The Importance of Audit Quality Standards[1]

Romana Autrey
UIUC Department of Accountancy

Richard Crowley
UIUC Department of Accountancy

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The Importance of Audit Quality Standards

Abstract Audit quality is critical for financial markets to function smoothly. This paper theoretically examines how auditing standards requiring a minimum level of audit quality affect the public company audit industry. We present a parsimonious model of audit market competition that analyzes the differing impact of auditing standards on Big 4 and non-Big 4 firms, as well as the social welfare implications of imposing minimum audit standards. The results suggest that when auditing standards intensify competition, increases in minimum audit quality actually lead to lower billing rates, transferring wealth from Big 4 and non-Big 4 audit firms to their clients. We derive a socially optimal level of auditing standards that balances the additional cost of increasing quality against the additional benefit of lower reputational costs for clients choosing non-Big 4 auditors.

Keywords: Audit quality, audit pricing, auditing standards

1 Introduction

Audit quality is critical for financial markets to function smoothly. This paper theoretically examines how auditing standards requiring a minimum level of audit quality affect a competitive market for public company audits. In our setting, audit quality refers to qualitatively better auditing, rather than more auditing. Natural examples include the independence or competence of audit firm personnel; both examples have numerous related standards that lay out the minimum acceptable quality levels. We model a setting in which the demand for audit quality is driven strictly by client demand (i.e., there is no auditor liability). Higher audit quality attracts investors more cheaply via improved assurance as to the client's true financial position; the higher the audit quality, the lower the risk of nasty surprises.

We use auditor independence, which requires that audit firm personnel have no interest in any client that would prejudice the firm's audit opinion, as a motivating example throughout this paper. Without independence, audit quality is deemed unacceptably low irrespective of hours. As with audit quality, independence lies on a continuum. For example, audit standards prohibit any member of an engagement team for a given client from owning shares of that client's stock, but the standards do not prohibit *all* personnel at a given audit firm from owning shares in every one of the firm's audit clients. Generally, the standards phase in more stringently as an individual becomes more likely to influence a given audit opinion (based on some combination of closer proximity to the client and higher position in the firm). Some audit firms choose to impose stricter internal policies (i.e., higher quality) than the minimum required by audit standards, at a substantial cost. For example, some audit firms hold personnel with less proximity (e.g., in a different office) or lower rank (e.g., managers versus partners) to the same standard as the actual engagement team, and implement costly tracking systems to ensure compliance with firm policy.

The audit industry is widely recognized to be dominated by four firms (referred to as the Big 4 or Big N): Deloitte, Ernst & Young, KPMG, and PwC; however, outside the Big 4 there are numerous audit firms that vary greatly in size. We present a parsimonious model that analyzes the differing impact of auditing standards on Big 4 ("Big") and non-Big 4 ("Small") audit firms (collectively, "firms"). To represent the Big 4's stronger competitive position, we model the Big firm and Small firm respectively as a Stackelberg leader and follower. This formulation is equivalent to modeling the Small firm as a price-taking competitive fringe, as in the dominant firm models of Carlton and Perloff (2005) [5].

This paper also explores the impact of auditing standards on social welfare. In our model, Big firms optimally provide higher quality auditing than the minimum set forth by auditing standards in exchange for higher fees; prior research generally concludes that this is descriptive of the Big 4 firms (e.g., Moizer 1997 [19]). A natural question is, if market forces determine the level of audit quality, should regulators ever mandate a minimum level of auditing quality? Easterbrook and Fischel (1984) argue that there are merits to both market and regulatory controls [11], and indeed we find in our setting that there is an optimal, minimum level of auditing standards.

This paper presents an analytical model of duopoly competition among the Big and Small audit firms. To this setting we add four main assumptions. First, consistent with the longstanding audit industry history, we assume there is no entry into Big firm status. Second, due to mandatory audits for public companies ("clients"), we assume that demand is perfectly inelastic. Third, we assume that to the extent that the Big firm provides higher quality audits than the Small firm, they can charge clients a higher billing rate. We model the price premium as a non-cash reputational cost borne by clients of a Small firm; when the client's need for a high-quality audit is sufficiently high, paying the cash premium is preferred to the reputational cost. Fourth, for tractability, we assume that the Small firm chooses the minimum audit quality.

This relatively simple model generates some intriguing results. The first set of results relates to an increase in the minimum audit quality required by the auditing standards. Raising the standard does not decrease the auditing quality for any client, but it strictly decreases billing rates and firm profits for all audit firms. Intuitively, increasing the standard reduces the Big firm's product differentiation with respect to the the Small firm (who chooses the quality specified in the standard). Improving quality to maintain its existing rate premium is too costly, so the Big firm instead lowers its billing rate, which pressures the Small firm into lowering its rates as well. This result is consistent with a recent report from Audit Analytics which shows that audit fees per $1 million in revenue for accelerated filers actually dropped after the implementation of the Sarbanes-Oxley Act (which went into effect in 2004) [13]. We posit that revenue is a noisy measure of our model's demand for audit hours, and thus the audit fees per million is a rough measure of billing rates. Further, Sarbanes-Oxley decidedly raised minimum independence standards. Notably, a contemporaneous report by BTI Consulting Group in June 2013 based on a survey of 259 global CFOs documented a perception that the Big 4's value is declining [14]. We believe that taken together these facts provide preliminary evidence for one of the paper's main results.

The second set of results concerns an increase in the cost of quality for all audit firms. When quality becomes more expensive, not only do auditing costs increase for both firms, but clients for both firms pay *less*. Intuitively, when quality becomes more costly, the Big firm supplies less of it; the Small firm cannot match this quality reduction because it is constrained by the minimum standard. Less differentiation relative to the Small firm lowers the Big firm's premium. Additionally, the Big firm's lower billing rate also pressures rates downward for the Small firm.

Our final result addresses social welfare. In our setting, there is a socially optimal level of auditing standards. In all cases, increasing auditing standards makes audit firms worse off. There is no total welfare impact on the Big firm and its clients, merely a transfer of wealth from firm to clients. The welfare result relates solely to the Small firm and is a result of the tension between the cost of quality (welfare reduction) and the non-cash reputational cost (welfare increase). At a very low quality standard, providing quality is relatively inexpensive, so raising the minimum audit standard lowers the client's reputational cost more than it increases the Small audit firm's quality cost, and thus total welfare increases. However, extreme quality is prohibitively expensive, and in this case the welfare reduction from inefficiently high levels of auditing dominates the lower reputational cost.

The rest of the paper is structured as follows: Section 2 discusses prior literature in audit standards and pricing, Section 3 presents the model assumptions and solution, Section 4 presents the results and discusses intuition, and Section 5 summarizes and provides concluding remarks.

2 Prior Literature

Prior analytical studies have primarily explored the role of audit standards in ensuring the diligence of auditors, using the threat of litigation (auditor liability). For example, Newman, Patterson and Smith (2005) [20] show that higher liability leads to higher investment (also see Schwartz 1997 [23]), higher audit fees (also see Dye 1993 [10]), and higher audit quality (also see DeAngelo 1981 [7]).

Another stream of analytical research links audit quality and audit regulation to auditor effort. Beck and Wu (2006) model auditor learning via audit effort, which also generates spillovers on an client's business model, and causes a trade-off between audit quality and audit fees [3]. Both Pae and Yoo (2001) and Willekens and Simunic (2007) present models where audit effort is determined in part by regulation [21][26]. Lastly, Ye and Simunic (Forthcoming) present a model from the opposite perspective, looking at how preferences over effort (called toughness) and vagueness affect interest groups' positions on proposed auditing standards [27].

In a setting with auditor liability, however, it is perhaps not surprising that increasing auditing standards make audit firms worse off. Interestingly, this paper finds similar results even in the absence of auditor liability; the competitive threat of rivals is sufficient to produce this outcome, even when the auditing standards don't bind (i.e., Big audit firms opt for higher than minimum quality).

Our paper also relates to the auditor reputation literature. Most of the research work in this area is empirical (the Klein and Leffler (1981) model of more expensive audits as a costly signal of better market position is a notable exception [16]). We draw upon this literature to motivate our assumption regarding a client's reputational cost, which leads to their demand for audit quality. Cao, Myers and Omer (2012) show that the better a company's reputation, the more they pay for an audit [4], which is consistent with the predictions of Klein and Leffler (1981) [16]. Stakeholders also react to this signal. Both Mansi, Maxwell and Miller (2004) and Pittman and Fortin (2004) show that clients using a Big N auditor have lower costs of debt financing [18][22]. In addition, Barton (2005) presents evidence that the reputation of one's auditor is important to clients that are more visible to the market; companies with greater analyst and news coverage, institutional ownership, share turnover, and cash from securities issues were switched away from Arthur Anderson earlier than other firms around its scandal and conviction in 2002 [1].

Lastly, we draw on the empirical literature regarding the differences between Big and Small auditors. The audit quality literature generally documents that Big audit firms provide higher quality audits than Small audit firms. Firms audited by Big auditors have, on average, lower discretionary accruals (Dechow, Ge and Schrand 2010 [8]), larger earnings response coefficients (Teoh and Wong 1993 [25]), and lower IPO returns (Beatty 1989 [2]), all of which support higher quality audits.[1] Further, the audit fee literature suggests that Big auditors command a fee premium (although some papers, such as Simunic 1980, do not find significant evidence of a pricing difference [24]). Moizer (1997) finds that 13 of 19 such studies support an audit fee premium, with 8 of 12 countries exhibiting this behavior [19]. Ireland and Lennox (2002) find a premium for Big auditors of 53.4% [15].

We contribute to the literature on auditing standards, audit quality and audit fees by introducing a model in which *market competition* between audit firms is the primary determinant of both audit quality and fees. Our model also includes a regulator that can require a

[1] On the other hand, Lawrence, Minutti-Meza and Zhang (2011) suggest that these results could be due to self-selection by clients to Big and Small auditors [17].

minimum level of auditing quality. Most extant models consider the effect of auditor liability; our model demonstrates that market forces and auditing standards together can generate insights consistent with the empirically observed audit firm-client interactions documented above even absent auditor liability.

3 Model

3.1 Audit Market Structure

We model the auditing industry using a duopoly model with two auditors, Big and Small. To capture the Big audit firm's market advantage, we model the Big firm as a Stackelberg leader and the Small firm as a Stackelberg follower. The Big firm thus anticipates the standard billing rate that the Small firm will choose as a best response to its own choice of standard billing rate. We make the following additional assumptions, motivated by characteristics of the auditing industry noted in the previous section:

1. No entry into Big firm status. No firms have been able to enter the Big N since 1989 and, in fact, the number of large auditing firms has decreased from eight to four since 1989 [9]. As such, we assume there are large barriers to entry into this segment of the market.

2. Inelastic demand. In the United States, public companies are required to have an audit of their financial statements in order to remain listed on a stock exchange. Thus, demand is almost perfectly inelastic. Unless the audit fee is so high that a client would rather cease operations, it must obtain an audit.[2]

3. Differentiated products. In the audit quality literature, audits by Big firms are different than audit by Small firms. For example, Cohen (2010) shows that audit clients have

[2] We note that a public company could hypothetically go private to avoid a mandated audit. We assume that going private would be far more costly to a public company than the cost of a mandated audit.

less leniency in accounting choices when audited by a Big firm [6]. We assume that a Big firm can choose to offer an audit of a higher quality than a Small firm's audit, and if it does so, a Big firm can charge a higher price.

4. Small firm audit quality. We assume, primarily for tractability reasons, that the Small audit firm has fewer resources or less capability to choose its audit quality, and as such it chooses to comply with the minimum quality mandated by the audit standard.

3.2 Model Setup

Consider a market comprised of Q clients and two auditing firms, Big and Small. Each client chooses the auditor that minimizes its total cost. Each audit firm f charges its clients a standard hourly rate of $x + x_f$, where x is the actual hourly labor cost common to both audit firms and the rate premium x_f is chosen by each firm. The Big firm also chooses its desired audit quality level, λ_B. In our main model, we assume that both audit firms charge all of their respective clients the same hourly rate. We relax this assumption later on in Section 4.4, in which larger clients are charged progressively lower billing rates; specifically, billing rates decrease in the number of hours billed.

Denote client i's total cost for hiring a Big audit firm as c_i^B, and denote client i's total cost (including reputational costs) for hiring a Small audit firm as c_i^S. Client i requires γ_i hours of actual auditing, which is increasing in size, complexity, or any other situation requiring greater assurance to its investors (e.g., an upcoming IPO). Function $\alpha(\cdot)$ represents the non-cash reputational cost of hiring a Small audit firm, which increases in the client's demand for assurance, γ_i, and is also a function of the quality levels chosen by the audit firms, λ_B and λ_S. Then:

$$c_i^B = (x + x_B)\gamma_i, \tag{1}$$

$$c_i^S = (x + x_S)\gamma_i + \alpha(\gamma_i; \lambda_B, \lambda_S), \tag{2}$$

Client i will be indifferent between auditors when these costs are the same, i.e. $c_i^B = c_i^S$. Without loss of generality, we order the Q clients by their demand for auditing, γ_i; the threshold at which $c_i^B = c_i^S$ partitions the Q clients among the two types of auditors. The Big firm services the Q_B clients for whom $c_i^B \leq c_i^S$, and the Small firm services the $Q_S = Q - Q_B$ clients for whom $c_i^B > c_i^S$ (as the entire market must be served given perfectly inelastic demand). We assume a functional form for $\alpha(\cdot)$ of $\alpha(\gamma_i; \lambda_B, \lambda_S) = (\lambda_B - \lambda_S){\gamma_i}^2$. Thus, client i's reputational cost increases in the incremental quality of the Big audit firm over the Small audit firm and is convex in the client's demand for audit intensity. Setting equations (1) and (2) equal, we get the following equation for indifference:

$$x_B \gamma_i = x_S \gamma_i + (\lambda_B - \lambda_S){\gamma_i}^2 \tag{3}$$

Consequently we can then define Q_S and Q_B analytically as

$$Q_B = Q \cdot \mathbb{P}\left(c_i^B \leq c_i^S\right),$$
$$Q_S = Q \cdot \mathbb{P}\left(c_i^B > c_i^S\right).$$

We solve (3) for the γ_i cutoff that partitions Q into Q_B and Q_S, as shown in Figure 1. We

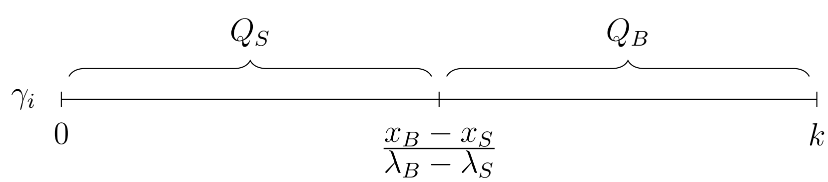

Figure 1: Partitioning of the Q clients in the Market

observe that this threshold closely resembles that of the Hotelling-style models of vertical differentiation, and the forces at work are indeed similar.

Next, we define the profit maximization problem for the audit firms. To do so we must specify each firm's cost for labor and audit quality. We assume that both audit firms have the same cost structures. The audit firms' cost of audit quality is convex of the form $\frac{{\lambda_f}^2}{2} p_f$, where $p_f > 0$ parameterizes the relative costliness of audit quality to each firm. The Small firm, to minimize its costs (and thus maximize market competitiveness), chooses the lowest level of quality permissible by the audit standards, and therefore λ_S is set exogenously by regulators.

From here, we can define the maximization problem for the audit firms as:

$$\max_{x_B|x_S} Q_B \cdot \mathbb{E}\left[\gamma_i | c_i^B \leq c_i^S\right] x_B - \frac{{\lambda_B}^2}{2} p_B, \tag{4}$$

$$\max_{x_S|x_B} Q_S \cdot \mathbb{E}\left[\gamma_i | c_i^B > c_i^S\right] x_S - \frac{{\lambda_S}^2}{2} p_S. \tag{5}$$

For tractability, we assume that γ_i is uniformly distributed between 0 and k. Now we can approach the maximization problem.

3.3 Model Solution

We start by solving the Small firm's problem, given in equation (5), which yields the following result:

Lemma 1. *The incremental rate charged by small firms, x_S, equals $\frac{x_B}{4}$.*

Proof: see Appendix A.

As expected, the Small audit firm charges lower billing rates than the Big firm, as $x_S < x_B$ given $x_B > 0$. The Big firm can charge more because it provides higher quality audits. Using this result, we next solve the maximization problem for the Big firm.

Lemma 2. *The equilibrium for the maximization problem (4) and (5) is given by:*

$$x_B = \frac{\left(384 - 384\sqrt{17}\right) k p_B \lambda_S + \left(487 - 79\sqrt{17}\right) k^3 Q}{2304 p_B}, \tag{6}$$

$$x_S = \frac{\left(384 - 384\sqrt{17}\right) k p_S \lambda_S + \left(487 - 79\sqrt{17}\right) k^3 Q}{9216 p_S}, \tag{7}$$

$$\lambda_B = \frac{\left(-107 + 51\sqrt{17}\right) k^2 Q}{768 p_B}. \tag{8}$$

Proof: see Appendix Section B.

We make the technical assumption that $k^2Q > \frac{384\sqrt{17}-384}{487-79\sqrt{17}}p_B\lambda_S$ ($\approx 7.44p_B\lambda_S$), to ensure this problem has a stable equilibrium and that there is an optimal choice for each firm's billing rate and the Big firm's audit quality. Both k (the upper bound of auditing hours demanded by a client) and Q (the number of clients requiring audits) should be relatively large. Admittedly, there is no "natural" scale for p_f (the cost parameter for audit quality) and λ_S (the minimum level of audit quality mandated by the regulator), but we have no reason to believe they would be remotely as large. Provided this assumption generally holds, this equilibrium obtains. As expected, the Big firm charges more for audits, and some of this higher billing rate is spent on increased audit quality.

4 Analysis

The equilibrium has some intuitively appealing properties: The Big firm charges a higher billing rate premium (i.e., $x_B > x_S$), and the chosen billing rate depends on audit standards (λ_S), the number of clients (Q), the expected size of audit engagements ($k/2$), and the relative costliness of audit quality (p_f). Next, we consider how the equilibrium outcomes change with changes in the auditing standards. Another interesting variation is how changes in the cost of audit quality affect the equilibrium. Finally, we compute social welfare and analyze the impact of changes in audit standards.

4.1 Changes in audit standards

How audit firms react to changes in auditing standards is an important question for regulators. Through our model, we can observe this analytically. The cutoff $\frac{x_B-x_S}{\lambda_B-\lambda_S}$ from Figure 1 that partitions clients into those serviced by the Big versus Small firm plays a central role in the analysis. Consider what happens if the regulator increases λ_S, the minimum audit quality required by auditing standards. We refer to this as "increasing the auditing standards."

Proposition 1. *An increase in the auditing standards lowers both the cash cost of an audit for all clients and the reputational cost of an audit for clients choosing the Small auditor.*

Proof.

$$\frac{\partial c_i^B}{\partial \lambda_S} = \frac{1-\sqrt{17}}{6} k\gamma_i < 0,$$

$$\frac{\partial c_i^S}{\partial \lambda_S} = \underbrace{\frac{1-\sqrt{17}}{24} k\gamma_i}_{\text{Cash}, < 0} \underbrace{- \gamma_i{}^2}_{\text{Reputation}, < 0} < 0.$$

∎

This result implies that audit standards are beneficial for all clients (and their investors), regardless of their demanded audit intensity or auditor choice. Furthermore, clients of the Small audit firm receive the benefit of reduced costs of both types. The intuition behind this result is that an increase in the auditing standard reduces the Big firm's reputational leverage for charging higher billing rates. The Big audit firm has essentially three options: (1) offer the same billing rate and quality as before and lose market share, letting the cutoff $\frac{x_B - x_S}{\lambda_B - \lambda_S}$ shift left (due to changes in λ_S alone), (2) raise its quality to sustain its billing rate premium, preserving the cutoff $\frac{x_B - x_S}{\lambda_B - \lambda_S}$ (i.e., offsetting the change in λ_S by a corresponding increase in λ_B), or (3) maintain its quality as before but drop its billing rate premium, preserving the cutoff $\frac{x_B - x_S}{\lambda_B - \lambda_S}$ (i.e., offsetting the change in λ_S by a corresponding decrease in x_B). Option (3) is the most cost-effective of these choices for the Big firm. From Lemma 1, the billing rate charged by Small firms is $x_S = \frac{x_B}{4}$, so if the Big firm lowers its billing rate, the Small firms must do so as well, or yield market share to the Big audit firms.

Two results immediately follow. The benefit to clients of both Big and Small audit firms does not come at the cost of lower audit quality for clients of Big audit firms.

Proposition 2. *Increasing auditing standards does not lower the audit quality chosen by either firm.*

Proof.

$$\frac{\partial \lambda_B}{\partial \lambda_S} = 0,$$

$$\frac{\partial \lambda_S}{\partial \lambda_S} = 1.$$

∎

The Small audit firm increases its audit quality as mandated, but the audit quality chosen by the Big firm is unaffected by the auditing standards. Therefore, clients who choose the Big auditor receive an audit of equal quality irrespective of audit standards.

Proposition 3. *An increase in the auditing standards lowers profit for both audit firms.*

Proof.

$$\frac{\partial \text{Profit}_B}{\partial \lambda_S} = \frac{1}{768}\left(107 - 51\sqrt{17}\right) k^2 Q < 0,$$

$$\frac{\partial \text{Profit}_S}{\partial \lambda_S} = \frac{\left(-49 + 9\sqrt{17}\right) k^2 Q)}{3072} < 0.$$

∎

Auditors have a significant interest in lobbying against such increases in the auditing standards. Note that we obtain this result purely with market forces, despite no role for auditor liability in our model. Instead, the Small audit firm spends more to comply with the higher quality standards. Higher audit quality by the Small firm reduces the reputational cost of choosing a Small firm, which in turn lowers the premium the Big firm can charge. For both auditors, higher auditing standards lead to lower hourly margins. However, requiring a higher minimum audit quality may be beneficial to society overall if the decreased reputational cost to audit clients were large enough.

4.2 Sensitivity to cost of quality

The cost of quality parameter, p_f, is empirically difficult to pinpoint. However, the model allows us to analytically derive some properties of p_f.

Lemma 3. *An increase in the cost of quality for the Big audit firm p_B lowers the audit quality it chooses, while an increase in the cost of quality for the Small firm p_S leads to no change in the audit quality chosen by the Big firm.*

Proof.

$$\frac{\partial \lambda_B}{\partial p_B} = \frac{\left(107 - 51\sqrt{17}\right) k^2 Q}{768 {p_B}^2} < 0,$$

$$\frac{\partial \lambda_B}{\partial p_S} = 0.$$

∎

Not surprisingly, as the cost of quality increases for the Big firm, it decreases its quality correspondingly. The net impact on clients of an increase in the cost of quality could go either way. On one hand, the Big audit firm could try to pass on the higher cost to its clients; on the other hand, the reduction in product differentiation may require the Big firm to accept lower profits.

Lemma 4. *An increase in the Big firm's cost of audit quality lowers profits for both audit firms, whereas an increase in the Small firm's cost of quality has no impact on the Big firm's profit and decreases the Small firm profits.*

Proof.

$$\frac{\partial \text{Profit}_B}{\partial p_B} = \frac{\left(-27833 + 5457\sqrt{17}\right) k^4 Q^2}{589824 {p_B}^2} < 0,$$

$$\frac{\partial \text{Profit}_B}{\partial p_S} = 0,$$

$$\frac{\partial \text{Profit}_S}{\partial p_B} = \frac{\left(6523 - 1731\sqrt{17}\right) k^4 Q^2}{1179648 {p_B}^2} < 0,$$

$$\frac{\partial \text{Profit}_S}{\partial p_S} = -\frac{{\lambda_S}^2}{2} < 0.$$

∎

The intuition behind this result is that the Big firm loses some of its leverage for charging clients a premium billing rate. Recall that the Small firm cannot lower its audit quality due to the binding minimum auditing standard. As in the case of an increasing audit standard, the Big firm loses market share if it attempts to charge the same hourly rate when its reputational advantage decreases. Therefore, it must lower its billing rates, and the Small firm must match the reduction. If λ_S is small compared to $\frac{k^4 Q^2}{{p_B}^2}$ then we see that the Big firm is more severely affected, as it pays more to provide its higher quality audit.

Proposition 4. *An increase in the Big firm's cost of audit quality lowers the audit cost for all clients, whereas an increase in the Small firm's cost of audit quality does not change the audit cost for any clients.*

Proof.

$$\frac{\partial c_i^B}{\partial p_B} = \frac{\left(-487 + 79\sqrt{17}\right) k^3 Q \gamma_i}{2304 {p_B}^2} < 0,$$

$$\frac{\partial c_i^S}{\partial p_B} = \underbrace{\frac{\left(-487 + 79\sqrt{17}\right) k^3 Q \gamma_i}{9216 {p_B}^2}}_{\text{Cash, } < 0} + \underbrace{\frac{\left(107 - 51\sqrt{17}\right) k^2 Q {\gamma_i}^2}{768 {p_B}^2}}_{\text{Reputation, } < 0} < 0.$$

$$\frac{\partial c_i^B}{\partial p_S} = \frac{\partial c_i^S}{\partial p_S} = 0$$

∎

Overall, the effect from less product differentiation dominates, decreasing every client's cash cost of an audit from either auditor. As in the prior subsection, the reputational cost of obtaining an audit from a Small audit firm also decreases due to the smaller quality difference between firms.

4.3 Welfare

In the previous two subsections, we showed that costs of audits decrease as regulation increases. However, it is not necessarily the case that social welfare increases from this action. If the increased cost to audit firms outweighs the decreased cash and reputational costs to audit clients, then such regulation would not be socially optimal. On the other hand, if the benefits to clients exceed the cost to audit firms, then such regulation would be socially beneficial.

Proposition 5. *There is a socially optimal level of auditing standards,*

$$\lambda_S^* = \frac{\left(5\sqrt{17} - 13\right) kQ}{256 p_S}. \tag{9}$$

Proof: see Appendix C.

This result demonstrates that it is socially optimal to have some level of audit regulation. The details of the proof shed light on which tension generates this effect. The sum of welfare for the Big audit firm (SW_B^f) and its clients (SW_B^c) is a zero-sum game, and $\frac{\partial}{\partial \lambda_S}\left(SW_B^f + SW_B^c\right) = 0$. Therefore, the welfare result is driven exclusively by the Small audit firm. In particular, the Small firm and its clients face two countervailing forces. First, an increase in auditing standards causes a net welfare loss due to higher investment in quality by the Small firm of $-p_S\lambda_S$. Second, an increase in auditing standards also reduces the reputation cost to Small firm audit clients by $\frac{\left(5\sqrt{17}-13\right)kQ}{256}$. If $\lambda_S < \lambda_S^*$, social welfare would be increased by increasing the amount of regulation, because in this region the incremental cost is outweighed by the reputational savings.

Francis (2004) reviews the empirical audit literature and concludes that little is known about the level of λ_S^* and its properties [12]. In our setting, the optimal auditing standard increases in k and Q, which correspond to larger audit engagements and more clients in the auditing market, as having larger and more clients provides a larger benefit from increasing the lower quality level. As expected, the optimal level of regulation is decreasing in the Small firm's cost p_s of providing audit quality, but unaffected by the Big firm's cost, p_B.

4.4 Extension: Discounted billing rates for large clients

In this subsection, we relax the assumption that audit firms charge the same standard hourly rate of $(x + x_f)$ to all clients irrespective of client size to demonstrate that the main results in our constant hourly rate model are not driven by this assumption. We instead assume that client i pays a discounted rate of $\delta_i(x + x_f)$, and that δ_i decreases in γ_i.

Consider the example of $\delta_i = \gamma_i^k$, where $k \in (-1, 0)$, and thus $\frac{\gamma_i}{\delta_i} = \gamma_i^{1-k}$. Let $\gamma_i^{1-k} \sim \beta\left(\frac{1+k}{1-k}, 1\right)$, which in turn implies that γ_i^{1+k} is uniformly distributed from 0 to 1. Next, we fix $k = -1/5$ to generate closed-form solutions. We solve the modified objective function for the $\frac{\gamma_i}{\delta_i}$ cutoff that partitions Q into Q_B and Q_S, which yields a cutoff of $\left(\frac{\gamma_i}{\delta_i}\right)^* = \frac{x_B - x_S}{\lambda_B - \lambda_S}$.

Solving as before yields qualitatively identical results as in the constant hourly rate model, with one exception. In this setting, an increase in the cost of audit quality parameter p_B now increases the profit of the Big audit firm (whereas the profit of Small audit firm decreases in p_B as before). This new result occurs even though the increased cost of audit quality similarly leads the Big firm to choose lower quality λ_B, which in turn reduces its billing rates (due to less quality differentiation). Intuitively, the discount mutes the impact of reducing the standard hourly rate on the Big firm's highest volume clients.

Interestingly, all audit clients still pay less for their audits when audit quality becomes more costly, despite the apparent contradiction of higher profit for the Big audit firm. In this setting, however, the Big audit firm actually saves more via quality cost reduction than it loses in client fees; the discount mutes the impact on client fees for the Big firm's largest clients, and the larger the client, the more muted the impact on client fees.

This example demonstrates that the results in Propositions $1-4$ are not driven solely by our assumption of each firm choosing a single standard hourly billing rate for all clients. Details and proofs paralleling the results in the main model are presented in Appendix D.

5 Observations and Concluding Remarks

Overall, this study documents an obvious – and previously neglected – incentive conflict between audit firms and their clients. We model a setting in which audit clients facing relatively higher reputational costs demand higher quality audits, consistent with evidence in the empirical literature. We examine the role of audit standards relating to qualitatively *better* auditing such as auditor independence or professional competence standards, as distinct from auditor liability or a minimum amount of auditing. Prior literature has established that audit firms are worse off as a result of standards mandating auditor liability. Our setting intentionally omits auditor liability and yet we find that audit quality standards also interact with an entirely different force, market competition, and this interaction also makes audit firms worse off.

We derive three main results. First, we demonstrate that both Big and Small audit firms benefit from a wider disparity in audit quality, because a larger reputational advantage over smaller audit firms increases the premium that Big N firms can charge. The higher billing rates at the Big N firms in turn increases the billing rates that smaller firms can charge. Audit clients, however, strictly benefit from minimum audit quality standards; any increase in audit standards lowers billing rates but not quality for all audit clients. An empirical implication of this result is that following a major increase in qualitative auditing standards, such as Sarbanes-Oxley, our model predicts decreases in the *billing rates* for all firms. We note, however, that *total audit fees* might increase if there is a concurrent increase in the number of audit hours required, as was the case with Sarbanes-Oxley.

Second, as audit quality becomes more expensive, Big N and smaller audit firms choose less differentiation in audit quality. An intriguing empirical implication of this result is that when quality becomes *less* costly (due to, say, an implementation of a new, lower-cost monitoring system), our model predicts that differentiation will increase, which in turn will increase billing rates even as quality costs decline.

Finally, we find that there exists a socially optimal level of minimum audit "quality" regulation. In our model, the compression of audit quality differentiation caused by audit standards is driven exclusively by small audit firms being required to comply with the binding minimum quality. The quality offered by Big N firms is unaffected because it exceeds the minimum standards, and those standards affect neither the marginal costs nor the marginal benefits relating to quality. In summary, audit standards can play a critical role in the market for audits, balancing audit prices and reputational consequences, as well as preventing audit quality from decreasing below the social optimum.

As with all models, we make several assumptions for tractability, and some of these assumptions are more limiting than others. For example, we model the distribution of audit clients as uniformly distributed because the partitioning of clients into Big N and small audit firms truncates the distribution, which introduces major tractability issues. Further, we assume just two types of auditors rather than the spectrum that no doubt exists, and even in this setting it is prohibitively difficult to solve for the quality level that the small audit firms would choose endogenously. Simulation researchers could potentially pursue addressing these issues of audit client distribution and apply numerical solution techniques for small audit firm optimal quality choices. Another possible extension we leave for future research includes extending the model to audit quality improvements that improve a firm's audit operational efficiency (i.e., relating to the number of audit hours required) but do not affect firm reputation (i.e., audit quality, the focus of the current study).

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A Proof of Lemma 1

Proof. Starting with the maximization problem from equation (5) and substituting in our assumptions, we obtain the following

$$\begin{aligned}
& \max_{x_S|x_B} Q_S \cdot \mathbb{E}\left[\gamma_i | c_i^B > c_i^S\right] x_S - \frac{{\lambda_S}^2}{2} p_S \\
= & \max_{x_S|x_B} Q \cdot \mathbb{P}\left(c_i^B > c_i^S\right) \mathbb{E}\left[\gamma_i | c_i^B > c_i^S\right] x_S - \frac{{\lambda_S}^2}{2} p_S \\
= & \max_{x_S|x_B} Q \cdot \frac{x_B - x_S}{k(\lambda_B - \lambda_S)} \left(\int_0^{\frac{x_B - x_S}{\lambda_B - \lambda_S}} \frac{t}{k} dt \right) x_S - \frac{{\lambda_S}^2}{2} p_S \\
= & \max_{x_S|x_B} Q \cdot \frac{x_B - x_S}{k(\lambda_B - \lambda_S)} \cdot \frac{(x_B - x_S)^2}{2k(\lambda_B - \lambda_S)^2} \cdot x_S - \frac{{\lambda_S}^2}{2} p_S \\
= & \max_{x_S|x_B} \frac{Q(x_B - x_S)^3 x_S}{2k^2(\lambda_B - \lambda_S)^3} - \frac{{\lambda_S}^2}{2} p_S. \qquad (10)
\end{aligned}$$

Then, using the first order condition with respect to x_S, we have:

$$\begin{aligned}
0 &= \frac{Q(x_B - x_S)^2 (x_B - 4x_S)}{2k^2(\lambda_B - \lambda_S)^3} \\
x_S &= \frac{x_B}{4}. \qquad (11)
\end{aligned}$$

(Note: there is a second root, $x_S = x_B$, but it is not optimal.) ∎

B Proof of Lemma 2

Proof. Start with the maximization problem for the Big firm, equation (4). From this we can see that

$$\begin{aligned}
&\max_{x_B|x_S} Q_B \cdot \mathbb{E}\left[\gamma_i | c_i^B \le c_i^S\right] x_B - \frac{{\lambda_B}^2}{2} p_B \\
= &\max_{x_B|x_S} Q \cdot \mathbb{P}\left(c_i^B \le c_i^S\right) \mathbb{E}\left[\gamma_i | c_i^B \le c_i^S\right] x_B - \frac{{\lambda_B}^2}{2} p_B \\
= &\max_{x_B|x_S} Q \cdot \frac{k - \frac{x_B - x_s}{\lambda_B - \lambda_S}}{k} \left(\int_{\frac{x_B - x_S}{\lambda_B - \lambda_S}}^{k} \frac{t}{k} dt \right) x_B - \frac{{\lambda_B}^2}{2} p_B.
\end{aligned}$$

Applying our solution to the Small firm maximization problem (11), we obtain

$$\begin{aligned}
= &\max_{x_B} Q \cdot \left(1 - \frac{3x_B}{4k(\lambda_B - \lambda_S)}\right) \left(\int_{\frac{3x_B}{4(\lambda_B - \lambda_S)}}^{k} \frac{t}{k} dt \right) x_B - \frac{{\lambda_B}^2}{2} p_B \\
= &\max_{x_B} Q \cdot \left(1 - \frac{3x_B}{4k(\lambda_B - \lambda_S)}\right) \left(\frac{k}{2} - \frac{9{x_B}^2}{32k(\lambda_B - \lambda_S)^2} \right) - \frac{{\lambda_B}^2}{2} p_B. \qquad (12)
\end{aligned}$$

Now we use the two first order conditions (with respect to x_B and λ_B). Using x_B,

$$0 = \frac{Q(27{x_B}^3 - 27k(\lambda_B - \lambda_S){x_B}^2 - 24k^2(\lambda_B - \lambda_S)^2 x_B + 16k^3(\lambda_B - \lambda_S)^3)}{32k^2(\lambda_B - \lambda_S)^3}. \qquad (13)$$

Using λ_B,

$$0 = \frac{3Q(4k(\lambda_B - \lambda_S) - 3x_B){x_B}^2(3k(\lambda_B - \lambda_S) + 9x_B)}{128k^2(\lambda_B - \lambda_S)^4} - \lambda_B p_B. \qquad (14)$$

Combing these (13) and (14) into a 2×2 system of equations and solving, we find that

$$x_B = \frac{\left(384 - 384\sqrt{17}\right) k p_B \lambda_S + \left(487 - 79\sqrt{17}\right) k^3 Q}{2304 p_B}, \qquad (15)$$

$$\lambda_B = \frac{\left(-107 + 51\sqrt{17}\right) k^2 Q}{768 p_B}. \qquad (16)$$

Going back to equation (11), this gives us:

$$x_S = \frac{\left(384 - 384\sqrt{17}\right) kp_B\lambda_S + \left(487 - 79\sqrt{17}\right) k^3 Q}{9216 p_B}. \tag{17}$$

∎

C Proof of Proposition 5

Proof. We will determine social welfare as a sum of five quantities, social welfare of the Big audit firm (SW_B^f), social welfare of the Small audit firm (SW_S^f), social welfare of clients choosing the Big firm (SW_B^c), cash-based social welfare of clients choosing the Small firm (SW_S^c), and reputational social welfare of clients choosing the Small firm ($\widetilde{SW}^c_S$). Then we have

$$SW = SW_B^f + SW_S^f + SW_B^c + SW_S^c + \widetilde{SW}^c_S.$$

We then consider the optimal choice of λ_S with respect to SW, i.e.

$$\max_{\lambda_S} SW.$$

We take first order condition of SW with respect to λ_S, beginning with SW_B^f and SW_B^c. From the solution in Appendix B, we determine that

$$\begin{aligned}
&\frac{\partial}{\partial \lambda_S}\left[SW_B^f + SW_B^c\right] \\
=&\frac{\partial}{\partial \lambda_S}\left[x_B Q_B \cdot \mathbb{E}\left[\gamma_i | c_i^B \leq c_i^S\right] - \frac{{\lambda_B}^2}{2} p_B - (x_B + x) Q_B \cdot \mathbb{E}\left[\gamma_i | c_i^B \leq c_i^S\right]\right] \\
=&\frac{\partial}{\partial \lambda_S}\left[-\frac{{\lambda_B}^2}{2} p_B - x Q_B \cdot \mathbb{E}\left[\gamma_i | c_i^B \leq c_i^S\right]\right].
\end{aligned}$$

Now, note that $\frac{\partial \lambda_B}{\partial \lambda_S} = 0$ from Appendix B, $\frac{\partial x}{\partial \lambda_S} = 0$ by assumption, and

$$\frac{\partial Q_B}{\partial \lambda_S} = \frac{\partial}{\partial \lambda_S}\left[Q \cdot \mathbb{P}\left(c_i^B \leq c_i^S\right)\right] = \frac{\partial}{\partial \lambda_S} Q \left[\frac{k - \frac{x_B - x_S}{\lambda_B - \lambda_S}}{k}\right] = \frac{\partial}{\partial \lambda_S}\left[\frac{Q}{8}\left(9 - \sqrt{17}\right)\right] = 0.$$

This implies that $\frac{\partial}{\partial \lambda_S}\left[\mathbb{E}\left[\gamma_i | c_i^B \leq c_i^S\right]\right] = 0$ as the clients choosing a large auditor remain constant as λ_S varies. As such, $\frac{\partial}{\partial \lambda_S}\left[SW_B^f + SW_B^c\right] = 0$.

Similarly, we can examine SW_S^f and SW_S^c.

$$\begin{aligned}
&\frac{\partial}{\partial \lambda_S}\left[SW_S^f + SW_S^c\right] \\
=&\frac{\partial}{\partial \lambda_S}\left[x_S Q_S \cdot \mathbb{E}\left[\gamma_i | c_i^B > c_i^S\right] - \frac{{\lambda_S}^2}{2} p_S - (x_S + x) Q_S \cdot \mathbb{E}\left[\gamma_i | c_i^B > c_i^S\right]\right] \\
=&\frac{\partial}{\partial \lambda_S}\left[-\frac{{\lambda_S}^2}{2} p_S - x Q_S \cdot \mathbb{E}\left[\gamma_i | c_i^B > c_i^S\right]\right].
\end{aligned}$$

Now, note that $\frac{\partial}{\partial \lambda_S}\left[-\frac{{\lambda_S}^2}{2} p_S\right] = -\lambda_S p_S$, $\frac{\partial x}{\partial \lambda_S} = 0$ by assumption, and $\frac{\partial}{\partial \lambda_S}\left[Q_S\right] = \frac{\partial}{\partial \lambda_S}\left[Q - Q_B\right] = 0 - 0 = 0$. As such, $\frac{\partial}{\partial \lambda_S}\left[SW_S^f + SW_S^c\right] = -\lambda_S p_S$.

Lastly, we must examine $\frac{\partial \widetilde{SW}_S^c}{\partial \lambda_S}$.

$$\begin{aligned}
&\frac{\partial \widetilde{SW}_S^c}{\partial \lambda_S} \\
=&\frac{\partial}{\partial \lambda_S}\left[(\lambda_B - \lambda_S) Q_S \cdot \mathbb{E}\left[\gamma_i | c_B^i > c_S^i\right]\right] \\
=&\frac{\partial}{\partial \lambda_S}\left[-(\lambda_B - \lambda_S) Q \cdot \frac{x_B - x_S}{k(\lambda_B - \lambda_S)} \cdot \frac{(x_B - x_S)^2}{2k\left(\lambda_B - \lambda_S\right)^2}\right]. \\
=&\frac{(5\sqrt{17} - 13) k Q}{256}.
\end{aligned}$$

Now, adding the three prior results together gives us the following result:

$$\begin{aligned}
\frac{\partial SW}{\partial \lambda_S} &= -\lambda_S^* p_S + \frac{(5\sqrt{17} - 13) k Q}{256} = 0, \\
\lambda_S^* &= \frac{(5\sqrt{17} - 13) k Q}{256 p_S}.
\end{aligned} \tag{18}$$

Thus, (18) provides the socially optimal point for the minimum amount of auditing quality.

∎

D Proof with discounted billing rates for large clients

Proof. Let $c_i^B = (x + x_B)\delta_i\gamma_i$ and let $c_i^S = (x + x_S)\delta_i\gamma_i + \alpha(\gamma_i; \lambda_B, \lambda_S)$, where δ_i is a firm specific pricing factor. We define this as $\delta_i = \delta(\gamma_i) = {\gamma_i}^k$, and we define α as $\alpha(\gamma_i; \lambda_B, \lambda_S) = (\lambda_B - \lambda_S){\gamma_i}^2$. Next, we solve for the cutoff,

$$
\begin{aligned}
(x + x_B)\delta_i\gamma_i &= (x + x_S)\delta_i\gamma_i + (\lambda_B - \lambda_S){\gamma_i}^2, \\
x_B\delta_i &= x_S\delta_i + (\lambda_B - \lambda_S)\gamma_i, \\
\frac{x_B - x_S}{\lambda_B - \lambda_S} &= \frac{\gamma_i}{\delta_i}.
\end{aligned}
$$

For ease of computation, let $\frac{\gamma_i}{\delta_i} = {\gamma_i}^{1-k} \sim \beta\left(\frac{1+k}{1-k}, 1\right)$. Note that this implies that ${\gamma_i}^{1+k} \sim U(0, 1)$.

To be able to solve the model, we must fix k. We demonstrate the model with $k = -\frac{1}{5}$ below. Now we can form the maximization problem:

$$
\begin{aligned}
&\max_{x_S|x_B} Q\mathbb{P}\left(c_i^B > c_i^S\right)\mathbb{E}\left[\gamma_i\delta_i | c_i^B > c_i^S\right](x + x_S) - \frac{{\lambda_S}^2}{2}p_S, \\
&\max_{x_B|x_S} Q\mathbb{P}\left(c_i^B < c_i^S\right)\mathbb{E}\left[\gamma_i\delta_i | c_i^B < c_i^S\right](x + x_B) - \frac{{\lambda_B}^2}{2}p_B.
\end{aligned}
$$

First, we solve the maximization problem for the small auditor in terms of x_B,

$$
\begin{aligned}
&\max_{x_S|x_B} Q\mathbb{P}\left(c_i^B > c_i^S\right)\mathbb{E}\left[\gamma_i\delta_i | c_i^B > c_i^S\right](x + x_S) - \frac{{\lambda_S}^2}{2}p_S \\
= &\max_{x_S|x_B} Q\left(\frac{x_B - x_S}{\lambda_B - \lambda_S}\right)^{\frac{1}{3}}\left(\frac{x_B - x_S}{2(\lambda_B - \lambda_S)}\right)(x + x_S) - \frac{{\lambda_S}^2}{2}p_S.
\end{aligned}
$$

Then, using the first order condition with respect to x_S, we see that

$$
\begin{aligned}
0 &= Q\left(\frac{x_B - x_S}{\lambda_B - \lambda_S}\right)^{\frac{1}{3}}\left(\frac{6_B - 16x_S + 2x}{12(x_B - x_S)}\right), \\
x_S &= \frac{3x_B - x}{4}. \qquad (19)
\end{aligned}
$$

Next, we solve the large audit firm's maximization problem.

$$
\begin{aligned}
&\max_{x_B,\lambda_B|x_S} Q\mathbb{P}\left(c_i^B < c_i^S\right)\mathbb{E}\left[\gamma_i\delta_i|c_i^B < c_i^S\right](x + x_B) - \frac{{\lambda_B}^2}{2}p_B \\
= &\max_{x_B,\lambda_B|x_S} Q\left(1 - \frac{x_B - x_S}{\lambda_B - \lambda_S}\right)^{\frac{2}{3}}\left(\frac{1}{2} + \frac{x_B - x_S}{2(\lambda_B - \lambda_S)}\right)(x + x_B) - \frac{{\lambda_B}^2}{2}p_B.
\end{aligned}
$$

Substituting in for x_S, this gives us

$$
\max_{x_B,\lambda_B} \frac{Q(x + x_B)(4\lambda_B - 4\lambda_S + x + x_B)}{2(\lambda_B - \lambda_S)\left(\frac{8\lambda_B - 8\lambda_S - 2x - x_B}{\lambda_B - \lambda_S}\right)^{\frac{2}{3}}} - \frac{{\lambda_B}^2}{2}p_B.
$$

Now we examine the first order conditions with respect to x_B and λ_B. With respect to x_B,

$$
0 = \frac{2^{\frac{1}{3}}Q\left(12{\lambda_B}^2 + \lambda_B(5(x + x_B) - 24\lambda_S) + 12{\lambda_S}^2 - 5\lambda_S(x + x_B) - (x + x_B)^2\right)}{3(\lambda_B - \lambda_S)^2\left(\frac{4\lambda_B - 4\lambda_S - x - x_B}{\lambda_B - \lambda_S}\right)^{\frac{5}{3}}}. \qquad (20)
$$

With respect to λ_B,

$$
0 = \frac{Q(x + x_B)^2(20\lambda_B - 20\lambda_S - x + -x_B)}{6(\lambda_B - \lambda_S)^2(-4\lambda_B + 4\lambda_S + x + x_B)\left(\frac{8\lambda_B - 8\lambda_S - 2x - 2x_B)}{\lambda_B - \lambda_S}\right)^{\frac{2}{3}}} - \lambda_B p_B. \qquad (21)
$$

Combining (20) and (21), we get the following result:

$$
\begin{aligned}
x_B &= \left(\frac{3\left(5 + \sqrt{73}\right)\left(1871 + 219\sqrt{73}\right)^{\frac{1}{3}}}{8}\right)\left(\frac{Q}{p_B}\right) - x - \left(\frac{5 + \sqrt{73}}{2}\right)\lambda_S, \\
\lambda_B &= \frac{3}{4}\left(1871 + 219\sqrt{73}\right)^{\frac{1}{3}}\left(\frac{Q}{p_B}\right).
\end{aligned}
$$

Combining these with (19) yields

$$x_S = 9\left(\frac{\left(581087+68011\sqrt{73}\right)^{\frac{1}{3}}}{16}\right)\left(\frac{Q}{p_B}\right) - x - \left(\frac{3(5+\sqrt{73})}{8}\right)\lambda_S.$$

∎

Therefore, we have a stable equilibrium with $x_B > x_S$ and $\lambda_B > \lambda_S$. We next verify that the equilibrium in this context acts similarly to the equilibrium in the primary model.

D.1 Proposition 1

We seek to show that $\frac{\partial c_i^B}{\partial \lambda_S} < 0$ and $\frac{\partial c_i^S}{\partial \lambda_S} < 0$.

$$\frac{\partial c_i^B}{\partial \lambda_S} = \frac{-1460-292\sqrt{73}}{584}\gamma_i^{\frac{4}{5}} < 0.$$
$$\frac{\partial c_i^S}{\partial \lambda_S} = \underbrace{\frac{-3(5+\sqrt{73})}{8}\gamma_i^{\frac{4}{5}}}_{\text{Cash, } <0} \underbrace{-\ \gamma_i^{2}}_{\text{Reputation, } <0} < 0.$$

D.2 Proposition 2

We seek to show that $\frac{\partial \lambda_B}{\partial \lambda_S} = 0$ and $\frac{\partial \lambda_S}{\partial \lambda_S} = 1$. Taking each derivative is straightforward and confirms the result.

D.3 Proposition 3

We seek to show that $\frac{\partial \text{Profit}_B}{\partial \lambda_S} < 0$ and $\frac{\partial \text{Profit}_S}{\partial \lambda_S} < 0$.

$$\frac{\partial \text{Profit}_B}{\partial \lambda_S} = \frac{-3\left(1871+219\sqrt{73}\right)^{\frac{1}{3}}}{4}Q < 0.$$

For the Small firm, it is tricky to sign the derivative directly. However, we can sign it indirectly. If the cutoff point for clients does not change, then since all clients pay less and costs from reputational quality increase for the Small firm when λ_S increases, it must be that the Small firm receives lower profits when λ_S increases (i.e. $\frac{\partial \text{Profit}_S}{\partial \lambda_S} < 0$).

$$\frac{\partial}{\partial \lambda_S}\left[\frac{x_B - x_S}{\lambda_B - \lambda_S}\right] = 0.$$

As such, $\frac{\partial \text{Profit}_S}{\partial \lambda_S} < 0$.

D.4 Lemma 3

We seek to show that $\frac{\partial \lambda_B}{\partial p_B} < 0$ and $\frac{\partial \lambda_B}{\partial p_S} = 0$.

$$\frac{\partial \lambda_B}{\partial p_B} = \left(\frac{-3\left(1871 + 219\sqrt{73}\right)^{\frac{1}{3}}}{4}\right)\left(\frac{Q}{p^2}\right) < 0,$$

$$\frac{\partial \lambda_B}{\partial p_S} = 0.$$

D.5 Lemma 4

If the model presented above agreed with the main model, we would find that $\frac{\partial \text{Profit}_B}{\partial p_B} < 0$ and $\frac{\partial \text{Profit}_S}{\partial p_B} < 0$. However, what we find is that as p_B increases, the Small auditor's profit generally decreases, but the Big auditor's profit generally increases (the reduced cost for the decreased audit quality dominates the impact of reduced billing rates).

$$\frac{\partial \text{Profit}_S}{\partial p_B} = -\frac{9Q^2\left(\frac{(31480952+3684568\sqrt{73})\lambda_S p_B - (3486522+408066\sqrt{73})^{\frac{4}{3}} Q}{4\lambda_S p_B - (5613+657\sqrt{73})^{\frac{1}{3}} Q}\right)^{\frac{1}{3}}}{64 p_B^2},$$

$$\frac{\partial \text{Profit}_S}{\partial p_S} = -\frac{{\lambda_S}^2}{2}.$$

Because Q should be larger than $\lambda_S p_B$ by definition, this expression is negative. Thus, the Small audit firm will generally see lower profit as the cost of quality increases. We can also see this intuitively: as cost to the Small firm's clients decreases (see Appendix D.6) and $\frac{\partial}{\partial p_B}\left[\frac{x_B - x_S}{\lambda_B - \lambda_S}\right] = 0$, the Small firm must see a decrease in profits as it receives less money from clients (who remain the same as p_B and p_S vary) and pay more for the level of audit quality required.

$$\begin{aligned}\frac{\partial \text{Profit}_B}{\partial p_B} = \frac{1}{2336{p_B}^2}\Bigg(\Bigg(&776\left(2\left(75623 + 8851\sqrt{73}\right)\right)^{\frac{1}{3}} p_B \\ &+ \left(2\left(555622257695306999 + 65030666448426403\sqrt{73}\right)\right)^{\frac{1}{3}}\Bigg) Q^2 \\ &- 24\left(487 + 57\sqrt{73}\right)^{\frac{1}{3}}\left(3149 + 377\sqrt{73}\right)\Bigg).\end{aligned}$$

$$\frac{\partial \text{Profit}_B}{\partial p_S} = 0$$

For any $Q > 1.17$, this expression is positive. Thus, the Big audit firm generally sees higher profits when its cost of quality increases, as it chooses lower audit quality which in turn lowers costs.

D.6 Proposition 4

We seek to show that $\frac{\partial c_i^B}{\partial p_B} < 0$, $\frac{\partial c_i^B}{\partial p_S} = 0$, $\frac{\partial c_i^S}{\partial p_B} < 0$, and $\frac{\partial c_i^S}{\partial p_S} = 0$.

$$\begin{aligned}\frac{\partial c_i^B}{\partial p_B} &= \left(\frac{-3\left(581087 + 68011\sqrt{73}\right)^{\frac{1}{3}}}{4}\right)\left(\frac{Q{\gamma_i}^{\frac{4}{5}}}{p^2}\right) < 0. \\ \frac{\partial c_i^S}{\partial p_B} &= \underbrace{\left(\frac{-9\left(581087 + 68011\sqrt{73}\right)^{\frac{1}{3}}}{16}\right)\left(\frac{Q{\gamma_i}^{\frac{4}{5}}}{p^2}\right)}_{\text{Cash, } < 0} + \underbrace{\left(\frac{-3\left(1871 + 219\sqrt{73}\right)^{\frac{1}{3}}}{4}\right)\left(\frac{Q{\gamma_i}^2}{p^2}\right)}_{\text{Reputation, } < 0} \\ &< 0, \\ \frac{\partial c_i^B}{\partial p_S} &= \frac{\partial c_i^S}{\partial p_S} = 0\end{aligned}$$