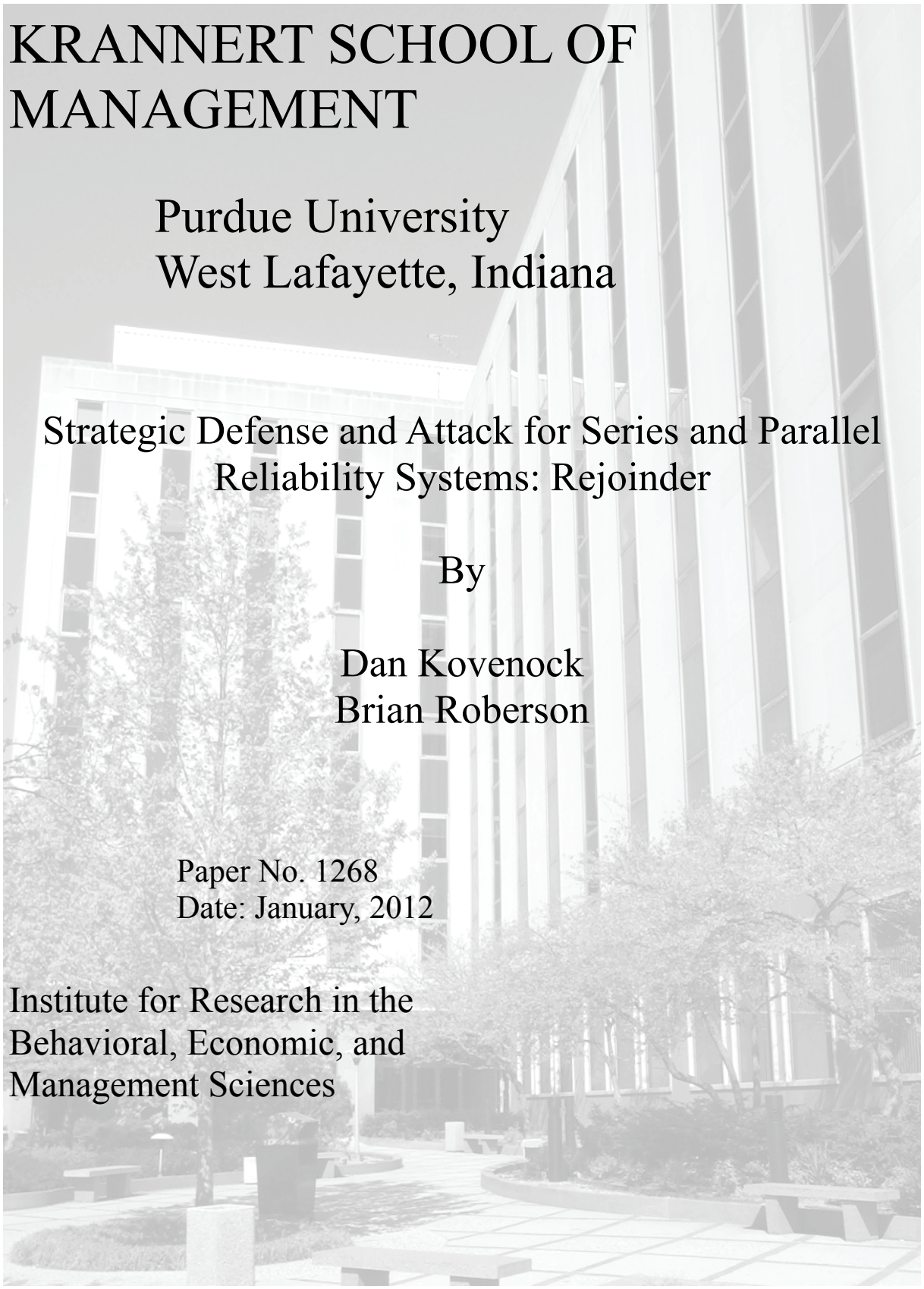

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Strategic Defense and Attack for Series and Parallel Reliability Systems: Rejoinder

By

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Strategic Defense and Attack for Series and Parallel Reliability Systems: Rejoinder

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Abstract

In our original comment, we showed that Hausken's characterization of Nash equilibrium is invalid for much of the parameter space examined and provided necessary conditions for his solution to hold. Most of the comments in his reply are either tangential or irrelevant. However, several of the claims made in the reply reveal continuing misunderstandings and gaps in his understanding. In this rejoinder, we briefly clarify the fundamental issues.

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Our original comment, Kovenock and Roberson (2012),[1] demonstrates that the supposed characterization of Nash equilibrium provided by Hausken is erroneous for much of the strategy space which he examines. In the reply to our comment, Hausken (2012) defends his characterization by stating that: "The term equilibrium is not present in Hausken's (2008) paper." This statement is correct. The approach in Hausken (2008), which appears in many of Hausken's contributions, is to use a slight of hand trick, involving the use of misleading statements and critical ad hoc behavioral assumptions that are not clearly identified as assumptions, that makes it appear as if the paper is appealing to a standard equilibrium concept, when this is not, in fact, the case.

Consider the following description, from Hausken (2008), regarding player objectives and the solution concept:

> The objective of this article is to extend the research where an entire system of independent components is defended by a fully strategic defender and attacked by a fully strategic attacker. The external threat is neither static, fixed, nor immutable. Series and parallel systems are considered. The defender and attacker adapt to each other optimally choosing continuous strategic variables for each component under defense and attack. The reliability of each component depends on the relative investments the defender and attacker direct into defending versus attacking that component. The defender seeks to maximize the reliability of the system, accounting for its assessment of the value of the system, while the attacker seeks to minimize the reliability, accounting for its often different assessment of the system value. (p. 857)

To most, this language would seem to imply that Hausken (2008) involves a fully game-theoretic analysis that appeals to a standard equilibrium concept — in this case Nash equilibrium. Indeed, the author himself seems to be confused over the issue. In attempting to respond to our critique Hausken (2012) claims, on the one hand, to have (#1) provided a characterization of pure-strategy Nash-equilibrium and identifed when it exists, so that our contribution is claimed to be unoriginal, and, on the other hand, to have (#2) never mentioned the word "equilibrium," so that our critique is misguided. Of course, it is not appropriate to claim to be solving for the set of Nash equilibria and at the same time claim to not be employing an equilibrium concept. This is simply a cover for sloppy analysis and ad hoc assumptions.

Apparently we, as does Hausken interpretation #1 above, took the discussion from page 857 of Hausken (2008) above to mean that the Nash equilibrium concept was being employed.

In this case, the prose following equation (17) in Hausken (2008) seems nonsensical:

> When (17) gives negative utilities, a corner solution emerges with zero utility and zero investment for that agent (either the defender or the attacker) which according to (17) would otherwise get negative utility. Using (12) or (13), the other agent gets a utility equal to the value of the system by investing arbitrarily small but positive amounts into defending or attacking the components. (p. 864)

Obviously, when negative utilities arise at least one of the players is not playing a best response to the other (actually in the pair of pure strategies mentioned above neither are) so this does not in fact constitute a Nash equilibrium. Moreover, the critique of our analysis that Hausken offers in his response, stating that because we are using the language of best responses, it must mean that we are discussing a sequential game is utter nonsense. In making this claim, Hausken appears to be unaware of the representation of Nash equilibrium as a fixed point of the best-response correspondence so that if the proposed strategy of some player does not represent a best response to the profile that profile is not an equilibrium.

Now consider for a moment the possibility that the wording on page 857 does not imply that Hausken (2008) is appealing to a standard equilibrium concept, and that Hausken interpretation #2 is valid. Given that the analysis relies on an ad hoc behavioral assumption, one would think that his idiosyncratic solution concept would be explicitly defined. However, the only hint that the reader is given appears on page 864 in the middle of the analysis after the first-order conditions have been solved and is presented as a fact not an assumption. Moreover, the only place prior to page 864 that the term solution is used is in the discussion following equation (11) on p. 862. Here, it is stated that,

> The characteristics of (11) are such that the ratio form which allows for general and often realistic interior solutions is more interesting to analyze, and is the main focus in this article. (p. 862)

Clearly, this statement is false under interpretation #1. For this statement to be correct under interpretation #2, the reader must somehow decipher that when the term "solution" is used here on page 862 Hausken (2008) is appealing to an ad hoc behavioral assumption that is stated two pages later, on page 864, as a fact not an assumption, and that by "solution" Hausken means that if a pure-strategy Nash equilibrium exists, then the solution is given by the set of pure-strategy Nash equilibria, but, if there exists no pure-strategy Nash equilibrium, then the players are assumed to play the arbitrary pair of pure strategies

mentioned on page 864. And, the reader is to figure all of this out even though the term equilibrium is not present in Hausken (2008).

We should also add that our comment provides a necessary condition for Hausken's characterization to hold, but makes no claims of providing a sufficient condition. In his response to our comment Hausken (2012) now claims:

> Hausken (2008) specifies correctly for which parts of the parameter space the solution is valid versus invalid, and provides necessary conditions. More specifically Hausken (2008) provides a correct solution which is a Nash equilibrium in pure strategies when both players' utilities are positive.

This response indicates a confusion regarding the difference between necessary and sufficient conditions for the existence of a pure-strategy equilibrium. In the first quoted sentence, which clearly involves interpretation #1, Hausken appears to confuse necessary and sufficient conditions with necessary conditions. In the second sentence, which switches to interpretation #2, he claims to provide a sufficient condition for existence of a pure-strategy equilibrium. We only claim to provide a necessary condition for existence of a pure-strategy equilibrium, meaning that parameter configurations that do not satisfy this condition cannot have pure-strategy Nash equilibria. Hausken's second quoted sentence claims that positive player utilities evaluated at the solution to the first-order conditions are sufficient for the existence of pure-strategy Nash equilibria. Although this may indeed be correct, no proof of this claim is provided either in the original article or his response.

Moving on to the tangential and irrelevant comments in Hausken's response, observe that in attempting to rehash basic game theory in general and contest theory in particular Hausken reveals continuing misunderstandings and gaps in his understanding. In particular, it is not immediate to appeal to existence of mixed strategy equilibria in games with discontinuous payoffs, which is why we cite Montiero and Page (2007) and Tian (2009) in our comment (see Baye and Kovenock (2008) for an economic example of non-existence of mixed-strategy equilibrium). Moreover, the Hirshleifer (1995) analysis that Hausken appeals to does not apply to the games that Hausken claims to be analyzing. Hirshleifer (1995), although it uses the same contest success function as does this paper, examines a steady state of a game where there is a common resource base that is contested by the players, so resources one player uses are denied to the rival. This is a different problem from the standard Tullock game where resources are obtained at a constant unit cost, as arises in the current paper. As a consequence, Hirshleifer's analysis and notions of dynamic stability do not apply

to this analysis and citing Hirshleifer's paper to justify a parameter restriction in this model is erroneous.

In our original comment, we note that many of the related articles in this literature examining multi-stage or sequential games assume that there exists an equilibrium in which each player evenly allocates forces among targets. We argue that these models also require corresponding parameter restrictions. In addressing this issue in his reply, Hausken focuses his discussion on the existence of pure-strategy equilibria in sequential-move games with finite action spaces. This is just one more example of a statement that is designed to mislead the reader. For this class of games, the issue is not with the existence of a pure-strategy equilibrium, but, as stated in our original comment, with "the existence of a pure-strategy equilibrium with an even allocation of forces."

To summarize, we stand by our claim that Hausken does not characterize the set of Nash equilibria of the game he examines. The scope of the results he claims to derive is misunderstood in the literature and continues to be misunderstood by the author. For the type of weakest-link network model that he examines Clark and Konrad (2007) have shown that pure-strategy Nash equilibria do not exist for a sufficiently high value of the attacker even for the prototypical case in which all players have identical costs and the exponent in the Tullock contest success function at each target is one. They provide an equilibrium in mixed strategies. Kovenock and Roberson (2010b) have characterized equilibrium when the outcome at each target is determined by an all-pay auction and show that nondegenerate mixed strategies are required. These two cases serve as useful benchmarks although a complete characterization of the set of mixed-strategy equilibria when no pure-strategy equilibrium exists remains an open problem.

Notes

[1]A working paper version of our comment first appeared in 2010 (Kovenock and Roberson 2010a) and elicited a response in the Munich Personal RePEc Archive (Hausken 2010).

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