - F =$$

(substituting (90))

$$\frac{1}{4t} z^4 - F. \tag{91}$$

The high-price solution is valid for $t \geq \frac{1}{2} z^2$. For $t \leq \frac{1}{2} z^2$, it is the low-price solution that is valid.

11.1.2 Unlicensed monopoly

We need an expression for firm A's payoff as an unlicensed monopolist serving a market with $1-\mu$ readers. The only change from the previous case is that the number of readers is reduced by the scale factor $1-\mu$. Payoffs are

$$\begin{array}{ll} (1-\mu)\left(z^2 - t\right) - F & \text{if} \;\; p^A \leq \alpha\left(1-\gamma^A\right) - t \\ \frac{1-\mu}{4t} z^4 - F & \text{if} \;\; p^A \geq \alpha\left(1-\gamma^A\right) - t \end{array}. \tag{92}$$

11.1.3 Duopoly, both firms licensed

The first-order conditions for firm A's profit maximization problem, (16) with $i = A$, are

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A \frac{\partial \pi_R^A}{\partial p^A} + \pi_R^A \frac{\partial n_R^A}{\partial p^A} = 0 \tag{93}$$

and

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A \frac{\partial \pi_R^A}{\partial \gamma^A} + \pi_R^A \frac{\partial n_R^A}{\partial \gamma^A} = 0, \tag{94}$$

where from (15)

$$\frac{\partial \pi_R^A}{\partial p^A} = 1 \tag{95}$$

$$\frac{\partial \pi_R^A}{\partial \gamma^A} = \left(\gamma^A - c\right)(-1) + 1 - \gamma^A = 1 + c - 2\gamma^A. \tag{96}$$

and from (9)

$$\frac{\partial n_R^A}{\partial p^A} = -\frac{1}{2t} \tag{97}$$

$$\frac{\partial n_R^A}{\partial \gamma^A} = -\frac{\alpha}{2t}. \tag{98}$$

Substituting (95), (96), (97), and (98) into (93) and (94) gives the first-order conditions

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A - \frac{1}{2t}\pi_R^A \equiv 0 \tag{99}$$

and

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A \left(1 + c - 2\gamma^A\right) - \frac{\alpha}{2t}\pi_R^A \equiv 0. \tag{100}$$

If (99) holds, which it will in equilibrium,

$$\pi_R^A = 2tn_R^A. \tag{101}$$

It follows that in equilibrium, firm A's payoff is

$$\pi^A = n_R^A \pi_R^A = 2t\left(n_R^A\right)^2 - F. \tag{102}$$

Substitute (101) into (99) to eliminate n_R^A, obtaining

$$\frac{\partial \pi^A}{\partial \gamma^A} = \frac{\pi_R^A}{2t}\left(1 + c - 2\gamma^A - \alpha\right) = 0. \tag{103}$$

For a positive equilibrium profit per reader, $\pi_R^A > 0$, (103) gives the equilibrium value of firm A's price-per-reader per advertisement:

$$\gamma^A = \frac{1}{2}\left[1 - (\alpha - c)\right] = \gamma^*. \tag{104}$$

We assume that marginal utility per ad in a newspaper exceeds marginal cost per ad in a newspaper,

$$\alpha - c > 0. \tag{105}$$

The per-reader advertising rate, γ^A, cannot exceed advertisers' profit per reader, the maximum value of which is 1. This gives

$$0 \leq \gamma^A \leq 1,$$

which implies

$$0 \leq 1 - (\alpha - c) \leq 2 \tag{106}$$

as a pair of inequalities that must be satisfied by $\alpha - c$.

(105) and (106) give

$$1 \geq \alpha - c \geq 0. \tag{107}$$

In the same way, we obtain for firm B the first-order conditions

$$\frac{\partial \pi^B}{\partial p^B} = n_R^B - \frac{1}{2t}\pi_R^B \equiv 0 \tag{108}$$

and

$$\frac{\partial \pi^B}{\partial \gamma^B} = n_R^B\left(1 + c - 2\gamma^B\right) - \frac{\alpha}{2t}\pi_R^B \equiv 0, \tag{109}$$

and the equilibrium price-per-reader of an advertisement,

$$\gamma^B = \frac{1}{2}\left[1 - (\alpha - c)\right] = \gamma^*. \tag{110}$$

From (8), the equilibrium number of advertisements (the same for both newspapers) is

$$n_a^A = n_a^B = 1 - \gamma^* = \frac{1}{2}\left(1 + \alpha - c\right). \tag{111}$$

The first-order conditions for p^A and p^B are (99) and (108), respectively. Substituting the equilibrium values of γ^i into (9) and (10) gives expressions for the numbers of readers per newspaper as functions of prices per reader, when $\gamma^A = \gamma^B = \gamma^*$:

$$n_R^A = \frac{1}{2t}\left(t - p^A + p^B\right), \tag{112}$$

$$n_R^B = \frac{1}{2t}\left(t - p^B + p^A\right). \tag{113}$$

Profit-per-reader of newspapers A and B are

$$\pi_R^A = p^A + z\left(\gamma^* - c\right) \tag{114}$$

and

$$\pi_R^B = p^B + z\left(\gamma^* - c\right), \tag{115}$$

respectively.

Using (114) and (115), the first-order conditions (99) and (108) become

$$2p^A - p^B = t - z\left(\gamma^* - c\right) \tag{116}$$

for p^A and

$$-p^A + 2p_R^B = t - z\left(\gamma^* - c\right) \tag{117}$$

for p^B.

The system of first-order equations, which we write in this form to permit comparison with (151) and (207), is

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} p^A \\ p^B \end{pmatrix} = \left[t - z\left(\gamma^* - c\right)\right]\begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{118}$$

Equilibrium prices are

$$p^A = p^B = t - z\left(\gamma^* - c\right). \tag{119}$$

From (19), for the market to be covered for these prices requires that t not be too great,

$$2\alpha - t \geq \alpha\left(\gamma^A + \gamma^B\right) + p^A + p^B,$$

$$t \leq \frac{2}{3}z\left(\gamma^* + \alpha - c\right), \tag{120}$$

or, using $\gamma^* + \alpha - c = 1 - \gamma^* = z$,

$$t \leq \frac{2}{3}z^2. \tag{121}$$

From (102), in equilibrium

$$\pi^A = 2t\left(n_R^A\right)^2 - F.$$

But if the market is covered in symmetric equilibrium, $n_R^A = \frac{1}{2}$ (see also (112)). Hence

$$\pi^A = \pi^B = \frac{1}{2}t - F. \tag{122}$$

11.1.4 Duopoly, A & B unlicensed

The only change from the previous case is that the number of readers is scaled down by the factor $1 - \mu$. Equilibrium payoffs per firm are

$$\pi^A = \pi^B = \frac{1-\mu}{2}t - F. \tag{123}$$

11.1.5 Duopoly, A licensed, B not

If $p^A \leq \alpha n_a^A - t$, objective functions are

$$\pi^A = n_R^A \pi_R^A - F \tag{124}$$

and

$$\pi^B = n_R^B \pi_R^B - F. \tag{125}$$

$p^A \leq \alpha n_a^A - t$ First analyze the outcome on the assumption that equilibrium values place demand in the low-p^A case. Analyze firm A's profit-maximization problem without imposing

$$p^A \leq \alpha n_a^A - t \tag{126}$$

as a constraint. Find equilibrium prices, and find conditions for (126) to be satisfied.

The number of readers of each firm are

$$n_R^A = (1-\mu)\left[\frac{1}{2} + \frac{\alpha\left(\gamma^B - \gamma^A\right) - p^A + p^B}{2t}\right] + \mu \tag{127}$$

$$= \frac{1}{2}\left[(1+\mu) + (1-\mu)\frac{\alpha\left(\gamma^B - \gamma^A\right) - p^A + p^B}{t}\right]. \tag{128}$$

$$n_R^B = \frac{1}{2}(1-\mu)\left[1 + \frac{\alpha\left(\gamma^A - \gamma^B\right) - p^B + p^A}{t}\right]. \tag{129}$$

The following comparative static derivatives will be used later. For the numbers of readers,

$$\frac{\partial n_R^A}{\partial p^A} = \frac{\partial n_R^B}{\partial p^B} = -\frac{1-\mu}{2t} \tag{130}$$

$$\frac{\partial n_R^A}{\partial \gamma^A} = \frac{\partial n_R^B}{\partial \gamma^B} = -\alpha\frac{1-\mu}{2t} \tag{131}$$

For profitability per reader,

$$\pi_R^A = p^A + \left(\gamma^A - c\right)\left(1 - \gamma^A\right)$$

$$\pi_R^B = p^B + \left(\gamma^B - c\right)\left(1 - \gamma^B\right)$$

$$\frac{\partial \pi_R^A}{\partial p^A} = \frac{\partial \pi_R^A}{\partial p^B} = 1 \tag{132}$$

$$\frac{\partial \pi_R^A}{\partial \gamma^A} = 1 + c - 2\gamma^A \tag{133}$$

$$\frac{\partial \pi_R^B}{\partial \gamma^B} = 1 + c - 2\gamma^B. \tag{134}$$

Firm A Firm A's first-order conditions are
p^A:

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A \frac{\partial \pi_R^A}{\partial p^A} + \pi_R^A \frac{\partial n_R^A}{\partial p^A} = 0$$

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A - \frac{1-\mu}{2t}\pi_R^A = 0. \tag{135}$$

From (135), in equilibrium

$$\pi_R^A = p^A + \left(\gamma^A - c\right)\left(1 - \gamma^A\right) = \frac{2t}{1-\mu} n_R^A. \tag{136}$$

Hence firm A's equilibrium profit satisfies

$$\pi^A = \frac{2t}{1-\mu}\left(n_R^A\right)^2 - F. \tag{137}$$

γ^A:

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A \frac{\partial \pi_R^A}{\partial \gamma^A} + \pi_R^A \frac{\partial n_R^A}{\partial \gamma^A} = 0$$

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A \left(1 + c - 2\gamma^A\right) - \alpha \frac{1-\mu}{2t} \pi_R^A = 0. \tag{138}$$

Substituting (136) into (138), in equilibrium

$$n_R^A \left[\left(1 + c - 2\gamma^A\right) - \alpha\right] = 0 \tag{139}$$

and since $n_R^A > 0$, in equilibrium

$$\gamma^A = \frac{1}{2}\left[1 - (\alpha - c)\right] = \gamma^*. \tag{140}$$

Firm B Firm B's payoff function is

$$\pi^B = n_R^B \pi_R^B - F.$$

The first-order condition with respect to p^B is

$$\frac{\partial \pi^B}{\partial p^B} = n_R^B - \frac{1-\mu}{2t} \pi_R^B = 0. \tag{141}$$

From (141), in equilibrium

$$p^B + \left(\gamma^B - c\right)\left(1 - \gamma^B\right) = \frac{2t}{1-\mu} n_R^B \tag{142}$$

Hence firm B's equilibrium profit satisfies

$$\pi^B = \frac{2t}{1-\mu}\left(n_R^B\right)^2 - F. \tag{143}$$

The first-order condition with respect to γ^B is

$$\frac{\partial \pi^B}{\partial \gamma^B} = n_R^B \left(1 + c - 2\gamma^B\right) - \alpha \frac{1-\mu}{2t} \pi_R^B = 0. \tag{144}$$

Substituting (142) into (144), in equilibrium

$$n_R^B \left(1 + c - 2\gamma^B\right) - \alpha \frac{1-\mu}{2t} \frac{2t}{1-\mu} n_R^B = 0$$

$$n_R^B \left[1 + c - 2\gamma^B - \alpha\right] = 0,$$

and for $n_R^B > 0$ we have

$$\gamma^B = \frac{1}{2}\left[1 - (\alpha - c)\right] = \gamma^*. \tag{145}$$

Equilibrium n_R^A, n_R^B (I) Use the equilibrium values of γ^A and γ^B to rewrite (127) and (129) as

$$n_R^A = \frac{1}{2}\left[(1+\mu) - (1-\mu)\frac{p^A - p^B}{t}\right] \tag{146}$$

and

$$n_R^B = \frac{1}{2}(1-\mu)\left(1 + \frac{p^A - p^B}{t}\right). \tag{147}$$

Equilibrium p^A, p^B Using (146), firm A's first-order condition for p^A, (135), becomes

$$2p^A - p^B = \frac{1+\mu}{1-\mu}t - z(\gamma^* - c). \tag{148}$$

This is firm A's equilibrium price best-response equation — "equilibrium" because the γs are set at their equilibrium values.

Using (147), firm B's first-order condition for p^B, (141), becomes

$$-p^A + 2p^B = t - z(\gamma^* - c). \tag{149}$$

This is firm B's equilibrium price best-response equation.

(148) and (149) can be solved for equilibrium prices.

Before doing so, subtract (149) from (148) to obtain an expression for $p^A - p^B$, which is what is needed to find the equilibrium number of readers for each newspaper:

$$p^A - p^B = \frac{2}{3}\frac{\mu}{1-\mu}t. \tag{150}$$

Now write the system of first-order equations in matrix form as

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} p^A \\ p^B \end{pmatrix} = \begin{pmatrix} \frac{1+\mu}{1-\mu} \\ 1 \end{pmatrix} t - z(\gamma^* - c)\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{151}$$

from which

$$\begin{pmatrix} p^A \\ p^B \end{pmatrix} = \frac{1}{3}\frac{1}{1-\mu}\begin{pmatrix} 3+\mu \\ 3-\mu \end{pmatrix} t - z(\gamma^* - c)\begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{152}$$

$$p^A = \frac{1}{3}\frac{3+\mu}{1-\mu}t - z(\gamma^* - c) \tag{153}$$

$$p^B = \frac{1}{3}\frac{3-\mu}{1-\mu}t - z\left(\gamma^* - c\right). \tag{154}$$

Subtracting (154) from (153) gives (150).

In a conventional oligopoly model, it would be taken for granted that $p^A \geq 0$, $p^B \geq 0$. In general in a model of a platform market, we cannot assume this. However, if the unconstrained model implies negative prices for newspapers, we would wish to impose zero prices as a constraint and pursue the implications. We therefore assume $p^A \geq 0$, $p^B \geq 0$. See the discussion of Armstrong and Wright (2007, p. 356), who make the same assumption.

Since

$$\frac{\partial p^A}{\partial \mu} = \frac{t}{3}\frac{\partial}{\partial \mu}\left(\frac{3+\mu}{1-\mu}\right) = \frac{4t}{3}\frac{1}{\left(1-\mu\right)^2} > 0 \tag{155}$$

$$\frac{\partial p^B}{\partial \mu} = \frac{t}{3}\frac{\partial}{\partial \mu}\left(\frac{3-\mu}{1-\mu}\right) = \frac{2t}{3}\frac{1}{\left(1-\mu\right)^2} > 0, \tag{156}$$

p^A and p^B are increasing in μ. It follows that platforms' profits per reader,

$$\pi_R^A = p^A + z\left(\gamma^* - c\right) = \frac{1}{3}\frac{3+\mu}{1-\mu}t \tag{157}$$

and

$$\pi_R^B = p^B + z\left(\gamma^* - c\right) = \frac{1}{3}\frac{3-\mu}{1-\mu}t \tag{158}$$

are also increasing in μ.

Consistency condition Now examine conditions under which (126), $p^A \leq \alpha n_a^A - t$, will be satisfied.

A preliminary remark is that considering the group that does not regard comics as essential, it must also be that consumers at the boundary location have nonnegative net utility,

$$\alpha n_a^A - p^A - tx \geq 0,$$

for x the boundary distance from the left end of the line. But if a reader at the right end of the line would have nonnegative net utility,

$$0 \leq \alpha n_a^A - p^A - t,$$

then so would a reader located closer to the left end of the line,

$$\alpha n_a^A - p^A - tx = \alpha n_a^A - p^A - t + (1-x) > 0,$$

and this is true in particular if x is the boundary location.

Now examine conditions for (126) to be satisfied:

$$\alpha n_a^A - p^A - t \geq 0.$$

$$\alpha z - \left[\frac{1}{3}\frac{3+\mu}{1-\mu}t - z\left(\gamma^* - c\right)\right] - t \geq 0$$

Omitting several steps, this the consistency condition becomes

$$t \leq \frac{3}{2}\frac{1-\mu}{3-\mu}z^2. \tag{159}$$

The right-hand side of (159) goes to 0 as $\mu \to 1$. It follows that there is a critical value μ^*, $0 < \mu^* < 1$, such that the consistency condition is satisfied exactly. μ^* is defined by

$$\frac{3}{2}\frac{1-\mu}{3-\mu}z^2 = t. \tag{160}$$

From (160),

$$\mu^* = \frac{3z^2 - 6t}{3z^2 - 2t}. \tag{161}$$

By the argument we made about starting at $\mu = 0$ and increasing μ, μ^* must lie between 0 and 1.

$$p^A = \frac{1}{3}\frac{3+\mu}{1-\mu}t - z\left(\gamma^* - c\right).$$

For $\mu = \mu^*$,

$$p^A = \alpha z - t.$$

$$p^B = \frac{1}{3}\frac{3-\mu}{1-\mu}t - z\left(\gamma^* - c\right).$$

Evaluate this for $\mu = \mu^*$; the result will be used below. Omitting several steps,

$$p^B = \frac{1}{3}\frac{3-\mu^*}{1-\mu^*}t - z\left(\gamma^* - c\right) = \frac{1}{2}z^2 - z\left(\gamma^* - c\right). \tag{162}$$

Equilibrium n_R^A, n_R^B (II) Substituting the expression for $p^A - p^B$, (150), into (146) and (147), the equilibrium numbers of readers per newspaper in the low-p^A regime are

$$n_R^A = \frac{1}{6}(3+\mu) \tag{163}$$

$$n_R^B = \frac{1}{6}(3-\mu). \tag{164}$$

In the unconstrained low-p^A regime, n_R^A rises from $\frac{1}{2}$ and n_R^B falls from $\frac{1}{2}$ as μ rises from 0.

Payoffs Substitute from (163) and (164) into (137) and (143), respectively, equilibrium payoffs are

$$\pi^A = \frac{t}{18}\frac{(3+\mu)^2}{1-\mu} - F \tag{165}$$

and

$$\pi^B = \frac{t}{18}\frac{(3-\mu)^2}{1-\mu} - F. \tag{166}$$

Comparative static derivatives with respect to μ are

$$\frac{\partial \pi^A}{\partial \mu} = \frac{t}{18}\frac{(3+\mu)(5-\mu)}{(1-\mu)^2} > 0 \tag{167}$$

$$\frac{\partial \pi^B}{\partial \mu} = \frac{t}{18}\frac{(1+\mu)(3-\mu)}{(1-\mu)^2} > 0. \tag{168}$$

As μ increases, in a comparative static sense, π_R^A and n_R^A both rise, so π^A, their product, certainly rises.

As μ increases, π_R^B rises and n_R^B falls. In the low-p^A regime, the former effect outweighs the latter, and π^B rises as μ rises from 0 to μ^*.

Constrained The equilibrium value

$$p^A = \frac{1}{3}\frac{3+\mu}{1-\mu}t - z(\gamma^* - c),$$

which is obtained by solving firm A's profit maximization problem for the low-p^A case without explicitly imposing the low-p^A constraint,

$$p^A \leq \alpha z - t, \tag{169}$$

satisfies the low-p^A constraint for $\mu \leq \mu^*$. For $\mu \geq \mu^*$, to obtain an equilibrium value of p^A consistent with the condition that defines n_R^A for the low-p^A case requires imposing (169) as an explicit constraint on firm A's profit-maximization problem.

In the low-p^A case, n_R^A is given by (128). A Lagrangian for firm A's constrained optimization problem is

$$\mathcal{L} = n_R^A \pi_R^A - F + \lambda \left[\alpha \left(1 - \gamma^A\right) - t - p^A\right]. \tag{170}$$

Assuming an interior solution, the Kuhn-Tucker first-order conditions are p^A:

$$n_R^A \frac{\partial \pi_R^A}{\partial p^A} + \pi_R^A \frac{\partial n_R^A}{\partial p^A} - \lambda = 0. \tag{171}$$

γ^A:

$$n_R^A \frac{\partial \pi_R^A}{\partial \gamma^A} + \pi_R^A \frac{\partial n_R^A}{\partial \gamma^A} - \alpha\lambda = 0. \tag{172}$$

λ:

$$\alpha \left(1 - \gamma^A\right) - t - p^A = 0. \tag{173}$$

Substituting (130), (131), (132), (133), and (134), (171) and (172) become

$$n_R^A - \frac{1 - \mu}{2t} \pi_R^A - \lambda = 0. \tag{174}$$

and

$$n_R^A \left(1 + c - 2\gamma^A\right) - \alpha \frac{1 - \mu}{2t} \pi_R^A - \alpha\lambda = 0, \tag{175}$$

respectively.

$$\alpha \left(1 - \gamma^A\right) - t - p^A = 0. \tag{176}$$

From (174),

$$\frac{1 - \mu}{2t} \pi_R^A = n_R^A - \lambda,$$

leading to

$$\pi_R^A = \frac{2t}{1 - \mu} \left(n_R^A - \lambda\right). \tag{177}$$

Substitute (177) into (175) to obtain

$$\gamma^A = \frac{1}{2}\left[1 - (\alpha - c)\right] = \gamma^*. \tag{178}$$

Firm B's problem is unaffected by the constraint imposed on firm A. Thus we have

$$\gamma^B = \frac{1}{2}\left[1 - (\alpha - c)\right] = \gamma^*,$$

as before. It follows that the expressions (146) and (147) for n_R^A and n_R^B, respectively, are valid for the constrained optimization case.

Since we know γ, we now have two equations, (174) and (176).

From (174),

$$\lambda = n_R^A - \frac{1-\mu}{2t}\pi_R^A, \tag{179}$$

and substituting for n_R^A and π_R^A, this becomes (omitting several steps)

$$\lambda = \frac{1}{2}(1+\mu) + \frac{1-\mu}{2t}\left[p^B - 2p^A - z(\gamma^* - c)\right]. \tag{180}$$

Rewriting (180) in a form that highlights the relationship to the first-order condition for p^A when the constraint does not bind, (148), gives

$$\lambda = \frac{1-\mu}{2t}\left[\frac{1+\mu}{1-\mu}t - z(\gamma^* - c) - \left(2p^A - p^B\right)\right]. \tag{181}$$

There is further analysis of the equilibrium value of λ below.

From the binding constraint, we get the value of p^A:

$$p^A = \alpha z - t. \tag{182}$$

Firm B's best-response equation is unchanged by the fact that the constraint on firm A's problem is binding; it is

$$-p^A + 2p^B = t - z(\gamma^* - c). \tag{183}$$

Substituting (182) into (183), firm B's equilibrium price when firm A's price is determined by the constraint is

$$p^B = \frac{1}{2}z(\alpha - \gamma^* + c). \tag{184}$$

By definition of μ^*,

$$\frac{1}{3}\frac{3-\mu^*}{1-\mu^*}t = \frac{1}{2}z^2.$$

Hence if $\mu = \mu^*$, firm B's equilibrium price per newspaper when firm A's optimization problem is unconstrained is

$$p^B = \frac{1}{2}z^2 - z\left(\gamma^* - c\right).$$

This is identical to (162); firm B's equilibrium price is continuous in μ at the value of μ for which the constraint on firm A's low-p^A optimization problem becomes binding.

When the low-p^A constraint is binding, the difference in equilibrium prices is

$$p^A - p^B = \frac{1}{2}z^2 - t.$$

Above, (159), for consistency in the low-p^A regime with $\mu = 0$, we assumed

$$\frac{1}{2}z^2 \geq t.$$

This implies that in equilibrium in the constrained case

$$p^A - p^B = \frac{1}{2}\left(1-\gamma^*\right)^2 - t > 0. \tag{185}$$

Find the equilibrium numbers of readers per platform,

$$n_R^A = \frac{1}{2}\left[1 + \mu + (1-\mu)\frac{p^B - p^A}{t}\right]$$

$$n_R^B = \frac{1}{2}(1-\mu)\left(1 + \frac{p^A - p^B}{t}\right).$$

Using (185), (omitting several steps)

$$n_R^A = 1 - (1-\mu)\frac{z^2}{4t}. \tag{186}$$

$$n_R^B = (1-\mu)\frac{z^2}{4t}. \tag{187}$$

Thus

$$n_R^A + n_R^B = 1.$$

In the low-p^A case, the market is covered.

Find equilibrium firm payoffs.

$$\pi^A = n_R^A \pi_R^A - F$$

$$\pi^B = n_R^B \pi_R^B - F.$$

For firm B, we have as before

$$\pi^B = \frac{2t}{1-\mu} \left(n_R^B\right)^2 - F. \tag{188}$$

When the low-p^A constraint is binding,

$$\pi_R^A = \frac{2t}{1-\mu} \left(n_R^A - \lambda\right),$$

and firm A's equilibrium payoff satisfies

$$\pi^A = \frac{2t}{1-\mu} n_R^A \left(n_R^A - \lambda\right) - F. \tag{189}$$

One of the expressions for λ is (181),

$$\lambda = \frac{1-\mu}{2t} \left\{ \frac{1+\mu}{1-\mu} t - \left[2p^A - p^B + z\left(\gamma^* - c\right)\right] \right\}.$$

Consider the expression in brackets; substituting (182) and (184), it is

$$2p^A - p^B + z\left(\gamma^* - c\right) =$$

$$2\left[\alpha z - t\right] - \frac{1}{2} z \left(\alpha - \gamma^* + c\right) + z\left(\gamma^* - c\right) =$$

(omitting several steps)

$$\frac{3}{2} z^2 - 2t.$$

Then

$$\lambda = \frac{1}{2} \left[3 - \mu - \frac{1-\mu}{t} \frac{3}{2} z^2 \right] \tag{190}$$

From (186)

$$n_R^A = 1 - (1-\mu) \frac{z^2}{4t}.$$

Then

$$n_R^A - \lambda = \frac{1-\mu}{2}\left[\frac{z^2}{t} - 1\right]. \tag{191}$$

Firm A's payoff in the low-p^A regime when the low-p^A constraint is binding is

$$\pi^A = \frac{2t}{1-\mu} n_R^A \left(n_R^A - \lambda\right) - F = \left[1 - (1-\mu)\frac{z^2}{4t}\right]\left[\frac{z^2}{t} - 1\right] t - F. \tag{192}$$

π^A rises as μ rises.

Firm B's equilibrium payoff is

$$\pi^B = \frac{2t}{1-\mu}\left(n_R^B\right)^2 - F = \frac{1-\mu}{8t} z^4 - F. \tag{193}$$

$p^A \geq \alpha n_a^A - t$ The underlying expressions for n_R^B and π_R^B are unchanged from the previous case. Firm B's choice of γ^B is given by (145), and its first-order conditions are as in the low-p^A regime.

Firm A's profit per reader,

$$\pi_R^A = p^A + \left(\gamma^A - c\right)\left(1 - \gamma^A\right),$$

is also as in the low-p^A regime. But in the high-p^A regime (from (40)), platform A's number of readers is

$$n_R^A = (1-\mu)\left[\frac{1}{2} + \frac{\alpha\left(\gamma^B - \gamma^A\right) - p^A + p^B}{2t}\right] + \mu\frac{\alpha n_a^A - p^A}{t}$$

$$= (1-\mu)\left[\frac{1}{2} - \frac{\alpha\left(1 - \gamma^B\right) - p^B}{2t}\right] + (1+\mu)\frac{\alpha\left(1-\gamma^A\right) - p^A}{2t}. \tag{194}$$

Firm A's first-order condition with respect to p^A is

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A - \frac{1+\mu}{2t}\left[p^A + \left(\gamma^A - c\right)\left(1 - \gamma^A\right)\right] \equiv 0 \tag{195}$$

(compare with (135) for the low-p^A regime).

From (195), in equilibrium

$$\pi_R^A = p^A + \left(\gamma^A - c\right)\left(1-\gamma^A\right) = \frac{2t}{1+\mu} n_R^A \tag{196}$$

and firm A's equilibrium payoff satisfies

$$\pi^A = \frac{2t}{1+\mu}\left(n_R^A\right)^2 - F. \tag{197}$$

Firm A's first-order condition with respect to γ^A is

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A \frac{\partial \pi_R^A}{\partial \gamma^A} + \pi_R^A \frac{\partial n_R^A}{\partial \gamma^A} \equiv 0 \tag{198}$$

or

$$\frac{\partial \pi^A}{\partial \gamma^A} = n_R^A\left(1 + c - 2\gamma^A\right) - \alpha\frac{1+\mu}{2t}\pi_R^A \equiv 0. \tag{199}$$

Substituting (196) into (199), in equilibrium

$$n_R^A\left(1 + c - 2\gamma^A - \alpha\right) \equiv 0$$

and

$$\gamma^A = \frac{1}{2}\left(1 + c - \alpha\right) = \gamma^*. \tag{200}$$

Equilibrium n_R^A, n_R^B (I) Substituting $\gamma^A = \gamma^B = \gamma^*$ in (194) and (129), the equilibrium numbers of readers satisfy

$$n_R^A = \frac{1}{2}\left(1-\mu\right) + \mu\frac{\alpha\left(1-\gamma\right)}{t} - \left(1+\mu\right)\frac{p^A}{2t} + \left(1-\mu\right)\frac{p^B}{2t} \tag{201}$$

and

$$n_R^B = \frac{1}{2}\left(1-\mu\right)\left(1 + \frac{p^A - p^B}{t}\right). \tag{202}$$

Equilibrium p^A, p^B Using (201), firm A's first-order condition for p^A, (195),

$$\frac{\partial \pi^A}{\partial p^A} = n_R^A - \frac{1+\mu}{2t}\left[p^A + z\left(\gamma^* - c\right)\right] \equiv 0,$$

becomes (omitting several steps)

$$2\left(1+\mu\right)p^A - \left(1-\mu\right)p^B = t - z\left(\gamma^* - c\right) - \left\{t - 2\alpha z + z\left(\gamma^* - c\right)\right\}\mu. \tag{203}$$

The first-order condition for p^B is

$$-p^A + 2p^B = t - z\left(\gamma^* - c\right). \tag{204}$$

Write the system of equations is

$$\begin{pmatrix} 2(1+\mu) & -(1-\mu) \\ -1 & 2 \end{pmatrix} \begin{pmatrix} p^A \\ p^B \end{pmatrix} = [t - z(\gamma^* - c)] \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$- \{t - 2\alpha z + z(\gamma^* - c)\} \mu \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{205}$$

The system of first-order conditions can be solved for prices,

$$(3+5\mu) \begin{pmatrix} p^A \\ p^B \end{pmatrix} = [t - z(\gamma^* - c)] \begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$- \{t - 2\alpha z + z(\gamma^* - c)\} \mu \begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{206}$$

Instead of looking at the solutions written in this form, it is useful to multiply both sides of (206) by

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix},$$

obtaining a transformed system of equations

$$(3+5\mu) \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} p^A \\ p^B \end{pmatrix} = [t - z(\gamma^* - c)] \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$- \{t - 2\alpha z + z(\gamma^* - c)\} \mu \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Coefficient matrices on the right are

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3-4\mu \\ 3+5\mu \end{pmatrix} = \begin{pmatrix} 3+5\mu-9\mu \\ 3+5\mu \end{pmatrix}$$

and

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1-\mu \\ 1 & 2(1+\mu) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}.$$

The transformed system of equations is (omitting several steps)

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} p^A \\ p^B \end{pmatrix} = [t - z(\gamma^* - c)] \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{12\mu}{3+5\mu} \left[t - \frac{1}{2} z\right] \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{207}$$

The first equation in (207) is a linear combination of the first-order conditions of the two platforms. It is clear from (207) that if $\mu = 0$, the system of first-order conditions of the essential component model corresponds to the system of first-order conditions of the basic model.

Solving (207) gives equilibrium prices

$$\begin{pmatrix} p^A \\ p^B \end{pmatrix} = [t - z(\gamma^* - c)] \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{4\mu}{3+5\mu}\left[t - \frac{1}{2}z^2\right]\begin{pmatrix} 2 \\ 1 \end{pmatrix}. \tag{208}$$

$$p^A = t - z(\gamma^* - c) - \frac{8\mu}{3+5\mu}\left[t - \frac{1}{2}z^2\right]. \tag{209}$$

$$p^B = t - z(\gamma^* - c) - \frac{4\mu}{3+5\mu}\left[t - \frac{1}{2}z\right]. \tag{210}$$

Numbers of readers We use (209) and (210) to evaluate the numbers of readers of each platform, (201) and (202).

Considering first platform B, from (209) and (210),

$$p^A - p^B = -\frac{4\mu}{3+5\mu}\left[t - \frac{1}{2}z^2\right] < 0. \tag{211}$$

Substituting (211) into (202) and rearranging terms gives

$$n_R^B = \frac{1-\mu}{2t}\frac{(3+\mu)\,t + 2\mu z^2}{3+5\mu}. \tag{212}$$

Now turn to platform A. We need to evaluate

$$-(1+\mu)\,p^A + (1-\mu)\,p^B = -\left(p^A - p^B\right) - \mu\left(p^A + p^B\right). \tag{213}$$

From (208),

$$-(1+\mu)\,p^A + (1-\mu)\,p^B =$$

(omitting several steps)

$$-2\mu\left[\frac{1-\mu}{3+5\mu}t - z(\gamma^* - c) + \frac{1+3\mu}{3+5\mu}z^2\right]. \tag{214}$$

Then

$$n_R^A = \frac{1}{2}(1-\mu) + \mu\frac{\alpha z}{t} + \frac{-(1+\mu)\,p^A + (1-\mu)\,p^B}{2t} =$$

(omitting several steps)

$$= \frac{1+\mu}{3+5\mu}\left[\frac{3}{2}(1-\mu) + \frac{2z^2}{t}\mu\right]. \tag{215}$$

The total number of readers is

$$n_R^A + n_R^B = \frac{1}{3+5\mu}\left[(1-\mu)(3+2\mu) + \mu(3+\mu)\frac{z^2}{t}\right]. \tag{216}$$

Consistency The consistency condition is

$$p^A \geq \alpha n_a^A - t.$$

Rewrite (209) to collect terms in t to obtain

$$p^A = 3\frac{1-\mu}{3+5\mu}t - z(\gamma^* - c) + \frac{4\mu}{3+5\mu}z^2.$$

Then

$$p^A - \alpha n_a^A + t = 2\frac{3+\mu}{3+5\mu}\left[t - \frac{1}{2}z^2\right].$$

In the high-p^A case, consistency requires

$$t \geq \frac{1}{2}z^2.$$

In the unconstrained low-p^A case, consistency requires the opposite relationship (see (159) for $\mu = 0$):

$$\frac{1}{2}z^2 \geq t.$$

Payoffs From (197) and (143), equilibrium payoffs are

$$\pi^A = \frac{2t}{1+\mu}\left(n_R^A\right)^2 - F$$

and

$$\pi^B = \frac{2t}{1-\mu}\left(n_R^B\right)^2 - F.$$

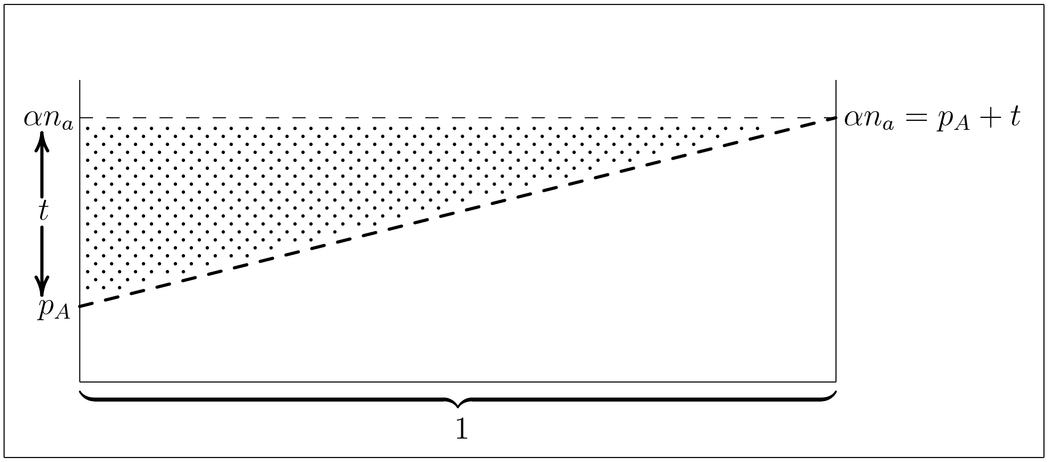

Figure 2: Consumer surplus, monopoly, low-t regime.

Using (215), firm A's equilibrium payoff is

$$\pi^A = 2t \frac{1+\mu}{(3+5\mu)^2} \left[\frac{3}{2}(1-\mu) + \frac{2z^2}{t}\mu \right]^2 - F. \tag{217}$$

Using (212), firm B's equilibrium payoff is

$$\pi^B = \frac{1-\mu}{2t} \left[\frac{(3+\mu)\, t + 2\mu z^2}{3+5\mu} \right]^2 - F. \tag{218}$$

As $\mu \to 1$, π^B becomes negative.

11.2 Welfare

11.2.1 Monopoly, low-t

In the low-price regime, the monopoly supplier sets a price that makes consumers at the right end of the line indifferent between purchasing and not purchasing a newspaper. Consumers whose preferences place them closer to the left end of the line enjoy positive surplus if they buy. Consumer surplus is the area of the shaded triangle in Figure 2,

$$\frac{1}{2}t. \tag{219}$$

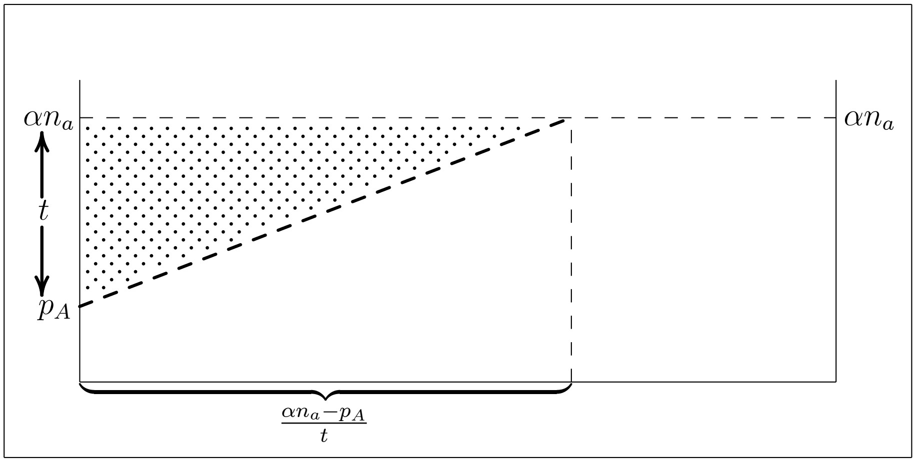

Figure 3: Consumer surplus, monopoly, high-t regime.

Economic profit generated by newspaper A in the low-t regime is $\pi_{l1}^m = z^2 - t - F$.[21] Adapting equation (11) to the present case, advertisers' profit in the low-price licensed-monopoly regime is

$$\frac{1}{2} n_R^A \left(1 - \gamma^A\right)^2 = \frac{1}{2} z^2. \tag{220}$$

Net social welfare in the low-price regime is the sum of profits and consumer surplus,

$$z^2 - t - F + \frac{1}{2} z^2 + \frac{1}{2} t = \frac{3}{2} z^2 - \frac{1}{2} t - F. \tag{221}$$

11.2.2 Monopoly, high t

Consumer surplus in the high-t regime is the area of the triangle in Figure 3,

$$\frac{\left(\alpha n_a - p_A\right)^2}{2t}. \tag{222}$$

Substituting $\alpha n_a = \alpha z$ and $p^{AM} = \frac{1}{2} z \left(\alpha + c - \gamma^*\right)$, and using $z = \gamma^* + \alpha - c$ gives

$$\alpha n_a - p_A = \frac{1}{2} z^2. \tag{223}$$

[21] The amount of the license fee determines the division of this profit between firm A and the syndicate; this division does not affect net social welfare.

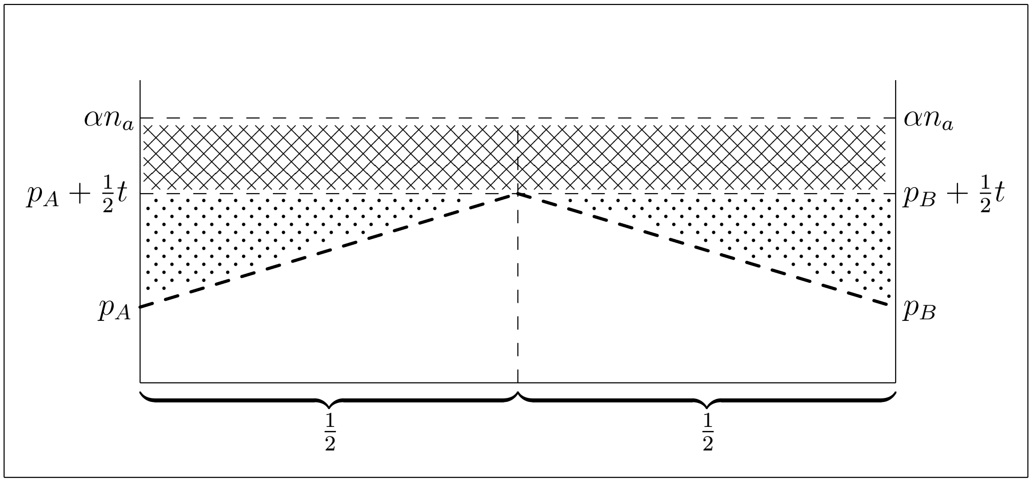

Figure 4: Consumer surplus, duopoly, both firms licensed.

Then consumer surplus in the firm A monopoly, high-price regime is

$$\frac{1}{2t}\left(\alpha n_a - p_A\right)^2 = \frac{1}{8t}z^4. \tag{224}$$

Economic profit from the operation of newspaper A in the high-price regime is $\pi_{l2}^m = \frac{1}{4t}z^4 - F$. Advertisers' profit is (using $n_R^A = \frac{1}{2t}z^2$)

$$\frac{1}{2}n_R^A z^2 = \frac{1}{4t}z^4. \tag{225}$$

Net social welfare in the high-t licensed-monopoly regime is

$$\frac{1}{4t}z^4 - F + \frac{1}{4t}z^4 + \frac{1}{8t}z^4 = \frac{5}{8t}z^4 - F. \tag{226}$$

11.2.3 Licensed-firm Duopoly

Net utility at either extreme of the line (the location for which transportation cost is zero) is (omitting superscripts since we consider symmetric equilibrium values)

$$\begin{gathered} \alpha n_a - p = \\ \alpha z - t + \left(\gamma^* - c\right) z = z\left(\gamma^* + \alpha - c\right) - t = \\ z^2 - t. \end{gathered} \tag{227}$$

Net utility at the center of the line (transportation cost $\frac{1}{2}t$) is

$$\alpha n_a - p - \frac{1}{2}t = z\left(\gamma^* + \alpha - c\right) - t - \frac{1}{2}t = z^2 - \frac{3}{2}t. \tag{228}$$

By the assumption that the market is covered, (121), we have assumed (228) is nonnegative.

Consumer surplus is the shaded area shown in Figure 4, one-half of which is

$$\int_{x=0}^{1/2} \left[\alpha n_a - (p + tx)\right] dx = \int_{x=0}^{1/2} \left[(\alpha n_a - p) - tx\right] dx =$$

$$\left[(\alpha n_a - p)\, x - \frac{1}{2}tx^2\right]_{x=0}^{1/2} = (\alpha n_a - p)\left(\frac{1}{2}\right) - \frac{1}{2}t\left(\frac{1}{2}\right)^2. \tag{229}$$

Total consumer surplus is twice (229),

$$\alpha n_a - p - t\left(\frac{1}{2}\right)^2 = z^2 - t - \frac{1}{4}t = z^2 - \frac{5}{4}t. \tag{230}$$

From equation (11), advertisers' profits are

$$\frac{1}{2}\left(n_R^A + n_R^B\right) z^2,$$

and in equilibrium for this model, $n_R^A = n_R^B = \frac{1}{2}$; hence advertisers' equilibrium profits are

$$\frac{1}{2}z^2, \tag{231}$$

Equilibrium net social welfare is the sum of the profits generated by the two newspapers, advertisers' profits, and consumer surplus,

$$2\left(\frac{1}{2}t - F\right) + \frac{1}{2}z^2 + z^2 - \frac{5}{4}t$$

$$2\left(\frac{1}{2}t - F\right) + \frac{1}{2}z^2 + z^2 - \frac{5}{4}t = \frac{3}{2}z^2 - \frac{1}{4}t - 2F. \tag{232}$$

11.3 High-t

Here we show that Theorem 1 holds for the high-t case. The inequalities

$$\max\left(\pi_{nlnl}^{d}, \pi_{nl2}^{m}, \pi_{nl3}^{Bd}\right) < 0 \leq \min\left(\pi_{ll}^{d}, \pi_{l2}^{m}, \pi_{l3}^{Ad}\right) \tag{233}$$

correspond to

$$\max\left\{\frac{1-\mu}{2}t, \frac{1-\mu}{4t}z^4, \frac{1-\mu}{2t}\left[\frac{(3+\mu)\,t+2\mu z^2}{3+5\mu}\right]^2\right\} < F \leq$$

$$\min\left\{\frac{1}{2}t, \frac{1}{4t}z^4, \frac{1+\mu}{2t}\left[\frac{3\,(1-\mu)\,t+4\mu z^2}{3+5\mu}\right]^2\right\}. \tag{234}$$

As $\mu \to 1$, (234) approaches

$$\max\left(0,0,0\right) = 0 < F \leq \min\left(\frac{1}{2}t, \frac{1}{4t}z^4, \frac{1}{4t}z^4\right). \tag{235}$$

By subtraction,

$$\frac{1}{4t}z^4 - \frac{1}{2}t = \frac{1}{2t}\left(\frac{1}{2}z^4 - t^2\right) = \frac{1}{2t}\left(\frac{1}{\sqrt{2}}z^2 - t\right)\left(\frac{1}{\sqrt{2}}z^2 + t\right), \tag{236}$$

which is positive in the high-t case since

$$\frac{1}{\sqrt{2}}z^2 - t > \frac{2}{3}z^2 - t \geq 0. \tag{237}$$

Then (235) reduces to (49). From this point, the argument is as for the low-t, high-μ case.