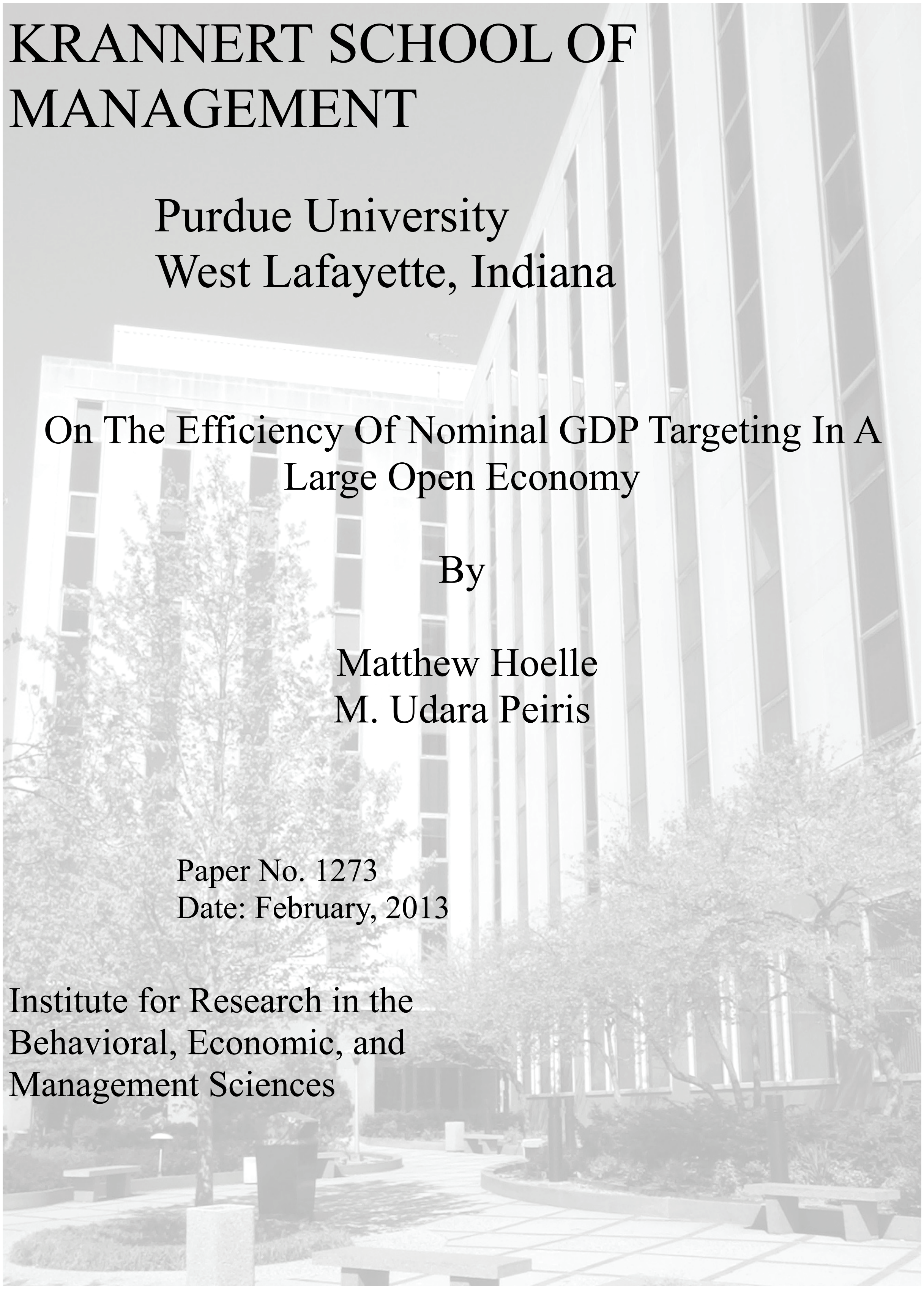

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On The Efficiency Of Nominal GDP Targeting In A Large Open Economy

By

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On the Efficiency of Nominal GDP Targeting in a Large Open Economy[1]

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[4]The latest version of this paper and an extended working paper is available on the authors' websites: http://www.matthew-hoelle.com/ and http://www.udarapeiris.org/

Abstract

Since 2007 there have been increasing calls to abandon a regime of Stabilizing Inflation (SI) in favor of Nominal GDP (NGDP) targeting. One argument in favor of NGDP targeting is that it allows inflation to redistribute resources among bond holders efficiently. Here we examine this claim in a large open monetary economy and show that, in contrast to SI, NGDP targeting is in fact (Pareto) efficient in a world with stochastic real uncertainty, and in the absence of complete insurance markets (only nominally risk free bonds are available). However this result is ultimately fragile and breaks down once we attempt to deviate from the simplistic setting necessary for the result to hold

Key words: monetary policy; open economy; uncertainty; incomplete markets; Pareto efficiency.

JEL classification numbers: D50; E31; E40; E50; F41.

1 Introduction

The abrupt end of the Great Moderation in 2007 has raised calls to abandon a regime of Stabilizing Inflation (SI) and the Taylor Rule[1] in favor of a regime of nominal GDP (NGDP) targeting whereby nominal income, rather than nominal prices, follows a stationary path[2]. We examine the claim that the path of inflation consistent with a NGDP target policy provides additional risk-sharing opportunities to households. We study the role that inflation plays in allocating resources between bondholders across countries in a world with incomplete financial markets and the implications for the optimal conduct of monetary policy and show that, even if agents are unable to insure against risk via existing asset markets (a complete set of asset markets, in an Arrow-Debreu sense, is unavailable) it is true that, under specific conditions, a global policy of NGDP targeting provides natural risk sharing opportunities to households to the extent that they can completely hedge risk (achieving the Pareto optimal allocation). However we find this result is ultimately fragile and breaks down once we attempt to deviate from the simplistic world necessary for the result to hold.

The most common arguments in favor of the NGDP target are that it (i) stabilizes employment as wage bills depends more on nominal income (than on the rate of inflation), (ii) limits asset market instability as asset bubbles tend to form when NGDP growth is higher than average, and (iii) stabilizes demand more effectively as an adverse supply shock is automatically divided between inflation and real GDP, in contrast to the excessive tightening in response to adverse supply shocks by stabilizing inflation.[3]

[1]SI, while optimal in economies with pricing frictions in the goods market, may be suboptimal in economies with financial frictions such as credit constraints or the possibility of default, since these frictions are persistent and propagate across markets. An example of recent work on monetary models with credit frictions includes Christiano et al. (2010). To fully analyze the welfare implications of SI, information and credit frictions should also be formally modeled. See Faia and Monacelli (2007), Carlstrom et al. (2010) and Fiore and Tristani (2012).

[2] Notable recent proponents include Crook (2011), Krugman (2011), Romer (2011),Frankel (2012), Sumner (2012) and Woodford (2012). See Hall and Mankiw (1994) for an excellent overview of the historical debate on nominal income targeting.

[3]As Koenig (2012*a*) argues:

> Suitably implemented, nominal-GDP targeting is the more-familiar Taylor rule's close cousin. The main difference between the two policy approaches is that nominal-GDP targeting takes a longer-term view of inflation than does the Taylor rule when pointing to a loose or tight policy setting. Consequently, nominal-GDP targeting is less forgiving of near-term expected deviations of inflation from long-run target inflation than is the Taylor rule: It requires that any near-term expected inflation shortfall be offset by expected forward inflation overages so that expected average inflation over the medium term holds steady. This unwillingness to let bygones be bygones can be an advantage if the zero bound limits current policy stimulus. Under nominal-GDP targeting, the monetary authority promises future accommodation to offset the current tighter-than-desired policy stance.

The other claim in favour of NGDP targeting stems from the Fisher debt-deflation story, whereby inflation redistributes resources among bond holders. The argument, briefly, is that there exists an optimal stochastic rate of inflation, consistent with that obtained under NGDP targeting, that supports an efficient allocation of resources (i.e. optimal risk sharing) in the absence of complete asset markets. To our knowledge, examinations of this claim have required the endowments of households to be perfectly correlated and have focused entirely on closed economies. As a result, they have ignored the role of the current account in distributing resources across countries exposed to *idiosyncratic* fluctuations in real income. Our purpose is to disentangle the effects of inflation in redisributing resources, from the specific endowment structure necessary to obtain effiiciency. An open economy provides a natural setting to do this.

We study the role that inflation plays in redistributing income across international bond holders who are subject to *idiosyncratic* and possibly also *aggregate* risk and show that even in the absence of any insurance markets (only nominal risk-free bonds are available), NGDP targeting by all countries in a large open economy can support the Pareto optimal allocation. In an economy with the same (real) fundamentals, switching to an SI regime by at least one country destroys these risk-sharing opportunities. This is because in a large open economy, optimal policy that would support the Pareto optimal allocation under SI requires a combination of active monetary (interest rate) policy and exchange rate policy which would violate the implicit debt constraint of the infinite horizon model. Our results are similar in spirit to Koenig (2012*b*) and Sheedy (2012) though we consider a large open economy, with an explicitly modeled monetary structure, and our results do not depend on the transfer of resources between creditors and debtors. Rather, our results describe how inflation affects the efficiency of the transfers of resources between bond holders across countries; the distinction between debtors and creditors is inconsequential in our analysis.

We consider a monetary economy of large open economies inhabited by homogeneous households. Money has value via a cash-in-advance constraint. The only assets are nominally risk-free bonds denominated in each currency. Monetary policy and exchange rate intervention affect either (i) the money supplies to change the price level and/or (ii) interest rates to affect the real value of domestic output. Under a global policy of nominal GDP targeting, the nominal value of endowments remains constant across states of uncertainty. This implies that the endowments are inversely related to the nominal price level. This setting naturally achieves the desired change in the real value of the nominal assets, as the real payouts span both the individual and the aggregate endowments. There is no role for active exchange rate policy and monetary policy is completely characterized by the monetary growth rate. On the other hand, if inflation is to be stabilized, monetary policy must be contingent on the idiosyncratic endowment realizations. This affects both the real value of the bond payoffs and the real value of endowments via the inflation tax on money balances. Pareto efficiency requires an additional degree of (policy) freedom which is obtained through exchange rate intervention. However such policy violates the debt constraint and hence Pareto efficiency cannot be an equilibrium outcome by stabilizing inflation (over a generic subset of endowments).[4]

This paper proceeds as follows. Section 2 presents a stylized economy that captures the essence of the risk sharing argument under different monetary policies. We then present a formal model with an explicit monetary structure in Section 3 and the efficiency of monetary policies in Section 4. Finally in Section 5 we give some concluding remarks about the robustness of the arguments in favour of NGDP targeting and how extensions and generalization to the model affect the results.

2 Motivating Example

The debt-deflation story of Irving Fisher requires, in a rational expectations world, the inability to fully insure for inflation to have real effects. Incompleteness of asset markets is a realistic description of the functioning of modern asset markets, and avoids the extreme assumption of the standard Arrow-Debreu model that all agents meet at one moment in time and trade assets that allow for every conceivable contingency in the infinitely long future. The absence of complete contingent markets may arise for several reasons.[5] First, asymmetric information may imply that certain events are unobservable to all market participants. Second, households may not have access to all asset markets, possibly for reasons of moral hazard. Third, transactions costs may prohibit the opening of additional markets.

When there are missing asset or insurance markets (incomplete markets), competitive allocations are generically inefficient[6] and one may envisage a role for active monetary policy and exchange rate management that would otherwise be unnecessary under complete markets. Our purpose in such a setting is to characterize policy that supports the (Pareto) efficient allocation of resources in a large open economy. Such policy deviates from (current) monetary orthodoxy[7], but is consistent with the arguments in the theory of incomplete markets that proceeded the seminal works of Arrow and Debreu.

In the open economy context we consider here, the incompleteness of markets may arise quite naturally. We can rationalize the domestic representative agent as the outcome of

[4]In an extended working paper we show that an optimal combination of monetary and exchange rate policy supports the Pareto optimal allocation under SI in a 2-period model. An optimal policy exists because the terminal period provides an additional degree of freedom for central banks.

[5]See Geanakoplos (1990) for an excellent overview of the general equilibrium literature on incomplete markets.

[6]See Stiglitz (1982) and Geanakoplos and Polemarchakis (1986).

[7]Nominal income targeting has been formally discussed since the late 1970's in monetary economics. Early contributions to the contemporary literature on nominal income targeting include Meade (1978), von Weizsacker (1978), Tobin (1980) and Bean (1983).

domestic markets being complete, whereas if there are limited risk sharing opportunities internationally, then *global markets* would be incomplete. Under the appropriate conditions, complete markets is observationally equivalent to a representative agent. In this sense, our global market incompleteness captures the inability of households to fully hedge foreign income risk. Though not formally modeled here, reasons that households may be unable to fully hedge foreign risk are incomplete information about foreign markets or even regulations that prohibit foreign capital flows into certain domestic markets.

Efficiency under incomplete markets requires (following Geanakoplos and Shubik (1990) and LeRoy and Werner (2001)) that the real payouts of the available nominal assets span both the individual and the aggregate endowment. This span condition can be achieved in one of two ways: (i) change the nominal payouts themselves or (ii) change the real value by changing the price level. Any economy with nominal contracts is characterized by its degrees of indeterminacy, nominal and possibly real.[8] This indeterminacy can be removed by formally modeling money and, more precisely, the actions of a central bank. If the underlying indeterminacy is real, as in the incomplete markets economy we consider, then these actions determine the equilibrium allocation.[9] In this motivating example we present only the efficiency of prices in equilibrium. In contrast, the formal model that follows in Section 3 will be determinate and will allow us to explicitly characterize the efficiency of monetary policy.

The proceeding analysis shows that when markets are incomplete, optimal monetary policy in a large open economy must take the form of nominal income targets *globally*. That is the price level should vary inversely with future output to allow inflation to redistribute resources among bond holders. We illustrate our argument with a stylized 2-period economy which captures the essence of the argument concisely. Importantly, the analysis of this 2-period economy dictates that our results are not specific to the particular monetary structure that we later impose.

2.1 Stylized Large Open Nominal Economy

Consider a 2-period large open economy with two countries, incomplete markets, and nominal assets with S states of uncertainty in the final period. In the initial period, households trade assets, but do not consume nor receive endowments. In the final period, one of S possible states occurs. In each state, one physical commodity is traded. There exist 2 countries: home (h) and foreign (f). Each country is populated by homogeneous households of equal measure, with real GDP (endowment) vectors given by $y_h = (y_h(1), ..., y_h(S))$

[8]There is an extensive literature on indeterminacy of monetary equilibria. An important example is Geanakoplos and Mas-Colell (1989).

[9]See Bloise and Polemarchakis (2006) for an overview on formally modeling money in a general equilibrium setting.

and $y_f = (y_f(1), ..., y_f(S))$. Total world output is denoted $Y = (Y(1), ..., Y(S))$ where $Y = y_h + y_f$. As the goods are identical, the law-of-one-price prevails and the real exchange rate is trivially one. The assets available to the households are nominal bonds, one bond issued by each country, with risk-free nominal payouts in the issuing country's currency.

Denote the equilibrium commodity price for the home country as $p = (p(1), ..., p(S))$ and that for the foreign country as $p^* = (p^*(1), ..., p^*(S))$. When solving the households' problem, we typically express the budget constraints in terms of real payouts, so the real asset payouts of the two assets (the home bond and the foreign bond) in each of the S states of uncertainty in the final period are expressed by the following payout matrix:

$$R = \begin{bmatrix} \frac{1}{p(1)} & \frac{1}{p^*(1)} \\ \vdots & \vdots \\ \frac{1}{p(S)} & \frac{1}{p^*(S)} \end{bmatrix}.$$

Define the span of the payout matrix as $\langle R \rangle$. Mathematically, the span is the $2-$dimensional linear subspace of the $S-$dimensional Euclidean space that can be achieved through a vector of bond holdings (home and foreign): $\langle R \rangle = \left\{ x \in \mathbb{R}^S : x = R\theta \text{ for some } \theta \in \mathbb{R}^2 \right\}$.

Definition. Span Condition
The endowment vectors (y_h, y_f) both lie in the span of the payout matrix $R : y_h \in \langle R \rangle$ and $y_f \in \langle R \rangle$.

If this *span condition* is met, the household in the home country can use the home bond to hedge against domestic output risk and the foreign bond to hedge against aggregate output risk. The span condition implies that the total endowment Y will also lie in the span, $Y \in \langle R \rangle$. Provided that the preferences of the representative agents in each country are identical and homothetic, the Pareto optimal allocation is such that all households consume a constant fraction of the total world output in all states: θ_h for home households and θ_f for foreign households with $\theta_h + \theta_f = 1$.

At the optimal allocation, the excess demand for the home households is $\theta_h Y - y_h \in \langle R \rangle$, which belongs to the span since both domestic output y_h and aggregate endowment Y do. Thus, this excess demand can be supported by an appropriate vector of bond holdings. The same conclusion is reached for the foreign households. Market clearing for the bonds holds trivially.

2.2 Policies and the Span Condition

We now turn to what equilibrium prices and monetary policies must prevail to ensure Pareto efficiency. Provided that velocity is state-independent (though not necessarily identical across countries), the quantity theory of money states that the money supply in each country is linearly related to both the commodity price and domestic output. Assuming this relation holds, the *span condition* holds for any economy if and only if the money supplies are stable, i.e., the money supplies are state-independent. This invariance is how we define nominal GDP targeting. Formally, the real asset payoff matrix is equivalent to:

$$\hat{R} = \begin{bmatrix} y_h(1) & y_f(1) \\ \vdots & \vdots \\ y_h(S) & y_f(S) \end{bmatrix}.$$

With constant domestic and foreign money supplies, the matrix $\hat{R}$ is a linear transformation of the matrix R, meaning that the spans are identical: $\left\langle \hat{R} \right\rangle = \langle R \rangle$. And since $y_h \in \left\langle \hat{R} \right\rangle$ and $y_f \in \left\langle \hat{R} \right\rangle$ by definition, the span condition is satisfied.

Notice that the result permits both aggregate risk ($Y(s)$ can vary across states) and idiosyncratic risk (y_h and y_f are not perfectly correlated).

With an arbitrary number of states of uncertainty (consider $S > 2$), the span $\langle R \rangle$ is a measure zero subset of the $S-$dimensional Euclidean space. Thus, if policies result in a span $\left\langle \hat{R} \right\rangle \neq \langle R \rangle$, then the Pareto optimal allocation is typically unable to be supported as an equilibrium allocation. For any policies other than global nominal GDP targeting, $\hat{R}$ is not a linear transformation of R, meaning $\left\langle \hat{R} \right\rangle \neq \langle R \rangle$. Thus, a policy of stabilizing inflation by at least one country suffices to destroy the risk-sharing opportunities of the financial structure.

Consider the consequences if the domestic households are not homogeneous, but heterogeneous with endowments that are not perfectly correlated. For this economy, all household endowments (both the heterogeneous domestic households and the homogeneous foreign households) must lie in the span, but the set of household endowments is at least a $3-$dimensional subspace,[10] while the span remains at most a $2-$dimensional subspace. Thus, the appropriate span condition for within-country heterogeneity cannot be satisfied.

In conclusion, prices that are consistent with both the quantity theory of money and Pareto optimality exist if and only if (i) global policies of nominal GDP targeting are adopted and (ii) domestic financial markets are complete.

The risk-sharing implications of this motivating example are the central message from Koenig (2012*b*) and Sheedy (2012). In fact, the Koenig (2012*b*) result is a special case of the above result for a closed economy with aggregate risk only (the individual household risk is perfectly correlated in Koenig (2012*b*)). However, in a monetary economy, the role of policy and its effect on the determinacy of prices in sustaining Pareto optimality is nontrivial and an issue to which we soon turn.[11]

[10]The subspace will be equal to, in general, the number of households in the world economy whose endowments are not perfectly correlated.

[11]Determinacy is an important result in our chosen monetary structure. In a closed economy with an OLG monetary structure, Sheedy (2012) also verifies the efficiency of nominal GDP targeting, but not the determinacy of such a policy (see Kehoe and Levine (1990) and Feng (2012) for the difficulties of obtaining

2.3 Supporting the Pareto Efficient Equilibria

Knowing now that prices exist that support the Pareto efficient allocation, what can be said about the *implementability* of these prices and which ones are supported by monetary policy (i.e. the determinacy of the equilibrium)? In general, the equilibrium above displays both nominal and real indeterminacy: setting either money supplies or interest rates is not enough to determine a locally unique optimal policy.[12] These questions cannot be answered from the above analysis, which only explores the risk-sharing implications of a nominal economy. To answer these questions, we must formally model a monetary structure. Such analysis provides not only an incentive for households to hold money, but also makes prices determinate and a function of central bank policy. Thus, the task for the remainder of the paper is to introduce a proper monetary structure and investigate if the risk-sharing implications of this example continue to hold in a determinate monetary economy where central bank policy has real effects.[13]

3 The Model

Consider an economy with an infinite time horizon. In each discrete period $t \geq 0$, one of S possible shocks $s \in \mathcal{S}$ is realized. Denote the shock occurring in period t as s_t. We represent the resolution of uncertainty by an event tree Σ, with a given date-event $\sigma \in \Sigma$. Each date-event σ is characterized by the history of shocks up to and including the current period $s^t = (s_0, ..., s_t)$.[14] The root of Σ is the date-event σ_0 with realization s_0, where $s_0 \in \mathcal{S}$ is a fixed state of the economy. Each $\sigma \in \Sigma$ has S immediate successors that are randomly drawn from $\mathcal{S}$ according to a Markov process with transition matrix Π. Each $\sigma \in \Sigma$ has a unique predecessor, where the unique predecessor of the date-event s^t is s^{t-1}. Finally, for a variable $x(s^t)$ at date-event s^t, the variables in the set of successor date-events are $(x(s^t, s))_{s \in \mathcal{S}}$.

3.1 Households

There are two countries: home (h) and foreign (f).[15] In each country there is a continuum of homogeneous households of measure 1. At each date-event, households are endowed with an

determinacy in an OLG framework).

[12] There is a long standing literature in the general equilibrium with incomplete markets literature on the indeterminacy of equilibria with financial contracts. For an important example in the context of an open economy see Polemarchakis (1988).

[13] The conditions for determinacy of monetary equilibria with nominal assets and incomplete markets in a closed setting are provided by Dubey and Geanakoplos (2006), while the conditions for a deterministic setting of an open economy are provided by Tsomocos (2008).

[14] The vector s^t is equivalently interpreted as an ordered set, so that $s \in s^t$ refers to a particular shock in the history of shocks up to period t.

[15] We have adopted the notational conventions of Svensson (1985).

amount of the single, homogeneous good, which is identical in both countries. In date-event s^t, the endowment of the home household is $y_h(s_t) > 0$ and the endowment of the foreign household is $y_f(s_t) > 0$; that is, endowments only depend on the current realization s_t. The economy does not contain any aggregate risk, meaning that $y_h(s_t) + y_f(s_t) = Y$ for any shock $s_t \in \mathcal{S}$. This assumption is made for expositional convenience, meaning that our results remain valid without it.

The home households in date-event s^t choose the consumption vector $(c_h(s^t), c_f(s^t))$, where $c_h(s^t) \geq 0$ is the consumption of the home commodity and $c_f(s^t) \geq 0$ is the consumption of the foreign commodity. The goods are identical across countries, so the home household preferences are only defined over the sum $c(s^t) = c_h(s^t) + c_f(s^t)$. Similarly, the foreign household consumption vector in date-event s^t is defined by $\left(c_h^*(s^t), c_f^*(s^t)\right)$ with $c^*(s^t) = c_h^*(s^t) + c_f^*(s^t)$.

The probability that the date-event characterized by s^t is realized is defined recursively as

$$\pi(s^t) = \pi(s^{t-1}) \cdot \pi(s_t|s_{t-1}),$$

where the initial probabilities $\pi(s_0)$ are parameters of the model and $\pi(s_t|s_{t-1})$ are elements of the Markov transition matrix Π.

The household preferences are identical and characterized by the utility function

$$\sum_{t,s^t} \beta^t \pi(s^t) u\left(c(s^t)\right), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor. The utility function, $u(c)$, satisfies standard conditions:

Assumption 1. *The utility function, $u : \mathbb{R}_{++} \to \mathbb{R}$, is C^1, differentiably strictly increasing, and strictly concave. The Inada condition holds:*

$$\lim_{c\to 0} u_1(c) = \infty.$$

3.2 Monetary Structure

We follow the monetary cash-in-advance structure of Nakajima and Polemarchakis (2005) and Peiris and Polemarchakis (2012).[16] Our timing convention is such that transactions occur after uncertainty is realized so that there is only a transactions demand for money. We assume a unitary velocity of money.

Consider the initial date-event s_0. The home household and the foreign household begin this date-event with nominal assets $w(s_0)$ and $w^*(s_0)$, respectively, where each is valued in terms of the local currency.

[16] These models are closely related to the open economy models of Lucas (1982) and Geanakoplos and Tsomocos (2002) and the open economy model with incomplete markets of Peiris and Tsomocos (2012).

The timing proceeds as follows. First, the asset market opens, in which cash and the bonds, one from each country, are traded. Additionally, the currency market opens, in which cash denominated in one currency is traded for cash denominated in another currency. Let $e^*(s_0)$ be the nominal exchange rate for the foreign country (number of units of home currency for each unit of foreign currency) and $e(s_0)$ is the exchange rate for the home country, where $e(s_0) = \frac{1}{e^*(s_0)}$. Let $r(s_0)$ and $r^*(s_0)$ denote the nominal interest rates for the home and foreign country, respectively, implying that $\frac{1}{1+r(s_0)}$ is the price of the nominal bond in the home currency and $\frac{1}{1+r^*(s_0)}$ is the price of the nominal bond in the foreign currency. Accounting for the foreign exchange markets, the budget constraint for the home household in terms of the home currency (and similarly for the foreign household) is given by:

$$\hat{m}_h(s_0) + e^*(s_0)\hat{m}_f(s_0) + \frac{b_h(s_0)}{1+r(s_0)} + e^*(s_0)\frac{b_f(s_0)}{1+r^*(s_0)} \leq w(s_0). \tag{2}$$

The variables $\hat{m}_h(s_0)$ and $\hat{m}_f(s_0)$ are the amounts of the home and foreign currency held, while $b_h(s_0)$ and $b_f(s_0)$ are the home and foreign bond positions (net savings). Cash amounts are nonnegative variables, while the bond holdings can take any values.

The market for goods opens next. Denote $p(s_0)$ and $p^*(s_0)$ as the commodity prices in the home and foreign country, respectively. The purchase of consumption goods is subject to the cash-in-advance constraints:

$$\begin{aligned} p(s_0)c_h(s_0) &\leq \hat{m}_h(s_0), \\ p^*(s_0)c_f(s_0) &\leq \hat{m}_f(s_0). \end{aligned} \tag{3}$$

The home household also receives cash by selling its endowment, $y_h(s_0)$. Hence, the amount of cash that it carries over to the next period is

$$\begin{aligned} m_h(s_0) &= p(s_0)y_h(s_0) + \hat{m}_h(s_0) - p(s_0)c_h(s_0). \\ m_f(s_0) &= \hat{m}_f(s_0) - p^*(s_0)c_f(s_0). \end{aligned} \tag{4}$$

Given (4), the cash-in-advance constraints (3) are equivalent to

$$\begin{aligned} m_h(s_0) &\geq p(s_0)y_h(s_0), \\ m_f(s_0) &\geq 0. \end{aligned} \tag{5}$$

In equilibrium, the Law of One Price must hold, meaning that $p(s_0) = e^*(s_0)p^*(s_0)$. Using this fact and substituting for $m_h(s_0)$ and $m_f(s_0)$ from (4) into (2) yields the budget constraint in date-event s_0 :

$$p(s_0)c(s_0) + \frac{b_h(s_0)}{1+r(s_0)} + e^*(s_0)\frac{b_f(s_0)}{1+r^*(s_0)} \leq w(s_0). \tag{6}$$

The transactions of the home household in all date-events s^t for $t > 0$ are similar. Maintain the notational convention with nominal exchange rates as $e(s^t)$ and $e^*(s^t)$ for home and foreign respectively, nominal interest rates as $r\left(s^t\right)$ and $r^*\left(s^t\right)$ for home and foreign respectively, and commodity prices $p(s^t)$ and $p^*(s^t)$ for home and foreign respectively. The budget constraint that the home household faces in date-event s^t is given by

$$\begin{aligned} p(s^t)c(s^t) + \frac{b_h(s^t)}{1+r(s^t)} + e^*(s^t)\frac{b_f(s^t)}{1+r^*\left(s^t\right)} & \\ \leq p(s^{t-1})y_h(s^{t-1}) + b_h(s^{t-1}) + e^*(s^t)b_f(s^{t-1}). & \end{aligned} \tag{7}$$

Similar to (5), the cash-in-advance constraints are given by:

$$\begin{aligned} m_h(s^t) &\geq p(s^t)y_h(s^t), \\ m_f(s^t) &\geq 0. \end{aligned} \tag{8}$$

The debt constraint requires that the real value of the portfolios are uniformly bounded:

$$\inf_{t,s^t}\left[\frac{b_h(s^t)}{\left(1+r(s^t)\right)p(s^t)} + \frac{b_f(s^t)}{\left(1+r^*\left(s^t\right)\right)p^*(s^t)}\right] > -\infty. \tag{9}$$

These debt constraints suffice to rule out Ponzi schemes and guaranteee an optimal solution to the household problem.[17] We consider the real values of the portfolios as we allow for inflation in the economy resulting from the selected monetary policy.

Define the choice vectors as $c, m_h, m_f \in \ell_+^\infty$ and $b_h, b_f \in \ell^\infty$ for the home household (and $c^*, m_h^*, m_f^* \in \ell_+^\infty$ and $b_h^*, b_f^* \in \ell^\infty$ for the foreign household), where $c = \left(c\left(s^t\right)\right)_{t\geq 0, s^t}$ is the infinite sequence of consumption choices for all date-events, with similar definitions for all other choice vectors. Define the home equilibrium price vectors as $e, r, p \in \ell_+^\infty$ and the foreign equilibrium price vectors as $e^*, r^*, p^* \in \ell_+^\infty$, where $r = \left(r\left(s^t\right)\right)_{t\geq 0, s^t}$ is the infinite sequence of nominal interest rates.[18]

The problem for the home household is given as follows (and symmetrically for the foreign household):

$$\begin{array}{ll} \max\limits_{c,m_h,m_f,b_h,b_f} & \sum_{t,s^t}\beta^t\pi(s^t)u\left(c(s^t)\right) \\ \textit{subj. to} & \text{budget and CIA constraints (6) and (5) in } s_0, \\ & \text{budget and CIA constraints (7) and (8) in } s^t, \\ & \text{debt constraint (9).} \end{array} \tag{10}$$

3.2.1 The Monetary-fiscal Authority

Each country contains a monetary-fiscal authority whose responsibilities include monetary (interest rate) policy and exchange rate policy.

[17] See Hernandez and Santos (1996) and Beker and Chattopadhyay (2010) for the sufficiency of such a condition.

[18] ℓ^∞ is the set of bounded infinite sequences, with respect to the sup norm. It is a metric space and more specifically a Banach space (one can apply the contraction mapping theorem to verify that the value function is well-defined and the Bellman equation is valid). ℓ_+^∞ is the set of nonnegative bounded infinite sequences. Further details on this can be found in Bertsekas (1976).

The parameters $W(s_0)$ and $W^*(s_0)$ are the nominal payments owed to the household in the home and foreign country, respectively, where the debt is owed by the monetary-fiscal authority in each country. In the initial date-event s_0, the monetary-fiscal authority in the home country chooses the domestic money supply $M(s_0)$, the domestic debt obligations $B_h(s_0)$, and the foreign debt obligations $B_f(s_0)$. The money supplies are nonnegative, while the debt obligations can be either positive (net borrow) or negative (net save). The similar choices for the monetary-fiscal authority in the foreign country are $M^*(s_0)$, $B_h^*(s_0)$, and $B_f^*(s_0)$. The constraint in s_0 for the monetary-fiscal authority in the home country (and similarly for the foreign country) is given by:

$$M(s_0) + \frac{B_h(s_0)}{1+r(s_0)} + e^*(s_0)\frac{B_f(s_0)}{1+r^*(s_0)} = W(s_0). \tag{11}$$

Similarly, the constraint in date-event s^t for any $t > 0$ is given by:

$$M(s^t) + \frac{B_h(s^t)}{1+r(s^t)} + e^*\left(s^t\right)\frac{B_f(s^t)}{1+r^*(s^t)} = M(s^{t-1}) + B_h(s^{t-1}) + e^*\left(s^t\right)B_f(s^{t-1}). \tag{12}$$

Define the choice vectors as $M \in \ell_+^\infty$ and $B_h, B_f \in \ell^\infty$ for the home household (and $M^* \in \ell_+^\infty$ and $B_h^*, B_f^* \in \ell^\infty$ for the foreign household), where $M = (M(s^t))_{t\geq 0, s^t}$ is the infinite sequence of money supplies for all date-events, with similar definitions for all other choice vectors.

3.3 Sequential Competitive Equilibria

The market clearing conditions are such that $w(s_0) = W(s_0)$ and $w^*(s_0) = W^*(s_0)$ hold in the initial date-event and for all date-events s^t :

$$\begin{aligned}
c_h(s^t) + c_h^*(s^t) &= y_h(s_t), \\
c_f(s^t) + c_f^*(s^t) &= y_f(s_t), \\
m_h(s^t) + m_h^*(s^t) &= M(s^t), \\
m_f(s^t) + m_f^*(s^t) &= M^*(s^t) \\
b_h(s^t) + b_h^*(s^t) &= B_h\left(s^t\right) + B_h^*(s^t), \\
b_f(s^t) + b_f^*(s^t) &= B_f\left(s^t\right) + B_f^*(s^t).
\end{aligned}$$

A sequential competitive equilibrium is defined as follows.

Definition. Given initial nominal obligations $W(s_0)$ and $W^*(s_0)$, a sequential competitive equilibrium consists of an allocation (c, c^*), household money holdings $\left(m_h, m_f, m_h^*, m_f^*\right)$, household portfolios $\left(b_h, b_f, b_h^*, b_f^*\right)$, money supplies (M, M^*), monetary-fiscal authority debt positions $\left(B_h, B_f, B_h^*, B_f^*\right)$, interest rates (r, r^*), commodity prices (p, p^*), and exchange rates (e, e^*) such that:

1. the monetary-fiscal authorities satisfy their constraints (11) and (12);
2. given interest rates (r, r^*), commodity prices (p, p^*), and exchange rates (e, e^*), households solve the problem (10);
3. all markets clear.

3.4 Exchange Rate Policy

We define moneary-fiscal policy without and with foreign exchange intervention. The two policies differ in that foreign exchange intervention permits monetary-fiscal authorities to hold debt obligations in the foreign bond.

Definition. Monetary-fiscal policy without foreign exchange intervention

Zero international debt positions, $B_f(s^t) = B_h^*(s^t) = 0$ for all date-events s^t.

Definition. Monetary-fiscal policy with foreign exchange intervention

Non-zero international debt positions, $B_f(s^t) \in \mathbb{R}$ and $B_h^*(s^t) \in \mathbb{R}$ for some date-events s^t.

We now consider the existence and determinacy of our equilibria.

3.5 Existence

This paper focuses attention on two policies of monetary-fiscal authorities: nominal GDP targeting (NGDP) and stabilizing inflation (SI), though the results can equivalently be stated in terms of global NGDP and the complementary set of all other policy combinations. The policy of NGDP does not require foreign exchange intervention, while the policy of SI does. Thus, rather than specify existence and determinacy results in terms of the target values for the money supplies (for NGDP) and the target values for the interest rates (for SI, since the rate of inflation is directly related to the interest rate via the Fisher equation), the specified policies of the monetary-fiscal authorities will be the debt positions $\left(B_h, B_f, B_h^*, B_f^*\right)$.

An appropriately chosen vector of debt positions $\left(B_h, B_f, B_h^*, B_f^*\right)$ can support NGDP and SI under the constraints (11) and (12). Thus, the global policy choice is characterized by the vector $\left(B_h, B_f, B_h^*, B_f^*\right)$, which can be viewed as parameters to the households in the economy.

The existence of a sequential competitive equilibrium follows standard arguments. The existence of a truncated equilibrium is known from Theorem 3 of Bai and Schwarz (2006) for the case of initial fiscal transfers $W(s_0)$ and $W^*(s_0)$ and zero fiscal transfers in all future period $t > 0$. The existence of the infinite-horizon analogue of the aforementioned truncated equilibrium follows the standard approach of Levine and Zame (1996) or Magill and Quinzii (1994), as the debt constraints (9) suffice to obtain the necessary uniform bounds.

The existence claim is stated as follows, where the proof follows standard arguments with the key steps discussed above.

Claim 1. *For any given policy vector* $\left(B_h, B_f, B_h^*, B_f^*\right)$, *there exists a sequential competitive equilibrium.*

3.6 Determinacy and Monetary Policy

As we are concerned not only about the efficiency of different monetary policies, but also the *implementability* of these policies, it is important that we verify that the model allows the policy vector $\left(B_h, B_f, B_h^*, B_f^*\right)$ to select a determinate allocation. It is no use to claim that efficiency arises under a particular policy vector if such a policy is also consistent with other equilibria that are inefficient.

The Fiscal Theory of the Price Level (see, e.g., Woodford (1994)) argues that the specification of fiscal transfers suffices to pin down the initial price level and the vector of state prices, provided that the fiscal transfers are fixed. Fixed fiscal transfers imply in a 2-period model that the terminal constraints are not satisfied for off-equilibrium vectors of money supplies, interest rates, and monetary-fiscal authority debt positions. This is the exact setting of our infinite horizon model, as our model only contains strictly positive nominal transfers in the initial period, $W(s_0)$ and $W^*(s_0)$, and zero fiscal transfers in all other date-events. Further, the infinite horizon analogue of the terminal constraint is the debt constraint, which need only be satisfied in equilibrium and not for all possible off-equilibrium vectors.

Theorems 3 and 4 from Bai and Schwarz (2006) verify the determinacy of cash-in-advance equilibria in a closed 2-period economy. Their model notably limits monetary-fiscal authorities to only holding riskless domestic debt positions. Thus, our determinacy result, for the infinite horizon model with the possibility for foreign exchange intervention, subsumes Bai and Schwarz (2006) as special cases. We first describe the special cases and then provide intuition for our determinacy result.

Consider the policy vector $\left(B_h, B_f, B_h^*, B_f^*\right)$ that is consistent with *nominal GDP targeting* for both countries. As previously discussed, the monetary-fiscal authorities can support both the nominal GDP targets and the optimal allocation by only holding domestic debt positions (no foreign exchange intervention). This is the setting of Bai and Schwarz (2006). Thus, Theorem 4 of Bai and Schwarz (2006) is directly applicable and for every policy vector $\left(B_h, B_f, B_h^*, B_f^*\right)$ consistent with a global policy of *nominal GDP targeting*, the set of sequential competitive equilibria is determinate.

Consider the policy vector $\left(B_h, B_f, B_h^*, B_f^*\right)$ that is consistent with *stabilizing inflation* for both countries. It would be straightforward for the monetary-fiscal authorities to support the inflation targets using only domestic debt positions, but as previously mentioned, we provide them with additional policy tools to improve risk-sharing. Specifically, monetary-fiscal authorities can inject or withdraw money from their respective domestic economies in a state-contingent manner by holding foreign debt positions. This additional policy tool allows the monetary-fiscal authorities to target the nominal exchange rate to support a stationary path of inflation. As previously discussed, this additional policy can achieve the Pareto efficient allocation in a 2-period model in which the terminal condition must bind for all monetary-fiscal authority choice vectors. Thus, the results of Bai and Schwarz (2006) are not applicable, but the monetary-fiscal authorities have an additional policy tool (international debt positions) and an additional target (nominal exchange rate).

Extending Bai and Schwarz (2006) to an open economy, we argue that for any combination of policies for the two monetary-fiscal authorities, $\left(B_h, B_f, B_h^*, B_f^*\right)$, the set of sequential competitive equilibria is determinate. This encompasses a global policy of *nominal GDP targeting*, a global policy of *stabilizing inflation*, and all other policy combinations, including asymmetric ones.

Claim 2. *For any given policy vector* $\left(B_h, B_f, B_h^*, B_f^*\right)$, *the set of sequential competitive equilibria is determinate.*

3.7 Stationary Markov Equilibria

In addition to the set of sequential competitive equilibria, we are also interested in the set of stationary Markov equilibria, a subset of the former set.[19] If one exists, a stationary Markov equilibrium can be numerically approximated using standard computational methods. Additionally, such an equilibrium allows the monetary-fiscal authority policy to depend on the smallest subset of observations, avoiding any (unmodeled) problems of incomplete information.

Definition. A stationary Markov equilibrium is a sequential competitive equilibrium in which functions $F_b, F_{b^*}, F_B, F_{B^*}, F_r$, and F_{r^*} exist such that for any date-event s^t and all conditional realizations in the following period $\left(s^t, s\right)_{s \in \mathcal{S}}$:

$$\begin{aligned}
\left(b_h\left(s^{t+1}\right), b_f\left(s^{t+1}\right)\right) &= F_b\left(b_h\left(s^t\right), b_f\left(s^t\right), s_{t+1}\right), \\
\left(b_h^*\left(s^{t+1}\right), b_f^*\left(s^{t+1}\right)\right) &= F_{b^*}\left(b_h^*\left(s^t\right), b_f^*\left(s^t\right), s_{t+1}\right), \\
\left(B_h\left(s^{t+1}\right), B_f\left(s^{t+1}\right)\right) &= F_B\left(B_h\left(s^t\right), B_f\left(s^t\right), s_{t+1}\right), \\
\left(B_h^*\left(s^{t+1}\right), B_f^*\left(s^{t+1}\right)\right) &= F_{B^*}\left(B_h^*\left(s^t\right), B_f^*\left(s^t\right), s_{t+1}\right), \\
r\left(s^{t+1}\right) &= F_r\left(s_{t+1}\right), \\
r^*\left(s^{t+1}\right) &= F_{r^*}\left(s_{t+1}\right).
\end{aligned}$$

[19] Stationary Markov equilibria are also referred to as recursive equilibria by Ljungqvist and Sargent (2012).

Given the above financial variables specified by stationary transition functions, the remaining variables can also be defined by stationary transition functions using the budget constraints (7), the monetary-fiscal authority constraints (12), and the price equations $M\left(s^{t+1}\right) = p\left(s^{t+1}\right) y_h\left(s_{t+1}\right)$ and $e^*\left(s^{t+1}\right) = \frac{p\left(s^{t+1}\right)}{p^*\left(s^{t+1}\right)}$.

4 Efficiency of Policies

This section considers the normative effects of the following policies under incomplete markets: Theorem 1 considers nominal GDP targeting by both countries; Theorem 2 considers stabilizing inflation by both countries; and Theorem 3 considers all global policies in between those just mentioned.

4.1 Nominal GDP Targeting

Definition. Nominal GDP Targeting
If the monetary policy of the home country satisfies $p\left(s^{t+1}\right) y_h\left(s_{t+1}\right) = p\left(s^t, s\right) y_h\left(s\right) = p\left(s^t, 1\right) y_h\left(1\right)$ for all possible realizations $s \in \mathcal{S}$, then the home country is implementing *nominal GDP targeting.*

Define the nominal GDP target growth rates as μ and μ^* for the home and foreign country, respectively[20]. Define the largest expected endowment growth in the home country (similarly for the foreign country) as:

$$G = \max_{s \in \mathcal{S}} \frac{\sum_{s' \in \mathcal{S}} \pi\left(s'|s\right) y_h\left(s'\right)}{y_h(s)}.$$

To ensure nonnegative interest rates and a consistent equilibrium definition, we impose the following bounds on the growth rates.

Assumption 2. *The nominal GDP target growth rates* (μ, μ^*) *are such that* $\beta G \leq 1 + \mu$ *and* $\beta G^* \leq 1 + \mu^*$.

Policies of *nominal GDP targeting* do not require either monetary-fiscal authority to engage in foreign exchange intervention. This implies that $B_f\left(s^t\right) = B_h^*\left(s^t\right) = 0$ for all date-events s^t.

Under *nominal GDP targeting,* the monetary-fiscal authority in the home country chooses a monetary policy consisting only of the initial money supply $M(s_0)$ and the monetary growth rate μ. Given these choices, the infinite sequence of money supplies is trivially determined. Further, the infinite sequence of debt positions B_h is determined to satisfy the monetary-fiscal authority constraints (12). Thus, the entire policy is automatic given $M(s_0)$ and μ (and similarly for the monetary-fiscal authority in the foreign country).

[20] See proof of Theorem 1 for a precise definition of this.

Theorem 1. *If both countries adopt **nominal GDP targeting,** then the stationary Markov equilibrium allocations are Pareto efficient.*

The recursive structure suffices to guarantee the existence of a stationary Markov equilibrium, since Theorem 1 ensures that the welfare theorems are applicable. The proof of Theorem 1 is a constructive argument and ultimately verifies two things: (i) that the Pareto optimal allocation can be supported in the sequential equilibrium budget constraints and (ii) that the transition functions implied from the budget constraints are stationary for all price variables, household choice variables, and monetary-fiscal authority choice variables. Standard arguments from dynamic programming (see Bertsekas (1976)) guarantee that the constructed equilibrium with Pareto optimal allocation is a stationary Markov equilibrium.

Remark

There is a long standing view that the main cost of inflation lies in the distortion it induces in households' intertemporal savings decisions. The meaningful deviation of our approach from others in the literature is that we do not consider costs of inflation such as inefficient long-term planning (Clarida et al. (1999)) that arises from *unanticipated* inflation. In our framework inflation is perfectly anticipated, in a rational expectations sense, but the incompleteness of markets means that agents cannot insure against this risk. More importantly, under a global policy of NGDP targeting, the fluctuations of inflation are not random but rather correspond exactly to fluctuations in domestic GDP and so inflation does not necessarily inhibit the ability to make long term planning decisions.[21] In the open economy context we consider, the fluctuations in inflation provide a natural hedge against fluctuations in the current account which would occur for reasons outside of the remit of domestic monetary-fiscal authorities. In contrast, the next section shows that policies which attempt to stabilize fluctuations in the current account by targeting the nominal exchange rate are not sustainable in the long run.

4.2 Stabilizing Inflation

We next address the implications of policies of *stabilizing inflation* by both countries.

Definition. Stabilizing Inflation

If the monetary policy of the home country satisfies $p\left(s^{t+1}\right) = p\left(s^t, s\right) = p\left(s^t, 1\right)$ for all possible realizations $s \in \mathcal{S}$, then the home country is implementing *stabilizing inflation.*

Define the expected inflation rates for both the home and foreign country, respectively, as a Markov process that only depends on the current period realization $s \in \mathcal{S} : \{\tau\left(s\right)\}_{s\in\mathcal{S}}$ and $\{\tau^*\left(s\right)\}_{s\in\mathcal{S}}$.

[21] See Sheedy (2012) for an extended discussion on this point.

Assumption 3. *The inflation rates* $\{\tau(s)\}_{s\in\mathcal{S}}$ *and* $\{\tau^*(s)\}_{s\in\mathcal{S}}$ *are such that* $\beta \leq 1+\tau(s)$ *and* $\beta \leq 1+\tau^*(s)$ *for all shocks* $s \in \mathcal{S}$.

Unlike with *nominal GDP targeting*, we permit the monetary-fiscal authorities to engage in foreign exchange intervention. Formally, this implies $B_f(s^t) \in \mathbb{R}$ and $B_h^*(s^t) \in \mathbb{R}$ for all date-events s^t. In a purely nominal economy, such as the one presented in the motivating example, we showed that only a global policy of nominal GDP targeting could support the Pareto efficient allocation. However, the cash-in-advance technology we consider provides an additional mechanism through which stabilitizing inflation may possibly do the same.

The cash-in-advance constraint effectively taxes nominal money balances at the rate of inflation and, in equilibrium, effectively causes agents to defer their nominal incomes by a period. In a present value sense, this is equivalent to discounting *future* income by the nominal interest rate. If real income is subject to stochastic shocks, then nominal interest rates could potentially be chosen in such a way so as to offset the real shock, leaving the present discounted value of current period nominal income state-independent or stationary.

In order to support both the Pareto efficient allocation and a stationary path of prices, the central bank needs an additional mechanism to adjust the nominal money supply. This can be done in two ways. The most obvious is for the monetary-fiscal authority to provide nominal lump sum transfers to agents. Such transfers would, however, be given only to domestic households, which would create wealth effects that offset the stationarity of income obtained through having a policy of state contingent nominal interest rates.

The second option for policy makers, and the policy we consider, is that the monetary-fiscal authority intervenes in the currency markets by purchasing and selling foreign bonds, thereby effectively injecting or withdrawing money from the economy. From the law of one price, this would imply that to maintain stable prices, the exchange rates would be constant over time. That is, the policy we consider is one where the monetary-fiscal authority uses nominal interest rates to affect domestic aggregate demand and targets a fixed exchange rate to support a stationary path of inflation.

Our result shows that under *stabilizing inflation*, not only is monetary policy insufficient to obtain a Pareto efficient allocation, but so is monetary policy combined with foreign exchange intervention. The equilibrium household bond holdings that are required to support the Pareto efficient allocation violate the debt constraint under a policy of *stabilizing inflation*.

Under a policy of *stabilizing inflation*, the monetary-fiscal authority in the home country chooses a monetary policy consisting only of the initial price level $p(s_0)$ and the Markov process for inflation rates $\{\tau(s)\}_{s\in\mathcal{S}}$. The implementation of the sequence of money supplies and debt positions is automatic given that the constraints (12) must be satisfied for all date-events s^t.

Theorem 2. *If both countries* ***stabilize inflation****, there do not exist policies that result in* *a Pareto efficient sequential competitive equilibrium allocation.*

From the statement of the theorem, notice that we do not restrict ourselves to only stationary Markov equilibria. The inefficiency result holds more generally.

The argument nowhere requires the variables to be specified in a stationary Markov manner. The stationary structure is imposed from the Markov endowment process and the Markov process of *stabilizing inflation*. The remaining variables of interest are the bond holdings of households and we do not impose any restrictions on the transition functions for these variables. Thus, inefficiency occurs under *stabilizing inflation* for all sequential competitive equilibria.

The proof proceeds by verifying that the Pareto efficient allocation cannot be supported as an optimal solution to the households' problem (10). The monetary-fiscal authority policy vectors $\left(B_h, B_f, B_h^*, B_f^*\right)$ can take any values such that the inflation targets are met, markets clear, and the constraints (11) and (12) are met. This obviously subsumes the case of foreign exchange intervention.

4.3 Mixed Global Policies

This section generalizes the result in Theorem 2 to show that for any global policies other than *nominal GDP targeting* by both countries, Pareto efficiency is not supportable under the stationary Markov equilibrium concept. Obviously, both countries *stabilize inflation* as in Theorem 2 is a special case of global policies other than *nominal GDP targeting* by both countries. While we only show that the Pareto inefficiency holds for the more restrictive definition of equilibrium (stationary Markov), it is this stationary Markov equilibrium concept that is typically considered in the theory and practice of monetary policy.

The definition of a stationary Markov equilibrium does impose minimal structure on the monetary policies that can be implemented. Namely, the definition requires that $\sum_{s\in\mathcal{S}} \pi(s|s_t)\frac{M\left(s^t\right)}{M(s^t,s)}$ only depends on the current shock s_t. As special cases are (i) *nominal GDP targeting* in which $\sum_{s\in\mathcal{S}} \pi(s|s_t)\frac{M\left(s^t\right)}{M(s^t,s)} = \frac{1}{1+\mu}$ and (ii) *stabilizing inflation* in which $\sum_{s\in\mathcal{S}} \pi(s|s_t)\frac{M\left(s^t\right)}{M(s^t,s)} = \frac{y_h(s_t)}{1+\tau(s_t)}\sum_{s\in\mathcal{S}}\frac{\pi(s|s_t)}{y_h(s)}$.

From the Fisher equation, the interest rates are in fact stationary functions and defined for the home country (and similarly for the foreign country) as:

$$r\left(s^t\right) = F_r\left(s_t\right) = \frac{y_h\left(s_t\right)}{\beta\sum_{s\in\mathcal{S}}\pi(s|s_t)y_h(s)\frac{M(s^t)}{M(s^t,s)}} - 1.$$

The function $F_r\left(s_t\right)$ is stationary iff $\sum_{s\in\mathcal{S}} \pi(s|s_t)\frac{M\left(s^t\right)}{M(s^t,s)}$ only depends on the current shock s_t.

This implies that the growth rates for any stationary monetary policy must be of the form:

$$M\left(s^t, s\right) = \left(1+\mu\left(s_t, s\right)\right) M\left(s^t\right)$$

for all date-events s^t, where the growth rate $\mu(s_t, s)$ only depends on the two most recent shock realizations.

We must impose assumptions to guarantee that the interests rates are nonnegative.

Assumption 4. *The growth rates* $\{\mu(s_t, s)\}_{s_t, s\in\mathcal{S}}$ *and* $\{\mu^*(s_t, s)\}_{s_t, s\in\mathcal{S}}$ *are such that* $\beta\sum_{s\in\mathcal{S}}\pi(s|s_t)y_h(s)\frac{M(s^t)}{M(s^t,s)} \leq y_h(s_t)$ *and* $\beta\sum_{s\in\mathcal{S}}\pi(s|s_t)y_f(s)\frac{M^*(s^t)}{M^*(s^t,s)} \leq y_f(s_t)$ *for all shocks* $s_t \in \mathcal{S}$.

The state-contingent definition of *nominal GDP targeting* for the home country specifies that $\mu(s_t, s) = \mu(s_t, 1)\ \forall s \in \mathcal{S}$. We allow for this general definition in the following result, rather than the original definition (state-independent) in which $\mu(s_t, s) = \mu\ \ \forall s_t, s \in \mathcal{S}$.

Theorem 3. *If one of the countries adopts a policy other than* ***nominal GDP targeting****, there do not exist policies that result in a Pareto efficient stationary Markov equilibrium allocation.*

As previously discussed, this result nests Theorem 2 when we agree to only focus on the policy-relevant equilibrium concept of a stationary Markov equilibrium.

5 Concluding Remarks

This paper analyzed the conditions under which a global policy of NGDP targeting permits optimal risk-sharing by households. The key points in this analysis are twofold: (i) while the risk-sharing implications hold for any nominal economy (recall the motivating example), the formal modeling of an appropriate monetary structure is crucial and (ii) the efficiency of NGDP targeting is fragile and not robust to simple extensions and generalizations.

Regarding the delicate nature of the efficiency result under NGDP targeting, we offer our thoughts for each of the following extensions. Each result is either derived from the classical literature in general equilibrium, or is formally proven by the authors and available upon request.

1. If multiple commodities are traded and consumed in each date-event, the assumption that preferences are identical and homothetic suffices to guarantee that a global policy of NGDP targeting remains efficient. In contrast, if preferences are no longer identical, then the presence of multiple commodities suffices to destroy the efficiency properties.

2. If domestic households are heterogeneous with endowment vectors that are not perfectly correlated with the domestic or aggregate output,[22] then in order for a global policy of NGDP targeting to remain efficient, domestic households must be able to completely share risk domestically through either an equity market or a complete set

[22] This lies in contrast to Koenig (2012*b*) and Sheedy (2012).

of assets in zero domestic net supply. Without these complete within-country insurance opportunities, a model containing within-country heterogeneity (i.e. individual real incomes are not perfectly correlated) is unable to support an efficient allocation.

3. Velocity must be state-independent for a global policy of NGDP targeting to remain efficient. If velocity depends on domestic real output, for example, then this condition is violated. However, NGDP targeting is efficient if velocity is modeled along the lines of Cagan (1956) in which velocity depends on the future path of money supplies, which is stationary under NGDP.

4. Our results are robust to a model with any finite number of countries. Provided that all countries adopt nominal GDP targeting, then efficiency is achieved. However, if just one country deviates, then efficiency is broken as the shocks to the current account can no longer be hedged.

5. We have not Pareto ranked different equilibria corresponding to different combinations of monetary policy, but we can provide some initial insights on how to study these issues. Using the simple parametric specifications in Neumeyer (1998), in particular mean-variance preferences, the distance between the complete markets allocation and the marketed subspace (the asset span) is a well-defined welfare measure. This distance is smaller when one country adopts NGDP targeting compared to when neither country adopts NGDP targeting. This implies that the equilibrium allocation under the former Pareto dominates the equilibrium allocation under the latter (see Proposition A1 of Neumeyer (1998) for the formal treatment of Pareto dominance).

While there is merit to the argument that NGDP targeting allows inflation to redistribute resources efficiently among bond holders, the extent to which these arguments carry weight in practice remains unclear. It remains to be shown, through characterizing equilibria under different policy choices and Pareto ranking their respective allocations, whether or not NGDP targeting can allocate resources more efficiently than other monetary policies.

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6 Appendix

Proof of Theorem 1

Proof. The money supply $M\left(s^{t}\right)$ for the date-event s^{t} only depends upon the period $t:$

$$M\left(s^{t}\right)=(1+\mu)^{t}\,M\left(s_{0}\right),$$

where $M\left(s_{0}\right)$ is the initial money supply. For simplicity, we define the scaled money supplies as

$$\hat{M}\left(s^{t}\right)=\frac{M\left(s^{t}\right)}{(1+\mu)^{t}}=M\left(s_{0}\right).$$

Prices are given by

$$p\left(s^{t}\right)=\frac{M\left(s^{t}\right)}{y_{h}\left(s_{t}\right)},$$

where the scaled prices are Markov processes that only depend on the current realization $s_{t}:$

$$\hat{p}\left(s^{t}\right)=\frac{p\left(s^{t}\right)}{(1+\mu)^{t}}=\frac{M\left(s_{0}\right)}{y_{h}\left(s_{t}\right)}.$$

The same definitions hold for the foreign country.

At a Pareto optimal equilibrium allocation, the Fisher equation under the *nominal GDP target* is given by:

$$\frac{M\left(s^{t}\right)}{M\left(s^{t-1}\right)}=\beta\left(1+r(s^{t-1})\right)\frac{\sum_{s\in\mathcal{S}}\pi(s|s_{t-1})y_{h}\left(s\right)}{y_{h}\left(s_{t-1}\right)}.$$

Thus, the interest rates that satisfy the Fisher equations are a Markov process that only depends on the current realization $s_{t}\in\mathcal{S}:$

$$r\left(s^{t}\right)=F_{r}\left(s_{t}\right)=\frac{(1+\mu)\,y_{h}\left(s_{t}\right)}{\beta\sum_{s\in\mathcal{S}}\pi(s|s_{t})y_{h}\left(s\right)}-1. \tag{13}$$

Using (13), $r\left(s^{t}\right)\geq 0$ provided that the expected endowment growth never exceeds the discounted monetary growth:

$$\frac{1+\mu}{\beta}\geq\frac{\sum_{s\in\mathcal{S}}\pi(s|s_{t})y_{h}\left(s\right)}{y_{h}\left(s_{t}\right)}$$

The inequality holds given Assumption 2. The same properties hold for the foreign country.

The exchange rates are also a Markov process if we assume that the nominal GDP growth rates are equal across countries. Without such an assumption, the exchange rates can drift over time:

$$e\left(s^{t}\right)=\frac{(1+\mu)^{t}}{(1+\mu^{*})^{t}}\frac{M\left(s_{0}\right)}{M^{*}\left(s_{0}\right)}\frac{y_{f}\left(s_{t}\right)}{y_{h}\left(s_{t}\right)}.$$

The only remaining variables that must be shown to be stationary are the bond holdings of households and debt positions of monetary-fiscal authorities.

Specify the following growth rates for household bonds: $\gamma_h(s^t)$, $\gamma_f(s^t)$, $\gamma_h^*(s^t)$, and $\gamma_f^*(s^t)$. Specify the following growth rates for the monetary-fiscal authority debt obligations: $\Gamma_h(s^t)$ and $\Gamma_f^*(s^t)$. The growth rate definitions are such that

$$b_h\left(s^t\right) = \left(1+\gamma_h\left(s^t\right)\right) b_h\left(s^{t-1}\right), \tag{14}$$

and similarly for each of the 5 other variables. Define the scaled financial variables as

$$\hat{b}_h\left(s^t\right) = \frac{b_h\left(s^t\right)}{(1+\mu)^t}, \tag{15}$$

and similarly for each of the other 5 variables. The exposition is simplified by expressing all budget constraints and debt constraints in terms of these scaled variables. Consequently, the transition equations for the scaled variables are given by

$$\hat{b}_h\left(s^t\right) = \frac{1+\gamma_h\left(s^t\right)}{1+\mu}\hat{b}_h\left(s^{t-1}\right), \tag{16}$$

and similarly for each of the other 5 variables.

Part (i): Market clearing

Observe that for any date-event s^t, market clearing in scaled variables, $\hat{b}_h(s^t)+\hat{b}_h^*(s^t) = \hat{B}_h(s^t)$, is satisfied iff market clearing in the original variables, $b_h(s^t)+b_h^*(s^t) = B_h(s^t)$, is satisfied.

To support a Pareto efficient equilibrium allocation, the budget constraint for the home household in date-event s^t (and similarly for the foreign household) is:

$$\begin{aligned} &\theta\left(y_h(s_t)+y_f(s_t)\right) + \frac{b_h(s^t)}{1+r(s_t)}\frac{y_h(s_t)}{M(s^t)} + \frac{b_f(s^t)}{1+r^*(s_t)}\frac{y_f(s_t)}{M^*(s^t)} \\ = \;& \frac{b_h(s^{t-1})+M(s^{t-1})}{M(s^t)}y_h(s_t) + \frac{b_f(s^{t-1})}{M^*(s^t)}y_f(s_t) \end{aligned}$$

for some $\theta \in (0,1)$, where θ is the Pareto weight for the home household. Given (16), the budget constraint can be re-written in terms of scaled variables:

$$\begin{aligned} \theta\left(y_h(s_t)+y_f(s_t)\right) &= \frac{1}{1+\mu}\left(1+\frac{\hat{b}_h\left(s^{t-1}\right)}{M(s_0)}\frac{r(s_t)-\gamma_h\left(s^t\right)}{1+r(s_t)}\right)y_h(s_t) \\ &\quad + \frac{1}{1+\mu^*}\left(\frac{\hat{b}_f\left(s^{t-1}\right)}{M^*(s_0)}\frac{r^*(s_t)-\gamma_f\left(s^t\right)}{1+r^*(s_t)}\right)y_f(s_t). \end{aligned} \tag{17}$$

Sufficient conditions are:

$$\begin{array}{l} \theta = \frac{1}{1+\mu}\left(1+\frac{\hat{b}_h\left(s^{t-1}\right)}{M(s_0)}\frac{r(s_t)-\gamma_h\left(s^t\right)}{1+r(s_t)}\right) \text{ and} \\ \theta = \frac{1}{1+\mu^*}\left(\frac{\hat{b}_f\left(s^{t-1}\right)}{M^*(s_0)}\frac{r^*(s_t)-\gamma_f\left(s^t\right)}{1+r^*(s_t)}\right) \end{array}. \tag{18}$$

In exactly the same manner, we obtain the sufficient conditions for the foreign household:

$$\begin{array}{l} 1-\theta = \frac{1}{1+\mu}\left(\frac{\hat{b}_h^*\left(s^{t-1}\right)}{M(s_0)}\frac{r(s_t)-\gamma_h^*\left(s^t\right)}{1+r(s_t)}\right) \text{ and} \\ 1-\theta = \frac{1}{1+\mu^*}\left(1+\frac{\hat{b}_f^*\left(s^{t-1}\right)}{M^*(s_0)}\frac{r^*(s_t)-\gamma_f^*\left(s^t\right)}{1+r^*(s_t)}\right) \end{array}. \tag{19}$$

The sufficient conditions (18) and (19) are satisfied provided that for all date-events s^t, there exists constant variables $\left(\chi_h, \chi_f, \chi_h^*, \chi_f^*\right) \in \mathbb{R}^4$ such that

$$\chi_h = \hat{b}_h\left(s^{t-1}\right)\frac{r(s_t)-\gamma_h\left(s^t\right)}{1+r(s_t)}, \tag{20}$$

and similarly for the remaining three variables.

For any initial period transfers $w(s_0)$ and $w^*\left(s_0\right)$, there exists a range of Pareto weights θ such that the corresponding Pareto optimal allocation can be supported in a stationary Markov equilibrium. For any growth rates μ and μ^*, initial money supplies $M(s_0)$ and $M^*(s_0)$, and any θ in the range discussed above, the constant values $\left(\chi_h, \chi_f, \chi_h^*, \chi_f^*\right) \in \mathbb{R}^4$ are defined as

$$\begin{array}{l} \chi_h = \theta M(s_0)\left(1+\mu\right) - M(s_0) \text{ and} \\ \chi_f = \theta M^*(s_0)\left(1+\mu^*\right), \end{array} \tag{21}$$

and similarly for the foreign household.

The realization s_t determines the nominal interest rate $r(s^t) = F\left(s_t\right)$, so we simply write the interest rate as $r(s_t)$. From (16), (20), and (21), the transition functions for the bond holdings are stationary:

$$\begin{aligned} \hat{b}_h\left(s^t\right) &= \frac{1+r(s_t)}{1+\mu}\left(\hat{b}_h\left(s^{t-1}\right) - \theta M(s_0)\left(1+\mu\right) + M(s_0)\right), \\ \hat{b}_f\left(s^t\right) &= \frac{1+r^*(s_t)}{1+\mu^*}\left(\hat{b}_f\left(s^{t-1}\right) - \theta M^*(s_0)\left(1+\mu^*\right)\right), \end{aligned} \tag{22}$$

and similarly for the foreign household. The transition function is of the form required for a stationary Markov equilibrium:

$$\left(\hat{b}_h\left(s^t\right), \hat{b}_f\left(s^t\right)\right) = F_b\left(\hat{b}_h\left(s^{t-1}\right), \hat{b}_f\left(s^{t-1}\right), s_t\right).$$

Given (16), the monetary-fiscal authority constraint in the home country for any history s^t can be written in terms of scaled variables:

$$\mu M(s_0) = \hat{B}_h\left(s^{t-1}\right)\frac{r(s_t)-\Gamma_h\left(s^t\right)}{1+r(s_t)} \tag{23}$$

and similarly for the foreign country. Exactly as above for household bond holdings, the transition function is

$$\hat{B}_h\left(s^t\right) = \frac{1+r(s_t)}{1+\mu}\left(\hat{B}_h\left(s^{t-1}\right) - \mu M(s_0)\right) \tag{24}$$

for the home country with stationary transition function defined as $\hat{B}_h(s^t) = F_B\left(\hat{B}_h(s^{t-1}), s_t\right)$ (and similarly for the foreign country).

Market clearing is verified using an inductive argument. The market clearing conditions are $w(s_0) = W(s_0)$ and $w^*(s_0) = W^*(s_0)$ and in the date-event s_0, the scaled bond holdings are identical to the unscaled holdings. The bond holdings $\left\{\hat{b}_h(s_0), \hat{b}_f(s_0), \hat{b}_h^*(s_0), \hat{b}_f^*(s_0)\right\}$ are set such that the following excess demand condition is satisfied:

$$y_h(s_0) - c(s_0) = \frac{\hat{b}_f(s_0)}{p^*(s_0)(1 + r^*(s_0))} - \frac{\hat{b}_h^*(s_0)}{p(s_0)(1 + r(s_0))}.$$

As a result, the debt positions $\hat{B}_h(s_0)$ and $\hat{B}_f^*(s_0)$ then trivially satisfy the market clearing conditions $\hat{b}_h(s_0) + \hat{b}_h^*(s_0) = \hat{B}_h(s_0)$ and $\hat{b}_f(s_0) + \hat{b}_f^*(s_0) = \hat{B}_f^*(s_0)$ as

$$\frac{\hat{B}_h(s_0)}{p(s_0)(1 + r(s_0))} = \frac{W(s_0)}{p(s_0)} - y_h(s_0)$$

from the monetary-fiscal authority constraint of the home country (11) and

$$\frac{\hat{b}_h(s_0)}{p(s_0)(1 + r(s_0))} + \frac{\hat{b}_f(s_0)}{p^*(s_0)(1 + r^*(s_0))} = \frac{w(s_0)}{p(s_0)} - c(s_0)$$

from the budget constraint for the home household (and similarly for the foreign household). Thus the initialization step of the argument is confirmed: $\hat{b}_h(s_0) + \hat{b}_h^*(s_0) = \hat{B}_h(s_0)$ and $\hat{b}_f(s_0) + \hat{b}_f^*(s_0) = \hat{B}_f^*(s_0)$.

Given these date-event s_0 variables and the monetary growth rates μ and μ^*, we show that appropriate paths for the bond holdings and debt positions can be specified to support the Pareto optimal allocation while satisfying market clearing.

For the induction step, fix the date-event $s^{t-1}$and assume that $\hat{b}_h(s^{t-1}) + \hat{b}_h^*(s^{t-1}) = \hat{B}_h(s^{t-1})$ and $\hat{b}_f(s^{t-1}) + \hat{b}_f^*(s^{t-1}) = \hat{B}_f^*(s^{t-1})$. For the home country in any date-event s^t, (22) implies

$$\hat{b}_h(s^t) + \hat{b}_h^*(s^t) = \frac{1 + r(s_t)}{1 + \mu}\left(\hat{b}_h(s^{t-1}) + \hat{b}_h^*(s^{t-1}) - \mu M(s_0)\right)$$

for any $s^t = \left(s^{t-1}, s\right)_{s \in \mathcal{S}}$. Using the assumption $\hat{b}_h(s^{t-1}) + \hat{b}_h^*(s^{t-1}) = \hat{B}_h(s^{t-1})$:

$$\hat{b}_h(s^t) + \hat{b}_h^*(s^t) = \frac{1 + r(s_t)}{1 + \mu}\left(\hat{B}_h(s^{t-1}) - \mu M(s_0)\right).$$

Given (24), markets clear in all date-events $s^t = \left(s^{t-1}, s\right)_{s \in \mathcal{S}}$:

$$\hat{b}_h(s^t) + \hat{b}_h^*(s^t) = \hat{B}_h(s^t).$$

Part (ii): Debt constraint

The scaled bond holdings $\left\{\hat{b}_h(s^t), \hat{b}_f(s^t), \hat{b}_h^*(s^t), \hat{b}_f^*(s^t)\right\}$ are uniformly bounded random variables. Suppose otherwise, that is, $\lim_{t\to\infty} \sup_{s^t} \left|\hat{b}_h(s^t)\right| = \infty$. From the sufficient condition

(20), it must be that

$$\liminf_{t\to\infty}\inf_{s^t}\left|\frac{r(s_{t+1})-\gamma_h\left(s^{t+1}\right)}{1+r(s_{t+1})}\right|=0. \tag{25}$$

With $r(s_{t+1})\geq 0$ and only dependent on the period $t+1$ realization s_{t+1}, the limit condition (25) is only satisfied if $\lim_{t\to\infty}\inf_{s^t}\left[\gamma_h\left(s^{t+1}\right)-r(s_{t+1})\right]=0$. This requires that $\gamma_h\left(s^{t+1}\right)$ only depends on the period $t+1$ realization $s_{t+1}:\gamma_h\left(s^{t+1}\right)=\gamma_h\left(s_{t+1}\right)$. If $\gamma_h\left(s_{t+1}\right)$ is independent of the shock s_{t+1}, then (20) cannot be satisfied as $r(s_{t+1})$ varies with the current endowment $y_h\left(s_{t+1}\right)$ according to the Fisher equation (13). If $\gamma_h\left(s_{t+1}\right)$ varies with the shock s_{t+1}, then $\hat{b}_h\left(s^{t+1}\right)=\frac{1+\gamma_h(s_{t+1})}{1+\mu}\hat{b}_h\left(s^t\right)$ varies across the realization s_{t+1}. Taken together, $\hat{b}_h\left(s^{t+1}\right)$ varies with s_{t+1}, while $\frac{r(s_{t+2})-\gamma_h\left(s^{t+2}\right)}{1+r(s_{t+2})}$ varies (only) with s_{t+2}. Thus, (20) cannot be satisfied for all possible histories s^{t+2}. This shows that $\hat{b}_h(s^t)$ is uniformly bounded. The same argument holds for the other 3 financial variables.

In terms of the scaled bond holdings, the debt constraint for the home household is written as:

$$\inf_{t,s^t}\left[\frac{\hat{b}_h(s^t)}{\left(1+r(s^t)\right)p(s_0)}+\frac{\hat{b}_f(s^t)}{\left(1+r^*\left(s^t\right)\right)p^*(s_0)}\right]>-\infty.$$

Since $\left\{\hat{b}_h(s^t),\hat{b}_f(s^t),\hat{b}^*_h(s^t),\hat{b}^*_f(s^t)\right\}$ are uniformly bounded random variables, then the debt constraints for both households are satisfied.

The nominal GDP targets impose the Markov structure on equilibrium prices as discussed leading up to the statement of the theorem. Given that a Pareto efficient allocation can be supported as a stationary Markov equilibrium, then any stationary Markov equilibrium allocation must be Pareto efficient (or else households would not be optimizing). Existence is trivial since a stationary Pareto efficient allocation exists for the specified endowments and preferences of the economy. □

Proof of Theorem 2

Proof. The price $p\left(s^t\right)$ for any date-event s^t is independent of the period t realization s_t :

$$p\left(s^{t-1},s\right)=\left(1+\tau\left(s_{t-1}\right)\right)p\left(s^{t-1}\right)\quad\forall s\in\mathcal{S}. \tag{26}$$

For simplicity, we define the scaled money supplies as:

$$\hat{M}\left(s^t\right)=\frac{M\left(s^t\right)}{\prod_{s'\in s^{t-1}}\left(1+\tau\left(s'\right)\right)}.^{23} \tag{27}$$

By definition,

$$\begin{aligned}p\left(s_0\right)&=\frac{p\left(s^t\right)}{\prod_{s'\in s^{t-1}}\left(1+\tau\left(s'\right)\right)}\text{ and}\\ M\left(s^t\right)&=p\left(s^t\right)y_h\left(s_t\right),\end{aligned}$$

[23] For example, $\hat{M}\left(s^2\right)=\hat{M}\left(s_0,s_1,s_2\right)=\frac{M\left(s^2\right)}{(1+\tau(s_0))(1+\tau(s_1))}$.

meaning that the scaled money supplies $\hat{M}\left(s^t\right)$ are Markov processes that depend only upon the current realization s_t :

$$\hat{M}\left(s^{t}\right)=F_{M}\left(s_{t}\right)=p\left(s_{0}\right)y_{h}\left(s_{t}\right). \tag{28}$$

The same properties hold for the foreign country.

At a Pareto efficient equilibrium allocation, the Fisher equation for the home country under the *inflation rate target* is given by:

$$1+\tau\left(s_{t-1}\right)=\beta\left(1+r(s^{t-1})\right). \tag{29}$$

Thus, the interest rate that satisfies the Fisher equation in any date-event s^t is a Markov process that only depends on the current realization

$$r\left(s^{t}\right)=F_{r}\left(s_{t}\right)=\frac{1+\tau\left(s_{t}\right)}{\beta}-1. \tag{30}$$

The interest rates are nonnegative provided that:

$$1+\tau\left(s_{t}\right)\geq\beta$$

for all shocks $s_t \in \mathcal{S}$. The inequality holds given Assumption 3. The same properties hold for the foreign country.

The exchange rates are also Markov processes if we assume that the contingent inflation rates are identical for both countries. At the optimal allocation, the uncovered interest rate parity condition is derived from the Fisher equation (29):

$$e^{*}\left(s^{t}\right)=\frac{\left(1+\tau\left(s_{t-1}\right)\right)}{\left(1+\tau^{*}\left(s_{t-1}\right)\right)}e^{*}\left(s^{t-1}\right). \tag{31}$$

Define the scaled financial variables as

$$\hat{b}_{h}\left(s^{t}\right)=\frac{b_{h}\left(s^{t}\right)}{\prod_{s'\in s^{t-1}}\left(1+\tau\left(s'\right)\right)}, \tag{32}$$

and similarly for each of the 7 other variables.

Part (i): Market clearing

With a Pareto efficient equilibrium allocation, the budget constraint for the home household in date-event s^t (and similarly for the foreign household) is given by:

$$\begin{aligned}&\theta\left(y_{h}\left(s_{t}\right)+y_{f}\left(s_{t}\right)\right)+\frac{b_{h}(s^{t})}{p(s^{t})\left(1+r\left(s_{t}\right)\right)}+\frac{b_{f}(s^{t})}{p^{*}(s^{t})\left(1+r^{*}\left(s_{t}\right)\right)}\\=&\frac{b_{h}(s^{t-1})}{p(s^{t})}+\frac{b_{f}(s^{t-1})}{p^{*}(s^{t})}+\frac{y_{h}\left(s_{t-1}\right)}{1+\tau\left(s_{t-1}\right)}.\end{aligned} \tag{33}$$

The real bond payouts are collinear. With dependent assets, a continuum of possible optimal household bond holdings exists, meaning that it is innocuous to assume $b_f(s^t) = b_f(s^{t-1}) = 0$

and $b_h^*(s^t) = b_h^*(s^{t-1}) = 0$. In words, households are indifferent and only trade domestic bonds. Given (32),

$$\frac{b_h(s^{t-1})}{p(s^t)} = \frac{\hat{b}_h\left(s^{t-1}\right)}{p(s_0)\left(1+\tau\left(s_{t-1}\right)\right)}. \tag{34}$$

From the Fisher equation (29):

$$\frac{b_h(s^t)}{\left(1+r\left(s_t\right)\right)p(s^t)} = \frac{\beta\hat{b}_h\left(s^t\right)}{p(s_0)\left(1+\tau\left(s_t\right)\right)}. \tag{35}$$

Consequently, the budget constraints can be updated from (33):

$$\theta\left(y_h\left(s_t\right)+y_f\left(s_t\right)\right) = \frac{y_h\left(s_{t-1}\right)}{1+\tau\left(s_{t-1}\right)} + \frac{1}{p(s_0)}\left(\frac{\hat{b}_h\left(s^{t-1}\right)}{1+\tau\left(s_{t-1}\right)} - \frac{\beta\hat{b}_h\left(s^t\right)}{1+\tau\left(s_t\right)}\right). \tag{36}$$

The transition function for the home household is then an implicit function of the equation:

$$p(s_0)\left(\theta\left(y_h\left(s_t\right)+y_f\left(s_t\right)\right) - \frac{y_h\left(s_{t-1}\right)}{1+\tau\left(s_{t-1}\right)}\right) = \frac{\hat{b}_h\left(s^{t-1}\right)}{1+\tau\left(s_{t-1}\right)} - \frac{\beta\hat{b}_h\left(s^t\right)}{1+\tau\left(s_t\right)}. \tag{37}$$

Mathematically, the difference between the transition function under *stabilizing inflation* and under *nominal GDP targeting* is that the transition function under *stabilizing inflation* must account for the endowment variation $y_h\left(s_{t-1}\right)$. Not only will this prevent a stationary transition function from being defined, but more importantly will lead to a sequence of scaled bond holdings that violate the debt constraint (9).

An implicit transition function for the monetary-fiscal authorities can be similarly specified.

Part (ii): Debt constraint

Consider any fixed time period $\tau > 0$. It cannot be that both

$$\begin{aligned}
\theta\left(y_h\left(s_\tau\right)+y_f\left(s_\tau\right)\right) - \frac{y_h\left(s_\tau\right)}{1+\tau\left(s_\tau\right)} &= 0 \\
(1-\theta)\left(y_h\left(s_\tau\right)+y_f\left(s_\tau\right)\right) - \frac{y_f\left(s_\tau\right)}{1+\tau^*\left(s_\tau\right)} &= 0
\end{aligned}$$

for all $s_\tau \in \mathcal{S}$. If so, then $1+\tau\left(s\right) \geq 1\ \forall s \in \mathcal{S}$ iff $\min_{s\in\mathcal{S}} y_h\left(s\right) \geq \theta\left(y_h\left(s_\tau\right)+y_f\left(s_\tau\right)\right)$ (and similarly, $1+\tau^*\left(s\right) \geq 1\ \forall s \in \mathcal{S}$ iff $\min_{s\in\mathcal{S}} y_f\left(s\right) \geq (1-\theta)\left(y_h\left(s_\tau\right)+y_f\left(s_\tau\right)\right)$). Taken together, this implies that $y_h\left(s\right) = \theta\left(y_h\left(s_\tau\right)+y_f\left(s_\tau\right)\right)\ \forall s \in \mathcal{S}$ (and $y_f\left(s\right) = (1-\theta)\left(y_h\left(s_\tau\right)+y_f\left(s_\tau\right)\right)$ $\forall s \in \mathcal{S}$) meaning that the endowments are perfectly correlated and equal to the Pareto optimal allocation. Such an economy is ruled out by our assumption that endowments are not perfectly correlated. We then specify, without loss of generality, that $\left(\theta\left(y_h\left(s_\tau\right)+y_f\left(s_\tau\right)\right) - \frac{y_h\left(s_\tau\right)}{1+\tau\left(s_\tau\right)}\right)$ can be either positive or negative.

Let the realization s_τ be such that $s_{\tau+k} = s_\tau$ for $k \in \mathbb{N}$ and

$$\theta\left(y_h\left(s_\tau\right)+y_f\left(s_\tau\right)\right) - \frac{y_h\left(s_\tau\right)}{1+\tau\left(s_\tau\right)} > 0 \tag{38}$$

and let the history of shocks s^τ be such that $\hat{b}_h(s^\tau) < 0$.

Transition function (37) then implies

$$\frac{\hat{b}_h(s^\tau)}{1+\tau(s_\tau)} - \frac{\beta^k \hat{b}_h(s^{\tau+k})}{1+\tau(s_{\tau+k})} > 0 \tag{39}$$

for $k \in \mathbb{N}$. Given that $s_{\tau+k} = s_\tau \ \forall k \in \mathbb{N}$:

$$\beta^k \hat{b}_h(s^{\tau+k}) < \hat{b}_h(s^\tau) < 0 \tag{40}$$

for $k \in \mathbb{N}$.

The debt constraint (9) for the lone independent asset in the portfolio and specified in terms of the scaled variable is given by:

$$\inf_{k, s^{\tau+k}} \left[\frac{\hat{b}_h(s^{\tau+k})}{(1+r(s_{\tau+k}))\, p(s_0)} \right] > -\infty. \tag{41}$$

The interest rate $1+r(s_{\tau+k})$ is only a function of the current realization $s_{\tau+k}$ and is uniformly bounded. From the strict inequality (40), for any $k \in \mathbb{N}$:

$$\beta^{\tau+k+1} \hat{b}_h(s^{\tau+k+1}) < \beta^{\tau+k} \hat{b}_h(s^{\tau+k}) < 0. \tag{42}$$

Given that $\beta < 1$, then the debt constraint (41) is violated for the home household. If, on the other hand, $\hat{b}_h(s^\tau) > 0$, the realizations $s_{\tau+k} = s_\tau \ \forall k \in \mathbb{N}$ are chosen such that the inequality in (38) is reversed and we obtain a similar expression as (42):

$$\beta^{\tau+k+1} \hat{b}_h(s^{\tau+k+1}) > \beta^{\tau+k} \hat{b}_h(s^{\tau+k}) > 0. \tag{43}$$

In this second case, market clearing dictates that the debt constraint (41) is violated for the foreign household.

This contradiction completes the argument. □

Proof of Theorem 3

Proof. Suppose, without loss of generality, that the home country adopts a policy other than *nominal GDP targeting*. Suppose, in order to obtain a contradiction, that the stationary Markov equilibrium allocation is Pareto efficient. The budget constraint for the home household in any given date-event s^t is:

$$\theta(y_h(s_t) + y_f(s_t)) + \tfrac{1}{p(s^t)} \left(-p(s^{t-1})\, y_h(s_{t-1}) + \tfrac{b_h(s^t)}{1+r(s_t)} - b_h(s^{t-1}) \right) \\ + \tfrac{1}{p^*(s^t)} \left(\tfrac{b_f(s^t)}{1+r^*(s_t)} - b_f(s^{t-1}) \right) = 0.$$

As the home country monetary policy is something other than nominal GDP targeting, then $M(s^{t-2}, s) \neq M(s^{t-2}, \sigma)$ for some $s, \sigma \in \mathcal{S}$, implying that $p(s^{t-2}, s)\, y_h(s) \neq p(s^{t-2}, \sigma)\, y_h(\sigma)$ for the different shocks $s_{t-1} = s$ and $s_{t-1} = \sigma$.

By the definition of a stationary Markov equilibrium, the bond positions are only functions of three variables:

$$\left(b_h(s^t), b_f(s^t)\right) = F_b\left(b_h(s^{t-1}), b_f(s^{t-1}), s_t\right).$$

Without aggregate risk, $\theta\left(y_h\left(s_t\right) + y_f\left(s_t\right)\right)$ is constant. This implies that the net portfolio gain

$$\frac{1}{p\left(s^t\right)}\left(\frac{b_h(s^t)}{1+r\left(s_t\right)} - b_h(s^{t-1})\right) + \frac{1}{p^*\left(s^t\right)}\left(\frac{b_f(s^t)}{1+r^*\left(s_t\right)} - b_f(s^{t-1})\right)$$

must be a function of the shock s_{t-1} and must vary $1:1$ with the variation in $\left(p\left(s^{t-2}, s\right) y_h\left(s\right)\right)_{s\in\mathcal{S}}$. Household optimization implies that the net portfolio gain defined above is a nonincreasing function of the portfolio $\frac{b_h(s^{t-1})}{p(s^t)} + \frac{b_f(s^{t-1})}{p^*(s^t)}$. Given this fact and the definition of the stationary transition function

$$\left(b_h(s^t), b_f(s^t)\right) = F_b\left(b_h(s^{t-1}), b_f(s^{t-1}), s_t\right),$$

the requisite variation in the net portfolio gain is possible iff $\frac{b_h(s^{t-1})}{p(s^t)} + \frac{b_f(s^{t-1})}{p^*(s^t)}$ are functions only of the shock s_{t-1}, and independent of the previous period portfolio $\left(b_h(s^{t-2}), b_f(s^{t-2})\right)$.

Now apply the same analysis to the budget constraint in the date-event s^{t-1}. If the portfolio gain in s^{t-1},

$$\frac{1}{p\left(s^{t-1}\right)}\left(\frac{b_h(s^{t-1})}{1+r\left(s_{t-1}\right)} - b_h(s^{t-2})\right) + \frac{1}{p^*\left(s^{t-1}\right)}\left(\frac{b_f(s^{t-1})}{1+r^*\left(s_{t-1}\right)} - b_f(s^{t-2})\right),$$

is a constant function of $\frac{b_h(s^{t-2})}{p(s^{t-1})} + \frac{b_f(s^{t-2})}{p^*(s^{t-1})}$, then it is unable to match the variation in $\left(p\left(s^{t-3}, s\right) y_h\left(s\right)\right)_{s\in\mathcal{S}}$. The only remaining option is that the portfolio gain in s^{t-1} is a strictly decreasing function of $\frac{b_h(s^{t-2})}{p(s^{t-1})} + \frac{b_f(s^{t-2})}{p^*(s^{t-1})}$. Thus, matching the variation in $\left(p\left(s^{t-3}, s\right) y_h\left(s\right)\right)_{s\in\mathcal{S}}$ in the s^{t-1} budget constraint results in a contradiction of the conclusion that $\frac{b_h(s^{t-1})}{p(s^t)} + \frac{b_f(s^{t-1})}{p^*(s^t)}$ are functions only of the shock s_{t-1}, and independent of the previous period portfolio $\left(b_h(s^{t-2}), b_f(s^{t-2})\right)$ (which is required to match the variation in $\left(p\left(s^{t-2}, s\right) y_h\left(s\right)\right)_{s\in\mathcal{S}}$ in the s^t budget constraint). This contradiction finishes the argument.

$\square$