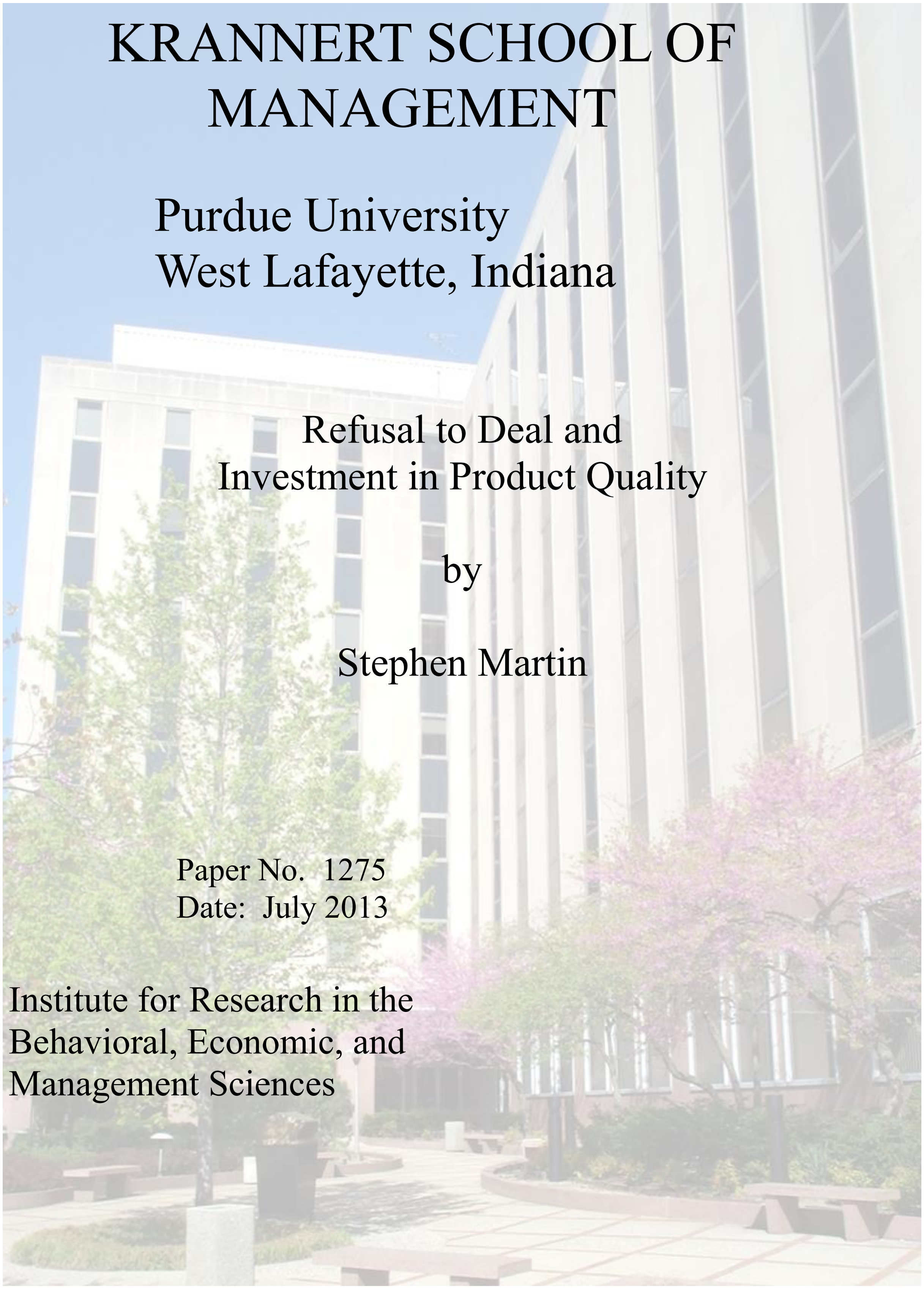

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Refusal to Deal and Investment in Product Quality

by

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Refusal to Deal and Investment in Product Quality*

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Abstract

Comments welcome.

Recent U.S. Supreme Court decisions have taken the views that monopoly profit is "incentive to innovate" and that obliging a vertically-integrated antitrust monopolist to deal with downstream rivals "may lessen the incentive for the monopolist, the rival, or both to invest in ... economically beneficial facilities." In a model of endogenous product quality, refusal to deal increases the payoff of the integrated firm and reduces equilibrium investment in quality, consumer surplus, and net social welfare if varieties are moderate or good substitutes. If varieties are poor substitutes, the integrated firm maximizes its payoff setting a wholesale price that allows the downstream rival a small economic profit.

JEL categories: L13, L12, L22, L41.

Keywords: refusal to deal, vertical exclusion, endogenous sunk cost.

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Contents

1 Introduction

U.S. antitrust policy long regarded competition as the essential mechanism for promoting good market performance.[1] In nuanced contrast to this principle of competition, in upholding the right of a vertically-integrated firm that controls an essential facility to refuse to deal with nonintegrated downstream rivals,[2] the U.S. Supreme Court has cast the role of monopoly profit in a market system in a positive light:[3]

> The mere possession of monopoly power, and the concomitant charging of monopoly prices, is not only not unlawful; it is an important element of the free-market system. The opportunity to charge monopoly prices—at least for a short period—-is what attracts "business acumen" in the first place; it induces risk taking that produces innovation and economic growth. . . .
>
> Firms may acquire monopoly power by establishing an infrastructure that renders them uniquely suited to serve their customers. Compelling such firms to share the source of their advantage is in some tension with the underlying purpose of antitrust law, since it may lessen the incentive for the monopolist, the rival, or both to invest in those economically beneficial facilities.

The two approaches to conduct-performance relationships need not be incompatible: one might argue that competition delivers good static market performance, while transient monopoly delivers good dynamic market performance. An issue implicit in such an argument, however, is that if refusal to deal is licit, monopoly may not be transient. Another issue, and the topic of this paper, is that refusal to deal, which shields vertically-integrated

[1] Northern Securities Company *v.* U.S. 193 U.S. 197 (1904) at 337-338: "in the judgment of Congress the public convenience and the general welfare will be best subserved when the natural laws of competition are left undisturbed by those engaged in interstate commerce."

[2] Under EU competition policy, for a dominant firm to refusal to deal or to engage in a vertical price squeeze is an abuse of a dominant position (See *Wanadoo* (France Télécom SA v Commission Case C-202/07 P), *Deutsche Telekom* (Case C-280-08 P 14 October 2010 (ECJ), *TeliaSonera* Case C-52/09 17 February 2011 (ECJ) (TeliaSonera), as well as the European Commission's 1988 *Notice on the application of the competition rules to access agreements in the telecommunications sector* OJ 98/C 265/02).

[3] Verizon Communications, Inc. *v.* Law Offices of Curtis V. Trinko, LLP 540 U.S. 398 (2004) at 407-408. See also Pacific Bell Telephone Co. *et al.* *v.* Linkline Communications, Inc., *et al.* 555 U. S. 438 (2009).

firms from downstream competition, completely eliminates the possibility that downstream rivals invest in innovation at all, by excluding them from the market, and may reduce the incentive of vertically-integrated firms to invest in innovation,[4] so reducing consumer welfare by reducing both equilibrium product quality and horizontal product differentiation.

In this paper, I use Sutton's (1991, 1998) endogenous sunk cost framework to model equilibrium investment in product quality by two firms, one the vertically-integrated supplier of an essential input in the production of a variety of a differentiated final good, one a downstream supplier of a competing variety of the final good. Depending on the degree of horizontal product differentiation, the vertically-integrated firm may find it most profitable to exclude the downstream firm entirely, or to set a wholesale price that permits the downstream firm at least a normal rate of return on investment. Equilibrium quality, consumer surplus, and net social welfare are reduced if the downstream firm is excluded from the market. If the downstream firm operates, all these equilibrium characteristics fall as the wholesale price of the essential input rises.

The paper is organized as follows. Section 2 places this paper in the context of the literature. Section 3 outlines the model and explores the impact of cost differences on market performance in horizontal duopoly with endogenous quality. Section 4 analyzes the impact of vertical integration and refusal to deal on market performance. Section 5 concludes. The Appendix contains an outline of proofs.[5]

2 Literature review

Economists as well as courts have reflected on the relative merits of competition and monopoly in promoting innovation and delivering good market performance. In *The Theory of Economic Development* (1934), which Winter (1984) calls Schumpeter Mark I, Joseph Schumpeter came down on the side of successive monopoly as the driver of innovation. This is the gale of creative destruction, and here the possibility of entry is a prerequisite for technological progress. In *Capitalism, Socialism, and Democracy* (1943), on the other hand — Schumpeter Mark II — Schumpeter saw persistently

[4] Following Hicks' (1935, p. 8), "The best of all monopoly profits is a quiet life."

[5] Full proofs are given in a separate appendix that is available on request from the author.

dominant firms as the source of technological advance, and of technological advance so great that it would overwhelm static welfare losses.

It is perhaps to Villard (1958) that we owe the first formulation of a hypothesized inverted-U relation between competition and innovation, among the most recent formalizations of which is Aghion *et al.* (2005).[6] If there is an inverted-U relationship between competition and innovation across industries, technological advance requires that incumbents not be able to block entry and hold product-market competition below what would otherwise be its equilibrium level.

The theoretical literature on product differentiation is surveyed by Neven (1986) and Waterson (1989).[7] Wauthy (1996) examines the choice of quality levels in duopoly in the Mussa-Rosen (1978) framework, if the selection of quality levels is costless. Motta (1993) compares outcomes with Cournot and Bertrand product-market competition and, alternatively, fixed and variable costs of quality. But the work presented here is an adaptation of the linear demand, quadratic cost-of-quality endogenous sunk cost model of Sutton (1991, 1998). Sutton's purpose is to explain equilibrium market structure when technology and quality are related as specified in the model. My purpose is to examine the impact of vertical conduct on market performance in the same type of market.

3 Cost differences and investment in quality

3.1 Demand

I assume that at most two varieties are supplied to a downstream market.[8] Inverse demand equations are

$$p_1 = \rho_1 - \frac{1}{N}\left(\rho_1 q_1 + \sigma\sqrt{\rho_1\rho_2}q_2\right) \tag{1}$$

[6] Aghion *et al.* emphasize, and find empirical support for, the inverted-U case. Their Proposition 2 identifies parameter ranges for which innovation rises continuously with competition, parameter ranges for which there is an inverted-U competition-innovation relationship, and parameter ranges for which innovation falls continuously with competition.

[7] See also Chapter 8 of Anderson, de Palma, and Thisse (1992).

[8] By appropriate assumptions about the cost of quality scale parameter ε that is introduced below, one can ensure that at most two firms will find it profitable to be in the market.

$$p_2 = \rho_2 - \frac{1}{N}\left(\sigma\sqrt{\rho_1\rho_2}q_1 + \rho_2 q_2\right), \tag{2}$$

where reservation prices ρ_1 and ρ_2 are measures of quality, the number of consumers N is a measure of market size, and σ, which lies between 0 and 1, is a measure of horizontal product differentiation.

Following Spence (1976), inverse demand equations of this form may be derived from a representative consumer welfare function

$$U = H + \rho_1 q_1 + \rho_2 q_2 - \frac{1}{2}\frac{1}{N}\left(\rho_1 q_1^2 + 2\sigma\sqrt{\rho_1\rho_2}q_1 q_1 + \rho_2 q_2^2\right), \tag{3}$$

where H is a Hicksian composite good produced under conditions of constant returns to scale by a perfectly competitive industry, the cost and price of which is normalized to be 1. Consumer surplus in the differentiated variety submarket is then the excess of utility in the differentiated good submarket minus what consumers pay for the products,

$$S = (\rho_1 - p_1) q_1 + (\rho_2 - p_2) q_2 - \frac{1}{2}\frac{1}{N}\left(\rho_1 q_1^2 + 2\sigma\sqrt{\rho_1\rho_2}q_1 q_1 + \rho_2 q_2^2\right). \tag{4}$$

3.2 Supply and quality

In this first model, I consider the case of quantity competition between two firms, each producing one variety of the product at marginal cost

$$\rho_i c_i. \tag{5}$$

Vertical relationships play no role in this version of model. In the second model, to which we turn in Section 4, firm 1 produces an essential input, and c_i is firm i's marginal cost, per unit of quality, of transforming one unit of the essential input into one unit of the final good.

In stage 2 of the basic model, firms' product-market objective functions, taking qualities as given, are

$$\pi_i = (p_i - \rho_i c_i) q_i, \tag{6}$$

(here and below, for $i = 1, 2$).

Noncooperative equilibrium product-market payoffs are[9]

$$\widehat{\pi}_i (\rho_1, \rho_1, c_1, c_2) = \frac{\rho_i}{N}\widehat{q}_i (\rho_1, \rho_1, c_1, c_2)^2. \tag{7}$$

[9] For notational compactness, in what follows I omit the list of arguments where it is possible to do so without confusion.

That the stage 2 payoff is proportional to the square of equilibrium output ($\widehat{q}$) follows immediately from the first-order conditions, and is the usual result for linear demand, constant marginal cost Cournot oligopoly.

3.3 Equilibrium relationships

In stage 1, firms noncooperatively select qualities to maximize the excess of the product-market payoff over the quadratic cost of quality:

$$\Pi_i = \frac{\rho_i}{N}\widehat{q}_i^2 - \frac{1}{2}\varepsilon\rho_i^2, \tag{8}$$

where ε is a scale parameter for the cost of quality.

The first-order conditions to maximize stage 1 objective functions (8) with respect to own quality can be written

$$2\left(1-c_1\right)^2 - \sigma\sqrt{\frac{\rho_2}{\rho_1}}\left(1-c_1\right)\left(1-c_2\right) - \frac{\left(4-\sigma^2\right)^2}{2N}\varepsilon\rho_1 = 0. \tag{9}$$

$$2\left(1-c_2\right)^2 - \sigma\sqrt{\frac{\rho_1}{\rho_2}}\left(1-c_1\right)\left(1-c_2\right) - \frac{\left(4-\sigma^2\right)^2}{2N}\varepsilon\rho_2 = 0. \tag{10}$$

Using the first-order conditions, equilibrium payoffs are

$$\widehat{\Pi}_1 = \frac{\left(1-c_1\right) - \sigma\sqrt{\frac{\rho_2}{\rho_1}}\left(1-c_2\right)}{4-\sigma^2}\rho_1\widehat{q}_1 \tag{11}$$

$$\widehat{\Pi}_2 = \frac{\left(1-c_2\right) - \sigma\sqrt{\frac{\rho_1}{\rho_2}}\left(1-c_1\right)}{4-\sigma^2}\rho_2\widehat{q}_2 \tag{12}$$

(when evaluated at equilibrium quality levels).

The conditions for both payoffs to be nonnegative are

$$\frac{1}{\sigma} \geq \frac{1-c_2}{1-c_1}\sqrt{\frac{\rho_2}{\rho_1}} \geq \sigma. \tag{13}$$

If the left-hand relationship holds with equality, $\Pi_1 = 0$. If the right-hand relationship holds with equality, $\Pi_2 = 0$.

In general, the system of quality first-order conditions has no analytic solution. As one would expect from Sutton's work, for the identical-marginal

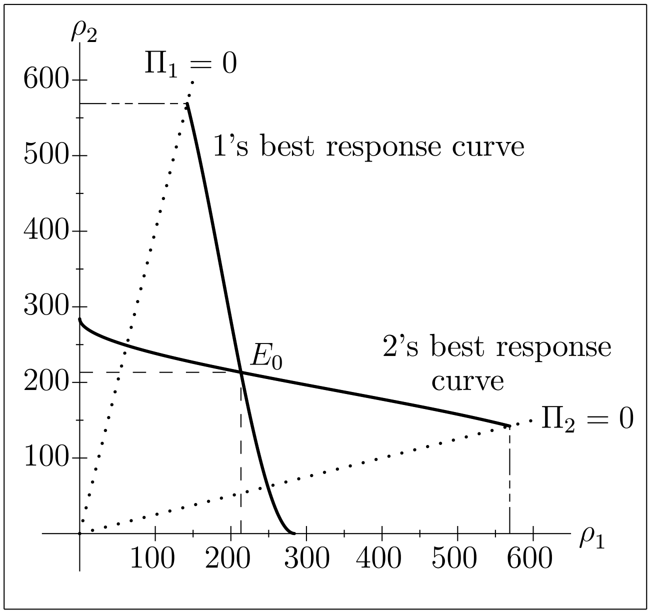

Figure 1: Quality best-response curves, $c_1 = c_2 = 0$, $\sigma = 1/2$, $N = 1000$, $\varepsilon = 1$.

cost case there is a unique equilibrium pair of qualities for which one can obtain the expression

$$\rho = \frac{2N}{(2-\sigma)(2+\sigma)^2}\frac{(1-c)^2}{\varepsilon}. \tag{14}$$

To illustrate symmetric equilibrium quality choices, let $\varepsilon = 1$ and $N = 1000$ (these are costless normalizations), and use a value of σ, $1/2$, that represents an intermediate degree of horizontal product differentiation. Figure 1 shows zero-payoff lines and quality best-response lines for the case that $c_1 = c_2 = 0$. Qualities are strategic substitutes in the neighborhood of equilibrium. The straight lines $\Pi_1 = 0$ and $\Pi_2 = 0$ bound the region within which both firms' participation constraints are satisfied. As marginal costs increase, maintaining equality, equilibrium qualities retreat along the 45-degree line toward the origin.

One can establish the comparative static properties of the model if marginal costs differ:

Lemma 1 *(a) As firm 2's marginal cost increases, $\widehat{\rho}_1$ rises and $\widehat{\rho}_2$ falls, all* *else equal:*

$$\frac{\partial \widehat{\rho}_1}{\partial c_2} > 0 \qquad \frac{\partial \widehat{\rho}_2}{\partial c_2} < 0. \tag{15}$$

(b) quality best-response curves are downward sloping:

$$\left.\frac{d\rho_1}{d\rho_2}\right|_{1's\ brf} < 0 \qquad \left.\frac{d\rho_1}{d\rho_2}\right|_{2's\ brf} < 0, \tag{16}$$

Proof: see Appendix.

If firm 2's marginal cost is greater than firm 1's marginal cost, both boundary lines rotate in a counterclockwise direction, compared with the identical marginal cost case. The higher marginal cost firm has lower equilibrium quality. If firm 2's marginal cost is sufficiently high,[10] the two best response lines intersect on the $\Pi_2 = 0$ line, and firm 2 just breaks even. This case is illustrated in Figure 2. For higher values of c_2, firm 2's participation constraint is not satisfied.

Lemma 1 is a harbinger of results from the vertical market model, when changes in the wholesale price of the essential input change the marginal cost of the nonintegrated firm.

4 Vertical relationships and investment in quality

4.1 Exclusion

Keeping all other aspects of the specification unchanged, suppose now that firm 1 produces an essential input, one unit of which is required for production of one unit of the final good. Marginal transformation costs per unit of quality are c_1 and c_2, respectively. I assume that the input is produced at constant marginal cost, which includes a normal rate of return on investment. For simplicity, normalize this marginal cost to be 0.

We need exclusion values for comparison with outcomes if both varieties have positive output in the downstream market. If firm 1 excludes firm 2, it

[10] To be precise, if $c_2 = 1 - \sigma^{1/2}\left(2 - \sigma^2\right)^{1/4}(1 - c_1)$.

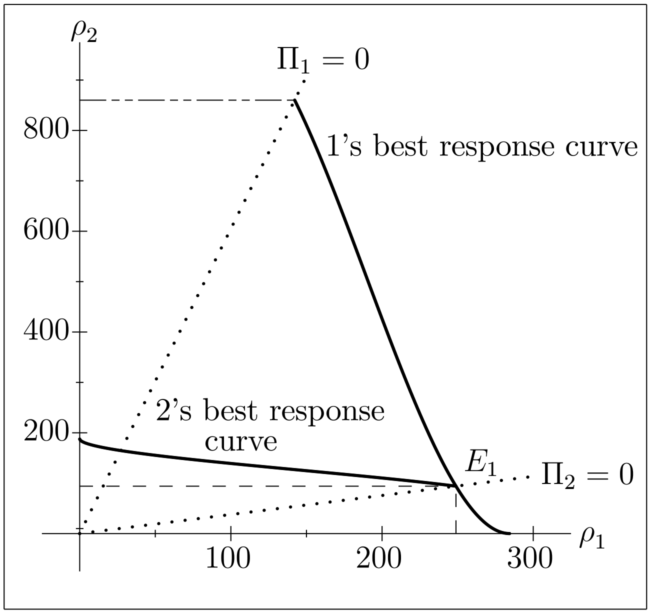

Figure 2: Quality best-response curves, $c_1 = 0$, $c_2 = 0.18671$, $\sigma = 1/2$, $N = 1000$, $\varepsilon = 1$.

picks its quality, then sets output. The monopoly objective function in stage 2 is

$$\pi_1^m = (p_1 - \rho_1 c_1)\, q_1 \tag{17}$$

Monopoly output is

$$q_1^m = \frac{N}{2}(1 - c_1). \tag{18}$$

The monopoly stage 2 payoff is

$$\pi_1^m = \frac{\rho_1}{N}(q_1^m)^2 \tag{19}$$

When firm 1 sets quality, its objective function is

$$\Pi_{1.}^m = \frac{N}{4}\rho_1 (1 - c_1)^2 - \frac{1}{2}\varepsilon \rho_1^2. \tag{20}$$

It is straightforward to show that

Lemma 2 *Exclusion equilibrium quality, monopoly profit, consumer surplus, and net social welfare are*

$$\rho_1^m = \frac{N}{4\varepsilon}(1 - c_1)^2 \tag{21}$$

$$\Pi_{1.}^{m} = \frac{N^2}{32\varepsilon}\left(1 - c_1\right)^4 \tag{22}$$

$$S^m = \frac{N^2}{32\varepsilon}\left(1 - c_1\right)^4. \tag{23}$$

and

$$u^m = \Pi_{1.}^{m} + S^m = \frac{N^2}{16\varepsilon}\left(1 - c_1\right)^4, \tag{24}$$

respectively.

4.2 Downstream duopoly

If firm 2 has positive output, stage 2 objective functions are

$$\pi_1^{dd} = \left(p_1 - \rho_1 c_1\right) q_1 + \omega \rho_2 q_2 \tag{25}$$

and

$$\pi_2^{dd} = \left[p_2 - \left(c_2 + \omega\right) \rho_2\right] q_2, \tag{26}$$

respectively, where the superscript dd denotes downstream duopoly, the case that both firms are active in the final good market. ω is the wholesale price of the essential input, set by firm 1 in what we call, for consistency with the basic model, stage 0.

Firm 1's stage 2 first-order condition is unchanged from the basic model, (32). The stage 2 first-order conditions yield explicit solutions for equilibrium outputs, as functions of (among other parameters) ω.

Firm 1's stage 1 objective function is

$$\Pi_1^{dd} = \frac{1}{N}\rho_1 \widehat{q}_1^2 + \omega \rho_2 \widehat{q}_2 - \frac{1}{2}\varepsilon \rho_1^2. \tag{27}$$

Firm 2's stage 1 objective function is (8), for $i = 2$. The two first-order conditions are incapable of explicit solution, but implicitly determine stage 1 equilibrium qualities $\widehat{\rho}_1\left(c_1, c_2, \omega\right)$ and $\widehat{\rho}_2\left(c_1, c_2, \omega\right)$. We can show

Lemma 3 *As ω increases, $\widehat{\rho}_1$ rises and $\widehat{\rho}_2$ falls, all else equal:*

$$\frac{\partial \widehat{\rho}_1}{\partial \omega} > 0 \qquad \frac{\partial \widehat{\rho}_2}{\partial \omega} < 0. \tag{28}$$

The proof is similar to the proof of Lemma 1 (given in the Appendix), and is omitted.

Firm 1's stage 0 problem is then (with some abuse of notation)

$$\max_{\omega} \Pi_1^{dd}(\omega) \text{ s.t. } \Pi_2^{dd}(\omega) \geq 0. \tag{29}$$

We analyze possible solutions by looking at the shapes of the payoff functions $\Pi_1^{dd}(\omega)$ and $\Pi_2^{dd}(\omega)$.

Theorem 4 *Higher wholesale prices reduce firm 2's payoff,*

$$\frac{d\Pi_2^{dd}}{d\omega} < 0; \tag{30}$$

and provided that both firms have positive output for $\omega = 0$*, either (a)* $\frac{d\Pi_1^{dd}}{d\omega} > 0 \; \forall \Pi_2^{dd} \geq 0$*, so firm 1 excludes firm 2 from the downstream market; or*
(b) $\frac{d\Pi_1^{dd}}{d\omega} > 0$ *for* $\omega = 0$*, declining in magnitude and achieving an internal maximum at a value of* ω *that allows firm 2 nonnegative profit. Proof: See Appendix.*

Numerical examples show that both cases may occur.

4.3 $\sigma = 1/4$

Figure 3 shows payoff functions for the parameter values of Figure 1, with the exception that $\sigma = 1/4$, so varieties are relatively poor horizontal substitutes. For $\omega = 0$, the outcome is that of the basic model; the firms have identical payoffs. As ω rises, firm 2's payoff falls. Firm 1's payoff first rises, then falls, as a function of ω. Its payoff is maximized for a value of ω that allows firm 2 to operate with positive profit. Further, firm 1's downstream duopoly payoff exceeds its exclusion payoff.

Figure 4 shows the corresponding quality values. In downstream duopoly equilibrium, $\rho_2 < \rho_1$ and ρ_1 is less than firm 1's exclusion quality. Despite this, however, as shown in Figure 5, consumer surplus and net social welfare are both greater with both firms active in the downstream market than if firm 1 excludes firm 2. Because variety 2 is a weak horizontal substitute for variety 1, firm 1 finds it profitable to set ω so firm 1 stays in the market and purchases a relatively large amount of the essential input. Because variety 2 is a weak horizontal substitute for variety 1, firm 2's sales do not much cut into firm 1's sales.

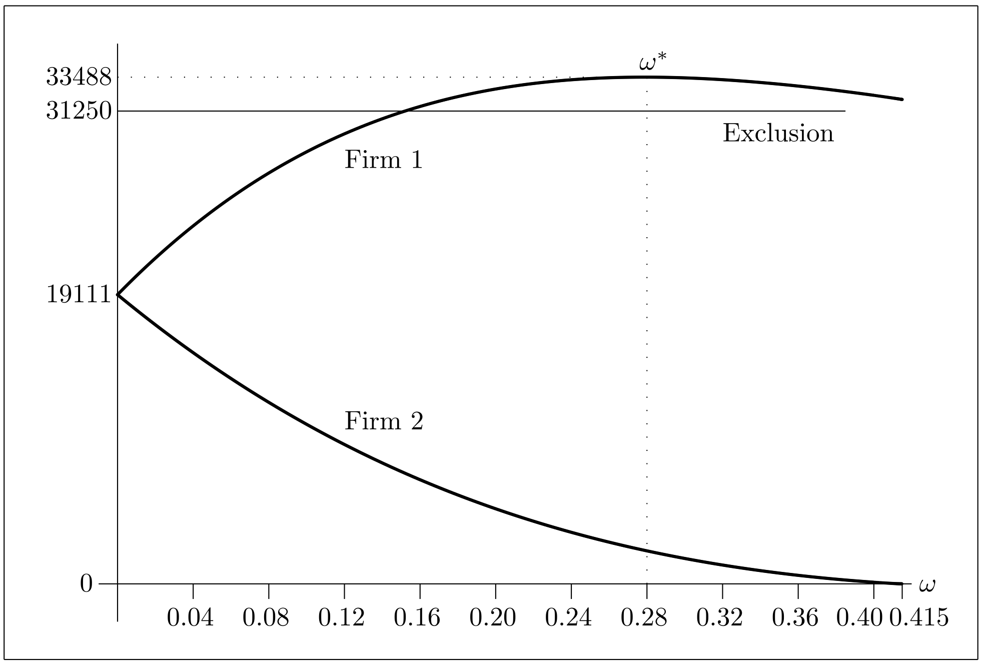

Figure 3: Exclusion payoff, firm 1, and downstream duopoly payoffs, as functions of ω. $\varepsilon = 1$, $\sigma = 1/4$, $N = 1000$, $c_1 = c_2 = 0$. ω^* indicates firm 1's profit-maximizing wholesale price.

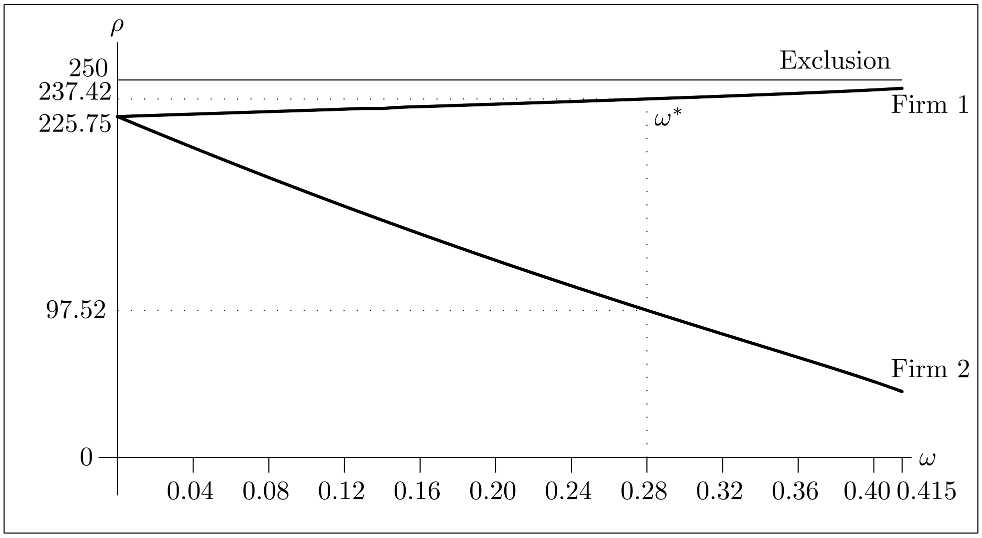

Figure 4: Reservation prices, exclusion and downstream duopoly, as functions of ω. $\varepsilon = 1$, $\sigma = 1/4$, $N = 1000$, $c_1 = c_2 = 0$. ω^* indicates firm 1's profit-maximizing wholesale price.

4.4 $\sigma = 1/2$

Now turn to the case that varieties are closer horizontal substitutes. Figure 6 shows payoffs as functions of ω for the parameter values of Figure 1. For $\omega = 0$, the outcome is that of the basic model; the firms have identical payoffs. As ω rises, Π_1 increases and Π_2 decreases, throughout the range for which $\Pi_2 \geq 0$; for higher values of ω, firm 2's participation constraint would not be met.

But for the intermediate product differentiation level indicated by $\sigma = 1/2$, firm 1's exclusion payoff exceeds its maximum duopoly payoff. Consumers' taste for variety is great enough for variety to maximize welfare, but the two varieties are close enough substitutes that firm 1's profit is greater if it excludes firm 2 from the market.

Figure 7 shows equilibrium quality levels for different wholesale prices, for the parameter values of Figure 6. As we expect from Lemma 2, higher wholesale prices induce firm 1 to invest more in quality, and firm 2 less. But even for the value of ω for which firm 2's participation constraint binds, firm 1's quality choice (242.31) is less than the quality it would choose (250) if it excludes firm 2. If one measures market performance by quality, horizontal

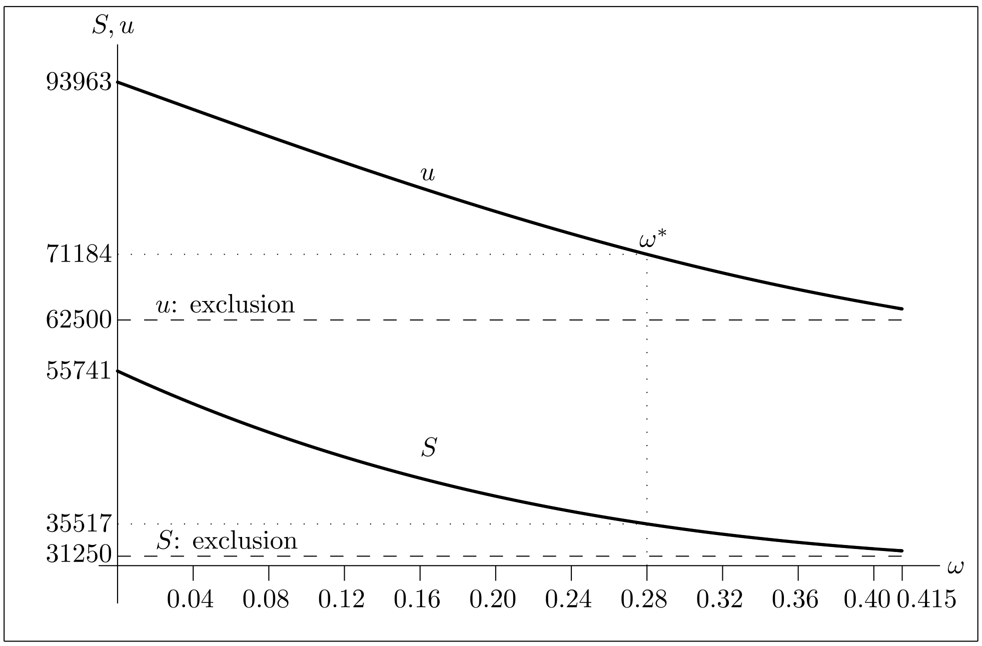

Figure 5: Consumer surplus (S) and net social welfare (u), exclusion and downstream duopoly, as functions of ω. $\varepsilon = 1$, $\sigma = 1/4$, $N = 1000$, $c_1 = c_2 = 0$. ω^* indicates firm 1's profit-maximizing wholesale price.

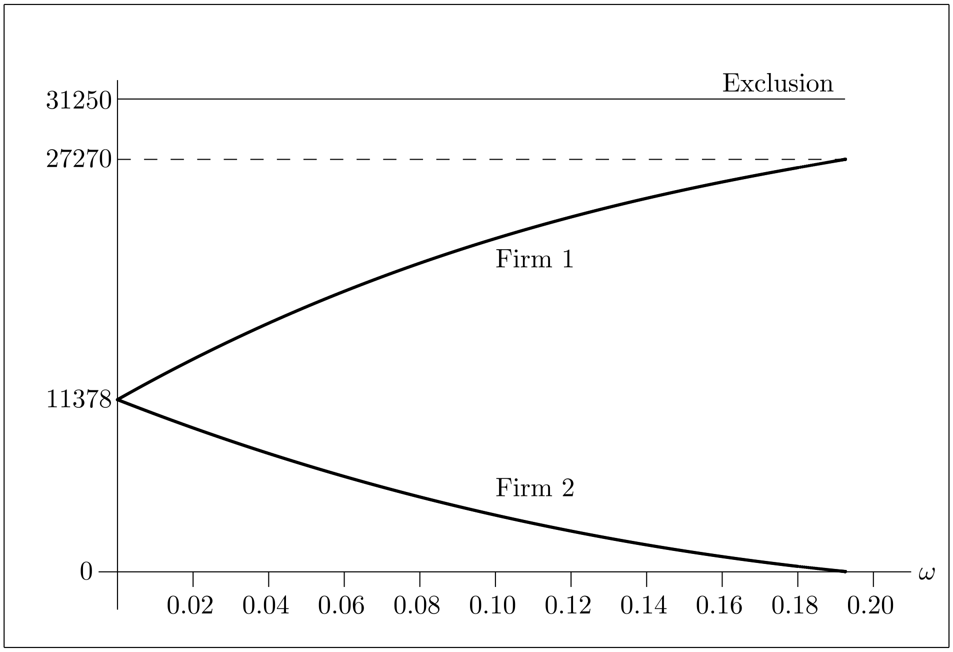

Figure 6: Exclusion payoff, firm 1, and downstream duopoly payoffs, as functions of ω. $\varepsilon = 1$, $\sigma = 1/2$, $N = 1000$, $c_1 = c_2 = 0$.

exclusion is best.

But neither economists nor antitrust authorities are wont to measure market performance by quality alone. Rather, they focus on consumer surplus or net social welfare, the sum of consumer surplus and firms' profits.[11] Figure 8 shows net social welfare (u) and consumer surplus (S) for the parameter values of the second case. In this instance, there is no need to choose between consumer surplus and net social welfare as measures of market performance; they yield the same result. Net social welfare and consumer surplus both peak for $\omega = 0$ and fall steadily as ω rises. Further, net social welfare and consumer surplus with two active firms are always greater than the corresponding exclusion values.

The difference between the quality rankings of Figure 7 and the welfare rankings of Figure 8 reflect the welfare gains from horizontal product dif-

[11]There is of course a debate whether consumer surplus or net social welfare is the appropriate index of market performance for policy purposes, but the terms of this debate are well known, and we do not enter into it here.

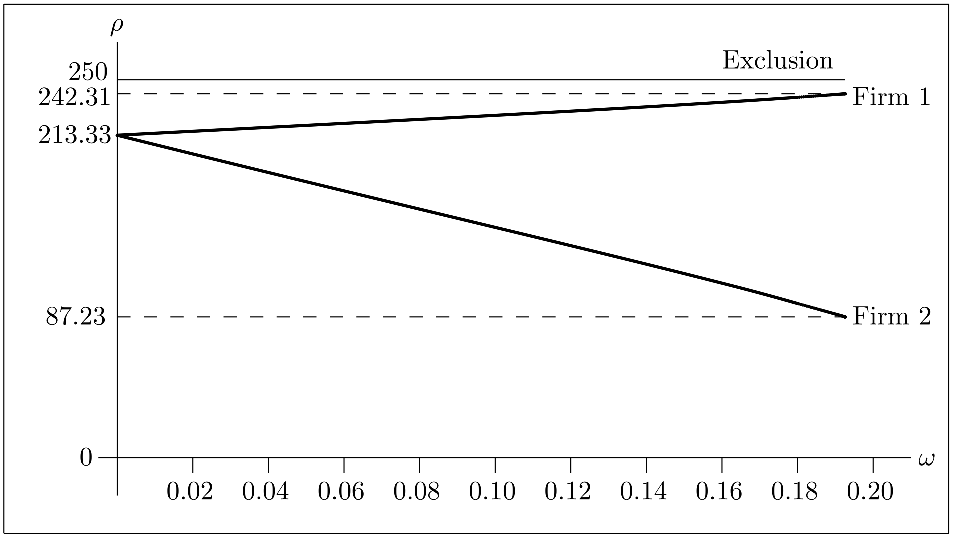

Figure 7: Reservation prices, exclusion and downstream duopoly, as functions of ω. $\varepsilon = 1$, $\sigma = 1/2$, $N = 1000$, $c_1 = c_2 = 0$.

ferentiation if both varieties are present. For this set of parameter values, market performance is best, assuming production by firms, if both firms are active and both firms have access to the essential input at its marginal cost of production (which allows firm 1 to earn a normal rate of return on investment).

5 Conclusion

In *Trinko*, the U.S. Supreme Court focused on quality competition — vertical product differentiation — as a driver of market performance. But, as the saying goes, "variety is the spice of life," and horizontal and vertical product differentiation both affect market performance. In the model explored here, if product varieties are weak horizontal substitutes, the vertically-integrated supplier of an essential input will set the price of its intermediate good so its nonintegrated downstream competitor stays in the market. Because of the market power exercised by the vertically-integrated firm, the independent downstream firm will invest less in quality, but horizontal competition increase consumer surplus and net social welfare, compared with exclusion. If, on the other hand, varieties are close horizontal substitutes, the most prof-

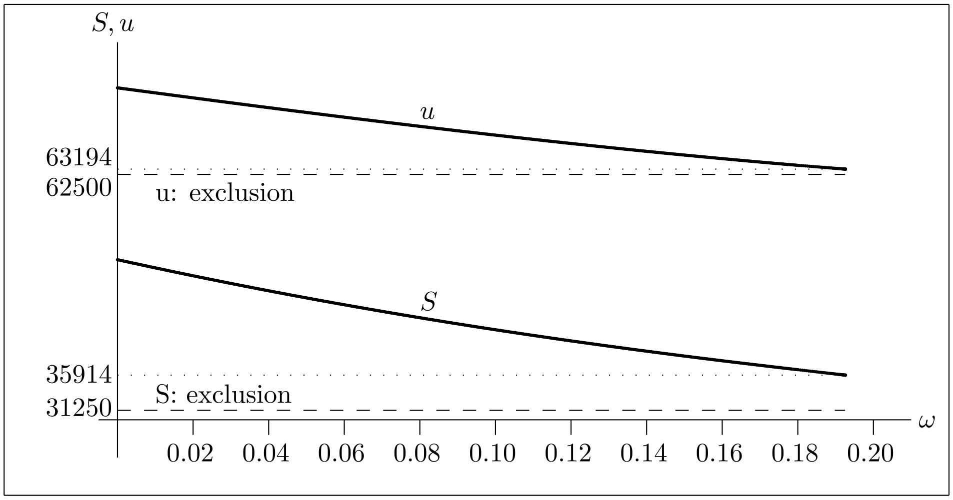

Figure 8: Consumer surplus (S) and net social welfare (u), exclusion and downstream duopoly, as functions of ω. $\varepsilon = 1$, $\sigma = 1/2$, $N = 1000$, $c_1 = c_2 = 0$.

itable path for the vertically-integrated supplier is to refuse to deal with the nonintegrated downstream firm. The vertically-integrated firm's monopoly quality is greater than its duopoly quality, if it were to price the intermediate good so the downstream rival just breaks even. But the loss of horizontal variety entailed by exclusion again reduces consumer surplus and net social welfare.

What is meant by the term "competition" in discussions of antitrust policy is often unclear, encompassing competitive market structure, potential competition, actual rivalry in the product market, and competitive market performance. The upshot of the analysis presented here is that it is actual rivalry in the development of high-quality substitute varieties that promotes consumer welfare, and that such rivalry is ill-served by the exercise of market power in input markets and by the refusal of vertically-integrated upstream firms with their nonintegrated downstream rivals.

6 Appendix

6.1 Horizontal competition

6.1.1 Stage 2

Substituting the inverse demand equation (1) into the objective function (6), firm 1's product-market objective function is

$$\pi_1 = \rho_1 \left[1 - c_1 - \frac{1}{N}\left(q_1 + \sigma\sqrt{\frac{\rho_2}{\rho_1}}q_2\right)\right] q_1 \tag{31}$$

The first-order condition to maximize π_1 is

$$1 - c_1 - \frac{1}{N}\left(2q_1 + \sigma\sqrt{\frac{\rho_2}{\rho_1}}q_2\right) = 0, \tag{32}$$

from which we get (7).

The first-order condition can be rewritten as

$$2q_1 + \sigma\sqrt{\frac{\rho_2}{\rho_1}}q_2 = N\left(1 - c_1\right). \tag{33}$$

In the same way, firm 2's stage 2 first-order condition can be written

$$\sigma\sqrt{\frac{\rho_1}{\rho_2}}q_1 + 2q_2 = N\left(1 - c_2\right). \tag{34}$$

Solving the two first-order conditions, equilibrium outputs are

$$\widehat{q}_1 = N\frac{2\left(1 - c_1\right) - \sigma\sqrt{\frac{\rho_2}{\rho_1}}\left(1 - c_2\right)}{4 - \sigma^2} \tag{35}$$

$$\widehat{q}_2 = N\frac{2\left(1 - c_2\right) - \sigma\sqrt{\frac{\rho_1}{\rho_2}}\left(1 - c_1\right)}{4 - \sigma^2}. \tag{36}$$

(35) and (36) are substituted in (8) to obtain expressions for stage 1 objective functions.

From (35) and (36), the conditions for both outputs to be nonnegative are

$$\frac{2}{\sigma} \geq \frac{1 - c_2}{1 - c_1}\sqrt{\frac{\rho_2}{\rho_1}} \geq \frac{\sigma}{2}, \tag{37}$$

where the first inequality is the condition for q_1 to be nonnegative and the second inequality is the condition for q_2 to be nonnegative.

6.1.2 Stage 1

6.1.3 Comparative statics

We are interested in comparative statics with respect to c_2. Differentiating the quality first-order conditions $\frac{\partial \Pi_1}{\partial \rho_1} \equiv 0$ and $\frac{\partial \Pi_2}{\partial \rho_2} \equiv 0$ with respect to c_2 gives the system of equations

$$\begin{pmatrix} \frac{\partial^2 \Pi_1^{dd}}{\partial \rho_1^2} & \frac{\partial^2 \Pi_1^{dd}}{\partial \rho_1 \partial \rho_2} \\ \frac{\partial^2 \Pi_2^{dd}}{\partial \rho_1 \partial \rho_2} & \frac{\partial^2 \Pi_2^{dd}}{\partial \rho_2^2} \end{pmatrix} \begin{pmatrix} \frac{\partial \rho_1}{\partial c_2} \\ \frac{\partial \rho_2}{\partial c_2} \end{pmatrix} = - \begin{pmatrix} \frac{\partial^2 \Pi_1^{dd}}{\partial c_2 \partial \rho_1} \\ \frac{\partial^2 \Pi_2^{dd}}{\partial c_2 \partial \rho_2} \end{pmatrix}. \tag{38}$$

Assuming stability gives that the trace of the coefficient matrix on the left is negative, and its determinant positive. Sufficient conditions for the trace to be negative are that the diagonal elements of the coefficient matrix be negative, which we would have in any event from the assumption that the second-order condition is met.

Tedious evaluation of the derivatives in (38) establishes the following sign pattern if the equation is multiplied by the inverse of the coefficient matrix on the right:

$$\begin{pmatrix} \frac{\partial \rho_1}{\partial c_2} \\ \frac{\partial \rho_2}{\partial c_2} \end{pmatrix} = \begin{pmatrix} - & + \\ + & - \end{pmatrix} \begin{pmatrix} - \\ + \end{pmatrix}, \tag{39}$$

Then the signs of the derivatives satisfy

$$\frac{\partial \rho_1}{\partial c_2} = (-)(-) + (+)(+) > 0 \tag{40}$$

and

$$\frac{\partial \rho_2}{\partial c_2} = (+)(-) + (-)(+) < 0, \tag{41}$$

and this establishes the first part of Lemma 1. As c_2 rises, equilibrium ρ_1 rises and ρ_2 falls.

For the slope of the firm 1's best-response equation, the first-order condition can be written

$$\frac{\partial \Pi_1^{dd}(\rho_1, \rho_2)}{\partial \rho_1} = 2N \frac{2(1-c_1) - \sigma \sqrt{\frac{\rho_2}{\rho_1}}(1-c_2)}{4-\sigma^2} \frac{1-c_1}{4-\sigma^2} - \varepsilon \rho_1 = 0. \tag{42}$$

Then

$$\frac{\partial^2 \Pi_1^{dd}}{\partial \rho_1^2} = N \frac{\sigma \left(1 - c_1\right) \left(1 - c_2\right)}{\left(4 - \sigma^2\right)^2} \rho_1^{-3/2} \rho_2^{1/2} - \varepsilon < 0, \tag{43}$$

where the sign follows from the second-order condition, and

$$\frac{\partial^2 \Pi_1^{dd}}{\partial \rho_1 \partial \rho_2} = -N \frac{\sigma \left(1 - c_1\right) \left(1 - c_2\right)}{\left(4 - \sigma^2\right)^2} \rho_1^{-1/2} \rho_2^{-1/2} < 0 \tag{44}$$

Differentiate the first-order condition to obtain the slope of the best-response function:

$$\frac{\partial^2 \Pi_1^{dd}}{\partial \rho_1^2} \left. \frac{d\rho_1}{d\rho_2} \right|_{brf} + \frac{\partial^2 \Pi_1^{dd}}{\partial \rho_1 \partial \rho_2} = 0$$

$$\left. \frac{d\rho_1}{d\rho_2} \right|_{brf} = \frac{\frac{\partial^2 \Pi_1^{dd}}{\partial \rho_1 \partial \rho_2}}{-\frac{\partial^2 \Pi_1^{dd}}{\partial \rho_1^2}} < 0. \tag{45}$$

This is the second part of Lemma 1.

6.2 Vertical competition

6.2.1 Quality

For the case that firm 1 is vertically integrated, stage 2 equilibrium outputs are

$$\widehat{q}_1 = N \frac{2\left(1 - c_1\right) - \sigma \sqrt{\frac{\rho_2}{\rho_1}} \left(1 - c_2 - \omega\right)}{4 - \sigma^2} \tag{46}$$

and

$$\widehat{q}_2 = N \frac{2\left(1 - c_2 - \omega\right) - \sigma \sqrt{\frac{\rho_1}{\rho_2}} \left(1 - c_1\right)}{4 - \sigma^2}. \tag{47}$$

These may be compared with (35) and (36). Substitution of these expressions for equilibrium outputs into (27) and into (8) (for $i = 2$) give the stage 1 objective functions of firm 1 and firm 2, respectively.

Theorem 4:

$$\begin{aligned} \frac{d\Pi_2^{dd}}{d\omega} &= \frac{\partial \Pi_2^{dd}}{\partial \rho_1} \frac{d\rho_1}{d\omega} + \frac{\partial \Pi_2^{dd}}{\partial \rho_2} \frac{d\rho_2}{d\omega} + \frac{\partial \Pi_2^{dd}}{\partial \omega} \\ &= \frac{\partial \Pi_2^{dd}}{\partial \rho_1} \frac{d\rho_1}{d\omega} + \frac{\partial \Pi_2^{dd}}{\partial \omega}, \end{aligned} \tag{48}$$

since $\frac{\partial \Pi_2^{dd}}{\partial \rho_2} = 0$ (an application of the envelope theorem). From Lemma 3, $\frac{d\rho_1}{d\omega} > 0$.

Firm 2's quality objective function is

$$\Pi_2^{dd} = \frac{1}{N}\widehat{\rho}_2 q_2^2 - \frac{1}{2}\varepsilon\rho_2^2, \tag{49}$$

where output is given by (47). Then

$$\frac{\partial \Pi_2^{dd}}{\partial \rho_1} = \frac{2}{N}\rho_2 q_2 \frac{\partial q_2}{\partial \rho_1} < 0,$$

$$\frac{\partial \Pi_2^{dd}}{\partial \omega} = \frac{2}{N}\rho_2 q_2 \frac{\partial q_2}{\partial \omega} < 0,$$

and for $\frac{d\Pi_2^{dd}}{d\omega}$ we have

$$sign\frac{d\Pi_2^{dd}}{d\omega} = (-)(+) + (-) < 0.$$

This is the first part of Theorem 4.

For the second part of Theorem 4, again using the envelope theorem and Lemma 3,

$$\frac{d\Pi_1^{dd}}{d\omega} = \frac{\partial \Pi_1^{dd}}{\partial \rho_2}\frac{d\rho_2}{d\omega} + \frac{\partial \Pi_1^{dd}}{\partial \omega} = \frac{\partial \Pi_1^{dd}}{\partial \rho_2}(-) + \frac{\partial \Pi_1^{dd}}{\partial \omega}.$$

From (27),

$$\frac{\partial \Pi_1^{dd}}{\partial \rho_2} =$$

$$\frac{2}{N}\rho_1 q_1 N \frac{-\frac{1}{2}\sigma\rho_1^{-1/2}\rho_2^{-1/2}(1 - c_2 - \omega)}{4-\sigma^2} + \omega\left(q_2 + \rho_2 N\frac{\frac{1}{2}\sigma\rho_1^{1/2}\rho_2^{-3/2}(1-c_1)}{4-\sigma^2}\right)$$

(and collecting terms in ω on the right)

$$= -q_1\frac{\sigma\rho_1^{1/2}\rho_2^{-1/2}(1-c_2)}{4-\sigma^2} + \omega\left\{q_2 + \left[q_1 + \frac{1}{2}N\frac{(1-c_1)}{4-\sigma^2}\right]\frac{\sigma\rho_1^{1/2}\rho_2^{-1/2}}{4-\sigma^2}\right\}.$$

The coefficient of ω is positive. Hence

$$\frac{\partial \Pi_1^{dd}}{\partial \rho_2} < 0$$

for $\omega = 0$, and $\frac{\partial \Pi_1^{dd}}{\partial \rho_2}$ falls in magnitude as ω rises. This gives us

$$sign \frac{d\Pi_1^{dd}}{d\omega} = \frac{\partial \Pi_1^{dd}}{\partial \rho_2} (-) + \frac{\partial \Pi_1^{dd}}{\partial \omega} = (-)(-) + \frac{\partial \Pi_1^{dd}}{\partial \omega},$$

for ω near 0, so that the first term on the right is positive for $\omega = 0$, eventually turning negative.

Taking the derivative of (27) with respect to ω gives,

$$\frac{1}{N} \frac{\partial \Pi_1^{dd}}{\partial \omega} =$$

$$2\sigma \rho_1^{1/2} \rho_2^{1/2} N \frac{2(1-c_1) - \sigma (1-c_2) \rho_1^{-1/2} \rho_2^{1/2}}{(4-\sigma^2)^2} + \rho_2 N \frac{2(1-c_2) - \sigma \rho_1^{1/2} \rho_2^{-1/2} (1-c_1)}{4-\sigma^2}$$

$$- \frac{2\omega \rho_2}{4-\sigma^2} \frac{8-3\sigma^2}{4-\sigma^2}.$$

The fractions on the right in the first two terms of the expression for $\frac{\partial \Pi_1^{dd}}{\partial \omega}$ are $q_1(0)$ and $q_2(0)$, respectively. If $q_2 > 0$ for $\omega > 0$, it is positive for $\omega = 0$. If we assume as well $q_1(0) \geq 0$, then

$$sign \frac{d\Pi_1^{dd}}{d\omega} = (-)(-) + (+)$$

for ω near 0, with the first term on the right positive for $\omega = 0$, eventually turning negative. The slope of the profit function may be positive throughout the range of ω where firm 2's participation constraint is satisfied, or it may turn negative within this region. This is the second part of Theorem 4.

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