The effects of public information with asymmetrically informed short-horizon investors*

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Abstract

This paper analyzes effects of public information in a perfect competition trading model populated by asymmetrically informed short-horizon investors with different levels of private information precision. We first show that information asymmetry reduces the amount of private information revealed by price in equilibrium (i.e., price informativeness) and can lead to multiple linear equilibria. We then demonstrate that the presence of both information asymmetry and short horizon provides a channel through which public information influences price informativeness and equilibrium uniqueness. We identify conditions under which public information increases or decreases price informativeness, and when multiple equilibria may arise. Our analysis shows that public information not only influences price efficiency directly by endowing prices with more (public) information, it can also have an important indirect effect on the efficiency with which prices reveal private information.

1 Introduction

That prices can aggregate and reveal diverse private information held by individual traders is a cornerstone for well functioning financial markets (Hayek (1945)). Aggregation occurs via the trading process when investors condition their trades on the information available to them (Grossman and Stiglitz (1980); Glosten and Milgrom (1985); Kyle (1985)). Given the amount of private information traders hold, the trading process serves an information transmission and production role by enabling prices to reveal traders' private information that is otherwise not available to the general public. In so doing, it enhances stock price's role as a public signal to guide investors' resource allocation decisions. Price informativeness, which refers to the amount of private information revealed by price in equilibrium, reflects the efficiency of this transmission process and directly contributes to the overall stock market efficiency. Prior studies suggest that price informativeness can have significant impact on the real economy.[1] For example, it can affect firms' investment decisions (e.g., Luo (2005); Chen, Goldstein, and Jiang (2007)), cross-listing decisions (Foucault and Fresard (2012)), and governance choices (Ferreira, Ferreira, and Raposo (2011)). That prices contain valuable private information also underlies proposals for policy makers and regulators to base their actions on prices (e.g., Bond, Goldstein, and Prescott (2010)).

In this study we examine how public information affects price informativeness in a two-period overlapping trading model with perfect competition, where individual traders have short investment horizons and are endowed with heterogenous private information with different precision levels. Our purpose is to identify conditions under which public information improves market efficiency not only directly by endowing price with more (public) information, but also indirectly by affecting price's ability in aggregating and revealing private information (i.e., price informativeness).[2] To the

[1] See, e.g., Holmstrom and Tirole (1993), Dow and Gorton (1997), Subrahmanyan and Titman (1999), Goldstein and Gumbel (2008), and Dow, Goldstein, and Gumbel (2011) for theoretical models. Bond, Edman, and Goldstein (2012) provide an excellent literature review.

[2] Throughout the paper, we use price efficiency and market efficiency interchangeably, both referring to the ability of stock prices to reflect all fundamental-relevant information, including both public and private information. We focus on prices' ability to reveal *private* information because unlike public information (which is by definition a common knowledge) private information may not be available to the economy without the price aggregation process.

extent that price informativeness affects the real economy, understanding whether and how public information affects price informativeness can shed light on alternative mechanisms through which public information affects the real economy.

The idea that public information can affect price informativeness is implicit in the conventional wisdom that public information can improve price efficiency by alleviating the adverse impact of information asymmetry among investors.[3] However, prior theoretical studies find that in stock markets with perfect competition, price informativeness depends only on the average precision of investors' private information; it does not depend on either information asymmetry or public information (Verrecchia (1982), Lambert, Leuz, and Verrecchia (2012)). A key assumption behind these findings is that investors' investment horizons are the same as the operating horizon of the firms that they own.[4] That is, investors hold stocks until the firms' final liquidation dates. This assumption can be restrictive to the extent that investors trade for various reasons and often close their positions before the final liquidation dates, either because they are subject to exogenous liquidity shocks, or because they are simply outlived by the firms they invest in.

In this paper, we relax this assumption and study the impact of public information in a two-period version of Allen, Morris, and Shin (2006) that allows information asymmetry among investors.[5] We first establish a mechanism through which information asymmetry affects price informativeness. We then show that how this mechanism enables public information to influence price informativeness.

Our first finding is that information asymmetry among short-horizon investors unambiguously

[3]The effect of information asymmetry on price informativeness is also at the center of the debate over insider trading. Proponents for insider trading argue that it allows prices to be more informative; whereas opponents argue that it reduces price informativeness by reducing market liquidity. See Easterbrook and Fischel (1991). We focus on how public information reduces the adverse impact of information asymmetry on price informativeness. Models of perfect competition (such as the one we study) assume that traders are price takers and therefore liquidity is not a concern.

[4]Another key assumption is perfect competition. It is well known that information asymmetry matters in models with imperfect competition (e.g., Kyle (1989); Diamond and Verrecchia (1991); and Lambert and Verrecchia (2012)).

[5]Similar models have been studied in Grundy and McNichols (1989) and Brown and Jennings (1989), and recently in Gao (2008). None of these papers allow investors to have differential precisions in their private information.

lowers price informativeness by exacerbating the information loss caused by short investment horizon. Unlike long-horizon investors, short-horizon investors face the uncertainty of the next period price, as opposed to the uncertainty of the fundamentals. Compared to long-horizon investors, their trades are thus less sensitive to their private information about the fundamentals, which reduces price informativeness. We find that information asymmetry exacerbates this information loss. The reason is that the sensitivity of investors' trades to their private information is not uniformly reduced when investors have different levels of private information precision. Whereas investors with less precise private information do not reduce their sensitivities as much (because their sensitivities are not high to begin with), the reduction is more pronounced among investors with more precise private information. Because price informativeness depends on the average of individual sensitivities, it follows from the Jensen's Inequality that the average of individual sensitivities is lower than the sensitivity of the average investor. This leads to an aggregation loss in price informativeness caused by information asymmetry, above and beyond the loss induced by short horizons.

Our second finding is that information asymmetry may give rise to multiple linear equilibria in situations where the equilibrium is otherwise unique (Gao (2008)). We show that short horizons can generate an "endogenous uncertainty effect" in that the sensitivities of short horizon investors' trades to their private information endogenously affect the uncertainty of their future payoff, which in turn affects how sensitive their trades should be to their private information. This endogenous uncertainty effect creates a positive feedback loop that gives rise to self-fulfilling multiple equilibria. More importantly, information asymmetry magnifies the endogenous uncertainty effect, because investors with different levels of precision respond to the endogenous uncertainty effect to different degrees.

Together, these two findings identify a mechanism through which public information affects price informativeness, and it does so only when both information asymmetry and short horizon are present. However, the effect is ambiguous and depends on the quality of the public information as well as investors' risk preferences and the amount of noise trading in the firm's stock. We show that the effect of public information operates by affecting the dispersion in the sensitivities of individual investors' trades to private information. When public information is very precise, all investors, regardless of their private information precision, place relatively small weights (close to 0 when public information is extremely precise) on their private information, resulting in less dispersion in sensitivities and limiting the loss of informativeness caused by information asymmetry. In this case, increasing public information further reduces the dispersion in sensitivities thus improving price informativeness. When public information is very imprecise, all investors place relatively large weights on private information (close to 1 when public information is extremely imprecise). In this case, increasing public information would increase dispersion in sensitivities, leading to more information aggregation loss. The result is thus a U-shaped relationship between the quality of public information and price informativeness.

We also find that the quality of public information plays an important role in equilibrium uniqueness. We show a sufficient condition for equilibrium uniqueness is that public information is precise enough; and a necessary condition for multiple equilibria is that the public information precision is low enough. The intuition is related to the "endogenous uncertainty effect" discussed earlier, which links the second period price to investors' perception of price informativeness in the first period. High quality public information eliminates multiple equilibria by weakening this link, in that when public information is very precise, the second period price will be mainly determined by public information, as opposed to be determined by private information, thus reducing the impact of self-fulfilling expectations. Only when public information is sufficiently noisy is there sufficient room for investors' self-fulfilling expectations to affect prices and generate multiple equilibria.

To further relate to the prior literature, we analyze the impact of public information on the discount in price investors demand to hold risky assets (typically referred to as the cost of capital). In our setting, public information affects price discount both directly as in the prior literature, and indirectly, as identified here, by affecting price informativeness. Both effects influence investors' average information precision which in turn determines price discounts. We show that holding the average *private* information precision in the economy constant, price discounts are higher with more information asymmetry and with less precise public information.

Lastly, we show that our main conclusion, that public information affects price informativeness and equilibrium uniqueness only when short horizon and information asymmetry are both present, remains robust when we extend our analysis to a setting with multiple risky assets. As long as asset payoffs are correlated, our results hold with respect to both public information that affects all firms and firm-specific public information.

Our paper contributes to the accounting literature on the role of public information and of information asymmetry (Lambert et al. (2007, 2012), Gao (2008)).[6] Our contribution lies in identifying a mechanism in which public information not only affects price efficiency directly, but also indirectly by influencing the ability of price to aggregate private information held by informed investors that otherwise would not be available to the general public. Prior literature has mostly focused on the direct role of public information. Our finding that information asymmetry is a necessary condition for public information to affect price informativeness provides a new justification for the claim that more public disclosure helps level the playing field and improves market efficiency. Furthermore, more public information in our model can help stabilize the market and reduce excess volatility in that precise public information can eliminate multiple equilibria.

Our analyses also reveal a dark side of public information in that small improvement on low quality public information may reduce price informativeness. While this message echoes that from Morris and Shin (2002) who also caution against the potential detrimental effect of public information, the underlying mechanism for our results is different from that in Morris and Shin (2002). We study a trading model where the key mechanism is investors' short horizon and information asymmetry, whereas Morris and Shin (2002) study a decision-making setting where the key mechanism is the externalities in individuals' actions. In addition, multiple equilibria do not arise in Morris and Shin (2002).

As to policy implications, our analysis suggests that public information needs to be credible and precise to achieve positive effect on price efficiency, particularly with short-horizon investors. Since the key assumptions for our findings are short-horizon and information asymmetry, two conditions

[6] Modeling wise, our paper belongs to the broad literature of dynamic noisy rational expectations model. A large literature has used this type of model to study price volatility, trading volume, and technical analyses (e.g., Wang (1994)). We refer readers to Brunnermeier (2001) and Vives (2010) for excellent and comprehensive reviews. To our best knowledge, our paper is the first to analyze the impact of heterogenous information precision among investors on price informativeness.

that empirically characterize a large cross-section of firms,[7] it is reasonable to believe our findings are highly relevant empirically. For example, our results help shed light on recent empirical findings that short-horizon investors magnify public news shock (e.g., Cella, Ellul, and Giannetti (2013)). Our analyses suggest that empirical efforts to identify the pricing effects of information asymmetry may be most fruitful in cases where short-horizon investors play a relatively important role in setting prices. This implication complements that from the prior literature that emphasizes the role of imperfect competition (e.g., Armstrong et al. (2011)). Lastly, to the extent that firms' voluntary disclosure is motivated by the concerns about how disclosure affects price efficiency, our results generate implications and empirical predictions regarding firms' disclosure choices and how they relate to price efficiency and investor composition (e.g., Bushee and Noe (2000)).

In what follows, we set up and solve the model in Section 2. Section 2 also contains the main result that public information influences price informativeness when information asymmetry exists among short-horizon investors. Sections 3 analyses in detail how public information affects price informativeness and equilibrium uniqueness. Section 4 extends the analysis to study the effects of information asymmetry and public information on price discounts and to settings with multiple assets. Section 5 concludes.

2 Model Setup and Solution

2.1 Model setup

To facilitate comparison with the prior literature, we study a two-period noisy rational expectations model with short-horizon investors, similar to those in Allen, Morris, and Shin (2006) and Gao (2008). Unlike these prior studies, however, our model allows the quality/precision of individual investors' private information to differ. We briefly describe the model below.

There are two periods (denoted by $t = 1$ and $t = 2$). In each period, a continuum of investors with a unit measure (indexed by $i \in [0, 1]$) choose their investments through trading between a

[7] See, for example, Hotchkiss and Strickland (2003); Bushee and Goodman (2007); Yan and Zhang (2009); and Cella et al. (2013).

risky asset (stock) and a riskless asset (cash) in a competitive market. Without loss of generality, the rate of return for cash is normalized to one. The per share liquidation value of the risky asset, θ, is random and will be realized at the end of the second period. Investors do not observe the average per capita supply of the risky asset (denoted as s_t) but understand that $s_t \sim N\left(\bar{s}_t, \frac{1}{\gamma_t}\right)$ and is independent across periods, with $\bar{s}_t > 0$. The supply noise is needed in this type of model to prevent the price from becoming fully revealing (Diamond and Verrecchia (1981)); and temporal independence in the supply noise is assumed to isolate the effect of trading volume (Brown and Jennings (1989)).

All investors are assumed to have a constant absolute risk aversion (CARA) utility function $U(c) = -\exp\left(-\frac{c}{\tau_t}\right)$ where τ_t is the risk tolerance parameter for investors born in period $t = 1$ or 2.[8] Investors born in the first period trade in the first period and unwind their holdings in exchange for consumption goods in the second period. The per share value they obtain from unwinding their positions is the price determined from the trades by the second generation investors. Investors born in the second period trade in the second period, and unwind their positions when the terminal value θ is realized at the end of the second period.

The common prior on θ is that it is diffusely distributed over the real line. Both generations of investors observe one common public signal z from the following distribution:[9]

$$z = \theta + \varepsilon_z \text{ where } \varepsilon_z \sim N\left(0, \frac{1}{\alpha}\right).$$

Each investor also observes a private signal x_{ti} prior to trading:

$$\begin{aligned} x_{ti} &= \theta + \varepsilon_{ti}, \text{ where} \\ \varepsilon_{ti} &\sim N\left(0, \frac{1}{\beta_{ti}}\right); \; cov\left(\varepsilon_{ti}, \varepsilon_{t\tilde{\imath}}\right) = 0, \; \forall t, i \neq \tilde{\imath}; \text{ and } cov\left(\varepsilon_{ti}, \varepsilon_{\tilde{t}\tilde{\imath}}\right) = 0, \; \forall t \neq \tilde{t}, i, \tilde{\imath}. \end{aligned}$$

Conditional on θ, private signals are independent across investors and periods. The precision of the private signal is β_{ti} for investor i in the t^{th} generation. We assume in each period, $\beta_i \in [\beta_{\min}, \beta_{\max}]$ is distributed according to a p.d.f. function of $g(\beta)$ (with the associated c.d.f. $G(\beta)$), $E(\beta_i) =$

[8] Allowing τ_t to differ across individual investors does not qualitatively change our results.

[9] Equivalently, both generations have the common prior that θ is normally distributed with mean z and variance $\frac{1}{\alpha}$.

$\int_i \beta_i dG(\beta) = \overline{\beta}$ and $E\left(\beta_i^2\right) < \infty$. We assume the cross-sectional distribution of β_i is identical and independent across periods. This assumption is without loss of generality: as will be shown shortly, the distribution of β_i among second generation investors does not affect the main results. We define the degree of information asymmetry among investors by the dispersion of β_i:

Definition: Let $F(\beta_i)$ and $G(\beta_i)$ be two distribution functions of β_i. We say the degree of information asymmetry among investors is higher under F than under G if $F(\beta_i)$ is a mean-preserving spread of $G(\beta_i)$.

A mean-preserving spread helps isolate the mean effect of β_i (i.e., $\overline{\beta}$) from the dispersion effect of β_i. This is important as the average private precision $\overline{\beta}$ plays an important role in this type of model (Lambert et al. (2012)).

Under CARA utility functions and normal distributions, the optimal demand for the risky asset by a first generation investor i is

$$D_{1i} = \tau_1 \frac{E_{1i}(p_2) - p_1}{Var_{1i}(p_2)}. \tag{1}$$

Similarly, the demand by a second generation investor is

$$D_{2i} = \tau_2 \frac{E_{2i}(\theta) - p_2}{Var_{2i}(\theta)}. \tag{2}$$

In both expressions, the subscripts denote that the expectations are taken with respect to the information set Φ_{ti} of investor i in the t^{th} generation. Specifically, $\Phi_{1i} \equiv \{z, p_1, x_{1i}\}$ where p_1 is the equilibrium price of the risky asset from the first round of trading, and x_{1i} is investor i's private information signal. Similarly, $\Phi_{2i} \equiv \{z, p_1, p_2, x_{2i}\}$. x_{1i} is not an element of Φ_{2i} because it is privately observed by the first generation investor i; although second generation investors will glean some information about the x_{1i}s from p_1. Also note that the payoff for second generation investors of holding the risky asset is its liquidation value θ, whereas the payoff for the first generation is the risky asset's price from the second round of trading, p_2.

In addition to facilitating technical tractability, a two-period model helps illustrate our main intuition and insight. It captures the key element that we are interested in, that is, price informativeness when investors' payoff from holding a risky asset is the future trading price, which depends on future investors' information as well as the firm's fundamental terminal payoff. The second period is an analytical tool to capture the time period between first generation investors' trading and the end of the firm's operating horizon, which can vary greatly depending on the type of investors and the type of the firm. The end of the second period is the firm's final liquidation date and should not be interpreted as the firm's next earnings report date. While the uncertainty about the firm's terminal payoff may be partially resolved by periodic earnings announcements or dividend payments, it will not be completely resolved until the firm's final liquidation date. We assume the firm does not pay interim dividends, although it is without loss of generality, as the main intuition holds as long as interim dividends do not reveal the terminal payoff completely. To the extent that interim dividends are informative about the terminal payoff, allowing dividends is equivalent to allowing additional amount of public information in the model.

2.2 Solution

2.2.1 Equilibrium and measure of price informativeness

The equilibrium concept and solution procedures used here are fairly standard. For brevity, we highlight the parts pertinent to our analysis and refer readers to Allen, Morris, and Shin (2006) for a detailed account. Following the literature, we focus on linear equilibria where period t price is given by

$$p_1 = b_1 z + c_1\theta - d_1 (s_1 - \bar{s}_1) - e_1\bar{s}_1 - f_1\bar{s}_2 \tag{3}$$

$$\text{and } p_2 = a_2 p_1 + b_2 z + c_2\theta - d_2 (s_2 - \bar{s}_2) - e_2\bar{s}_1 - f_2\bar{s}_2. \tag{4}$$

A key feature of (3) and (4) is that prices are linear functions of θ. This happens because the equilibrium prices are determined by the aggregate supply and demand for the risky asset. As will be shown next, individual investors' demand for the risky asset is linear in their private signals, and the aggregate demand is linear in the average of private signals, which, by the Law of Large Numbers, equals θ. Investors understand this feature and will take into account the information in prices about θ in their trades. Since the stock price in each trading round is affected by two random variables (θ and the supply shock s_t), investors view the observed price as a noisy signal of θ where the noise comes from the supply shock. Specifically, rearrange (3) and (4) to get

$$P_1^* \equiv \frac{p_1 - b_1 z + e_1 \bar{s}_1 + f_1 \bar{s}_2}{c_1} = \theta - \frac{d_1}{c_1}(s_1 - \bar{s}_1), \tag{5}$$

$$P_2^* \equiv \frac{p_2 - (a_2 p_1 + b_2 z) + e_2 \bar{s}_1 + f_2 \bar{s}_2}{c_2} = \theta - \frac{d_2}{c_2}(s_2 - \bar{s}_2), \tag{6}$$

where P_t^* measures θ with a noise term of $-\frac{d_t}{c_t}(s_t - \bar{s}_t)$, which is normally distributed with mean zero and variance of $1/\left[\left(\frac{c_t}{d_t}\right)^2 \gamma_t\right]$ (recall $s_t \sim N\left(\bar{s}_t, \frac{1}{\gamma_t}\right)$). Conditional on observing the pre-trading public information (i.e., $\{z\}$ for the first generation and $\{z, p_1\}$ for the second generation), observing p_t provides the same information content regarding θ as observing P_t^*. Thus, the informativeness of price can be measured by the inverse of the variance term as:

$$\rho_t \equiv \left(\frac{c_t}{d_t}\right)^2 \gamma_t. \tag{7}$$

The higher ρ_t is, the more informative p_t is with respect to θ. Since γ_t is exogenously given, we are interested in the endogenous part of ρ_t: the ratio $\frac{c_t}{d_t}$.

Note that price informativeness is different from the concept of market/price efficiency, which measures the extent to which price reflects all value relevant information, including both private and public information. In a similar setting to ours but without heterogenous private information quality, Gao (2008) analyzes the role of public information on price efficiency, which he measures as the reciprocal of the mean-squared error between the firm's fundamental and its stock price. *Ceteris paribus*, more price informativeness will increase price efficiency while the reverse is not true. This is because price can be close to the fundamental from incorporating public information alone without reflecting any private information. In contrast, price informativeness measures the amount of private information conveyed by price that is not otherwise available to the public.

2.2.2 Price informativeness in the second period

Since second generation investors hold the stock until θ is realized, they trade based on their expectations of θ and the equilibrium is determined similarly as that in a standard one-period model. This is confirmed and characterized below as Lemma 1.

Lemma 1: For a given p_1 and ρ_1, there is a unique linear equilibrium for the second period where price is given by (4) with

$$\begin{aligned}
a_2 &= \frac{\rho_1}{c_1}\frac{1}{M}, b_2 = \left(\alpha - \frac{\rho_1 b_1}{c_1}\right)\frac{1}{M}, \\
c_2 &= \left(\rho_2 + \overline{\beta}\right)\frac{1}{M}, d_2 = \frac{1}{\tau_2\overline{\beta}}c_2, \\
e_2 &= -e_1 a_2, f_2 = \left(\frac{1}{\tau_2} - \frac{\rho_1 f_1}{c_1}\right)\frac{1}{M}, \\
\rho_2 &= \left(\tau_2\overline{\beta}\right)^2\gamma_2,\ M = \alpha + \rho_1 + \rho_2 + \overline{\beta}, \text{ and } \overline{\beta} \equiv \int_{\beta_i} \beta_i dG\left(\beta_i\right).
\end{aligned}$$

Proof. See the Appendix of Allen, Morris, and Shin (2006). ■

Lemma 1 shows that $\rho_2 = \left(\tau_2\overline{\beta}\right)^2\gamma_2$, which implies that holding the average of private information precision $\overline{\beta}$ constant, price informativeness in period 2 does not depend on how the precision of investors' private information differs from each other. This is the same conclusion as that from a standard one-period model (Verrecchia (1982)). It is not surprising since second period investors hold assets till maturity and thus are in fact long-term investors, as are the investors in one-period models. In what follows, we make two comments on the intuition and implication in order to set the stage for later discussion.

First is the intuition. In a noisy rational expectations model, stock price becomes informative because it aggregates investors' demand, which depends on investors' private information. Therefore, the degree of price informativeness depends on how sensitive investors' trades are to their private signals and equals the average of all investors' individual sensitivities in equilibrium. To see this, notice that the equilibrium p_2 is set to equalize the aggregate demand with the aggregate supply for every realization of the supply shock and liquation value, i.e.,

$$\int_{\beta_i} D_{2i}\left(p_2\right) dG\left(\beta_i\right) = s_2.$$

Write (4) as $p_2 = c_2\theta - d_2 s_2 + W$ (where W is a constant term observable to all investors). By construction, p_2 would remain the same if one introduces a shock of ϵ to the fundamental θ and a simultaneous shock of $\frac{c_2}{d_2}\epsilon$ to the supply noise s_2.[10] This implies that

$$\begin{aligned}\epsilon\frac{\partial}{\partial\theta}\int_{\beta_i} D_{2i}dG\left(\beta_i\right) = \epsilon\frac{c_2}{d_2} &= \epsilon\sqrt{\frac{\rho_2}{\gamma_2}}\\ \Rightarrow \rho_2 = \gamma_2\left[\int_{\beta_i}\frac{\partial D_{2i}}{\partial x_{2i}}dG\left(\beta_i\right)\right]^2. &\qquad (8)\end{aligned}$$

In other words, ρ_2 monotonically increases in the average sensitivity of each individual's demand to his private signal.

Substituting

$$\begin{aligned} E_{2i}\left(\theta\right) &= \frac{\alpha z + \rho_1 P_1^* + \rho_2 P_2^* + \beta_i x_{2i}}{\alpha + \rho_1 + \rho_2 + \beta_i}, \\ \left[Var_{2i}\left(\theta\right)\right]^{-1} &= \alpha + \rho_1 + \rho_2 + \beta_i, \end{aligned}$$

into the demand function D_{2i} from (2), we have

$$\frac{\partial D_{2i}}{\partial x_{2i}} = \frac{\tau_2}{Var_{2i}\left(\theta\right)}\frac{\partial E_{2i}\left(\theta\right)}{\partial x_{2i}} = \tau_2\beta_i. \qquad (9)$$

Substituting (9) into (8) shows that ρ_2 depends only on the average precision and not on how β_i is distributed among investors. This is because the sensitivity is linear in the precision of investors' private information. The linearity arises because the denominator of $E_{2i}\left(\theta\right)$ is exactly cancelled out by the scaling factor ($\left[Var_{2i}\left(\theta\right)\right]^{-1}$) in the demand function. The linearity of the sensitivity in β_i implies that a unit increase in β_i will be offset by a unit decrease in β_j. As long as the average $\overline{\beta}$ is held constant, price informativeness does not change.

Second, it is worth pointing out that price informativeness does not depend on the precision of the public information (α). The result may appear counter-intuitive at the first glance, as more precise public information reduces investors' sensitivity to their private information, which would reduce price informativeness (recall $\frac{\partial E_{2i}(\theta)}{\partial x_{2i}} = \frac{\beta_i}{\alpha+\rho_1+\rho_2+\beta_i}$ which is decreasing in α). However, more precise public information also reduces trader i's residual uncertainty about the risky return. This induces investors to trade more aggressively, which increases price informativeness (recall $\left[Var_{2i}\left(\theta\right)\right]^{-1}$ is increasing in α). In equilibrium, these two effects exactly offset each other.

[10] Bond and Goldstein (2010) first introduce this intuitive illustration.

2.2.3 Price informativeness in the first period

The first period equilibrium can be solved in a similar fashion, except that the consumption value of the risky asset for first generation investors is now p_2 instead of θ. As a result, unlike second generation investors, first generation investors' trading sensitivities to their private information is no longer linear in the precision of their private information. To see this, substitute

$$\begin{aligned} E_{1i}(p_2) &= a_2 p_1 + b_2 z + c_2 E_{1i}(\theta) \qquad (10)\\ &= a_2 p_1 + b_2 z + c_2 \frac{\alpha z + \rho_1 P_1^* + \beta_i x_{1i}}{\alpha + \rho_1 + \beta_i}, \\ \text{and } Var_{1i}(p_2) &= c_2^2 Var_{1i}(P_2^*) = c_2^2 \left[Var_{1i}(\theta) + \frac{1}{\rho_2} \right], \qquad (11) \end{aligned}$$

into D_{1i} from (1). The sensitivity of investor $1i$'s demand to his private information is

$$\frac{\partial D_{1i}}{\partial x_{1i}} = \frac{\partial}{\partial x_{1i}} \left[\tau_1 \frac{E_{1i}(p_2)}{Var_{1i}(p_2)} \right] = \frac{c_2 Var_{1i}(\theta)}{Var_{1i}(p_2)} \tau_1 \beta_i. \qquad (12)$$

Note that $\tau_1 \beta_i$ is the sensitivity for a long-horizon investor (i.e., where θ is his payoff). The effect of short horizon is captured by the term $\frac{c_2 Var_{1i}(\theta)}{Var_{1i}(p_2)}$, which in general is a nonlinear function of β_i. The nonlinearity implies that the distribution of private information would matter in equilibrium.

Specifically, rewrite $c_2 = \frac{\rho_2 + \overline{\beta}}{\alpha + \rho_1 + \rho_2 + \overline{\beta}}$ from Lemma 1 as

$$c_2 = \frac{\rho_2 + \overline{\beta}}{\alpha + \rho_1 + \rho_2 + \overline{\beta}} = \frac{\rho_2 + \overline{\beta}}{\overline{\beta}} r\left(\overline{\beta}\right), \qquad (13)$$

where $\overline{\beta}$ is the average precision of investors' private information and the function $r(\beta)$ is defined as

$$r(\beta) = \frac{\beta}{\alpha + \rho_1 + \rho_2 + \beta}. \qquad (14)$$

$\alpha + \rho_1 + \rho_2 + \beta$ measures the precision of total information available to an investor with the private information precision β who observes both p_1 and p_2. Thus $r(\beta)$ captures the proportion of this investor's total information that is contributed by his private information.

Substituting (11) and (13) into (12) and integrating (12) over β_i yields the expression for the first period price informativeness (denoted as ρ_1^{asym}). The detailed derivation is shown in the appendix and the main results are summarized in Proposition 1 below.

Proposition 1 • (i) If θ were realized at the end of the first period (i.e., first generation investors have long investment horizons), there would exist a unique linear equilibrium where the first period price informativeness would be $\rho_1^{LH} = \left(\tau_1 \overline{\beta}\right)^2 \gamma_1$ where the superscript LH stands for long horizon.

- (ii) When investors have homogenous information precisions (i.e., $\beta_i = \overline{\beta}$, for all i), there exists a unique linear equilibrium where the first period equilibrium price informativeness ρ_1 is given by

$$\rho_1^{sym} = \rho_1^{LH} \left(\frac{\rho_2}{\rho_2 + \overline{\beta}} \right)^2 < \rho_1^{LH} \tag{15}$$

- (iii) When investors have heterogeneous information precisions, the first period equilibrium price informativeness is (implicitly) determined by

$$\rho_1^{asym} = \rho_1^{sym} \left[\frac{\int_{\beta_i} r\left(\beta_i\right) dG\left(\beta_i\right)}{r\left(\overline{\beta}\right)} \right]^2 = \rho_1^{sym} \left\{ \frac{E_{\beta_i}\left[r\left(\beta_i\right)\right]}{r\left(\overline{\beta}\right)} \right\}^2 < \rho_1^{sym}, \tag{16}$$

where $r\left(\beta_i\right) = \frac{\beta_i}{\alpha + \rho_1^{asym} + \rho_2 + \beta_i}$ and the subscript indicates the expectation is taken with respect to β_i.

Proof of Proposition 1 (See the appendix for details.)

Proposition 1 lays out price informativeness in three different cases. Part (i) and (ii) are from prior findings and are presented for comparison purposes. Part (i) characterizes a hypothetical scenario where θ were realized at the end of period 1. The expression for ρ_1^{LH} is identical to ρ_2 in Lemma 1, except for different time subscripts for the risk tolerance and the variance of the supply noise. This is not surprising because when first generation investors obtain θ as their terminal payoff, their trades are guided by their expectations of θ, much like second generation investors.

Part (ii) corresponds to the "Beauty Contest" settings studied in Allen, Morris, and Shin (2006) and Gao (2008) and shows that price informativeness is lower when investors have short horizons. Intuitively, short-horizon investors only care about the second period price, which is determined both by the risky asset's terminal payoff and by the second period random supply shock. Consequently, risk-averse first period investors face additional uncertainty and thus trade less upon their private information (compared to the long horizon case), reducing price informativeness.

Part (iii) summarizes the two new key results from our analysis. The first is that information asymmetry further reduces price informativeness and the second is that the equilibrium is not necessarily unique anymore. We discuss these results in turn.

The intuition for the first key result, that information asymmetry further reduces price informativeness, is that investors do not reduce the sensitivity of their trades to private information to the same degree: relative to investors with more precise private information, investors with less precise private information reduce their sensitivities less because their sensitivities are not very high to begin with.

To elaborate, consider two extreme cases. In the first case, an investor has $\beta_i = 0$ and hence optimally assigns zero weight to his private information, regardless of his investment horizon. In the second case, consider an investor with $\beta = \infty$. If this investor is a long-horizon investor, he will take an infinite position (i.e., maximum sensitivity) whenever $p_1 \neq \theta$ because he has no residual uncertainty about his payoff. However, if he is a short-horizon investor and has to close his position before θ is realized, he faces an uncertain second period price and hence no longer wishes to take an infinite position even when $p_1 \neq \theta$, resulting in a significant reduction in the sensitivity of his trade to his private information.

Different degrees of reduction in trading sensitivity in turn imply a concave relationship between trading sensitivity and private information precision. Since price informativeness is an average of all individual sensitivities, information asymmetry leads to overall reduction in price informativeness.[11]

The second key result from part (iii) is that unlike in parts (i) and (ii) where the equilibrium is unique, the equilibrium is not necessarily unique when both short horizon and information

[11] This can be seen from Proposition 1(iii) which shows that the effect of information asymmetry is completely captured by the $\frac{E_{\beta_i}[r(\beta_i)]}{r(\overline{\beta})}$ term in (16). Since $r(\beta_i) = \frac{\beta_i}{\alpha+\rho_1+\rho_2+\beta_i}$ is concave in β_i, by the Jensen's Inequality, $E(r(\beta_i)) \equiv \int_{\beta_i} r(\beta_i)\, dG(\beta_i) \leq r(E(\beta_i)) \equiv r(\overline{\beta})$ where the equality holds if and only if $\beta_i = \overline{\beta}$ for all i. Thus $\rho_1^{asym} < \rho^{sym}$ as long as $\beta_i \neq \beta_j$ for some $i \neq j$. Obviously, the larger the $\frac{E_{\beta_i}[r(\beta_i)]}{r(\overline{\beta})}$ term is, the less information is lost in the price aggregation process and the higher price informativeness is. As such, $\frac{E_{\beta_i}[r(\beta_i)]}{r(\overline{\beta})}$ represents the (inverse of) aggregation loss.

asymmetry are present. To see this, define the right hand side of (16) as a function of ρ_1:

$$R(\rho_1) \equiv \rho_1^{sym} \left\{ \frac{E_{\beta_i}\left[r(\beta_i, \rho_1)\right]}{r\left(\overline{\beta}, \rho_1\right)} \right\}^2, \tag{17}$$

where ρ_1^{sym} is a constant that does not depend on ρ_1. The intersections of the $R(\rho_1)$ curve and the 45° line determine the equilibrium ρ_1^{asym}. Since $R(0) > 0$, and $R(\rho_1)$ approaches ρ_1^{sym} from below as $\rho_1 \longrightarrow \infty$, an equilibrium always exists. However, $R(\rho_1)$ is not necessarily monotone in ρ_1 and its slope can be either positive or negative. Since $R(\rho_1)$ is continuous in ρ_1 and approaches ρ_1^{sym} from below as $\rho_1 \to \infty$, if $\frac{\partial R(\rho_1)}{\partial \rho_1}|_{\rho_1=\rho_1^{asym}} > 1$ when evaluated at an equilibrium point, there must exist at least one other equilibrium.

Figure 1 plots the various cases of equilibrium solutions in three panels. In each panel, the 45-degree straight line represents the left hand side of (16) and the curvy line represents $R(\rho_1)$. Panel A (B) corresponds to a unique equilibrium solution where $\frac{\partial R(\rho_1)}{\partial \rho_1}|_{\rho_1=\rho_1^{asym}} \in (0,1)$ ($\frac{\partial R(\rho_1)}{\partial \rho_1}|_{\rho_1=\rho_1^{asym}} < 0$). The solid line in Panel C illustrates the case of multiple equilibria where $R(\rho_1)$ intersects with the 45-degree line three times, yielding three solutions for ρ_1 with $\frac{\partial R(\rho_1)}{\partial \rho_1}|_{\rho_1=\rho_1^{asym}} > 1$ at the second equilibrium only.

The existence of multiple equilibria may sound counter-intuitive as the conventional wisdom seems to suggest an unique equilibrium. To see this, start with a linear equilibrium. If all investors deviate by conjecturing that price is more informative than the existing equilibrium level, they would rely less on their private information and more on price relative to the existing equilibrium. In standard models with long horizon and no information asymmetry, such a deviation would not be self-fulfilling as lower sensitivity toward private information would result in lower price informativeness, contradicting investors' initial conjecture.

However, the conventional wisdom doesn't take into account that short-horizon investors' payoff depends on the second period price, which endogenously depends on the conjectured first period price informativeness. This in turn gives rise to an "endogenous uncertainty effect": when the perceived first period price informativeness goes up, first generation investors will perceive the second period price to be less uncertain. Intuitively, this is because second generation investors (who determine the second period price) can resolve more uncertainty from a more informed first period price and thus have more capacity to absorb random supply shocks in the second period, making the second period price less sensitive to supply shocks and hence more predictable from the first generation's perspective.[12] The lower ex ante uncertainty induces first generation investors to trade more aggressively on their private information, which increases the equilibrium first period price informativeness, confirming the initial conjecture. More pertinent to our analysis here is that less informed investors do not increase their trading sensitivities as much because their private information is not that precise to begin with. In contrast, more informed investors trade more aggressively, and rely more on their private information. That is, information asymmetry magnifies the endogenous uncertainty effect. When the degree of information asymmetry is strong enough, the endogenous uncertainty effect can overwhelm the standard effect and result in higher price informativeness in another equilibrium. Similarly, when the conjectured first period informativeness decreases, the same "endogenous uncertainty effect" could again lead to a self-fulfilling prophecy and generates lower informativeness in a third equilibrium.

With multiple equilibria comes the issue of equilibrium selection. We note that any equilibrium with $\frac{\partial R(\rho_1)}{\partial \rho_1}|_{\rho_1=\rho_1^{asym}} \notin (-1,1)$ is unstable in the sense that a small deviation in investors' perceived ρ_1 will prevent the resulting values of $R(\rho_1)$ from converging back to the equilibrium (Stokey, Lucas, and Prescott (1989)). For this reason, our subsequent analyses focus only on the stable equilibria where $\frac{\partial R(\rho_1)}{\partial \rho_1}|_{\rho_1=\rho_1^{asym}} \in (-1,1)$. In Panel C of Figure 1 equilibrium #2 is unstable, while the other two equilibria are stable.

3 Effects of Public Information

Proposition 1 establishes the key result in our paper, that is, public information does not affect the equilibrium first period price informativeness (as α does not appear in the expression for ρ_1^{LH} and ρ_1^{sym}) or equilibrium uniqueness unless both short horizon and information asymmetry are present. This result stands in contrast with the prior literature where the equilibrium is unique and public

[12] To see this, note that in Lemma 1 the equilibrium coefficient for the supply shock in the second period price p_2 is $d_2 = \frac{1}{\tau_2 \overline{\beta}} \frac{(\rho_2+\overline{\beta})}{\alpha+\rho_1+\rho_2+\overline{\beta}}$ which is decreasing in ρ_1. This implies that, everything else equal, the ex ante variance of the second period price decreases as the first period price informativeness goes up.

information does not affect price informativeness. In this section, we analyze in detail how public information can affect both price informativeness and equilibrium uniqueness.

3.1 Effect of public information on price informativeness

The effect of public information on the equilibrium price informativeness can be seen from (17) which shows that α and ρ_1 affect $R(\rho_1)$ only through their sum. That is, *ceteris paribus*, a unit change of α has the same effect on $R(\rho_1)$ as a unit change of ρ_1 in the same direction. Hence, increasing α is equivalent to shifting the $R(\rho_1)$ curve to the left. Consequently, increasing α can either increase or decrease the equilibrium informativeness ρ_1 depending on the sign of slope of $R(\rho_1)$ at equilibrium (i.e., $\frac{\partial R(\rho_1)}{\partial \rho_1}|_{\rho_1=\rho_1^{asym}}$). Specifically, for any stable equilibrium where $\frac{\partial R(\rho_1)}{\partial \rho_1}|_{\rho_1=\rho_1^{asym}} \in (0,1)$, increasing public information strictly improves price informativeness of that equilibrium. In contrast, for any stable equilibrium where $\frac{\partial R(\rho_1)}{\partial \rho_1}|_{\rho_1=\rho_1^{asym}} \in (-1,0)$, increasing public information strictly reduces price informativeness. Proposition 2 provides sufficient conditions under which increasing α increases (reduces) price informativeness in the first period.

Proposition 2 (i) $\frac{d\rho_1^{sym}}{d\alpha} = 0$. (ii) For a given set of exogenous parameters, there exists a $\overline{\alpha}$ such that $\frac{d\rho_1^{asym}}{d\alpha} > 0$ for all $\alpha > \overline{\alpha}$. (iii) When $\beta_{\min} \neq 0$ and $\gamma_1\tau_1^2$ and $\gamma_2\tau_2^2$ are sufficiently small, there exists a $\underline{\alpha}$ such that $\frac{d\rho_1^{asym}}{d\alpha} < 0$ for all $\alpha < \underline{\alpha}$.

Proof of Proposition 2 (See the appendix for details.)

Proposition 2(i) can be shown from an inspection of ρ_1^{sym} in Proposition 1. It states that with homogeneous investors, public information has no effect on the ability of price to aggregate and reveal private information. Proposition 2(ii) shows that when public information is precise enough, further increasing its precision on the margin can enable the first period price to better aggregate investors' private information. Conversely, Proposition 2(iii) demonstrates that when public information is not precise to begin with, further increasing its precision on the margin can reduce the first period price informativeness.

The intuition behind Part (ii) and (iii) of the proposition is as follows. As discussed earlier, information asymmetry affects price informativeness due to the dispersion in sensitivities of individual investors' trades with respect to their private information: the higher the dispersion, the larger the aggregation loss that results from information asymmetry, the lower the price informativeness. Public information enters the picture by affecting the degree of dispersion in individual sensitivities. Specifically, when public information is very precise (imprecise), all investors, regardless of their private information precision, place a very small (large) weight, say, close to 0 (1) to their private information, hence leading to less dispersion, limiting the effect of information asymmetry on the price formation process and reducing the aggregation loss. Only with moderately precise public information is there significant dispersion in individual sensitivities. As a result, there can exist a U-shaped relationship between the quality of public information and price informativeness.

3.2 Effect of public information on equilibrium uniqueness

Proposition 1 shows that the presence of both information asymmetry and short horizon can lead to multiple equilibria. In this setting, an additional effect of public information is to affect equilibrium uniqueness via its effect on $\frac{E_{\beta_i}[r(\beta_i)]}{r(\overline{\beta})}$. Note that a sufficient condition for a unique equilibrium is $\frac{\partial R(\rho_1)}{\partial \rho_1} < 1$, $\forall \rho_1$. As α becomes large, the ratio $\frac{E_{\beta_i}[r(\beta_i)]}{r(\overline{\beta})}$ in (16) approaches 1, and the right-hand-side of (16) does not vary much with ρ_1, which helps obtaining a unique equilibrium solution of ρ_1. The proof for Part (ii) of Proposition 2 shows that as long as α is large enough, $\frac{\partial R(\rho_1)}{\partial \rho_1} \in (0, 1)$. This is stated formally as Proposition 3 below.

Proposition 3 When α is large enough, there exists a unique stable linear equilibrium.

Intuitively, multiple equilibria arise due to the "endogenous uncertainty effect" in which the perceived uncertainty in the second period price is linked to the first generation's conjectured price informativeness in the first period. Precise public information eliminates multiple equilibria by weakening this link. Specifically, when public information is very precise, the second period price will primarily be driven by the public information regardless of the conjectured first period price informativeness. Only when public information is sufficiently noisy is there much room for investors' conjectured price informativeness to impact their demand, possibly leading to multiple equilibria.

An immediate implication of Proposition 3 is that a necessary condition for multiple linear equilibria to exist is that public information cannot be too precise. The following observation in fact shows that there could exist a threshold such that multiple equilibria are obtained if and only if public information precision drops below the threshold.

Observation 1 Consider a binary distribution of β_i's, where $\beta_i \in \{\beta_h, \beta_l\}$ with $\beta_h = 100$, $\beta_l = 0$, $\Pr(\beta_h) = 0.01$, $\tau_1 = 10$, $\tau_2 = 1$, $\gamma_1 = 10$, and $\gamma_2 = 2$, there exists $\widehat{\alpha}$ such that there are multiple linear equilibria if and only if $\alpha \leq \widehat{\alpha}$.

Proof for Observation 1 (See the appendix for details.)

Observation 1 shows the possibility of a discontinuous effect of public information on price. The discontinuity would take place if investors start in the least informative equilibrium whenever multiple equilibria exist. If public information becomes precise enough such that the equilibrium becomes unique, price would jump from the least informative equilibrium to the surviving equilibrium.

Because a closed-form solution for ρ_1^{asym} is unavailable, Proposition 2 and 3 are stated in terms of sufficient conditions. An important implication of these results is that the threshold is a function of the exogenous model parameters including investors' private information quality, risk preferences, and the amount of noise trading in each period. Since these parameters vary significantly by firms, by the types of public information disclosure, and by the length of each time period (the model is silent on how long each period is), the threshold values can exhibit significant cross-sectional variations as well.[13]

We believe the insights from Propositions 2 and 3 can be informative to policy makers in charge of devising public disclosure requirements, as well as to empirical researchers interested in the informational and pricing effect of public disclosure and how the effect may differ across firms. On the policy and normative side, a key implication of these results is that public disclosure needs to

[13] In unreported analysis, we numerically simulate the relation between the exogenous model parameters (γ_t) and the critical threshold value below which α negatively affect ρ_1^{asym}. We find that the critical value varies significantly by these parameter values. For example, under the (fairly uncontroversial) parameterization of $\tau_1 = \tau_2 = 1$ and $\gamma_1 = \gamma_2 = 1$, $\frac{\partial \rho_1^{asym}}{\partial \alpha} < 0$ for all $\alpha < 1.55\overline{\beta}$. The critical value is higher when either γ_1 or γ_2 is lower. While we are not aware of any empirical evidence explicitly quantify the magnitude of private and public information precision, we interpret this result as suggesting that the non-monotone relation identified by Proposition 2 is not merely a theoretical possibility under extreme parameterizations.

be credible and precise to achieve its positive effects on price informativeness and market stability, especially with short-horizon investors.

To the extent that improving price efficiency and stabilizing markets are the objectives guiding firms' voluntary disclosure decisions in practice, our analyses generate empirical predictions consistent with firms' disclosure behaviors. For example, that disclosure needs to be of higher quality is consistent with the observation that firms refrain from disclosing information of speculative nature, and that when they do disclose (for example, by issuing earnings forecasts), their disclosures are deemed accurate, and viewed as informative by the market.[14] These results suggest that the effects of public information would differ by the type and nature of public information (insofar as they affect the quality of the information), as well as by the type of investors (insofar as they differ in trading horizons, risk preferences and in the amount of noise trading). To the extent that institutional investors differ from retail investors in these dimensions, these results provide a theoretical channel consistent with the empirical findings that institutional ownership affects firm disclosures (e.g, Bushee and Noe (2000)). Further, to the extent that multiple equilibria result in higher stock volatility, these results imply that when information quality is low (such as during times of market turmoil), firms with more short-horizon investors may exhibit more price volatility, consistent with the findings in Cella, Ellul, and Giannetti (2013).

Our analysis is by no means the only explanation for the aforementioned empirical findings. The empirical relevance of the forces analyzed in this paper is ultimately an empirical question. Nonetheless, our analysis is to our knowledge the first in the literature to highlight the effect of short horizon and information asymmetry on the effects of public information.

4 Extension and Discussion

4.1 Effect of information asymmetry and public information on price discounts

Our primary focus is to examine the effect of public information on price informativeness when investors have short horizons and are asymmetrically informed. Our results show that information

[14] We refer readers to Beyer et al. (2010) for a survey of the findings in the empirical disclosure literature.

asymmetry unambiguously decreases price informativeness, and that the effect of public information is non-monotone. A related issue that we now turn to is the effect of information asymmetry and public information on the discount investors demand to hold the risky security. We analyze this because prior research has used models similar to ours to analyze the effect of information asymmetry and public information on price discount (e.g., Easley and O'Hara (2004)).[15] Empirical researchers have also examined the relation between accounting disclosure quality and measures of firms' costs of capital (e.g., Francis et al. (2005)). Following the literature, we define the price discount in period t as $E(\theta - p_t)$. To illustrate the main intuition, Propositions 4 and 5 below establish the effects of information asymmetry and public information on price discounts in our base model with a single asset. We postpone a discussion for the case of multiple risky assets until the next subsection.

We first analyze the impact of information asymmetry on price discounts in both periods.

Proposition 4 If $F(\beta_i)$ is a mean-preserving spread of $G(\beta_i)$ and there exists a unique linear equilibrium under $G(\beta_i)$, then the price discount in each period under F is higher than under G.

Proof of Proposition 4 (See the appendix for details.)

As in a standard noisy rational expectations model, price discounts arise in our setting to induce risk averse investors to hold the risky asset. The more information investors have regarding their final payoff, the less uncertainty they face, and the lower the price discount they demand. Second generation investors rely on the first period price as a source of information regarding the risky asset's liquidation payoff. When price is less informative because of information asymmetry, they face more residual uncertainty about their payoff, thus demanding a higher discount. This implies that information asymmetry in the first period increases price discount in the second period without directly affecting the second period price informativeness (see Lemma 1).

More information asymmetry also contributes to a higher price discount in the first period.

[15] Bloomfield and Fischer (2012) analyze the effect of disagreement in an overlapping generation model with infinite horizon.

Decompose the first period discount as

$$E(\theta - p_1) = E(\theta - p_2) + E(p_2 - p_1).$$

As first generation investors anticipate that they would have to sell the asset to second generation investors at a discount, the second period price discount (which is increasing in the degree of information asymmetry) is carried forward into the first period price, giving rise to the first term in the right hand side of the expression above. The second term is the expected discount from the second period price, which is the final payoff for first generation investors. More information asymmetry reduces the first period price informativeness regarding θ. Since p_2 is a function of θ, more information asymmetry in turn translates into more uncertainty regarding p_2 and consequently a higher discount from p_2. Thus, both terms in $E(\theta - p_1)$ increase with more information asymmetry.

Lambert, Leuz, and Verrecchia (2012) show that in models with perfect competition the price discount depends on the average precision of all investors' information, which includes private information (β_i), public information (α), and prices (ρ_t). In Lambert et al. (2012), holding the average private information precision ($\overline{\beta}$) constant, the distribution of private information precision among investors does not affect price informativeness (ρ_t) and hence does not change the price discount. Our results complement Lambert et al. (2012)'s message in that the discount still depends on the average precision of all information. However, with short horizon, more information asymmetry decreases the first period price informativeness and reduces the average information precision for both generations of investors, increasing the discounts in both periods.

Next, we discuss the effect of public information on price discounts.

Proposition 5 Let Λ be an interval of α such that the equilibrium is unique and stable for all $\alpha \in \Lambda$. The price discount in each period decreases with α for all $\alpha \in \Lambda$.

Proof of Proposition 5 (See the appendix for details.)

Increasing the public information precision α has two effects on price discounts. The first is a direct effect in that more accurate public information directly reduces investors' uncertainty about the liquidation value of the risky asset. This effect is the driving force behind standard one-period models (with or without information asymmetry, e.g., Lambert et al. (2012)) and two-period models with no information asymmetry (Gao (2008)). When both information asymmetry and short horizon are present, public information has a second indirect effect via its effect on the first period price informativeness. While the indirect effect is ambiguous in general as shown in Proposition 2, Proposition 5 shows that in any stable unique equilibrium the direct effect dominates the indirect effect so that increasing the precision of public information unambiguously decreases price discounts on the margin, consistent with the findings in Bhattacharya et al. (2012).

4.2 Multiple risky assets

Our analyses up to this point are restricted to a single risky asset. In this section, we extend our analyses to a multiple-asset setting. In order to maintain a comparable information structure to the base model (i.e., investors possess diverse private information with differential precisions), we study a setting similar to that used in Admati (1985) and Lambert et al. (2007).[16] Specifically, we consider an economy with N risky assets and 1 risk-free asset. The risky assets' terminal payoff vector Θ has a common prior that is normal with an $N \times 1$ mean vector $\bar{\Theta}$ and an $N \times N$ symmetric positive definite precision matrix A. The noisy supply of shares in period t is S_t and is for simplicity assumed normal with mean $\bar{S}_t$ and precision $\Gamma_t I$, where $\bar{S}_t$ is a constant $N \times 1$ vector, Γ_t a scalar, and I an $N \times N$ identity matrix. Each investor ti observes a private information signal vector X_{ti} prior to trading:

$$X_{ti} = \Theta + \Xi_{ti}, \text{ where}$$
$$\Xi_{ti} \sim N\left(0, \frac{1}{\beta_i} I\right).$$

β_i is a scalar, representing the precision of private information. We continue to use $\bar{\beta}$ to denote the average β_i across all investors, i.e., $\bar{\beta} = \int_{\beta_i} \beta_i dG(\beta_i)$, where $G(\beta_i)$ is the c.d.f. of β_i. Before trading starts, a piece of public information Z is released and takes the following form:

$$Z = \Theta + \Lambda, \text{ where}$$

[16] Hughes et al. (2007) study a multi-asset setting using a linear factor pricing structure. In their model, there is only one piece of (private) information commonly observed by a subset of informed traders. They note that this is a key assumption that facilitates a tractable solution.

$$\Lambda \sim N\left(0, \Psi^{-1}\right).$$

We assume that Ψ is a positive definite diagonal matrix, representing the precision of public information. This is without loss of generality, as we place no restriction on the off-diagonal elements of the precision matrix of the common prior (A). As in the single asset case, we assume Θ, Ξ_{ti} $(\forall t, i)$, Λ, and S_t $(\forall t)$ are independent from each other for tractability.

As before, we focus on linear equilibria and conjecture a linear price for each of the two trading periods as

$$\begin{aligned} P_1 &= J_1\bar{\Theta} + H_1 Z + C_1\Theta - D_1\left(S_1 - \bar{S}_1\right) - E_1\bar{S}_1 - F_1\bar{S}_2; \\ \text{and } P_2 &= G_2 P_1 + J_2\bar{\Theta} + H_2 Z + C_2\Theta - D_2\left(S_2 - \bar{S}_2\right) - E_2\bar{S}_1 - F_2\bar{S}_2. \end{aligned}$$

Assuming $C_t's$ are invertible (which will be the case in equilibrium), prices can be rewritten as

$$\begin{aligned} P_1^* &\equiv C_1^{-1}\left(P_1 - J_1\bar{\Theta} - H_1 Z + E_1\bar{S}_1 + F_1\bar{S}_2\right) = \Theta - C_1^{-1}D_1\left(S_1 - \bar{S}_1\right); \\ P_2^* &\equiv C_2^{-1}\left(P_2 - G_2 P_1 - J_2\bar{\Theta} - H_2 Z + E_2\bar{S}_1 + F_2\bar{S}_2\right) = \Theta - C_2^{-1}D_2\left(S_2 - \bar{S}_2\right). \end{aligned}$$

Thus, we can define price informativeness in period t as

$$\Phi_t \equiv \Gamma_t C_t' D_t'^{-1} D_t^{-1} C_t.$$

The following proposition characterizes the equilibrium first period price informativeness with multiple risky assets. As the proposition can be established using a similar logic to Proposition 1, its proof is omitted and available upon request.

Proposition 6

- (i) If Θ were realized at the end of the first period (i.e., first generation investors have long investment horizons), there would exist a unique linear equilibrium where the first period price informativeness is given by $\Phi_1^{LH} = \Gamma_1\left(\overline{\beta}\tau_1\right)^2 I$ where the superscript LH stands for long horizon.
- (ii) When investors have homogenous information precisions (i.e., $\beta_i = \overline{\beta}$, for all i), there exists a unique linear equilibrium where the first period equilibrium price informativeness

Φ_1 is given by

$$\Phi_1^{sym} = \Phi_1^{LH} \left[\Phi_2 \left(\overline{\beta} I + \Phi_2\right)^{-1}\right]' \left[\Phi_2 \left(\overline{\beta} I + \Phi_2\right)^{-1}\right], \text{ where}$$
$$\Phi_2 = \Gamma_2 \left(\overline{\beta}\tau_2\right)^2 I.$$

- (iii) When investors have heterogeneous information precisions, the first period equilibrium price informativeness is (implicitly) determined by

$$\begin{aligned} \Phi_1^{asym} &= \Gamma_1 \left(D_1^{-1} C_1\right)' \left(D_1^{-1} C_1\right), \text{ where} \qquad (18) \\ D_1^{-1} C_1 &= \tau_1 \int_{\beta_i} \left[C_2'^{-1} \Phi_2 \left(A + \Psi + \beta_i I + \Phi_1^{asym} + \Phi_2\right)^{-1} \beta_i\right] dG(\beta_i), \\ \text{and } C_2 &= \left(A + \Psi + \overline{\beta} I + \Phi_1^{asym} + \Phi_2\right)^{-1} \left(\overline{\beta} I + \Phi_2\right). \end{aligned}$$

Proposition 6 bears a close resemblance to its single-asset counterpart. Specifically, note that the public information precision matrix (Ψ) appears only in Part (iii) of the proposition. This implies that a key insight from our single-asset analyses continues to hold *regardless of the number of risky assets*: for public information to affect price informativeness and equilibrium uniqueness, it is necessary to have both short horizons and information asymmetry.

Since (18) does not allow a closed-form solution, we rely on numerical examples to evaluate the effects of public information. All of the numerical examples that follow assume two risky assets with correlated returns.[17] We assume a binary distribution of β_i where $\Pr(\beta_i = \beta_h) = 1 - Q$ and $\Pr(\beta_i = \beta_l) = Q$. In all our examples, we keep $\overline{\beta}$ and β_l constant and vary Q and β_h. Because a mean preserving spread in β_i is equivalent to an increase in Q, Q captures the degree of information asymmetry.

In order to focus on prices' ability to reflect an *individual* asset's terminal payoff, we define the first period price informativeness for asset $j = 1$ or 2 as $\frac{1}{Var[\theta_j|P_1]}$. That is, price informativeness for asset j is the inverse of the residual uncertainty regarding j's terminal payoff conditional on just the price vector P_1 (and the prior). One can easily show that $Var[\theta_j|P_1]$ is simply the jjth

[17] The two-asset case is without loss of generality, as we can think of the second asset as the rest of the market portfolio. If the correlation between the two assets is zero, then we are back to the single asset case. The effects plotted are robust to various specifications on the sign or magnitude of the correlation.

element of the matrix $(A+\Phi_1)^{-1}$. It is worth emphasizing that $Var\left[\theta_j|P_1\right]$ is not conditional on public information. Thus, it does not reflect any direct effect of public information in reducing uncertainty and therefore is the multi-asset counterpart to ρ_1 in the single asset case.

Figure 2 assumes $\Psi = \alpha I$ where α is a positive scalar and I is a 2×2 identity matrix and studies the effect of varying α. This formulation captures situations where changes in public information are "uniform" in the sense that they affect both assets to the same degree.[18] In each panel of Figure 2, we present three cases representing different degrees of information asymmetry. In Figure 2A, the parameter values are chosen such that the equilibrium is always unique and stable. The bottom horizontal line in Panel A shows that when there is no information asymmetry, public information does not affect price informativeness. The middle solid line shows that public information monotonically decreases residual uncertainty (i.e., increases price informativeness) when the degree of information asymmetry is not very severe. However, the top dash-dot line shows that when information asymmetry is severe, the relation between public information and residual uncertainty is non-monotone: more precise public information increases residual uncertainty (i.e., decreases price informativeness) first and decreases residual uncertainty only when it's higher than some threshold value. Panels B-C show that more public information monotonically decreases price discounts in both periods, regardless of the degree of information asymmetry.

Figure 2B is similar to Figure 2A except that it focuses on cases where multiple equilibria are possible. To highlight the effect of public information on equilibrium uniqueness, we assume that whenever there are multiple equilibria, investors end up in the least informative one.[19] Parameter values are chosen such that all equilibria in Figure 2B are stable. Panel A of Figure 2B shows a discontinuity effect of public information on price informativeness. The discontinuity takes place when public information becomes precise enough that multiple equilibria are no longer sustained. The threshold α at which multiple equilibria are eliminated (the discontinuity point) is increasing in the degree of information asymmetry, suggesting that with more information asymmetry among investors, public information needs to be more precise to eliminate multiple equilibria. Similarly,

[18] We obtain similar (unreported) results when the change in public information is specific to asset 1 only (i.e. we vary only the public information precision for asset 1 while holding that for asset 2 constant).

[19] The plots would be similar to those in Figure 2A if the most informative equilibrium was chosen at all times.

Panels B and C show that the effect of public information on price discounts is also discontinuous. There exists a threshold point where a small increase in public information can result in significant drop in firms' discounts. This is expected, given that both price informativeness and public information help reduce price discounts, and given the discontinuous effect of public information on price informativeness.

That public information affects price discount with multiple assets is consistent with Lambert, Leuz and Verrecchia (2007) who show public disclosures of individual asset's terminal payoff can have a non-diversifiable cost of capital effect. The idea is that such disclosures can affect the covariance of the asset's return with the rest of the market portfolio. Since both Lambert et al. (2007) and our multi-asset setting follow the setup in Admati (1985), the effect we show follows the same intuition.

Hughes et al. (2007) argue that information about firm-specific, idiosyncratic risks can be diversified away in a large economy. Their analysis suggests that firm-specific public information may not have any pricing effect. It is worth noting that Hughes et al. (2007) reach their conclusion in a different setting than those used in Admati's (1985), Lambert et al. (2007) and ours. As such, their conclusion does not carry over to our setup.

The main difference is that Hughes et al. (2007) assume a linear factor model, where each asset's expected terminal payoff is a linear function of exogenous systematic risk factors, with the coefficients on the systematic risk factors (i.e., factor loadings) exogenously given and known. The uncertainty in the aggregate economy is only about the systematic risk factors. The factor structure model enables Hughes et al. (2007) to show that uncertainty about idiosyncratic risks has no pricing impact in a large economy. In contrast, in our setting assets' terminal payoffs follow a variance-covariance structure. Individual assets' returns are determined by their correlations with the market portfolio (i.e., factor loadings) and the market portfolio return (i.e., systematic risk). Both the correlations and the market portfolio return are endogenously determined in the model and are functions of the entire information structure of the economy. As Lambert et al. (2007) point out, while the variance risk of a firm-specific public information may be diversified away, the covariance risk is not. In this setting, the difference between systematic information and firm-specific information is not well defined because any changes to the public information matrix would lead to simultaneous changes in the market portfolio return and its correlation with individual assets' returns.

These modeling differences and their implications have been noted both by Admati (1985) and re-emphasized in Hughes et al. (2007) and Lambert et al. (2007). Whether firm-specific information is priced, and if so, through what mechanism is ultimately an empirical question. Our analysis, however, shows that within the framework where it is priced, the pricing effect is also a function of investors' investment horizons and information asymmetry.

5 Conclusion

The paper analyzes the effects of public information in a perfect competition trading model populated by asymmetrically informed short-horizon investors who have different levels of private information precision. We first show that information asymmetry reduces price informativeness and can lead to multiple linear equilibria, where price informativeness refers to the amount of private information revealed by price in equilibrium. We then demonstrate that the presence of both information asymmetry and short horizon provides a channel through which public information influences price informativeness and equilibrium uniqueness. Specifically, public information improves price informativeness only when it is of high quality. When the quality of public information is low, multiple equilibria can arise and increasing public information quality can reduce price informativeness.

A potentially fruitful extension of our paper is to take a closer look at short horizon investors' information endowment. In our base model, first generation investors are constrained to receive private information that is only informative about the risky asset's terminal payoff θ. Future research could endogenize these investors' information acquisition decisions. Conceivably, because short horizon investors have to liquidate their holdings before θ is realized, they have additional incentives to collect information regarding the second period supply noise which affects the second period price. A recent paper by Manzano and Vives (2011) shows that allowing investors to collect information regarding supply noise in a single period setting could lead to multiple linear equilibria. Future research may explore consequences of information collection regarding future noisy supply in a dynamic setting such as ours.

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6 Appendix

Proof of Proposition 1

Substitute (10) and (11) into (1), and apply the market clearing condition of $\int_{\beta_i} D_{1i} dG(\beta_i) = s_1$, we have

$$\int_{\beta_i} \frac{E_{1i}(p_2) - p_1}{Var_{1i}(p_2)} dG(\beta_i) - \frac{s_1}{\tau_1} = 0$$

$$\begin{aligned}
&(a_2 - 1) p_1 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) + b_2 z \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) \\
&+ c_2 \int_{\beta_i} \frac{E_{1i}(\theta)}{Var_{1i}(\theta)} \frac{Var_{1i}(\theta)}{Var_{1i}(p_2)} dG(\beta_i) - e_2 \bar{s}_1 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) \\
&- f_2 \bar{s}_2 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) - \frac{s_1}{\tau_1} = 0
\end{aligned}$$

Since $E_{1i}(\theta) = \frac{\alpha z + \rho_1 P_1^* + \beta_i x_{1i}}{\alpha + \rho_1 + \beta_i}$ and $Var_{1i}(\theta) = \frac{1}{\alpha + \rho_1 + \beta_i}$, we have

$$\begin{aligned}
&(a_2 - 1) p_1 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) + b_2 z \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) \\
&+ c_2 \int_{\beta_i} (\alpha z + \rho_1 P_1^* + \beta_i x_{1i}) \frac{Var_{1i}(\theta)}{Var_{1i}(p_2)} dG(\beta_i) - e_2 \bar{s}_1 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) \\
&- f_2 \bar{s}_2 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) - \frac{s_1}{\tau_1} = 0
\end{aligned}$$

Substitute in $P_1^* \equiv \frac{p_1 - b_1 z + e_1 \bar{s}_1 + f_1 \bar{s}_2}{c_1}$ and collect terms. We have

$$\begin{aligned}
&(a_2 - 1) p_1 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) + b_2 z \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) + c_2 \alpha z \int_{\beta_i} \frac{Var_{1i}(\theta)}{Var_{1i}(p_2)} dG(\beta_i) \\
&+ c_2 \rho_1 \frac{p_1 - b_1 z + e_1 \bar{s}_1 + f_1 \bar{s}_2}{c_1} \int_{\beta_i} \frac{Var_{1i}(\theta)}{Var_{1i}(p_2)} dG(\beta_i) + c_2 \left(\int_{\beta_i} \beta_i \frac{Var_{1i}(\theta)}{Var_{1i}(p_2)} dG(\beta_i) \right) \theta \\
&- e_2 \bar{s}_1 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) - f_2 \bar{s}_2 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG(\beta_i) - \frac{s_1}{\tau_1} = 0
\end{aligned}$$

and therefore

$$p_1 = k_2 \left[\begin{array}{c} \left(b_2 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG\left(\beta_i\right) + c_2 (a - \rho_1 \frac{b_1}{c_1}) \int_{\beta_i} \frac{Var_{1i}(\theta)}{Var_{1i}(p_2)} dG\left(\beta_i\right) \right) z \\ +c_2 \left(\int_{\beta_i} \beta_i \frac{Var_{1i}(\theta)}{Var_{1i}(p_2)} dG\left(\beta_i\right) \right) \theta + \left(\begin{array}{c} \frac{c_2 \rho_1 e_1}{c_1} \int_{\beta_i} \frac{Var_{1i}(\theta)}{Var_{1i}(p_2)} dG\left(\beta_i\right) \\ -e_2 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG\left(\beta_i\right) \end{array} \right) \bar{s}_1 \\ + \left(\frac{c_2 \rho_1 f_1}{c_1} \int_{\beta_i} \frac{Var_{1i}(\theta)}{Var_{1i}(p_2)} dG\left(\beta_i\right) - f_2 \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG\left(\beta_i\right) \right) \bar{s}_2 - \frac{s_1}{\tau_1} \end{array} \right] \quad (19)$$

$$\text{where } k_2 = \frac{1}{(1-a_2) \int_{\beta_i} \frac{1}{Var_{1i}(p_2)} dG\left(\beta_i\right) - \frac{c_2 \rho_1}{c_1} \int_{\beta_i} \frac{Var_{1i}(\theta)}{Var_{1i}(p_2)} dG\left(\beta_i\right)}.$$

Thus, the informativeness of p_1 is given by

$$\begin{aligned} \rho_1 &= \left(\frac{c_1}{d_1}\right)^2 \gamma_1 = \gamma_1 \tau_1^2 \left(c_2 \int_{\beta_i} \beta_i \frac{Var_{1i}(\theta)}{Var_{1i}(p_2)} dG\left(\beta_i\right) \right)^2 \\ &= \gamma_1 \tau_1^2 \left(\int_{\beta_i} \beta_i \frac{c_2 \frac{1}{\alpha+\rho_1+\beta_i}}{\frac{c_2^2}{\rho_2} \left(\frac{\rho_2}{\alpha+\rho_1+\beta_i} + 1 \right)} dG\left(\beta_i\right) \right)^2 \\ &= \gamma_1 \tau_1^2 \left(\int_{\beta_i} \frac{\rho_2 \beta_i}{c_2 \left(\alpha + \rho_1 + \rho_2 + \beta_i\right)} dG\left(\beta_i\right) \right)^2 \\ &= \gamma_1 \tau_1^2 \left(\frac{\rho_2}{c_2}\right)^2 \left(\int_{\beta_i} r\left(\beta_i\right) dG\left(\beta_i\right) \right)^2 = \gamma_1 \tau_1^2 \left(\frac{\rho_2 \bar{\beta}}{\rho_2 + \bar{\beta}} \frac{E\left[r\left(\beta_i\right)\right]}{r\left(\bar{\beta}\right)} \right)^2, \end{aligned}$$

where the final step follows after substituting c_2 from (13). For completeness, one can solve the pricing functions by setting up a system of equations where coefficient in (3) are equal to those in (19). These coefficients are

$$\begin{aligned} b_1 &= \frac{\alpha}{\alpha + \rho_1 + \rho_2 + \bar{\beta}} + \left[\int_{\beta_i} \frac{\frac{\alpha c_2}{\alpha+\rho_1+\beta_i}}{\frac{1}{\alpha+\rho_1+\beta_i} + \frac{1}{\rho_2}} dG\left(\beta_i\right) \right] / \left[\int_{\beta_i} \frac{1}{\frac{1}{\alpha+\rho_1+\beta_i} + \frac{1}{\rho_2}} dG\left(\beta_i\right) \right]; \\ c_1 &= 1 - b_1; d_1 = c_1 / \left[\tau_1 \bar{\beta} \left(\frac{\rho_2}{\rho_2 + \bar{\beta}} \right) \left(\frac{E_{\beta_i}\left[r\left(\beta_i\right)\right]}{r\left(\bar{\beta}\right)} \right) \right]; \\ e_1 &= \frac{c_2^2}{\tau_1 \left[\int_{\beta_i} \frac{1}{\frac{1}{\alpha+\rho_1+\beta_i} + \frac{1}{\rho_2}} dG\left(\beta_i\right) \right]}; f_1 = \frac{1}{\tau_2 \left(\alpha + \bar{\beta} + \rho_1 + \rho_2\right)}. \end{aligned}$$

where the coefficients for p_2 are given in Lemma 1. Clearly, for every solution of ρ_1, there is a unique set of equilibrium prices. Q.E.D.

Proof of Proposition 2

- Part (i) is immediate by inspecting (15).

- For part (ii), notice that $\frac{\partial R(\rho_1)}{\partial \alpha} = \frac{\partial R(\rho_1)}{\partial \rho_1}$. Apply implicit function theorem to the equilibrium condition for ρ_1^{asym}, we have

$$\frac{d\rho_1^{asym}}{d\alpha} = -\frac{-\frac{\partial R(\rho_1)}{\partial \alpha}}{1-\frac{\partial R(\rho_1)}{\partial \rho_1}} = \frac{\frac{\partial R(\rho_1)}{\partial \rho_1}}{1-\frac{\partial R(\rho_1)}{\partial \rho_1}},$$

where

$$\begin{aligned}\frac{\partial R(\rho_1)}{\partial \rho_1} \equiv\ & 2\gamma_1 \tau_1^2 \overline{\beta} \left(\frac{\rho_2}{\rho_2+\overline{\beta}}\right)^2 \left[\int_{\beta_i} \frac{\beta_i \left(\alpha+\rho_1+\rho_2+\overline{\beta}\right)}{\overline{\beta}\left(\alpha+\rho_1+\rho_2+\beta_i\right)} dG(\beta_i)\right] * \\ & \left[\int_{\beta_i} \frac{\left(\beta_i-\overline{\beta}\right)\beta_i}{\left(\alpha+\rho_1+\rho_2+\beta_i\right)^2} dG(\beta_i)\right].\end{aligned}$$

Thus, $\frac{d\rho_1^{asym}}{d\alpha} > 0$ iff $\frac{\partial R(\rho_1)}{\partial \rho_1} \in (0,1)$. Given the integrand in the first bracket is always non-negative, the sign of $\frac{\partial R(\rho_1)}{\partial \rho_1}$ depends on the sign of the second bracket. Because $\frac{\left(\beta_i-\overline{\beta}\right)\beta_i}{\left(\alpha+\rho_1+\rho_2+\beta_i\right)^2}$ evaluated at $\beta_i = \bar{\beta}$ is zero, the Jensen's Inequality implies that $\frac{\partial R(\rho_1)}{\partial \rho_1} > 0$ if $\frac{\left(\beta_i-\overline{\beta}\right)\beta_i}{\left(\alpha+\rho_1+\rho_2+\beta_i\right)^2}$ is convex in β_i. Straight forward algebra shows that $\frac{\partial^2 \frac{\left(\beta_i-\overline{\beta}\right)\beta_i}{\left(\alpha+\rho_1+\rho_2+\beta_i\right)^2}}{\partial \beta_i^2}$ has the same sign as

$$K_i \equiv (\alpha+\rho_1+\rho_2)(\alpha+\rho_1+\rho_2-2\beta_i)+\bar{\beta}\left[2(\alpha+\rho_1+\rho_2)-\beta_i\right] \tag{20}$$

Thus, a sufficient condition for $\frac{\left(\beta_i-\overline{\beta}\right)\beta_i}{\left(\alpha+\rho_1+\rho_2+\beta_i\right)^2}$ to be convex in β_i is $(\alpha+\rho_2) > 2\beta_{\max}$. When $\alpha+\rho_2 > 2\beta_{\max}$, holding other exogenous parameters constant, we have

$$\begin{aligned} 0 \ &\leq \ \frac{\beta_i\left(\alpha+\rho_1+\rho_2+\overline{\beta}\right)}{\overline{\beta}\left(\alpha+\rho_1+\rho_2+\beta_i\right)} \leq \frac{\beta_{\max}\left(\alpha+\rho_1+\rho_2+\overline{\beta}\right)}{\overline{\beta}\left(\alpha+\rho_1+\rho_2\right)} < \frac{\beta_{\max}\left(\alpha+\rho_2+\overline{\beta}\right)}{\overline{\beta}\left(\alpha+\rho_2\right)} \\ &< \ \frac{\beta_{\max}}{\bar{\beta}}+\frac{1}{2} = \frac{1}{2\overline{\beta}}\left(2\beta_{\max}+\bar{\beta}\right)\end{aligned}$$

and

$$0 < \int_{\beta_i} \frac{\left(\beta_i-\overline{\beta}\right)\beta_i}{\left(\alpha+\rho_1+\rho_2+\beta_i\right)^2} dG(\beta_i) < \frac{\beta_{\max}-\overline{\beta}}{2\left(\alpha+\rho_1+\rho_2\right)} < \frac{\beta_{\max}-\overline{\beta}}{2\alpha}.$$

Substitute these bounds into $\frac{\partial R(\rho_1)}{\partial \rho_1}$, we have

$$\frac{\partial R(\rho_1)}{\partial \rho_1} < \frac{\gamma_1 \tau_1^2 \overline{\beta}\left(\beta_{\max}^2-\bar{\beta}^2\right)}{2\alpha}$$

Thus, as long as α is large enough, $\frac{\partial R(\rho_1)}{\partial \rho_1}$ will be less than 1, and $\frac{d\rho_1^{asym}}{d\alpha} > 0$ follows. Finally, $\frac{\partial R(\rho_1)}{\partial \rho_1} \in (0, 1)$ also implies that the equilibrium is stable.

- For Part (iii), note $\frac{d\rho_1}{d\alpha} = \frac{\frac{\partial R(\rho_1)}{\partial \rho_1}}{1-\frac{\partial R(\rho_1)}{\partial \rho_1}} < 0$ if $\frac{\partial R(\rho_1)}{\partial \rho_1} < 0$, which is guaranteed if $\frac{(\beta_i - \bar{\beta})\beta_i}{(\alpha+\rho_1+\rho_2+\beta_i)^2}$ is concave in β_i. Note that (20) is negative if

$$(\alpha + \rho_1 + \rho_2)^2 - 2\left(\beta_{\min} - \bar{\beta}\right)(\alpha + \rho_1 + \rho_2) - \bar{\beta}\beta_{\min} < 0.$$

Thus, when

$$\alpha + \rho_1 + \rho_2 \in [0, \beta_{\min} - \bar{\beta} + \sqrt{\left(\bar{\beta} - \beta_{\min}\right)^2 + \bar{\beta}\beta_{\min}}),$$

$\frac{(\beta_i - \bar{\beta})\beta_i}{(\alpha+\rho_1+\rho_2+\beta_i)^2}$ is concave in β_i. Next, note

$$\rho_1 = \gamma_1 \overline{\beta}^2 \tau_1^2 \left(\frac{\rho_2}{\rho_2 + \overline{\beta}}\right)^2 \left\{\frac{E_{\beta_i}\left[r\left(\beta_i\right)\right]}{r\left(\overline{\beta}\right)}\right\}^2 < \gamma_1 \overline{\beta}^2 \tau_1^2.$$

Consequently, $\alpha + \rho_1 + \rho_2 \in [0, \beta_{\min} - \bar{\beta} + \sqrt{\left(\bar{\beta} - \beta_{\min}\right)^2 + \bar{\beta}\beta_{\min}})$ if

$$\alpha \in \left[0, \beta_{\min} - \bar{\beta} + \sqrt{\left(\bar{\beta} - \beta_{\min}\right)^2 + \bar{\beta}\beta_{\min}} - \gamma_1 \overline{\beta}^2 \tau_1^2 - \left(\tau_2 \overline{\beta}\right)^2 \gamma_2\right],$$

Clearly, the set of α is not empty when $\gamma_1 \tau_1^2$ and $\gamma_2 \tau_2^2$ are sufficiently small and $\beta_{\min} \neq 0$. Finally, we establish that under these conditions the identified equilibrium is stable, i.e., $\frac{\partial R(\rho_1)}{\partial \rho_1}|_{\rho_1 = \rho_1^{asym}} > -1$. Note

$$0 \leq \frac{\beta_i\left(\alpha + \rho_1 + \rho_2 + \overline{\beta}\right)}{\bar{\beta}\left(\alpha + \rho_1 + \rho_2 + \beta_i\right)} < \frac{\beta_{\max}\left(\alpha + \rho_1 + \rho_2 + \overline{\beta}\right)}{\bar{\beta}\left(\alpha + \rho_1 + \rho_2 + \beta_{\min}\right)} < \frac{\beta_{\max}}{\beta_{\min}},$$

and

$$0 > \int_{\beta_i} \frac{\left(\beta_i - \overline{\beta}\right)\beta_i}{\left(\alpha + \rho_1 + \rho_2 + \beta_i\right)^2} dG\left(\beta_i\right) > \frac{\left(\beta_{\min} - \overline{\beta}\right)\beta_{\max}}{\left(\alpha + \rho_1 + \rho_2 + \beta_i\right)^2} > \frac{\left(\beta_{\min} - \overline{\beta}\right)\beta_{\max}}{\beta_{\min}^2}.$$

Thus,

$$\frac{\partial R\left(\rho_1\right)}{\partial \rho_1} > 2\gamma_1 \tau_1^2 \overline{\beta} \frac{\left(\beta_{\min} - \overline{\beta}\right)\beta_{\max}^2}{\beta_{\min}^3}.$$

Clearly, when $\gamma_1 \tau_1^2$ is sufficiently small, $\frac{\partial R(\rho_1)}{\partial \rho_1} > -1$. Q.E.D.

Proof of Observation 1 Figure A1 plots $R(\rho_1)$ with $\alpha = 0$. Since α and ρ_1 affect $R(\rho_1)$ only through their sum, an increase in α would lead to a leftward shift of $R(\rho_1)$. Hence, as is obvious from the figure, increasing α would make the two least informative equilibria converge toward each other and disappear altogether when α is sufficiently big, while the most informative equilibrium would always exist. Q.E.D.

Proof of Proposition 4

Utilizing the expression for p_1 derived in the proof of Proposition 1, under $G(\beta_i)$ we obtain

$$E(\theta - p_1) = e_1\bar{s}_1 + f_1\bar{s}_2,$$

where

$$\begin{aligned} e_1 &= \frac{c_2^2}{\tau_1\left[\int_{\beta_i} \frac{1}{\frac{1}{\alpha+\rho_1+\beta_i}+\frac{1}{\rho_2}} dG(\beta_i)\right]}; f_1 = \frac{1}{\tau_2\left(\alpha+\bar{\beta}+\rho_1+\rho_2\right)}; \\ c_2 &= \frac{\rho_2+\bar{\beta}}{\alpha+\bar{\beta}+\rho_1+\rho_2}. \end{aligned}$$

Note that

$$\frac{1}{\frac{1}{\alpha+\rho_1+\beta_i}+\frac{1}{\rho_2}} = \frac{(\alpha+\rho_1+\beta_i)\rho_2}{\alpha+\rho_1+\rho_2+\beta_i}$$

which is concave in β_i. Thus, e_1 is higher under F than under G. In addition, Corollary 1 shows that $\rho_1(F) < \rho_1(G)$. This implies that f_1 is also higher under F than under G. Consequently, $E(\theta - p_1)$ is higher under F than under G.

Re-write the equilibrium second period price in Lemma 1 as

$$\begin{aligned} p_2 &= \frac{\rho_1}{\alpha+\rho_1+\rho_2+\bar{\beta}}P_1^* + \frac{\alpha}{\alpha+\rho_1+\rho_2+\bar{\beta}}z + \frac{\rho_2+\bar{\beta}}{\alpha+\rho_1+\rho_2+\bar{\beta}}\theta \\ &\quad -\frac{1}{\bar{\beta}\tau_2}\frac{\rho_2+\bar{\beta}}{\alpha+\rho_1+\rho_2+\bar{\beta}}(s_2-\bar{s}_2) - \frac{1}{\tau_2\left(\alpha+\rho_1+\rho_2+\bar{\beta}\right)}\bar{s}_2. \end{aligned}$$

Thus, the second period price discount

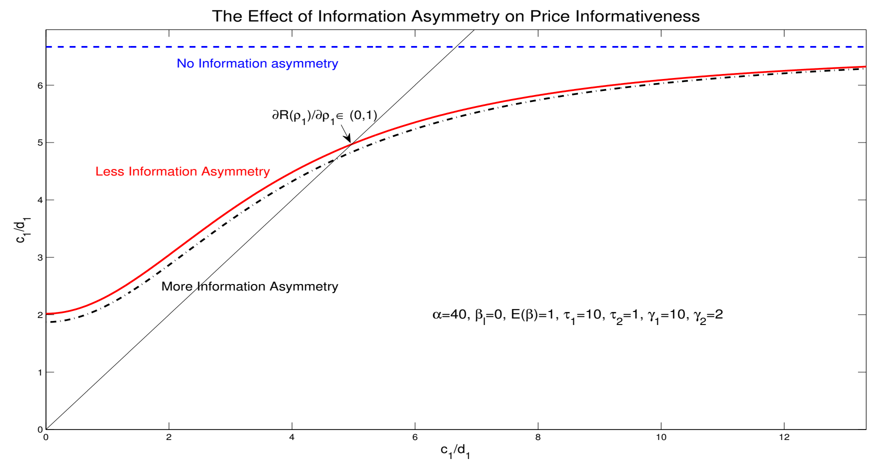
$$E(\theta - p_2) = \frac{1}{\tau_2\left(\alpha+\rho_1+\rho_2+\bar{\beta}\right)}\bar{s}_2.$$

Since $\rho_1(F) < \rho_1(G)$ by Corollary 1, the second period price discount is higher under F than under G. Q.E.D.

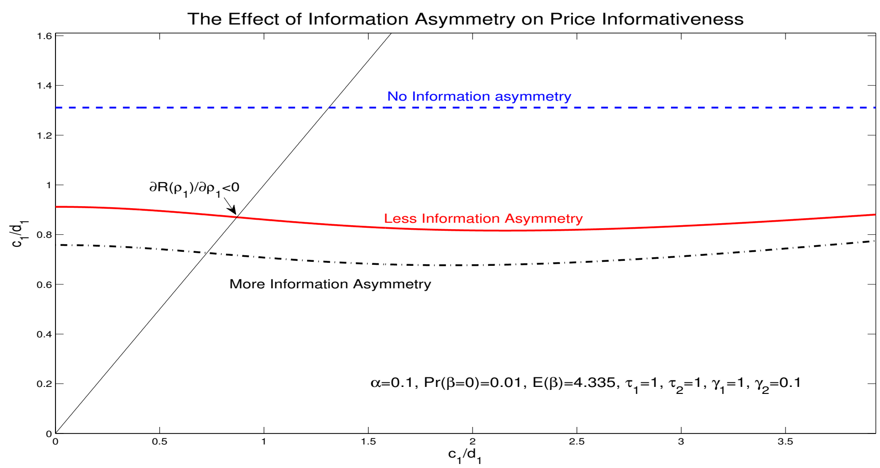
Proof of Proposition 5 By the proof of Proposition 4, the first period price discount is $E(\theta - p_1) = e_1 \bar{s}_1 + f_1 \bar{s}_2$. Notice that α and ρ_1 impact e_1 and e_2 only via their sum $\alpha + \rho_1$. Specifically, the larger $\alpha + \rho_1$, the smaller e_1 and e_2, and the smaller the first period price discount. Thus, $\frac{dE(\theta - p_1)}{d\alpha} < 0$ if and only if $\frac{d\rho_1^{asym}}{d\alpha} > -1$. Since $\frac{d\rho_1}{d\alpha} = \frac{\frac{\partial R(\rho_1)}{\partial \rho_1}}{1 - \frac{\partial R(\rho_1)}{\partial \rho_1}}$ (see the proof for Part (ii) of Proposition 2), $\frac{d\rho_1^{asym}}{d\alpha} > -1$ if and only if $\frac{\partial R(\rho_1)}{\partial \rho_1} < 1$, which is satisfied in any stable unique equilibrium. Q.E.D.

Figure 1: Equilibrium Price Informativeness

Panel A: Unique Equilibrium with $\partial R(\rho_1)/\partial\rho_1 > 0$



Panel B: Unique Equilibrium with $\partial R(\rho_1)/\partial\rho_1 < 0$



Panel C: Multiple Equilibria

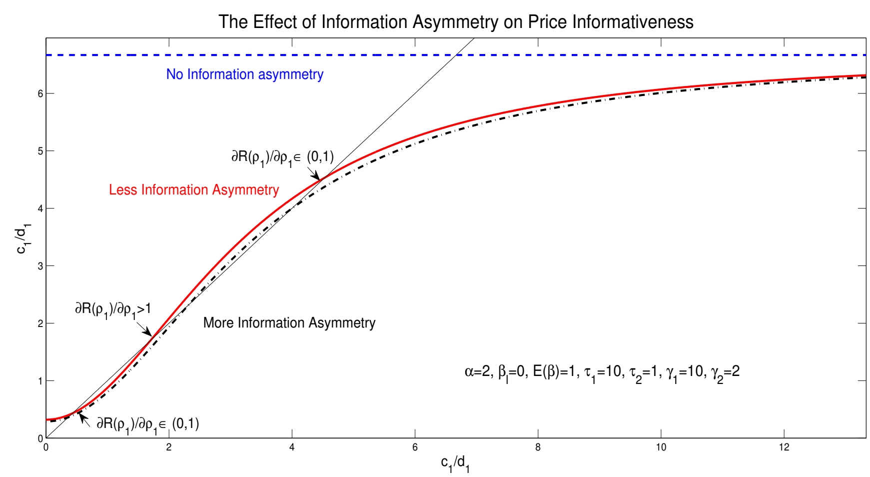


Figure 2: The Effects of Public Information in Multiple Assets Case

Figure 2A: The Effects of Public Information in Unique Equilibrium

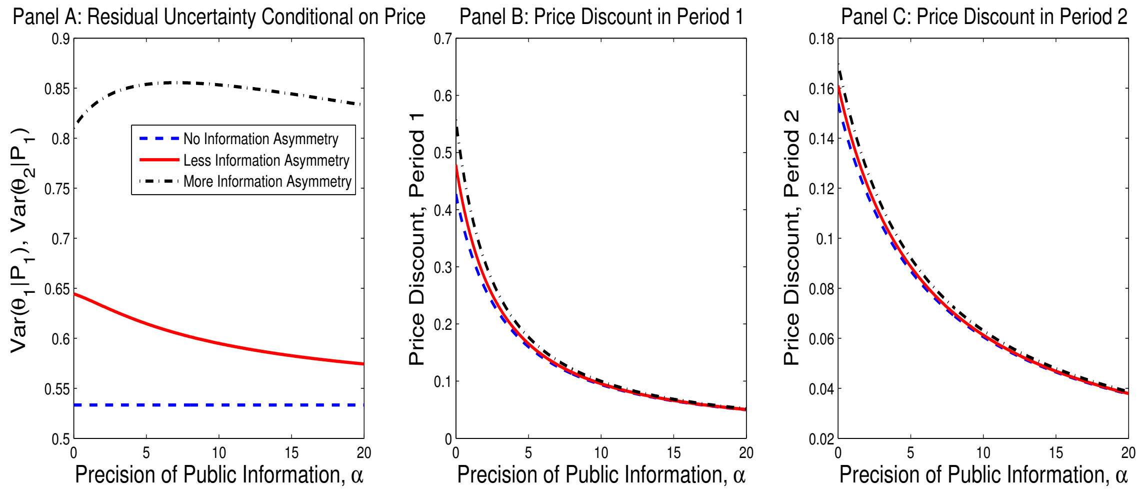


Figure 2B: The Effects of Public Information When Multiple Equilibria Exist

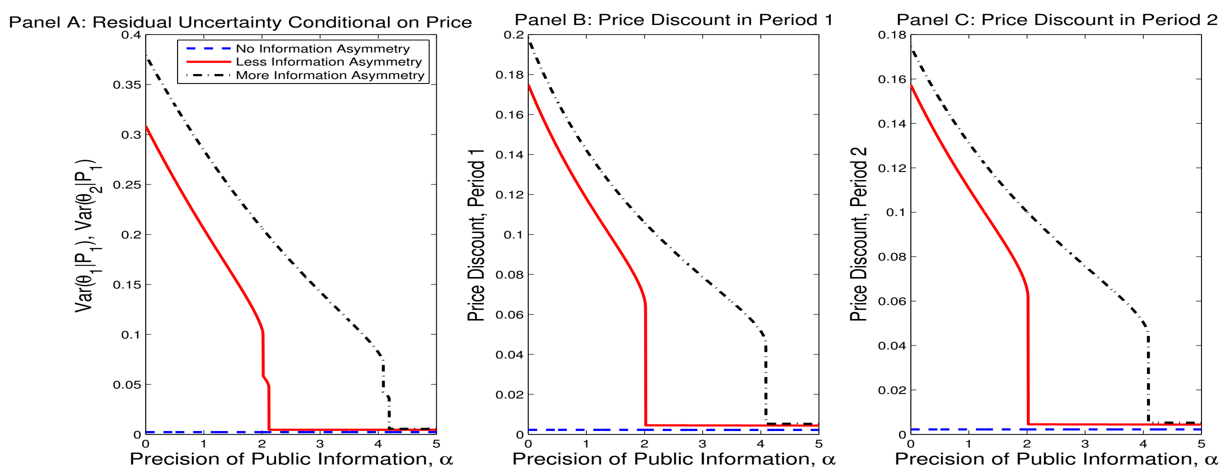


Figure A1: The Effect of Public Information on Equilibrium Uniqueness