Public Disclosure, Liquidity Risk, and the Parallel Banking System*

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Abstract

This paper proposes a theory why both the traditional and the recently emerged shadow banking systems become opaque when they co-exist. The theory endogenizes the disclosure decisions of the two banks when they both face liquidity risk stemming from runs by investors and, at the same time, compete against each other in attracting investment from the same group of investors. The paper first builds a simple model of disclosure in a coordination setting in which a single bank trades off the disclosure's coordination benefit against its cost of induced excess volatility. Then the disclosure model is embedded in an entry-deterrence game in which the entrant shadow bank competes with the incumbent traditional bank. The presence of liquidity risk shifts both the traditional bank's attitudes towards entry and disclosure. Competing with an opaque and fragile shadow bank can benefit the traditional bank through spurring favorable coordination by the investors. In equilibrium, the traditional bank accommodates entry and induces the shadow bank to be both opaque and fragile by making itself opaque.

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1 Introduction

The last three decades have witnessed a profound transformation in the U.S. banking sector. Since the mid-1980s, a shadow banking system has emerged and developed rapidly in parallel to the traditional banking system. Shadow banking is referred to as "the business of borrowing and lending money outside the traditional banking system" (Cox, 2010). As of March, 2008, this shadow system had grown to a gross size of $20 trillion, which is significantly above the total liabilities of the traditional banking system (Pozsar et al., 2010). Despite its massive size and economic significance, the shadow banking system is still known for its opacity (i.e., little public information disclosed) and fragility (i.e., vulnerable to runs and liquidity risk). The interplay between public disclosure and liquidity risk exposure arguably played a critical role in triggering the near collapse of the system itself and in bringing about the 2007-2009 financial crisis (Allen and Carletti, 2008; Plantin, Sapra, and Shin, 2008).

Coinciding with the growth of the shadow banking system in mid-1980s, the traditional banking system has also become increasingly opaque and less transparent (Figure 2, top, Morgan, 2002). The opacity of traditional and shadow banks has drawn considerable attention from regulators and academicians. For instance, Timothy Geithner, the former Secretary of the Treasury, called for "more extensive disclosure, including loan-level data for asset-back securities," in order to "ensure that investors have the information they need to make informed decisions" (Geithner, 2010). In this paper, I examine why the two banking systems become jointly opaque by studying the interaction between their disclosure incentives. More specifically, I focus on the roles of two factors in shaping banks' choices of disclosure transparency: liquidity risk stemming from runs by investors and competition between the two banking systems.

I first examine banks' disclosure incentive in light of the interaction between liquidity risk and public disclosure. In contrast to most other non-banking firms, banks are subject to liquidity risk due to liquidity mismatch between their assets and liabilities (Diamond and Dybvig, 1983). The root cause of liquidity risk is driven by the collective actions of individual investors who fund these banks, such as runs. While a bank's payoff depends critically on the beliefs and actions of individual investors, the bank can sway these beliefs and actions through its public disclosure decision. Public disclosure is extremely effective in coordinating individual beliefs because it is a source of information commonly available to all investors (Morris and Shin, 2002). Through coordinating individual beliefs, public disclosure has a *real* effect in altering individual investment decisions, which in turn determines the liquidity risk borne by the bank. My model examines the disclosure trade-off in the presence of liquidity risk by analyzing the disclosure decision of a single bank whose return to investment is contingent on successful coordination among a group of small investors.

To capture the role of competition, I place the same liquidity-risk-driven disclosure trade-off in an incumbent-entrant competition setting. The entry of the shadow banking system posts significant competitive threats to traditional banks (FCIC, 2011). Prior disclosure literature suggests that firms facing entry threats are reluctant to disclose, fearing this may attract new rivals. This paper revisits the role of competition on disclosure jointly with liquidity risk borne by banks. I find that the presence of liquidity risk shifts both traditional banks' attitudes towards entry of shadow banks and disclosure. In fact, an incumbent traditional bank may find it beneficial to accommodate entry because the presence of a shadow bank that is opaque and fragile spurs favorable coordination on the investments in the traditional bank. In equilibrium, the traditional bank may lower its own disclosure transparency in order to accommodate the entry of the shadow bank. Moreover, when the shadow bank enters, the traditional bank lowers its disclosure transparency further more in order to induce the opacity and fragility of the shadow bank.

Specifically, the paper first analyzes how liquidity risk affects the disclosure policy of a single traditional bank. The traditional bank is endowed with an investment project and raises funds from a group of investors. The return to the investment depends on not only the fundamentals of projects, but also liquidity risk, which is driven by the aggregate investment collectively made by the group of investors. In particular, the return on the traditional bank's investment is lower when the aggregate investment decreases. The traditional bank decides the precision to disclose public information about its fundamentals, which is used jointly with private information by individual investors in making their investment decisions. Summing individual investments gives rise to the total amount of investment made by the traditional bank.[1] As such, the traditional bank effectively uses disclosure to manage the bank-level investment. The disclosure decision in turn coordinates investment decisions at the individual level.

The key message from the analysis is that the presence of the liquidity risk leads to a trade-off for the bank's disclosure decision that is different from the ones in existing literature (Verrecchia, 1983; Dye, 1985). In equilibrium, the bank trades the benefit of spurring favorable coordination with the cost of heightened investment volatility stemming from noises in the disclosure.

On one hand, the benefit of disclosure comes from its role in mitigating coordination inefficiencies among the individual investors. In particular, since the return on investment depends on the aggregate investment by others, an investor needs to coordinate with others in making investments. Because others' strategies are motivated by their own beliefs, the investor must conjecture the beliefs held by other investors, other investors' beliefs about others and even higher-order beliefs. I find that there exist two coordination inefficiencies associated with individual investors' decisions. First, the investors do not internalize the positive spillover of successful coordination that fall on

[1] A key assumption of the paper is that the investment in the traditional bank is not directly controlled by the bank itself but is driven by individual investment decisions. I make this assumption to capture an important feature in the banking industry. Non-banking firms usually have almost perfect control over the size of their investments. Banks, however, do not enjoy the same degree of discretion. The reason is that, after a bank makes its investment, investors at the bank are allowed to withdraw their investments. These withdrawal requests in turn force the bank to liquidate some of its projects and shrink its investment size. It is these withdrawals (runs) that cause the bank to lose the absolute control over its investment.

others. Second, the investors overweight the common prior and underweight their information, relative to the weights under Bayesian updating. The reason is that public information, as a common information source, is more effective in guessing beliefs of others (Morris and Shin, 2002). In total, these coordination inefficiencies make investors less responsive to information, relative to the level preferred by the bank. In this regard, when the traditional bank increases its disclosure precision, the investors respond more promptly to this public information since the disclosure reduces both the fundamental uncertainty about the quality of the project and the strategic uncertainty about others' actions. The increase in the sensitivity to information in turn mitigates the coordination inefficiencies and benefits the traditional bank.

On the other hand, more precise disclosure also hurts the traditional bank because this prompts each investor to rely more on this public information, which has already been overweighted relative to its precision in the higher-order-beliefs context. Investors overreact to public information, which injects magnified noise in this public information into individual investments. At an aggregate level, the public noise is not diversified away as private noises, leading to heightened volatility in the bank-level investment and lowers the traditional bank's payoff.

When two banks compete, the disclosure decision by the traditional bank takes on a competitive role of influencing the shadow bank's decisions, which interacts with its role of affecting liquidity risk. Specifically, built on the analysis for a single traditional bank, I examine a two-bank setting in which a shadow bank decides whether to enter the banking market and compete with the traditional bank for a common group of investors. The analysis shows that the liquidity risk shifts the traditional bank's attitudes towards entry and disclosure. In particular, the coordination inefficiencies among investors create a role for the entry of the shadow bank to benefit the traditional bank. This is because competition between the two banks produces a "multiplier" effect that improves the investors' responsiveness to information. Specifically, from an individual investor's perspective, a favorable signal of the traditional bank indicates not only better fundamentals, but also that others believe the fundamentals to be better. Because of the competition, the investor conjectures that others are more likely to reallocate their investments from the shadow bank to the traditional bank. When the shadow bank is subject to liquidity risk, others' reallocations reduce the shadow bank's return, which in turn induces the investor to make the same reallocating decision, transferring more of her investment from the shadow bank to the traditional bank. Concisely, the competition-driven reallocations make the investors more responsive to information. In equilibrium, the traditional bank trades the costs of increased competition from the shadow bank with the benefit of mitigating the coordination inefficiencies. When the coordination inefficiencies are sufficiently severe, the traditional bank views the shadow bank as a partner in spurring favorable coordination and hence accommodates entry.

In light of the role that competition plays in facilitating coordination, I study the joint disclosure decisions of the traditional and the shadow bank. I find that the traditional bank lowers its disclosure transparency in order to induce entry, different from the conventional view that firms disclose less in order to deter entry. This difference is due to the effect of public disclosure in magnifying the volatility of the shadow bank's investment, which lowers its payoff. In addition, when the shadow bank enters, the traditional bank prefers the shadow bank to disclose less and be more vulnerable to runs. This is because the benefit of the shadow bank in improving coordination is greater when the shadow bank is more opaque and fragile. Through the same real effect of disclosure in altering individual investments, the traditional bank is able to induce the opacity and fragility of the shadow bank by lowering its own disclosure precision further more, leading to a banking sector with an endogenously opaque traditional bank and an endogenously opaque and fragile shadow bank.

It is important to notice that in order to focus on studying the disclosure incentive by banks, I don't consider certain important features of traditional and shadow banking. For example, I do not explicitly model the impact of regulation, which is emphasized in the studies on the role of regulatory arbitrage in boosting shadow banking (Calomiris and Mason, 2004; Acharya, Schnabl, and Suarez, 2013). I also don't examine the optimal contracting between banks and investors (De Marzo and Duffie, 1999). Nor does my model consider the specific structure of securitization and short-term funding (De Marzo, 2005; Brunnermeier, 2009). Despite these limitations, I believe that this paper makes an incremental contribution to understand the opacity of the banking sector by exploring the interplay between coordination and disclosure. My findings suggest that banks have incentives to choose to be opaque voluntarily because of their liquidity risk and competition among them.

1.1 Literature Review

This paper is mostly related to three streams of literatures on public disclosure, higher-order beliefs and shadow banking. The extant literature in accounting and economics has examined extensively the effects of public disclosure. In particular, Goldstein and Sapra (2012) review the extant literature on information disclosure by financial intermediaries. For a review of the related empirical literature, see Beatty and Liao (2014). Some other papers examine the role of public disclosure in the product market competition (Vives, 2006). For example, Bagnoli and Watts (2010) examine the interactions among firms' earnings managment decisions in a Cournot competition environment. They find that competition induces firms to add larger biases to their reports, which leads to lower total production and higher product price. Arya and Mittendorf (2013) illustrate that in the presence of dual distribution channels, retailers may be more willing to disclose favorable information to induce entry by competitors, which improves the supply terms for these retailers. My paper extends this literature of product-market effect of disclosure to a banking context. The banking feature I consider is the liquidity mismatch between banks' assets and liabilities, which helps to provide liquidity to the economy while, at the same time, exposing banks themselves to liquidity risk (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005; Allen and Carletti, 2008; Plantin, Sapra, and Shin, 2008). I find that the interaction between disclosure and liquidity risk creates a competitive role for disclosure that differs from previous literature. Specifically, in my setting, public disclosure is used to coordinate individual actions, which affects the aggregate liquidity risk borne by each bank. This change in the liquidity risk, in return, shifts competing strategies and competitive outcomes.

This paper is related to the literature on higher-order beliefs and global games (Carlsson and Van Damme, 1993; Morris and Shin, 2007). Several studies focus on the effect of public information in games with incomplete information. In a seminal work, Morris and Shin (2002) show that, in a Keynesian beauty contest economy, increasing public disclosure might reduce social welfare by magnifying volatility. In contrast, several other studies argue that more precise public disclosure is welfare-improving (Angeletos and Pavan, 2004, 2007). There are also some studies in the accounting literature that apply the theory of higher-order beliefs and global games. For example, Plantin, Sapra, and Shin (2008) study sales of securitized loans in an illiquid market and show that mark-to-market accounting injects artificial volatility into prices, which distorts real decisions. In addition, Gao (2008) examines the market efficiency of accounting disclosure in a beauty-contest economy and finds that more precise public disclosure always improves market efficiency in spite of its commonality role. More recently, Gigler, Kanodia and Venugopalan (2013) study a setting in which customers are concerned about the firm's total wealth and find that although fair-value accounting provides more precise information, it also magnifies the volatility of the firm's income and wealth. This increase in the volatility in turn distorts the firm's assets allocating decision and makes the firm worse off. Chen, Huang and Zhang (2014) examine effects of public information in a market setting populated by asymmetrically informed short-horizon investors with different levels of private information precision. They show that public information not only influences price informativeness but also equilibrium uniqueness. The study closest to mine is Angeletos and Pavan (2007), which examines the efficient use of information from a social welfare perspective. In particular, they focus on the optimal degree of coordination (i.e., the level of complementarity or substitutability) under which the equilibrium allocation would coincide with the socially efficient allocation. In a similar spirit, I study the use of information in a competitive environment. In my setting, two competing banks determine the degree of coordination and the information structure in order to maximize their respective profits.

This paper is also related to the extensive literature on shadow banking and securitization. Gennaioli, Shleifer and Vishny (2013)'s discussion section provides a comprehensive review for this literature. In particular, some earlier studies argue that the business of shadow banking, by pooling and tranching risky assets, helps generate liquid and safe assets for uninformed investors, which alleviates the adverse selection problems in the banking market (Gorton and Pennacchi, 1990). Several other studies focus on the "regulatory arbitrage" obtained by the business of shadow banking (Calomiris and Mason, 2004; Acharya, Schnabl, and Suarez, 2013). According to these studies, banks conduct off-balance-sheet shadow banking activities in order to circumvent regulatory capital requirements. Some recent studies examine what might have caused the fragility and collapse of the shadow banking system in the 2007-08 crisis. Gennaioli, Shleifer and Vishny (2013), for example, argue that investors' negligence of tail risks contributed to the fragility of the shadow banking system, which would have been stable under rational expectations. Brunnermeier (2009) stresses that the liquidity mismatch between assets and liabilities, combined with runs by short-term investors, might serve as triggers for the meltdown of the shadow banking system (see also Shin, 2009; Gorton and Metrick, 2012). My paper is related to these studies in the sense that

I also examine the fragility and opacity of the shadow banking system. Nevertheless, I depart from the literature by assuming that the shadow banking system has the discretion to choose its level of opacity and vulnerability to liquidity risk, in anticipation of competing with the traditional banking system. Therefore, in my setting, the opaque and fragile nature of shadow banking arises endogenously as a result of the competition.

The rest of the paper is organized as follows. In Section 2, I describe the model setup. Section 3 analyze a model with only the traditional bank. Section 4 presents a model with both the traditional bank the shadow bank. Section 5 concludes the paper.

2 Model Setup

I examine a four-date model in which an incumbent traditional bank, indexed by bank 1, occupies a banking market and a shadow bank, indexed by bank 2, decides whether to enter. Both the banks and the investors are risk neutral. The time line of the model is shown below.

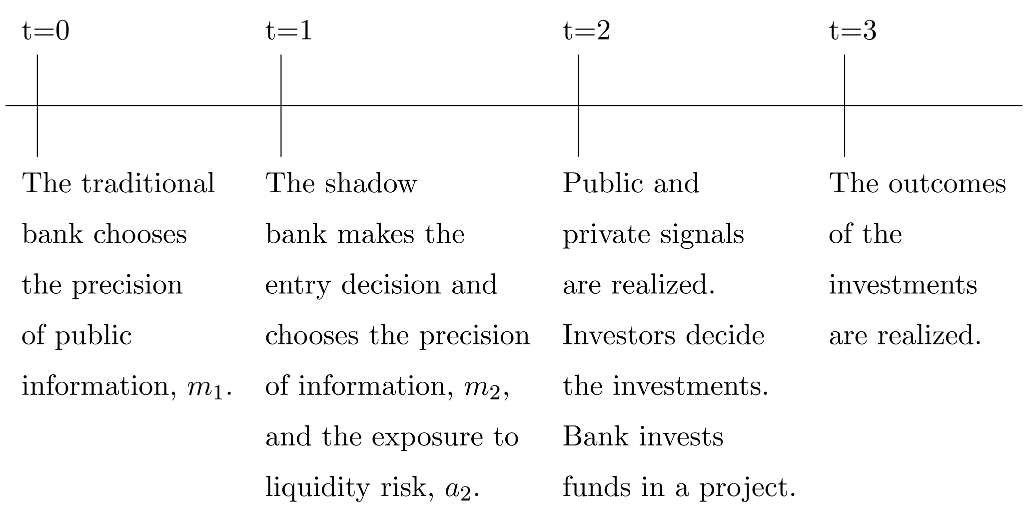

Figure 1: Time line.

I now describe and explain the decisions and events at each date in detail.

Date 0

At date 0, each bank is endowed with an investment project (a loan) that yields a stochastic per-unit return, R_i. The two banks finance the projects by competing for a common group of investors, indexed by the unit interval $[0, 1]$, in Bertrand competition. Each investor j's investment in the traditional bank is denoted as k_{1j}, and investment in the shadow bank is denoted as k_{2j}. I let $K_1 = \int_0^1 k_{1j} dj$ and $K_2 = \int_0^1 k_{2j} dj$ denote the aggregate level of investments in the two banks, respectively.

To account for the liquidity risk borne by the bank, following Angeletos and Pavan (2004) and Plantin, Sapra, and Shin (2008), I assume that the return to the investment, R_i, is linear in an exogenous shock, θ_i, represents the fundamentals of the bank's project, and the aggregate investment, K_i, such that:

$$R_i = \theta_i + a_i K_i, \tag{1}$$

where $a_i \in [0, \frac{1}{2}]$ is publicly observable and interpreted as a measure of the bank's exposure to liquidity risk.[2] The random shock θ_i is normally distributed with mean $\bar{\theta} > 0$ and variance, $\frac{1}{q} > 0$. I assume that θ_1 and θ_2 are independent of each other.[3]

An important assumption of my model is that the investment return depends on the aggregate amount of investment by investors. In particular, the investment return is lower when the aggregate investment decreases. In Appendix I, I show that this linear reduced-form representation of the return can be derived from a model in which banks conduct liquidity transformation and incur liquidity risk. One can think of the decrease in the bank's investment as equivalent to investors'

[2] Assuming $a_i \leq \frac{1}{2}$ is to ensure that the bank's payoff is concave in the investment, K_i. Otherwise, volatility in K_i would be desirable to the bank. This is also sufficient to make the equilibrium unique.

[3] In reality, there are overlaps between the assets held by the two banks and hence their fundamentals are correlated. I assume that the two banks' fundamentals are independent in order to focus on studying the interactions between the two banks through the channel of competing for funding. Furthermore, numerical analysis shows that this assumption doesn't affect my main results qualitatively.

withdrawal of previous investments from the bank.[4] These withdrawals result in liquidity risk for the bank as a result of the liquidity mismatch between the bank's assets and liabilities: when investors withdraw from the bank (decrease their investments), the bank is forced to liquidate a portion of its illiquid project at a loss to meet the withdrawal request, leading to a lower investment return. When $a_i = 0$, the project is infinitely liquid so that the bank can always liquidate a portion of the project without incurring liquidity losses to satisfy withdrawal requests. As a result, the investment return does not depend on the aggregate investment. When $a_i > 0$, the investment return is sensitive to the aggregate investment. The higher the a_i, the more illiquid is the bank's project. As the bank liquidates a portion of the project to meet investors' withdrawal requests, the bank incurs liquidity losses, which lower its investment return. In addition, following Morris and Shin (2002), I assume that investors' prior precision on the fundamentals, q, is sufficiently low such that the investors have little prior knowledge about the fundamentals before receiving private and public information.[5] I also assume that the fundamentals are not observable to either banks or investors until date 3.

At date 0, the traditional bank decides the precision of the public signal of θ_1, denoted by $m_1 > 0$, which the bank will disclose to investors at date 2.[6] Specifically, the public signal, z_1, is equal to:

$$z_1 = \theta_1 + \varepsilon_1, \tag{2}$$

[4]Different from traditional bank runs in which investors withdraw their funds from the bank, modern bank runs often occur as a result of investors' refusal to roll over their lending (repos, commercial papers, etc.) to the bank. In my model, equation (1) can also be used to think about these modern runs. When the investors refuse to roll over some of the investments (decrease their investments in the bank), the bank is forced to shrink its assets portfolio and sell some of its illiquid assets at a price lower than the fair value, leading to an erosion of the bank's net investment return. See Gorton and Mertrick (2012) for a discussion of the runs on repos.

[5]This assumption is made to maintain tractability; however, numerical analysis shows that most of my results hold qualitatively for $q > 0$.

[6]In practice, despite various disclosure rules imposed by regulators, banks can still manipulate the quality of financial information to a large extent. This is because many of these rules for banks are vaguely defined, leaving a substantial amount of discretion in reporting. For example, SFAS 159 grants banks a Fair Value Option, according to which, a reporting entity has a sufficient degree of discretion to choose between historical cost and fair value accounting when measuring its financial assets and liabilities.

where ε_1 is normally distributed, independent of θ_1, with mean 0 and variance $\frac{1}{m_1}$. In choosing m_1, the traditional bank incurs a disclosure cost $c(m_1) = \frac{c_m}{2} m_1^2$ where $c_m > 0$. This cost can be interpreted as the building costs of the information system that gathers data to generate the report, the internal control system that safeguards the faithfulness of the report, etc. The disclosure cost is not essential to my results and employed only to guarantee an interior equilibrium. I assume that the choice of m_1 is committed by the traditional bank and publicly observable by the shadow bank and the investors.[7]

Date 1

At date 1, the shadow bank decides whether to enter the banking market. If the shadow bank chooses not to enter, it earns a payoff, $\overline{U}$. Otherwise, the shadow bank earns the investment proceeds after paying investors. To focus on interesting cases, I assume that $\overline{U}$ is not too large such that entry is not blockaded, that is, the monopoly choice of m_1 by the traditional bank doesn't deter entry. I also assume that $\overline{U}$ is not too small such that deterrence of entry is feasible, that is, there exists some choice of m_1 that deters entry. Upon entering, the shadow bank makes two decisions: the precision of the public information that will be disclosed, m_2, and the exposure to liquidity risk, a_2. I assume that the choices of m_2 and a_2 are publicly observable by investors. Specifically, the shadow bank chooses the precision of the public signal z_2 about the fundamentals θ_2 such that

$$z_2 = \theta_2 + \varepsilon_2, \tag{3}$$

where ε_2 is normally distributed, independent of θ_2, with mean 0 and variance $\frac{1}{m_2}$. In choosing m_2, The shadow bank also incurs a disclosure cost, $c(m_2) = \frac{c_m}{2} m_2^2$, as with the traditional bank.

[7] The disclosure precision chosen by banks depends on the information system installed (Arya, Glover and Sivaramakrishnan, 1997), the internal control procedures established, etc. These systems and procedures, once chosen, cannot be easily adjusted in a short horizon, allowing banks to commit to their disclosure precision. Indeed, empirical studies (Healy, Hutton and Palepu, 1999; Bushee, Matsumoto and Miller, 2003) find that disclosure decisions are often sticky and firms rarely alter their earlier disclosure practices.

In addition, the shadow bank, different from the traditional bank, can freely choose its exposure to the liquidity risk, a_2, while the traditional bank is restrained from choosing its liquidity exposure a_1.[8] I assume this asymmetric structure to capture the incremental regulation on the traditional bank relative to the shadow bank, which I think is an important difference between the two. In reality, traditional banks are under intensive regulatory oversights, such as liquidity reserve requirement, capital requirement, etc., and hence are severely restrained in taking liquidity risk; on the contrary, most prudential regulations do not extend to shadow banks, allowing shadow banks to choose the desired exposure to liquidity risk.[9] In addition, endogenizing the liquidity risk decision by the shadow bank also allows me to analyze a competitive role of the traditional bank's disclosure in influencing the shadow bank's liquidity exposure to its advantage. To attain an interior solution, I assume that the shadow bank incurs a cost of taking liquidity risk $c(a_2) = \frac{c_a}{2}a_2^2$, where $c_a > 0$.[10] This cost can be interpreted broadly in a number of ways. For instance, choosing a higher liquidity exposure increases the risk of financial distress for the shadow bank, which entails additional costs, such as fire-sale costs (Shleifer and Vishny, 2011). In addition, to achieve a higher liquidity mismatch between its assets and the claims issued, the shadow bank needs to involve more liquidity enhancements in its securitization chain in order to transform ("polish") illiquid assets into more liquid claims (Pozsar et al., 2010). Lastly, if the shadow bank takes too much liquidity risk, it will attract more monitoring from regulators and may even trigger regulatory intervention, which

[8] In reality, the actual choice of a_2 may represent a set of long-term decisions that banks make as an integral part of their business plans. For example, a bank's exposure to liquidity risk depends critically on the bank's decisions on its financial structure (whether to rely primarily on short-term funding sources such as repos, asset-backed commercial securities, etc., or a mixture of long-term and short-term funding sources), assets portfolios (whether to hold a portfolio primarily consisted with illiquid mortgages and mortgage-backed securities or to hold a more diversified and liquid one), access to public and private liquidity support, etc. (Pozsar et al., 2010).

[9] Timothy Geithner, the former Secretary of the Treasury, once commented that "a principal cause of the crisis was the failure to provide legal authority to constrain risk in this parallel banking system" (Geithner, 2010). Henry Paulson, the former Secretary of the Treasury also said "(c)ompounding the problems at these financial institutions was a financial regulatory system that was archaic and outmoded" (Paulson, 2010). In fact, this view of the shadow banking system has been shared by many practitioners, government officials as well as academicians (Cox, 2010; Donaldson, 2010).

[10] Appendix I shows that this cost can be motivated from a model in which banks conduct liquidity transformation and incur liquidity risk. The analysis there suggests that a higher liquidity exposure can directly reduce the return to investment and the bank's payoff (the constant term $-a\bar{K}$ in the return function).

leads to regulatory compliance costs. The cost is not essential to my main results and employed to guarantee an interior equilibrium.

Date 2

At date 2, the two banks disclose the public signals, z_1 and z_2, in accordance with their choices of m_1 and m_2. Besides signals released by banks, each investor j also observes a pair of private signals, x_{1j} and x_{2j}:

$$\begin{aligned} x_{1j} &= \theta_1 + \eta_{1j}, \\ x_{2j} &= \theta_2 + \eta_{2j}, \end{aligned} \tag{4}$$

where the noises η_{1j} and η_{2j} are normally distributed with mean zero and variance, $\frac{1}{n_1}$ and $\frac{1}{n_2}$, respectively, and independent everywhere.

If the shadow bank enters, it competes with the traditional bank for investments from a common group of investors. Following Singh and Vives (1984) and Angeletos and Pavan (2004, 2007), I assume that the individual investors incur a quadratic cost of investment, $c_j(k_{1j}, k_{2j}|b)$, as follows:

$$c_j(k_{1j}, k_{2j}|b) = \frac{1}{2\,(1-b^2)}\,(k_{1j}^2 + k_{2j}^2 + 2\,b\,k_{1j}\,k_{2j}), \tag{5}$$

where $0 < b < 1$. The cost can be interpreted as the cost of raising the capital or as effort incurred in monitoring the investment, etc. Notice that the cross term, $2\,b\,k_{1j}\,k_{2j}$, makes each investor's investments in the two banks a strategic substitute to each other. For instance, when investor j is attracted to invest more in the shadow bank, this increases the marginal cost of investing in the traditional bank, which is given by $\frac{\partial c_j}{\partial k_{1j}} = \frac{k_{1j}+bk_{2j}}{1-b^2}$. As a result, investor j will decrease her investment in the traditional bank. As such, the shadow bank effectively competes some funds away from the traditional bank. The higher the b, the stronger the adverse impact of increasing k_{2j}

on k_{1j}. Thus I interpret b as a measure of competitiveness in the banking market. In $c_j(k_{1j}, k_{2j}|b)$, I set the coefficient in front of the quadratic terms to $\frac{1}{2\,(1-b^2)}$, in order to normalize the coefficients on $E_j\,[R_1]$ and $E_j\,[R_2]$ to 1 in the individual investment rules k_{1j} and k_{2j} respectively in equation (7).

Following Diamond and Dybvig (1983) and Goldstein and Pauzner (2005), I assume that each bank is mutually owned by the investors and hence each investor receives a pro rata share of the banks' final investment proceeds, $R_1\,k_{1j} + R_2\,k_{2j}$. The investor's payoff is thus given by

$$u_j(k_{1j}, k_{2j}) = R_1\,k_{1j} + R_2\,k_{2j} - \frac{1}{2\,(1-b^2)}\,(k_{1j}^2 + k_{2j}^2 + 2\,b\,k_{1j}\,k_{2j}). \tag{6}$$

Each investor chooses $\{k_{1j}, k_{2j}\}$ to maximize her expected payoff, which gives a pair of linear investment functions:

$$\begin{aligned} k_{1j} &= E_j\,[R_1] - b\,E_j\,[R_2]\,, \\ k_{2j} &= E_j\,[R_2] - b\,E_j\,[R_1]\,, \end{aligned} \tag{7}$$

where $E_j[\cdot]$ denotes investor j's expectation of investment returns conditional on her information set. That is, an individual's investment in bank i is increasing in her share of the expected return of bank i and decreasing in her share of the expected return of the other. On the other hand, when the shadow bank does not enter, without loss of generality, I set $k_{2j}, R_2, b \equiv 0$, which gives $k_{1j}^D = E_j\,[R_1]$.

The Banks thereafter invest the funds from the investors in the projects. I assume that there is no conflict of interest between the banks and the investors. Each bank benevolently maximizes the total cash flow generated from its project, $R_i K_i$, subject to a private cost from managing the project, $C_i(K_i) = \frac{1}{2}\,K_i^2$. In reality, this cost can arise because of banks' activities in monitoring borrowers, servicing mortgages, and supervising projects, etc. (Diamond, 1984). The objective function for bank i is then given by:

$$E\left[R_i K_i - \frac{1}{2}K_i^2\right]. \tag{8}$$

The bank's objective function merits additional discussions. As shown later in the single-bank analysis (Corollary 1), if there were no coordination inefficiencies among the investors ($a_i = 0$), the bank's investment incentive would be perfectly aligned with that of an individual investor, that is, the investor would adjust her investment only when the bank also prefers her to do so. It is the coordination inefficiencies that make the investment decisions made by the investors differ from the ones preferred by the bank which in turn seeks to shift the individual decisions through its disclosure.

Date 3

The investment returns are realized and the proceeds from the projects are distributed to the banks and investors. The table below summarizes the notations of the paper.

a_i	Liquidity risk of bank i	n_i	Private information precision about bank i
b	market competitiveness	q	Common prior precision
c_a	Cost of liquidity risk	R_i	Investment return of bank i
c_m	Cost of disclosure	θ_i	Fundamentals of bank i
k_{ij}	Investment by investor j in bank i	$\bar{\theta}$	Common prior mean
K_i	Aggregate investment in bank i	x_{ij}	Private signal of investor j about bank i
m_i	Disclosure precision of bank i	z_i	Public signal about bank i

Table 1: Notations.

3 A Model with a Traditional Bank

In this section, I analyze the choice of disclosure precision by a single traditional bank to highlight the disclosure trade-off in the presence of liquidity risk. This analysis also applies to the situation in which the traditional bank is the monopoly. I solve the model by backward induction and first characterize each investor's optimal investment conditional on the traditional bank's choice of disclosure precision m_1 and the available information set, $I_j = \{x_{1j}, z_1\}$, as summarized in the following proposition.

Proposition 1 *Given the traditional bank's decision, m_1, there exists a unique equilibrium of individual investment in which each investor makes the optimal investments $k_{1j}^{*D}(m_1)$ that satisfy*

$$k_{1j}^{*D} = \beta_1^{*D} x_{1j} + \gamma_1^{*D} z_1 + h_1^{*D}, \tag{9}$$

where:

$$\begin{aligned} \beta_1^{*D} &= \frac{n_1}{q + m_1 + (1-a_1)n_1}, \\ \gamma_1^{*D} &= \frac{m_1}{(1-a_1)[q + m_1 + (1-a_1)n_1]}, \\ h_1^{*D} &= \frac{q\bar{\theta}}{(1-a_1)[q + m_1 + (1-a_1)n_1]}. \end{aligned} \tag{10}$$

Proposition 1 characterizes the individual investors' responses to information in their investments. Relative to the private information, the public information (the public signal z_1 and the common prior $\bar{\theta}$) is given disproportionately high weights that are incommensurate with their respective precision (e.g., $\frac{\gamma_1^{*D}}{\beta_1^{*D}} = \frac{m_1}{n_1(1-a_1)} > \frac{m_1}{n_1}$). This result of overweighting public information often appears in the literature of higher-order beliefs; the reason is that public information, as a common information source, is more effective in guessing beliefs of others (Morris and Shin, 2002).

More importantly, an examination of the weights individual investors assign to their signals shows that the presence of the liquidity risk causes some coordination inefficiencies in individual decisions. As I will show later, these coordination inefficiencies create an incentive for the traditional bank to disclose. To see the coordination inefficiencies, it is illuminating to compare my model in which individual investors determine the investments with a "perfect-coordination" benchmark in which the traditional bank determines the investment by itself.[11] Thus in the benchmark, the bank-level investment is not plagued by the coordination inefficiencies among individual investors. I further assume that in the benchmark, the traditional bank has the same amount of information $\{x_{1j}, z_1\}$ as an individual investor. Therefore, examining this benchmark allows me to separate the effect of the coordination inefficiencies from other effects. First, when the investment decision is made by each individual investor, the equilibrium individual investment is:

$$k_{1j}^{*D} = E\left[R_1 | x_{1j}, z_1\right] = \beta_1^{*D}\, x_{1j} + \gamma_1^{*D}\, z_1 + h_1^{*D}. \tag{11}$$

Aggregating all the individual investments gives the equilibrium aggregate investment $K_1^{*D} = \int_0^1 k_{1j}^{*D} dj$. The total sensitivity to the set of the information $\{x_{1j}, z_1\}$ is equal to the sum of the weights on z_1 and x_{1j}:

$$\beta_1^{*D} + \gamma_1^{*D} = \frac{1}{1-a_1} \frac{m_1 + (1-a_1)n_1}{m_1 + (1-a_1)n_1 + q}. \tag{12}$$

On the other hand, in the perfect-coordination benchmark, the traditional bank determines the bank-level investment K_1, contingent on x_{1j} and z_1, to maximize its expected payoff:

$$E\left[R_1\, K_1 - \frac{1}{2}\, K_1^2 | x_{1j}, z_1\right]. \tag{13}$$

[11] The other interesting benchmark is to assume that the traditional bank can perfectly control how an investor's action depends on her own information, but cannot make her action depend on other investors' private information, as suggested by Angeletos and Pavan (2007). This benchmark thus identifies the best the traditional bank could do if its investors were to internalize their payoff interdependencies and appropriately adjust their use of available information without communicating with one another. I find that in this benchmark, the traditional bank's preferred level of informational sensitivity is $\frac{1}{1-2a_1}$, which is larger than the one chosen by the individual investors.

The first-order condition is:

$$K_1 = E\left[R_1|x_{1j}, z_1\right] + \frac{\partial E\left[R_1|x_{1j}, z_1\right]}{\partial K_1} K_1. \tag{14}$$

Comparing equation (14) with the first-order condition for the individual investor in equation (11) reveals a coordination inefficiency among the individual investors. In particular, each individual does not internalize the positive spillover of successful coordination that fall on the other investors, as measured by the term $\frac{\partial E\left[\frac{1}{2}R_1|x_{1j},z_1\right]}{\partial K_1} K_1 > 0$. Intuitively, when an investor decides to increase (decrease) her investment, this also results in gains (losses) to all the other investors which, however, are not fully accounted for by the investor.[12] Solving equation (14) gives the optimal investment K_1^p in the perfect-coordination benchmark:

$$K_1^p = \frac{E[\theta_1|x_{1j}, z_1]}{1-2a_1} = \frac{1}{1-2a_1}\frac{n_1 x_{1j} + m_1 z_1}{m_1 + n_1 + q} + \frac{1}{1-2a_1}\frac{q\bar{\theta}}{m_1 + n_1 + q}, \tag{15}$$

where the sensitivity of the investment to the set of the information $\{x_{1j}, z_1\}$ is $\frac{1}{1-2a_1}\frac{m_1+n_1}{m_1+n_1+q}$. Comparing this sensitivity with the one chosen by the individual investors, I have

$$\underbrace{\frac{1}{1-2a_1}\frac{m_1+n_1}{m_1+n_1+q}}_{\text{Bank's preferred informational sensitivity}} > \underbrace{\frac{1}{1-a_1}\frac{m_1+(1-a_1)n_1}{m_1+(1-a_1)n_1+q}}_{\text{Individual informational sensitivity}}. \tag{16}$$

That is, as long as $a_1 \neq 0$, the individual investors are less responsive to the information than the level preferred by the traditional bank. Equation (16) shows that the investors' low informational sensitivity is caused by two reasons. First, the individual investors do not internalize the positive spillover of successful coordination that fall on others. This can be seen in the coefficients multiplying the informational sensitivities, $\frac{1}{1-a_1}$ and $\frac{1}{1-2a_1}$. The second reason is the investors'

[12] Such coordination failures often occur in bank-run contexts such as in Diamond and Dybvig (1983).

overweighting of the public information in the higher-order-belief context. As previously discussed, to be better coordinated, the individual puts too much weight on the common prior and is less responsive to the private information (i.e., in the individual investment, the private information is discounted by a factor $1 - a_1$). Such low responsiveness in turn lowers the informational sensitivity in the individual investment further more. I summarize the coordination inefficiency result in the corollary below.

Corollary 1 *Compared to a perfect-coordination benchmark in which the traditional bank decides the investment itself, the investment chosen by the individual investors is less sensitive to the set of information $\{x_{1j}, z_1\}$, as characterized in equation (16).*

I now solve for the choice of disclosure precision m_1 by the traditional bank. Let Π_1^M denote the traditional bank's payoff given the optimal individual investment, where

$$\Pi_1^M (m_1) = E\left[R_1\, K_1^{*D} - \frac{1}{2}\left(K_1^{*D}\right)^2\right] - \frac{c_m}{2} m_1^2. \tag{17}$$

To see the disclosure trade-off for the traditional bank, substitute in the expression for K_1^{*D} and rewrite $\Pi_1^M (m_1)$ as

$$\Pi_1^M (m_1) = \underbrace{\left[\left(\beta_1^{*D} + \gamma_1^{*D}\right) - \frac{1-2a_1}{2}(\beta_1^{*D} + \gamma_1^{*D})^2\right]\frac{1}{q}}_{\text{Coordination effect}} - \underbrace{\frac{1-2a_1}{2}\frac{\left(\gamma_1^{*D}\right)^2}{m_1}}_{\text{Volatility effect}} + \bar{\Pi}_1^M - \frac{c_m}{2} m_1^2, \tag{18}$$

where $\bar{\Pi}_1^M = \frac{\bar{\theta}^2}{2(1-a_1)^2}$ is a constant. Equation (18) suggests that besides the disclosure cost $\frac{c_m}{2} m_1^2$, the disclosure precision chosen by the traditional bank has two endogenous effects on its payoff through affecting the investment K_1^{*D}. The first term in the equation represents a benefit of disclosure from spurring favorable coordination. As shown in equation (12), the total sensitivity to information $\beta_1^{*D} + \gamma_1^{*D}$ is strictly increasing in the disclosure precision m_1. This is because more precise disclosure by the traditional bank reduces both the fundamental uncertainty about the quality of the project and the strategic uncertainty about others' actions. As a result, the investors respond more promptly to the public information. This increase in the sensitivity to information benefits the traditional bank because the investors are less sensitive to information than the level preferred by the traditional bank, due to the coordination inefficiencies discussed previously. I call this benefit of disclosure a *coordination effect.* The second term in equation (18) represents an endogenous cost of disclosure from magnifying the volatility in the traditional bank's investment. This is due to the investors' overweighting of the public signal z_1 and hence the noise in z_1. Specifically, as the traditional bank's disclosure precision increases, this disclosed information is used more by the investors in estimating the fundamentals (γ_1^{*D} increases). Therefore, knowing that others use the public information more, an individual will assign an even larger weight to the public information since this information is more effective in second-guessing others' actions. As the public information is further overweighted relative to its precision, this magnifies the impact of the noise in this public information on the investment in the traditional bank. In aggregating the individual investments, this public noise is not diversified away as the noises in the private information, injecting additional volatility into the aggregate investment and lowering the traditional bank's payoff. I call this cost of disclosure a *volatility effect.*

In equilibrium, the traditional bank decides its disclosure precision by trading the coordination benefit with the volatility cost. In order to highlight the trade-off between the coordination effect and the volatility effect, I compute the comparative statics of Π_1^M on m_1 without adding the disclosure cost $\frac{c_m}{2}m_1^2$ in the lemma below.

Lemma 1 *Denote $B_1^M = \Pi_1^M|_{c_m=0}$ as the payoff to the traditional bank without adding the disclosure cost $\frac{c_m}{2}m_1^2$. When $a_1 \geq \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1}+9}$, B_1^M is strictly increasing in m_1; when $a_1 < \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1}+9}$, B_1^M is U-shaped in m_1, i.e., $\frac{\partial B_1^M}{\partial m_1} > 0$ if and only if $m_1 > (1-a_1)(1-4a_1)n_1 - q$.*

Lemma 1 suggests that the trade-off between the coordination effect and the volatility effect leads to a non-monotonic effect of the disclosure precision on the traditional bank's payoff. In particular, the liquidity exposure of the traditional bank plays an important role in weighting this trade-off. When the liquidity exposure is high (i.e., a_1 is close to $\frac{1}{2}$), equation (16) shows that the traditional bank's preferred level of informational sensitivity approaches infinity and thus the traditional bank *always* wants to increase the informational sensitivity. Intuitively, $a_1 = \frac{1}{2}$ is the case where the coordination gains that the investors fail to internalize are paramount. As a result, the coordination effect dominates the volatility effect and improving the disclosure precision always increases the traditional bank's payoff. However, when the liquidity exposure is low (i.e., a_1 is close to 0), equation (16) shows that the individual sensitivity to the information is almost equal to the one preferred by the traditional bank, making the benefit from mitigating the coordination inefficiencies small. Therefore, when the disclosure precision m_1 is small and the disclosure noise is large, the volatility effect dominates the coordination effect, making more precise disclosure undesirable to the traditional bank.

It is important to compare the disclosure trade-off elaborated in my model with others in the literature. First, the liquidity-risk-driven trade-off for disclosure in my model is different from the ones in the extant accounting literature built on information asymmetry between firms and market (Verrecchia, 1983; Rye, 1985). Second, this trade-off is also distinct from others developed in higher-order-belief contexts. For instance, Morris and Shin (2002) discuss the heightened volatility induced by public disclosure in contexts where coordination has no social value. Angeletos and Pavan (2004) examine a trade-off between individual dispersion and aggregate volatility. In the presence of liquidity risk and coordination inefficiencies, my model develops a new disclosure trade-off between spurring favorable coordination and magnifying investment volatility. This trade-off in turn determines the equilibrium disclosure precision m_1^{*M}, as given in the following first-order condition:

$$\frac{\partial \Pi_1^M}{\partial m_1} = \frac{\partial B_1^M}{\partial m_1} - c_m m_1 = 0. \tag{19}$$

I summarize the equilibrium disclosure precision in the proposition below.

Proposition 2 *Given the optimal individual investment k_{1j}^{*D}, the equilibrium disclosure precision by the traditional bank m_1^{*M} always exists and is unique. There exists a threshold on the disclosure cost, $\hat{c}_m^M$, such that:*

1. *When $c_m < \hat{c}_m^M$, there exists a unique interior equilibrium, m_1^{*M}, that solves equation (19);*
2. *When $c_m \geq \hat{c}_m^M$, if $a_1 \geq \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1} + 9}$, there exists a unique interior equilibrium, m_1^{*M}, that solves equation (19); if $a_1 < \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1} + 9}$, the unique equilibrium is $m_1^{*M} = 0$.*

Proposition 2 suggests that when either the cost of disclosure is sufficiently small or the liquidity risk is sufficiently large, the equilibrium disclosure precision by the traditional bank is unique and nonzero. Otherwise, the volatility effect of disclosure dominates and leads to a zero-disclosure equilibrium.

4 A Model with a Traditional and a Shadow Bank

In this section, I turn to analyze the entry-deterrence model with both the traditional and the shadow bank. I will show that when the two banks compete, the disclosure by the traditional bank plays a competitive role in influencing the shadow bank's decisions. This competitive role alters the disclosure trade-off in the single bank setting. In equilibrium, the traditional and the shadow bank both become more opaque than when they don't compete with each other.

I consider a rational expectation equilibrium that satisfies: at date 2, each investor chooses the optimal investments conditional on her information set; at date 1, the shadow bank chooses the entry decision as well as decisions on disclosure and liquidity risk to maximize its payoff; at date 0, the traditional bank chooses the precision of public disclosure to maximize its payoff. I solve the equilibrium by backward induction. At each date, I characterize the equilibrium and derive its properties.

4.1 Individual Investment Decisions

I first solve for each investor's optimal investments conditional on the two banks' actions and the available information set, $I_j = \{x_{1j}, x_{2j}, z_1, z_2\}$, at date 2. I summarize the individual investments in equilibrium, $\{k^*_{1j}(\cdot), k^*_{2j}(\cdot)\}$, in the following proposition.

Proposition 3 *Given the traditional bank's decision, m_1, and the shadow bank's decisions, $\{m_2, a_2\}$, there exists a unique equilibrium of individual investments in which,*

1. *When the entry of the shadow bank is deterred, each investor chooses $k^{*D}_{2j}(m_1, m_2, a_2) \equiv 0$ and $k^{*D}_{1j}(m_1)$ as given in Proposition 1.*

2. *When shadow bank enters, each investor chooses $\{k^*_{1j}(\cdot), k^*_{2j}(\cdot)\}$ that satisfy*

$$k^*_{ij} = (\beta^*_i\, x_{ij} + \gamma^*_i\, z_i) - b\,(\lambda^*_i\, x_{tj} + \omega^*_i\, z_t) + h^*_i,\ \ (i,t) \in \{(1,2),(2,1)\}, \tag{20}$$

   where: $\beta^*_i = \beta^{*D}_i(1+\kappa_i), \gamma^*_i = \gamma^{*D}_i(1+\nu_i), \lambda^*_i = \beta^{*D}_t(1+\kappa_t)$, and $\omega^*_i = \gamma^{*D}_t(1+\nu_t)$, with $\nu_i > \kappa_i > 0$. The expressions of $\{\beta^*_i, \gamma^*_i, \lambda^*_i, \omega^*_i, h^*_i\}$ are given in the proof.[13]

The second part of Proposition 3 suggests that the entry of the competing shadow bank has a multiplier effect on the investors' responses to information. That is, when the shadow bank enters, the investors assign larger weight to both their public and private signals in their investment and hence become more sensitive to information ($\nu_i, \kappa_i > 0$). In this regard, the multiplier effect of

[13] With some abuse of notation, I let $\beta^{*D}_2 = \frac{n_2}{q+m_2+(1-a_2)n_2}$ and $\gamma^{*D}_2 = \frac{m_2}{(1-a_2)[q+m_2+(1-a_2)n_2]}$.

competition is complementary to the coordination benefit of disclosure in mitigating the coordination inefficiencies the traditional bank faces. As I will show in later analysis, this multiplier effect thereby creates a channel for the entry of the shadow bank to benefit the traditional bank and may shift the traditional bank's attitudes towards entry and disclosure. For future references, I summarize the properties of the multiplier effect in the corollary below.

Corollary 2 *The entry of the competing shadow bank yields a multiplier effect on an individual investor's sensitivities to the signals about the traditional bank; the multiplier effect is stronger on the sensitivity to the public signal than the private one, i.e.,* $\frac{\gamma_1^*}{\gamma_1^{*D}} > \frac{\beta_1^*}{\beta_1^{*D}} > 1$. *In addition, the sizes of the multipliers are strictly increasing in the intensity of competition* b, *and the liquidity exposure of the shadow bank* a_2, *i.e.,* $\frac{\partial}{\partial y}\left(\frac{\gamma_1^*}{\gamma_1^{*D}}\right) > \frac{\partial}{\partial y}\left(\frac{\beta_1^*}{\beta_1^{*D}}\right) > 0$, *where* $y \in \{a_2, b\}$.

The multiplier effect is driven by the strategic complementarity between individual investments.[14] Recall that the presence of the liquidity risk causes the investment return for an individual to be lower when others invest less. This renders a strategic complementarity between individual investments and forces each individual to choose an investment strategy that matches the investments of others. Since others' investments are motivated by their beliefs, each individual must take account of the beliefs held by other investors, others' beliefs about others and even higher-order beliefs. The motive to form higher-order beliefs in turn gives rise to the multiplier effect of competition. To see this, rewrite the individual sensitivity to a signal $s_{1j} \in \{x_{1j}, z_1\}$ as:

$$\frac{dk_{1j}^*}{ds_{1j}} = \underbrace{\frac{\partial k_{1j}^*}{\partial E_j[\theta_1]}\frac{\partial E_j[\theta_1]}{\partial s_{1j}}}_{\text{Fundamentals}} + \underbrace{\frac{\partial k_{1j}^*}{\partial E_j[K_1]}\frac{\partial E_j[K_1]}{\partial E_j[\bar{E}[\theta_1]]}\frac{\partial E_j[\bar{E}[\theta_1]]}{\partial s_{1j}}}_{\text{Multiplier effect of TB's liquidity risk}} + \underbrace{\frac{\partial k_{1j}^*}{\partial E_j[R_2]}\frac{\partial E_j[R_2]}{\partial E_j[K_2]}\frac{\partial E_j[K_2]}{\partial E_j[\bar{E}[\theta_1]]}\frac{\partial E_j[\bar{E}[\theta_1]]}{\partial s_{1j}}}_{\text{Multiplier effect of competition}}, \tag{21}$$

[14] Cooper and John (1988) show that in a general class of games, strategic complementarity is both sufficient and necessary for multipliers.

where $E_j[\cdot]$ denotes individual j's expectation and $\bar{E}[\cdot]$ denotes the aggregate expectation. Equation (21) suggests that the signal s_{1j} affects the investment k_{1j}^* via three channels. The first term in the equation describes the channel related to the fundamental value of the signal. A higher s_{1j} improves the investor's posterior expectation about the fundamentals θ_1, which in turn induces the investor to invest more. This term determines the investor's initial response to s_{1j}. The second and the third terms describe the channels related to the strategic value of the signal, which creates the multiplier effect. Specifically, the second term represents the multiplier effect driven by the traditional bank's liquidity risk. From individual j's standpoint, a higher s_{1j} indicates not only better fundamentals of the traditional bank's project, but also that others believe the bank's fundamentals to be better (i.e., the higher-order belief $E_j[\bar{E}[\theta_1]]$ is also better). As a result, individual j forecasts a larger aggregate investment K_1 and chooses to invest more because of the strategic complementarity induced by the traditional bank's liquidity risk. The second term thereafter amplifies the investor's initial response to s_{1j} and gives her equilibrium response in the single bank setting.

The third term in equation (21) captures the multiplier effect caused by competition. Specifically, competition prompts investors to reallocate investments between banks. Therefore, a higher s_{1j} suggests that other investors hold more optimistic beliefs about the traditional bank's project and hence are more likely to reallocate their investments from the shadow bank to the traditional bank (i.e., a lower $E_j[K_2]$). When the shadow bank is subject to liquidity risk, these reallocations reduce the shadow bank's return, which in turn induces the individual j to make the same reallocating decision as others. That is, the combination of the shadow bank's liquidity risk and its competition with the traditional bank creates another degree of strategic complementarity between individual investments in the traditional bank. This competition-driven complementarity further magnifies the investors' informational sensitivities in the monopoly case and makes the investors more responsive to information in the entry case. Since this multiplier effect depends critically on the competition intensity and the shadow bank's liquidity exposure, improvements in these two measures will increase the sizes of the multipliers, κ_1 and ν_1.

It is also important to notice that the higher-order beliefs formed by the investors play a vital role in producing the multiplier effect of competition. In fact, if the investors were not concerned with guessing others' beliefs (e.g., $a_1 = a_2 = 0$), the entry of the shadow bank would not change the investors' responses to the signals of the traditional bank. This observation suggests that because the public signal has a larger impact on the higher-order beliefs $\left(\frac{\partial E_j[\bar{E}[\theta_1]]}{\partial z_1} > \frac{\partial E_j[\bar{E}[\theta_1]]}{\partial x_{1j}}\right)$, the multiplier effect is stronger on the sensitivity to the public signal than the private one, as shown in Corollary 2. As a result, relative to the monopoly case, the entry of the shadow bank further exacerbates the investors' overweighting of the public information. The public information is overweighted even more when the competition becomes more intense or the shadow bank takes more liquidity risk. This is because improvements in both measures increase the complementarity between individual investments and hence reinforce the role of the higher-order beliefs, inducing the investors to place a larger weight on the public signal.

4.2 Entry, Disclosure and Liquidity Risk Decisions by the Shadow Bank

At date 1, the shadow bank decides whether to enter the banking market. Upon entering, the shadow bank also decides the precision of public information, m_2, and the exposure to liquidity risk, a_2. To solve for the shadow bank's decisions, I substitute the investors' equilibrium investments into the banks' payoffs. Let Π_1 and Π_2 denote the traditional bank and the shadow bank's payoffs given the optimal investments respectively.

Disclosure and Liquidity Risk Decisions I first examine the equilibrium decisions of liquidity risk and disclosure precision, $\{m_2^*(m_1), a_2^*(m_1)\}$, by the shadow bank when it decides to enter, given the traditional bank's disclosure precision m_1. The pair $\{m_2^*(m_1), a_2^*(m_1)\}$ solves the two first-order conditions,

$$\begin{aligned} \frac{\partial \Pi_2}{\partial m_2} &= 0, \\ \frac{\partial \Pi_2}{\partial a_2} &= 0. \end{aligned} \tag{22}$$

In Appendix II, I show that the pair of equilibrium decisions always exists and is unique. In order to study the interactions between the decisions of the shadow and the traditional bank, I focus on the case in which $\{m_2^*(m_1), a_2^*(m_1)\}$ are interior throughout the paper. In Appendix II, I characterize the sufficient conditions for the equilibrium to be interior. With the equilibrium decisions $\{m_2^*(m_1), a_2^*(m_1)\}$ characterized, I now turn to the strategic interaction between the disclosure decision by the traditional bank and the liquidity risk and disclosure decisions by the shadow bank. This analysis helps us to understand the competitive role of the traditional bank's disclosure. That is, the traditional bank, as a Stackelberg leader, chooses its disclosure precision to influence the shadow bank's choices of liquidity exposure and disclosure precision to its advantage. This role of disclosure, together with liquidity-risk-driven disclosure trade-off characterized in the single bank setting, determine the traditional bank's disclosure precision. Proposition 4 summarizes the effect of the traditional bank's disclosure on the shadow bank's decisions.

Proposition 4 *For the private information precision n_1 and n_2 sufficiently large, there exist two thresholds, $0 < \hat{b}_1, \hat{b}_2 < 1$, such that,*

1. *If $b > \hat{b}_1$, the shadow bank's optimal choice of liquidity risk $a_2^*(m_1)$ is strictly decreasing in the traditional bank's choice of disclosure precision m_1 (i.e., $\frac{\partial a_2^*}{\partial m_1} < 0$);*

2. *If $b > \hat{b}_2$, the shadow bank's optimal choice of disclosure precision $m_2^*(m_1)$ is strictly increasing in the traditional bank's choice of disclosure precision m_1 (i.e., $\frac{\partial m_2^*}{\partial m_1} > 0$).*

Proposition 4 suggests that when the competition between the two banks is sufficiently intense, the traditional bank's disclosure precision is a strategic substitute to the shadow bank's decision of liquidity risk and a strategic complement to the shadow bank's decision of disclosure precision. The underlying intuition for this result depends on the interplay between disclosure and individual investments. I first explain the substitute relation between the disclosure decision by the traditional bank and the liquidity risk decision by the shadow bank. It is instructive to rewrite the shadow bank's payoff Π_2 as follows:

$$\begin{aligned}\Pi_2 &= E\left[R_2 K_2 - \frac{1}{2}K_2^2\right] \\ &= \bar{\Pi}_2 + Cov\left(R_2, K_2\right) - \frac{1}{2}Var(K_2),\end{aligned} \tag{23}$$

where $\bar{\Pi}_2 = E[R_2]E[K_2] - \frac{1}{2}(E[K_2])^2$ is the mean of Π_2 and is independent of the disclosure precision by the traditional bank. Given that $K_2 = (\beta_2^* \theta_2 + \gamma_2^* z_2) - b\left(\lambda_2^* \theta_2 + \omega_2^* z_2\right) + h_2^*$, the aggregate investment in the shadow bank varies with not only the signals of the shadow bank itself but also the signals of the traditional bank. Let $s_2' = (\beta_2^* \theta_2 + \gamma_2^* z_2)$ and $s_1' = b\left(\lambda_2^* \theta_1 + \omega_2^* z_1\right)$ denote the variation of K_2 driven by the signals of the shadow bank and the signals of the traditional bank, respectively. Hence, $K_2 = s_2' - s_1' + h_2^*$. Notice that since the two banks' fundamentals are independent, $Cov(s_1', s_2') = 0$ and thus I have

$$\begin{aligned}\Pi_2 &= \bar{\Pi}_2 + Cov(\theta_2 + a_2 s_2' + a_2 s_1', s_2' - s_1') - \frac{1}{2}Var(s_2' - s_1'), \\ &= \bar{\Pi}_2 + Cov(\theta_2 + a_2 s_2', s_2') - \frac{1}{2}Var(s_2') - \frac{1-2a_2}{2}Var(s_1').\end{aligned} \tag{24}$$

Equation (24) suggests that the disclosure by the traditional bank z_1 affects Π_2 only through injecting the additional variation s_1' into the volatility of K_2. To see how the volatility of K_2

changes, I rewrite $Var(s_1')$ as:

$$\begin{aligned} Var(s_1') &= b^2\, Var(\lambda_2^*\theta_1 + \omega_2^* z_1) \\ &= b^2\, \left[(\lambda_2^* + \omega_2^*)^2 Var(\theta_1) + (\omega_2^*)^2 Var(\varepsilon_1)\right]. \end{aligned} \tag{25}$$

This expression shows that the traditional bank's disclosure increases the volatility of K_2 for two reasons. The first term is related to the fundamental-driven volatility. As the traditional bank's disclosure precision improves, the set of the investors' signals $\{z_1, x_{1j}\}$ becomes a more accurate reflection of the bank's fundamentals than the prior. Therefore, the investors adjust their investments more promptly in response to changes in the signals. From an ex-ante perspective, these adjustments lead to higher volatility in the investment. The second term is related to the non-fundamental volatility. As explained in the single bank setting, more precise disclosure by the traditional bank induces the investors to overweight the public information z_1, which magnifies the impact of the noise ε_1 and increases the volatility in the aggregate investment.

The increase in the volatility of K_2 in turn affects the shadow bank's choice of liquidity risk in two ways. On one hand, notice that from equation (24), the relative weight on the volatility, $\frac{1-2a_2}{2}$, is strictly decreasing in a_2. This reflects the value of common-shock-driven volatility in facilitating coordination. In particular, the disclosure by the traditional bank injects two common shocks, the public signal z_1 and the fundamentals θ_1, into the individual investment in the shadow bank. These common shocks, while amplifying the volatility of the aggregate investment, also help to align the individual investments, which improves the coordination among the investors. The higher the a_2, the more valuable the coordination. As a result, the heightened volatility from the traditional bank's disclosure is less costly as a_2 increases. When the traditional bank discloses more precisely, the shadow bank can increase its liquidity risk to reduce the relative weight on the volatility, which helps to alleviate the damage by the heightened volatility. I call this effect a *weighting effect*. On the other hand, recall that more public disclosure amplifies volatility because it exacerbates the investors' overreaction to public information, which occurs because of the shadow bank's exposure to liquidity risk. This observation suggests that the shadow bank can dampen the overreaction by reducing its liquidity risk, which mitigates the increase in the volatility. I call this effect an *overreaction effect*. The relation between m_1 and a_2 is hence determined by the trade-off between the strategic complementarity induced by the weighting effect and the strategic substitutability induced by the overreaction effect. In particular, competition plays a critical role in weighting this trade-off because competition exacerbates the investors' overweighting of public information. When the competition is sufficiently intense (i.e., $b > \hat{b}_1$), the problem of overweighting becomes a central concern for the shadow bank, forcing the bank to reduce its liquidity risk exposure to dampen the investors' overreaction. As a result, the overreaction effect dominates and leads to the strategic substitutability between m_1 and a_2.

The intuition for the relation between the two banks' disclosure can be gleaned similarly. Notice that the disclosure decision of the traditional bank has no direct effect on the disclosure decision of the shadow bank ($\frac{\partial^2 \Pi_2}{\partial m_1 \partial m_2} = 0$), because the fundamentals θ_1 and θ_2 are independent of each other.[15] In my model, the traditional bank's disclosure decision m_1 influences the shadow bank's disclosure decision m_2 indirectly through affecting the shadow bank's liquidity risk a_2. Specifically, notice first that when b is large, m_2 and a_2 are strategic substitutes to each other. The intuition is similar to that for the relation between m_1 and a_2. Therefore, as the precision of the traditional bank's public information deteriorates, the shadow bank is motivated to take more liquidity risk (the strategic substitutability between m_1 and a_2), which in turn induces it to disclose less precise

[15] I check the robustness of my results by examining a model with correlated fundamentals. It is analytically intractable, but the numerical analysis suggests that when the fundamentals are correlated positively, the cross-partial derivative $\frac{\partial^2 \Pi^2}{\partial m_1 \partial m_2} > 0$, which strengthens the strategic complementarity between m_1 and m_2 as shown in my current setting.

information (the strategic substitutability between a_2 and m_2). Overall this chain of reasoning results in the strategic complementarity between the two banks' disclosure decisions.

Entry Decision I now derive the condition for the shadow bank to enter the banking market. In equilibrium, the shadow bank enters if and only if its expected payoff under its optimal decisions, $\Pi_2 \left(m_2^* \left(m_1\right), a_2^* \left(m_1\right), m_1\right)$ is higher than the payoff when it chooses not to enter, $\overline{U}$. Proposition 5 shows that there exists a single cutoff on the traditional bank's disclosure precision m_1, such that the shadow bank enters if and only if m_1 is below the cutoff.

Proposition 5 *Let m_1^D denote the unique solution to the equation $\Pi_2 \left(m_2^* \left(m_1\right), a_2^* \left(m_1\right), m_1\right) = \overline{U}$. The shadow bank decides to enter if and only if $m_1 < m_1^D$.*

Proposition 5 suggests that the traditional bank's disclosure decision has an additional competitive role of deterring or accommodating entry, besides affecting the shadow bank's disclosure and liquidity risk decisions. In particular, I find that the traditional bank actually needs to lower its disclosure transparency in order to induce entry, different from the conventional view that firms disclose less in order to deter entry. This difference is due to the effect of the traditional bank's disclosure in magnifying the volatility of the shadow bank's investment, as explained in the discussions for Proposition 4. Indeed, in the proof of Proposition 5, I show that $\Pi_2 \left(m_2^* \left(m_1\right), a_2^* \left(m_1\right), m_1\right)$ is strictly decreasing in m_1. Therefore, if the traditional bank discloses very precisely, $\Pi_2 \left(m_2^* \left(m_1\right), a_2^* \left(m_1\right), m_1\right)$ falls below $\overline{U}$ and hence the shadow bank decides not to enter.

4.3 Disclosure Decision by the Traditional Bank

To complete the analysis, I now characterize the traditional bank's disclosure decision m_1 at date 0. As suggested by Proposition 5, when the traditional bank's disclosure precision is low ($m_1 < m_1^D$), the shadow bank enters and the traditional bank maximizes a payoff given the shadow bank's optimal choices. Formally, the traditional bank solves the following optimization program:

$$\max_{m_1} \Pi_1 \left(m_1, m_2^* \left(m_1\right), a_2^* \left(m_1\right)\right), \tag{26}$$
$$s.t.\, m_1 < m_1^D.$$

When traditional bank's disclosure precision is high ($m_1 \geq m_1^D$), the entry is deterred and the traditional bank maximizes its payoff in the single bank case. Formally, the traditional bank solves the following optimization program:

$$\max_{m_1} \Pi_1^M \left(m_1\right), \tag{27}$$
$$s.t.\, m_1 \geq m_1^D.$$

In equilibrium, which of the two cases prevails depends on whether the traditional bank finds it advantageous to deter or accommodate entry. In the proposition below, I find that the traditional bank will accommodate entry when its liquidity exposure is sufficiently high.

Proposition 6 *There exists a threshold* $0 < \hat{a} < \frac{1}{2}$, *such that,*

1. *when* $a_1 > \hat{a}$, *the traditional bank always accommodates the shadow bank's entry. There exists a unique disclosure precision* $m_1^* < m_1^D$ *by the traditional bank that solves* $\frac{d\Pi_1}{dm_1} = 0$ *and the shadow bank chooses* $\{m_2^* \left(m_1^*\right), a_2^* \left(m_1^*\right)\}$;

2. *when* $a_1 < \hat{a}$, *the traditional bank deters entry, as long as the disclosure cost for the traditional bank is not too high. The traditional bank chooses a disclosure precision equal to the deterrence cutoff,* $m_1^{*D} = m_1^D$.

Proposition 6 suggests that the liquidity risk borne by the traditional bank plays an important role in determining the entry of the shadow bank. When the traditional bank's exposure to liquidity risk is low (i.e., $a_1 < \hat{a}$), the traditional bank is better off by deterring the shadow bank from entry. When the traditional bank is highly vulnerable to liquidity risk (i.e., $a_1 > \hat{a}$), the traditional bank chooses to accommodate the entry of the shadow bank. It is important to notice that the traditional bank's decision to accommodate entry is not due to the prohibitively high cost of deterrence, which is often the case in prior studies (see Chapter 8, Tirole, 1988, for a comprehensive review). In fact, in the corollary below, I show that when a_1 is sufficiently large, the traditional bank is better off when it shares the banking market with the shadow bank than when it is the monopoly.

Corollary 3 *When the liquidity risk of the traditional bank is sufficiently large, the traditional bank obtains a higher payoff when it accommodates entry than when it is the monopoly,*

$$\Pi_1(m_1^*, m_2^*(m_1^*), a_2^*(m_1^*)) > \Pi_1^M(m_1^{*M}). \tag{28}$$

The multiplier effect of competition create a role for the entry of the shadow bank to benefit the traditional bank. Recall that the entry of the shadow bank induces the investors to be more responsive to the information, which mitigates the coordination inefficiences the traditional bank faces. In equilibrium, the traditional bank trades the coordination benefit from the entry with the cost of increased competition. To see this trade-off, rewrite the traditional bank's expected payoff as follows:

$$\begin{aligned} \Pi_1 &= \bar{\Pi}_1 + Cov(\theta_1 + a_1 s_1 + a_1 s_2, s_1 + s_2) - \frac{1}{2}Var(s_1 + s_2) \qquad (29) \\ &= \underbrace{Cov(\theta_1 + a_1 s_1, s_1) - \frac{1}{2}Var(s_1) +}_{\text{Coordination effect}} \underbrace{\bar{\Pi}_1}_{\text{Competition effect}} \underbrace{- \frac{1-2a_1}{2}Var(s_2)}_{\text{Volatility effect}}. \end{aligned}$$

where $\bar{\Pi}_1 = E[R_1]E[K_1] - \frac{1}{2}(E[K_1])^2$ is the mean of Π_1, $s_1 = (\beta_1^*\theta_1 + \gamma_1^* z_1)$ and $s_2 = b\,(\lambda_1^*\theta_2 + \omega_1^* z_2)$ denote the variation of K_1 driven by the signals of the traditional bank and the signals of the shadow bank, respectively. Hence, $K_1 = s_1 - s_2 + h_1^*$.

Equation (29) shows that the entry of the shadow bank affects the traditional bank's payoff in three ways. The first term represents the coordination benefit stemming from the multiplier effect: the entry increases the investors' informational sensitivity towards the traditional bank's preferred level. Notice that this effect is complementary to the coordination effect of disclosure discussed in the single bank case and thus I also call this benefit of entry a coordination effect. The second and the third terms represent the costs of competition. Specifically, the second term describes the usual competition effect. The entry of the shadow bank reduces the expected level of investment in the traditional bank ($E[K_1]$) and erodes its expected profit. The third term captures the increase in the volatility of K_1 caused by the competition. That is, the entry of the shadow bank injects the additional variation s_2 into K_1, which lowers the traditional bank's payoff. I call this cost of entry a volatility effect.

Similar to the disclosure trade-off between the coordination and the volatility effect, the entry trade-off between the coordination benefit and the cost of competition also depends critically on the level of the traditional bank's liquidity risk. When the liquidity exposure is low (i.e., a_1 is close to 0), the coordination gains is limited relative to the costs of competition. Therefore, the entry of the shadow bank *always* reduces the traditional bank's profit. This result is an reminiscence of the results in standard entry models with strategic-risk-free firms. The traditional bank simply views the shadow bank as a conventional competitor whose entry erodes its profit. However, the traditional bank's attitude towards entry shifts when its liquidity exposure is of vital importance. In the extreme with a_1 close to $\frac{1}{2}$, the coordination gains are sufficiently large and dominates the cost of competition. As a result, the traditional bank views the shadow bank as a partner in spurring favorable coordination and hences prefers its entry.

The shadow bank's decisions on liquidity risk and disclosure also plays an important role in determining the traditional bank's preference for its entry. In the corollary below, I show that the entry is more beneficial when the shadow bank is more fragile (high a_2) and opaque (low m_2).

Corollary 4 *The traditional bank's payoff is strictly decreasing in the shadow bank's disclosure precision, i.e., $\frac{\partial \Pi_1}{\partial m_2} < 0$; for a_1 sufficiently large, its payoff is strictly increasing in the shadow bank's liquidity risk, i.e., $\frac{\partial \Pi_1}{\partial a_2} > 0$.*

The intuition behind Corollary 4 is related to the coordination and the volatility effects of entry. Specifically, a higher a_2 can benefit the traditional bank because it reinforces the multiplier effect of competition. The higher a_2, on the other hand, also exacerbates the overweighting of public information and increases the volatility. When a_1 is sufficiently high, the coordination effect dominates and the traditional bank prefers the shadow bank to be fragile. The traditional bank's preference for the opacity of the shadow bank is due to the concern of reducing the volatility. In short, the more precise disclosure by the shadow bank increases the sensitivity of investors to this public information and exacerbates the overweighting of the associated noises, both of which cause the aggregate investment in the traditional bank to be more volatile. Therefore, the traditional bank prefers the shadow bank to disclose less to avoid the increase in volatility.

4.4 Joint Opacity of the Traditional and the Shadow Bank

With the equilibrium characterized, I now can ask the question whether the traditional bank and the shadow bank become opaque jointly when they co-exist. On top of the liquidity-risk-driven disclosure trade-off in the single bank setting, the traditional bank also considers the competitive role of disclosure in influencing the decisions of the shadow bank on entry, liquidity risk and disclosure, given the traditional bank's preferences for these decisions. To see how this competitive role alters the disclosure trade-off, I compare the traditional bank's disclosure precision in the two-bank case to three benchmark cases: 1) the monopoly case; 2) the case when the traditional bank deters entry; 3) a simultaneous-move case where the traditional bank's disclosure decision is unobservable to the shadow bank, which makes the shadow bank's decisions independent of the traditional bank's actual disclosure decision.[16] In the proposition below, I show that because of the competitive role of disclosure, the traditional bank chooses to make itself more opaque in order to induce the entry, opacity and fragility of the shadow bank.

Proposition 7 *Denote the two banks' decisions in equilibrium as* $\{m_1^*, m_2^*(m_1^*), a_2^*(m_1^*)\}$*, in the monopoly case as* $\{m_1^{*M}, m_2^*(m_1^{*M}), a_2^*(m_1^{*M})\}$*, in the deterrence case as* $\{m_1^{*D}, m_2^*(m_1^{*D}), a_2^*(m_1^{*D})\}$*, and in the simultaneous-move case as* $\{m_1^c, m_2^*(m_1^c), a_2^*(m_1^c)\}$*. When both* a_1 *and* b *are sufficiently large,*

1. *The traditional bank discloses less precise public information than all the benchmarks,*

$$m_1^* < m_1^c < m_1^{*M} < m_1^{*D};$$

2. *The shadow bank discloses less precise public information and takes more liquidity risk than all the benchmarks,*

$$m_2^*(m_1^*) < m_2^*(m_1^c) < m_2^*(m_1^{*M}) < m_2^*(m_1^{*D}) \text{ and } a_2^*(m_1^*) > a_2^*(m_1^c) > a_2^*(m_1^{*M}) > a_2^*(m_1^{*D}).$$

Proposition 7 shows that the interactions between the two banks lead to a parallel banking system with an endogenously opaque traditional bank (low m_1) and an endogenously opaque and fragile shadow bank (low m_2 and high a_2), which is consistent with the observed empirical patterns.

[16] This benchmark was suggested by Tirole (1988). Tirole defines the equilibrium in which the first mover's actions are not observed by the second mover as an "open-loop solution" and the one in which the first mover's actions are observed by the second mover as a "close-loop solution."

In particular, I find that when the shadow bank enters, the traditional bank becomes more opaque relative to the monopoly case when the shadow bank doesn't exist $\left(m_1^* < m_1^{*M}\right)$. Proposition 7 suggests that there are two reasons for the increase in the opacity of the traditional bank. First, when its liquidity risk is sufficiently high, the traditional bank finds it beneficial to share the market with the shadow bank, because the presence of the shadow bank spurs favorable coordination. In order to induce entry, the traditional bank lowers its disclosure precision, relative to the case when it deters entry $\left(m_1^* < m_1^{*D}\right)$. Second, when the shadow bank enters, in order to make the shadow bank opaque and fragile, the traditional bank lowers its disclosure precision further more, relative to the simultaneous-move case when the traditional bank's disclosure doesn't affect the shadow bank's decisions $(m_1^* < m_1^c)$. To see this, I rewrite the first-order condition on m_1 as follows:

$$\frac{d\Pi_1}{dm_1} = \frac{\partial \Pi_1}{\partial m_1} + \frac{\partial \Pi_1}{\partial m_2}\frac{\partial m_2^*}{\partial m_1} + \frac{\partial \Pi_1}{\partial a_2}\frac{\partial a_2^*}{\partial m_1}. \tag{30}$$

The first term captures the traditional bank's disclosure incentive corresponds to the simultaneous-move benchmark. The remaining two terms reflect how the competitive role affects the traditional bank's disclosure. In particular, the second term in equation (30) characterizes the traditional bank's use of disclosure to affect the shadow bank's disclosure decision. From Proposition 4 and Corollary 4, this term is negative with $\frac{\partial \Pi_1}{\partial m_2} < 0$ and $\frac{\partial m_2^*}{\partial m_1} > 0$. Similarly, the third term in equation (30) describes how the traditional bank alters the shadow bank's choice of liquidity exposure through its disclosure. Observe that from Proposition 4 and Corollary 4, $\frac{\partial \Pi_1}{\partial a_2} > 0$ and $\frac{\partial a_2^*}{\partial m_1} < 0$, which makes the third term negative. Therefore, the incentives to decrease the disclosure and to increase the liquidity exposure of the shadow bank jointly induce the traditional bank to disclose less precise information, compared to the situation in which these incentives are absent.

5 Conclusion

This paper examines why both the traditional and the shadow banking systems become opaque when they co-exist. I build a model that endogenizes the disclosure decisions of the two banks when they both face liquidity risk stemming from runs by investors and, at the same time, compete against each other in attracting investment from the same group of investors. In a single-bank setting, the presence of liquidity risk leads to a disclosure trade-off between the benefit of spurring favorable coordination and the cost of induced excess volatility. Then the disclosure trade-off is embedded in an entry-deterrence game in which the entrant shadow bank competes with the incumbent traditional bank. The presence of liquidity risk shifts both the traditional bank's attitudes towards entry and disclosure. Competing with an opaque and fragile shadow bank can benefit the traditional bank through spurring favorable coordination by the investors. In equilibrium, the traditional bank accommodates entry and induces the shadow bank to be both opaque and fragile by making itself opaque.

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Appendix I: Discussions of the linear structure of the investment return

In the main setting, I assume that the investment return is strictly increasing in the aggregate investment. In this appendix, I examine a model that explicitly considers the bank's role of liquidity transformation to motivate this return structure. Specifically, consider a three-date model with one bank and a continuum of investors, indexed by the unit interval $[0, 1]$. The analysis for two banks is similar. The timeline is as follows. At date 0, each investor is endowed with an initial amount of wealth, $\bar{K}$. For simplicity, I assume that each investor j invests the endowment, $\bar{K}$, in the bank. The bank thereafter plunges all the investments it raised, $\bar{K} = \int_0^1 \bar{K} dj$, into a production technology that yields a stochastic return, $\theta \sim N(\bar{\theta}, \frac{1}{q})$, at date 2. To capture the bank's role in liquidity transformation, I assume that the investment is illiquid. That is, if the production is interrupted at date 1, the salvage value is λ per unit of interrupted investments, where $0 < \lambda < 1$. I also assume that the bank's liability is liquid, in the sense that the investors can freely withdraw any amounts of their investments at the face value. At date 1, the investor decides the amount of the investment she wants to withdraw. I denote investor j's remaining investment as k_j and the aggregate investment remaining in the bank as K, where $K = \int_0^1 k_j dj$. To satisfy the withdrawal request, the bank is forced to liquidate its illiquid project at the rate, λ. To fulfill the withdraw requests $\bar{K} - K$, the amount of the bank's assets that needs to be liquidated is $\frac{\bar{K}-K}{\lambda}$ and the remaining amount of the investments becomes $\bar{K} - \frac{\bar{K}-K}{\lambda}$. The total investment profits from the project then becomes $\left(\bar{K} - \frac{\bar{K}-K}{\lambda}\right)\theta$ and the per-unit investment return of the project, R, is:

$$\left[\frac{1}{\lambda} - \frac{(1-\lambda)\,\bar{K}}{\lambda\, K}\right]\theta. \tag{31}$$

Approximating the investment return, R, with a first-order Taylor expansion around $K = \bar{K}$ and $\theta = \bar{\theta}$, gives:

$$R = \bar{\theta} + \frac{\partial R}{\partial K}(K - \bar{K}) + \frac{\partial R}{\partial \theta}(\theta - \bar{\theta}), \tag{32}$$

where $\frac{\partial R}{\partial K} = \frac{1-\lambda}{\lambda}\frac{\bar{\theta}}{\bar{K}} > 0$ which is a positive constant, and $\frac{\partial R}{\partial \theta} = 1$. Denoting $\frac{\partial R}{\partial K}$ as a, I have,

$$R = \theta + a\,K - a\,\bar{K}. \tag{33}$$

This approximation differs from the one I used in the main setting only by a constant, $-a\,\bar{K}$, the inclusion of which doesn't affect the equilibrium characteristics qualitatively.

Appendix II: Proofs for equilibrium existence and uniqueness and sufficient conditions for interior equilibrium

In this appendix, I show that when the shadow bank enters, the equilibrium decisions by the traditional and the shadow bank, $\{m_1^*, a_2^*(m_1), m_2^*(m_1)\}$ always exist and is unique. In addition, I characterize the sufficient conditions for the equilibrium to be interior.

Proposition 8 *The pair of equilibrium decisions by the shadow bank $\{a_2^*(m_1), m_2^*(m_1)\}$ always exists and is unique. For a sufficiently small disclosure cost c_m and a sufficiently low liquidity cost c_a, $m_2^*(m_1)$ is interior such that $m_2^*(m_1) > 0$. For a sufficiently large liquidity cost c_a and $(1-b^2)(1-b^4)a_1^3 + (b^4+2b^2-3)a_1^2 + (3-b^2+b^4)a_1 < 1$, $a_2^*(m_1)$ is interior such that $0 < a_2^*(m_1) < \frac{1}{2}$.*

Proof. I first solve for the equilibrium disclosure precision m_2^* as a function of m_1 and a_2. The shadow bank's objective function can be written as:

$$\Pi_2 = \frac{1}{q}\left[(\beta_2^* + \gamma_2^*) - \frac{1-2a_2}{2}(\beta_2^* + \gamma_2^*)^2\right] - \frac{1-2a_2}{2}\left[\frac{(\gamma_2^*)^2}{m_2} + \frac{b^2(\omega_2^*)^2}{m_1} + \frac{b^2(\lambda_2^* + \omega_2^*)^2}{q}\right] + \bar{\Pi}_2 - \frac{c_m}{2}m_2^2 - \frac{c_a}{2}a_2^2, \tag{34}$$

where $\bar{\Pi}_2 = E[R_2^*]E[K_2^*] - \frac{1}{2}(E[K_2^*])^2$ is a constant independent of m_2. The FOC for m_2 becomes:

$$\frac{\partial \Pi_2}{\partial m_2} = \frac{1}{q}\left[1 - (1-2a_2)(\beta_2^* + \gamma_2^*)\right]\frac{\partial(\beta_2^* + \gamma_2^*)}{\partial m_2} - \frac{1-2a_2}{2}\left[\frac{2\gamma_2^*}{m_2}\frac{\partial \gamma_2^*}{\partial m_2} - \frac{(\gamma_2^*)^2}{m_2^2}\right] - c_m m_2 = 0. \tag{35}$$

Denote

$$B_2(m_2) = \frac{1}{q}\left[(\beta_2^* + \gamma_2^*) - \frac{1-2a_2}{2}(\beta_2^* + \gamma_2^*)^2\right] - \frac{1-2a_2}{2}\left[\frac{(\gamma_2^*)^2}{m_2} + \frac{b^2(\omega_2^*)^2}{m_1} + \frac{b^2(\lambda_2^* + \omega_2^*)^2}{q}\right]. \tag{36}$$

Notice that the FOC, $\frac{\partial \Pi_2}{\partial m_2} = 0$, doesn't include m_1 $\left(\frac{\partial^2 \Pi_2}{\partial m_1 \partial m_2} = 0\right)$. This is because the fundamentals θ_1 and θ_2 are independent of each other. As a result, m_2^* is solved as a function of only a_2. To see whether such solutions exist, I check that at $m_2 = \infty$, $\lim_{m_2 \to \infty} \frac{\partial \Pi_2}{\partial m_2} = -\infty < 0$. However, at $m_2 = 0$, the sign of $\lim_{m_2 \to 0} \frac{\partial \Pi_2}{\partial m_2} = \frac{\partial B_2(0)}{\partial m_2}$ is ambiguous due to the effect of disclosure in magnifying volatility. Consider first the case in which $\frac{\partial B_2(0)}{\partial m_2} > 0$. By the intermediate value theorem, there always exists an interior solution of m_2. There can be multiple local maximum but the shadow bank always chooses the one that maximizes its payoff globally. In addition, the corner solution $m_2 = 0$ cannot be an equilibrium since $\lim_{m_2 \to 0} \frac{\partial \Pi_2}{\partial m_2} > 0$ and the shadow bank wants to deviate at $m_2 = 0$. Thus when $\frac{\partial B_2(0)}{\partial m_2} > 0$, there exists a unique interior solution of m_2^*. Denote the solution as $m_2^{*\prime}(a_2)$.

Now consider the other case $\frac{\partial B_2(0)}{\partial m_2} \leq 0$. Notice that $\frac{\partial B_2(m_2)}{\partial m_2} > 0$ if and only if

$$m_2 > \frac{(1-2a_2)\,\gamma_2^* \frac{\partial \gamma_2^*}{\partial m_2}}{\frac{1}{q}\left[1-(1-2a_2)\left(\beta_2^*+\gamma_2^*\right)\right]\frac{\partial(\beta_2^*+\gamma_2^*)}{\partial m_2}+\frac{1-2a_2}{2}\left(\frac{\gamma_2^*}{m_2}\right)^2}. \tag{37}$$

At $m_2 = \infty$, the RHS of the inequality above is finite while the LHS goes to infinity. Therefore, there exists some sufficiently large m_2^p such that $\frac{\partial B_2\left(m_2^p\right)}{\partial m_2} > 0$. As long as c_m is sufficiently small, I have $\frac{\partial B_2\left(m_2^p\right)}{\partial m_2} > c_m m_2^p$ and hence at $m_2 = m_2^p$, $\frac{\partial \Pi_2}{\partial m_2} > 0$. As a result, there exists at least one local maximum in the interval (m_2^p, ∞) that solves $\frac{\partial \Pi_2}{\partial m_2} = 0$. In equilibrium, the shadow bank always chooses the one that maximizes its payoff globally. Denote the solution as $m_2^{*\prime}(a_2)$. For $m_2^{*\prime}(a_2)$ to be an equilibrium, I need the shadow bank attains a higher payoff at $m_2 = m_2^{*\prime}(a_2)$ than at $m_2 = 0$. This satisfies when the disclosure cost c_m is not too large and the liquidity cost c_a is not too large. To verify this, consider the extreme case $c_m = 0$. I first compare the shadow bank's payoff when it chooses $m_2 = 0$ and $m_2 = \infty$.

$$\begin{aligned} & \lim_{m_2\to\infty} \Pi_2 - \lim_{m_2\to 0} \Pi_2 \\ = & \lim_{m_2\to\infty}\left\{\frac{1}{q}\left[\left(\beta_2^*+\gamma_2^*\right)-\frac{1-2a_2}{2}\left(\beta_2^*+\gamma_2^*\right)^2\right]\right\} \\ & -\lim_{m_2\to 0}\left\{\frac{1}{q}\left[\left(\beta_2^*+\gamma_2^*\right)-\frac{1-2a_2}{2}\left(\beta_2^*+\gamma_2^*\right)^2\right]\right\}, \end{aligned} \tag{38}$$

where other terms in Π_2 are cancelled out. Notice that investors' total sensitivity to information $\beta_2^*+\gamma_2^*$ is strictly increasing in m_2 and the function $(\beta_2^*+\gamma_2^*)-\frac{1-2a_2}{2}(\beta_2^*+\gamma_2^*)^2$ is strictly increasing in $\beta_2^*+\gamma_2^*$ if and only if $\beta_2^*+\gamma_2^* < \frac{1}{1-2a_2}$. In addition, at $m_2 = \infty$,

$$\beta_2^*+\gamma_2^* = \frac{1-\left(1-b^2\right)a_1}{1-a_2-a_1\left[1-\left(1-b^2\right)a_2\right]}, \tag{39}$$

which could be higher than $\frac{1}{1-2a_2}$. That is, competition may increase investors' informational sensitivity up to a point that is even higher than the optimal level for the shadow bank, which is $\frac{1}{1-2a_2}$. In that situation, disclosure has an additional detrimental effect of "overshooting" informational sensitivities, which can lead to a zero disclosure equilibrium. To rule out such cases and make the equilibrium interior, I assume that c_a is not too large such that the equilibrium a_2^* is larger than $\frac{b^2 a_1}{1-a_1+b^2 a_1}$, which makes

$$\frac{1-\left(1-b^2\right)a_1}{1-a_2^*-a_1\left[1-\left(1-b^2\right)a_2^*\right]} < \frac{1}{1-2a_2^*}. \tag{40}$$

In this case, $\lim_{m_2\to\infty}\Pi_2 - \lim_{m_2\to 0}\Pi_2 > 0$. At the interior solution $m_2^{*\prime}(a_2)$, the shadow bank should attain a payoff not lower than $\lim_{m_2\to\infty}\Pi_2$, which is strictly higher than $\lim_{m_2\to 0}\Pi_2$. Therefore, $m_2 = 0$ cannot be an equilibrium.

Similarly, I now solve for the equilibrium liquidity risk a_2^* as a function of m_1 and m_2.

$$\begin{aligned}\frac{\partial \Pi_2}{\partial a_2} &= \frac{1}{q}\left[1-(1-2a_2)\left(\beta_2^*+\gamma_2^*\right)\right]\frac{\partial(\beta_2^*+\gamma_2^*)}{\partial a_2} \\ &\quad -(1-2a_2)\left[\frac{\gamma_2^*}{m_2}\frac{\partial\gamma_2^*}{\partial a_2}+b^2\frac{\omega_2^*}{m_1}\frac{\partial\omega_2^*}{\partial a_2}+b^2\frac{(\lambda_2^*+\omega_2^*)}{q}\frac{\partial(\lambda_2^*+\omega_2^*)}{\partial a_2}\right] \\ &\quad +\left[\frac{(\gamma_2^*)^2}{m_2}+\frac{b^2(\omega_2^*)^2}{m_1}+\frac{b^2(\lambda_2^*+\omega_2^*)^2}{q}+\frac{(\beta_2^*+\gamma_2^*)^2}{q}\right]-c_a a_2,\end{aligned} \tag{41}$$

at $a_2 = 0$, this becomes

$$\begin{aligned}\frac{\partial \Pi_2}{\partial a_2} &= \frac{1}{q}\left[1-\left(\beta_2^*+\gamma_2^*\right)\right]\frac{\partial(\beta_2^*+\gamma_2^*)}{\partial a_2}-\left[\frac{\gamma_2^*}{m_2}\frac{\partial\gamma_2^*}{\partial a_2}+b^2\frac{\omega_2^*}{m_1}\frac{\partial\omega_2^*}{\partial a_2}+b^2\frac{(\lambda_2^*+\omega_2^*)}{q}\frac{\partial(\lambda_2^*+\omega_2^*)}{\partial a_2}\right] \\ &\quad +\left[\frac{(\gamma_2^*)^2}{m_2}+\frac{b^2(\omega_2^*)^2}{m_1}+\frac{b^2(\lambda_2^*+\omega_2^*)^2}{q}+\frac{(\beta_2^*+\gamma_2^*)^2}{q}\right]+\frac{\partial\bar{\Pi}_2}{\partial a_2},\end{aligned} \tag{42}$$

which is positive if and only if

$$\begin{aligned}&\left[1-\left(\beta_2^*+\gamma_2^*\right)\right]\frac{\partial(\beta_2^*+\gamma_2^*)}{\partial a_2}+\left(\beta_2^*+\gamma_2^*\right)^2+b^2\left(\lambda_2^*+\omega_2^*\right)\left[\left(\lambda_2^*+\omega_2^*\right)-\frac{\partial(\lambda_2^*+\omega_2^*)}{\partial a_2}\right] \\ &> q\left\{\frac{\gamma_2^*}{m_2}\frac{\partial\gamma_2^*}{\partial a_2}+b^2\frac{\omega_2^*}{m_1}\frac{\partial\omega_2^*}{\partial a_2}-\frac{\partial\bar{\Pi}_2}{\partial a_2}\frac{(\gamma_2^*)^2}{m_2}-\frac{b^2(\omega_2^*)^2}{m_1}\right\}.\end{aligned} \tag{43}$$

When q is close to zero, this inequality is reduced into

$$\left[1-\left(\beta_2^*+\gamma_2^*\right)\right]\frac{\partial(\beta_2^*+\gamma_2^*)}{\partial a_2}+\left(\beta_2^*+\gamma_2^*\right)^2+b^2\left(\lambda_2^*+\omega_2^*\right)\left[\left(\lambda_2^*+\omega_2^*\right)-\frac{\partial(\lambda_2^*+\omega_2^*)}{\partial a_2}\right]>0, \tag{44}$$

and becomes

$$-\frac{\left(1-b^2\right)\left(1-b^4\right)a_1^3+\left(b^4+2b^2-3\right)a_1^2+\left(3-b^2+b^4\right)a_1-1}{\left(1-a_1\right)^3}>0, \tag{45}$$

which holds if and only if $\left(1-b^2\right)\left(1-b^4\right)a_1^3+\left(b^4+2b^2-3\right)a_1^2+\left(3-b^2+b^4\right)a_1<1$. At $a_2=\frac{1}{2}$, $\lim_{a_2\to\frac{1}{2}}\frac{\partial\Pi_2}{\partial a_2}<0$ if c_a is sufficiently large. By the intermediate value theorem, there exists at least one local maximum that solves $\frac{\partial\Pi_2}{\partial a_2}$. In equilibrium, the shadow bank always chooses the one that maximizes its payoff globally. Denote the solution as $a_2^{*\prime}(m_1,m_2)$.

The equilibrium $\left(a_2^*\left(m_1\right), m_2^*\left(m_1\right)\right)$ needs to satisfy that $a_2^*\left(m_1\right) = a_2^{*\prime}\left(m_1, m_2^*\left(m_1\right)\right)$ and $m_2^*\left(m_1\right) = m_2^{*\prime}\left(a_2^*\left(m_1\right)\right)$. These two conditions reduce into

$$a_2^*\left(m_1\right) = a_2^{*\prime}\left(m_1, m_2^{*\prime}\left(a_2^*\left(m_1\right)\right)\right). \tag{46}$$

A solution of $a_2^*\left(m_1\right)$ always exists as a result of the Kakutani's fixed point theorem. There can be multiple solutions but the shadow bank always chooses the pair of $\left(a_2^*\left(m_1\right), m_2^*\left(m_1\right)\right)$ that attains the highest Π_2. As a result, the equilibrium is always unique. ■

Proposition 9 *The equilibrium decision by the traditional bank m_1^* always exists and is unique. For a sufficiently small disclosure cost c_m and a sufficiently high liquidity risk a_1, m_1^* is interior such that $m_1^* > 0$.*

Proof. Following similar steps to the proof for m_2^*, I can show that there exists an unique interior solution of m_1^* to $\frac{d\Pi_1}{dm_1} = 0$ when c_m is sufficiently low and a_1 is sufficiently large. There can be multiple solutions but the traditional bank always chooses m_1^* that attains the highest Π_1. As a result, the equilibrium is always unique. ■

Appendix III: Proofs

Proof of Proposition 1

Proof. The proof is a special case of Proposition 3. ■

Proof Lemma 1

Proof. Notice that the payoff to the traditional bank without the disclosure cost, B_1^M, is equal to

$$B_1^M = \left[\left(\beta_1^{*D} + \gamma_1^{*D}\right) - \frac{1-2a_1}{2}(\beta_1^{*D} + \gamma_1^{*D})^2\right]\frac{1}{q} - \frac{1-2a_1}{2}\frac{\left(\gamma_1^{*D}\right)^2}{m_1} + \bar{\Pi}_1^M. \tag{47}$$

Taking the derivative with respect to m_1 gives:

$$\frac{\partial B_1^M}{\partial m_1} = \left[1-(1-2a_1)\left(\beta_1^{*D}+\gamma_1^{*D}\right)\right]\frac{\partial\left(\beta_1^{*D}+\gamma_1^{*D}\right)}{\partial m_1}\frac{1}{q} + \frac{1-2a_1}{2}\left[\frac{\left(\gamma_1^{*D}\right)^2}{m_1^2} - \frac{2\left(\gamma_1^{*D}\right)}{m_1}\frac{\partial\gamma_1^{*D}}{\partial m_1}\right]. \tag{48}$$

The expression can be further simplified into

$$\frac{\partial B_1^M}{\partial m_1} = \frac{m_1 + q - (1-a_1)(1-4a_1)n_1}{2(1-a_1)^2\left[q+m_1+(1-a_1)n_1\right]^3}. \tag{49}$$

Therefore, if $q-(1-a_1)(1-4a_1)n_1 > 0$, $\frac{\partial B_1^M}{\partial m_1} > 0$. Otherwise, $\frac{\partial B_1^M}{\partial m_1} > 0$ if and only if $m_1 > (1-a_1)(1-4a_1)n_1 - q$. Furthermore, it can be verified that $q-(1-a_1)(1-4a_1)n_1 > 0$ if and only if $a_1 \geq \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1}+9}$. ■

Proof of Proposition 2

Proof. Notice that the optimal disclosure precision m_1^{*M} is given by the following first-order condition:

$$\frac{\partial \Pi_1^M}{\partial m_1} = \frac{\partial B_1^M}{\partial m_1} - c_m m_1 = 0, \tag{50}$$

from the proof of Lemma 1, the FOC can be simplified into:

$$\frac{\partial \Pi_1^M}{\partial m_1} = \frac{m_1 + q - (1-a_1)(1-4a_1)n_1}{2(1-a_1)^2\left[q+m_1+(1-a_1)n_1\right]^3} - c_m m_1. \tag{51}$$

Notice that at $m_1 = \infty$, $\lim_{m_1\to\infty}\frac{\partial\Pi_1^M}{\partial m_1} = -\infty < 0$. At $m_1 = 0$, $\lim_{m_1\to 0}\frac{\partial\Pi_1^M}{\partial m_1} = \frac{q-(1-a_1)(1-4a_1)n_1}{2(1-a_1)^2[q+(1-a_1)n_1]^3}$ which is positive if and only $q-(1-a_1)(1-4a_1)n_1 > 0$ ($a_1 \geq \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1}+9}$). When $a_1 \geq \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1}+9}$, by the intermediate value theorem, there always exists an interior solution of m_1

to $\frac{\partial \Pi_1^M}{\partial m_1} = 0$. In addition, when $a_1 \geq \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1} + 9}$, it can be verified that the quartic equation $\frac{\partial \Pi_1^M}{\partial m_1} = 0$ has a unique positive solution for any $c_m > 0$. Notice also that in this case, the corner solution $m_1 = 0$ cannot be an equilibrium since $\lim_{m_1 \to 0} \frac{\partial \Pi_1^M}{\partial m_1} > 0$ and the traditional bank wants to deviate at $m_1 = 0$. Thus when $a_1 \geq \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1} + 9}$, the equilibrium m_1^{*M} is unique and interior.

Now consider the case when $a_1 < \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1} + 9}$. The equation $\frac{\partial \Pi_1^M}{\partial m_1} = 0$ can be reduced into a quartic equation. Applying the Descartes rule of signs, I find that when $c_m > \frac{1}{2(1-a_1)^2[q+m_1+(1-a_1)n_1]^3}$, $\frac{\partial \Pi_1^M}{\partial m_1} = 0$ has no positive root, which implies that the only equilibrium is $m_1^{*M} = 0$. When $c_m < \frac{1}{2(1-a_1)^2[q+m_1+(1-a_1)n_1]^3}$, $\frac{\partial \Pi_1^M}{\partial m_1} = 0$ has two positive roots. Denote the two roots as m_1^s and m_1^l, such that $m_1^l > m_1^s$. Since $\lim_{m_1 \to 0} \frac{\partial \Pi_1^M}{\partial m_1} < 0$ and $\lim_{m_1 \to \infty} \frac{\partial \Pi_1^M}{\partial m_1} < 0$, it is straightforward to verify that m_1^s is a local minimum of Π_1^M while m_1^l is a local maximum. For m_1^l to be an equilibrium, I need the traditional bank attains a higher payoff at $m_1 = m_1^l$ than at $m_1 = 0$, which holds when the cost c_m is smaller than a threshold $\hat{c}_m'$. Therefore, define the threshold $\hat{c}_m^M = \min\left(\frac{1}{2(1-a_1)^2[q+m_1+(1-a_1)n_1]^3}, \hat{c}_m'\right)$. When $c_m \geq \hat{c}_m^M$, the unique equilibrium is $m_1^{*M} = 0$; when $c_m < \hat{c}_m^M$, the unique equilibrium is $m_1^{*M} = m_1^l$.

To summarize, when $c_m < \hat{c}_m^M$, the equilibrium is interior and unique. When $c_m > \hat{c}_m^M$, if $a_1 \geq \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1} + 9}$, the equilibrium remains interior and unique while if $a_1 < \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n_1} + 9}$, the unique equilibrium is $m_1^{*M} = 0$. ■

Proof of Proposition 3

Proof. The general idea is that I let an individual investor form a conjecture on equilibrium investments, which is linear in all signals in the information set. The investor then decides her own optimal investments given this conjecture. In a rational expectation equilibrium, the investor's conjecture must be consistent with the individual optimal investments in equilibrium. Therefore, comparing the coefficients in the linear conjecture with the coefficients in the individual optimal investment determines the unknown coefficients in the investor's conjecture. I further demonstrate that this linear equilibrium is the unique equilibrium using the higher-order-belief approach developed in Morris and Shin (2002). Thus, as a first step, each individual forms a linear conjecture on equilibrium investments

$$
\begin{aligned}
k_{1j} &= (\beta_1 x_{1j} + \gamma_1 z_1) - b(\omega_1 z_2 + \lambda_1 x_{2j}) + h_1, \qquad (52)\\
k_{2j} &= (\beta_2 x_{2j} + \gamma_2 z_2) - b(\omega_2 z_2 + \lambda_2 x_{2j}) + h_2,
\end{aligned}
$$

then the aggregate investments become

$$
\begin{aligned}
K_1 &= (\beta_1 \theta_1 + \gamma_1 z_1) - b(\omega_1 z_2 + \lambda_1 \theta_2) + h_1, \\
K_2 &= (\beta_2 \theta_2 + \gamma_2 z_2) - b(\omega_2 z_2 + \lambda_2 \theta_1) + h_2,
\end{aligned} \tag{53}
$$

investor j's conditional expectations of the aggregate investment are

$$
\begin{aligned}
E_j[K_1] &= \beta_1 \left(\frac{m_1 z_1 + n_1 x_{1j} + q\bar{\theta}}{m_1 + n_1 + q} \right) + \gamma_1 z_1 - b \left[\lambda_1 \left(\frac{m_2 z_2 + n_2 x_{2j} + q\bar{\theta}}{m_2 + n_2 + q} \right) + \omega_1 z_2 \right] + h_1, \\
E_j[K_2] &= \beta_2 \left(\frac{m_2 z_2 + n_2 x_{2j} + q\bar{\theta}}{m_2 + n_2 + q} \right) + \gamma_2 z_2 - b \left[\lambda_2 \left(\frac{m_1 z_1 + n_1 x_{1j} + q\bar{\theta}}{m_1 + n_1 + q} \right) + \omega_2 z_1 \right] + h_2,
\end{aligned} \tag{54}
$$

thus the individual investments become

$$
\begin{aligned}
k_{1j} &= E_j R_1 - b E_j R_2 \\
&= E_j[\theta_1] + a_1 E_j[K_1] - b(E_j[\theta_2] + a_2 E_j[K_2]) \\
&= \left(\frac{m_1 z_1 + n_1 x_{1j} + q\bar{\theta}}{m_1 + n_1 + q} \right) + a_1 E_j[K_1] \\
&\quad - b \left[\left(\frac{m_2 z_2 + n_2 x_{2j} + q\bar{\theta}}{m_2 + n_2 + q} \right) + a_2 E_j[K_2] \right],
\end{aligned} \tag{55}
$$

and

$$
\begin{aligned}
k_{2j} &= E_j R_2 - b E_j R_1 \\
&= E_j[\theta_2] + a_2 E_j[K_2] - b(E_j[\theta_1] + a_1 E_j[K_1]) \\
&= \frac{m_2 z_2 + n_2 x_{2j} + q\bar{\theta}}{m_2 + n_2 + q} + a_2 E_j[K_2] \\
&\quad - \left[b \left(\frac{m_1 z_1 + n_1 x_{1j} + q\bar{\theta}}{m_1 + n_1 + q} \right) + a_1 E_j[K_1] \right],
\end{aligned} \tag{56}
$$

where $E_j[K_1]$ and $E_j[K_2]$ are given by equation (54). Comparing the coefficients in (52) and (55), I obtain[17]

$$
\begin{aligned}
\beta_i^* &= \beta_i^{*D}\left(1+\frac{\frac{n_i\left(1+\beta_i^{*D}a_i\right)}{q+m_i+(1-a_l)n_i}}{1-\frac{n_i^2}{[q+m_i+(1-a_i)n_i][q+m_i+(1-a_k)n_i]}a_ia_lb^2}a_lb^2\right), \qquad (57)\\
\gamma_i^* &= \gamma_i^{*D}\left(1+\frac{a_ia_l}{(1-a_i)(1-a_l)}b^2\right)\left(1+\frac{\frac{n_i[1+(1-a_l)\beta_i^{*D}a_i]}{(1-a_l)[q+m_i+(1-a_l)n_i]}+\frac{1}{1-a_l}}{1-\frac{n_i^2}{[q+m_i+(1-a_i)n_i][q+m_i+(1-a_l)n_i]}a_ia_lb^2}a_lb^2\right),\\
\lambda_i^* &= \beta_l^{*D}\left(1+\frac{\frac{n_l\left(1+\beta_l^{*D}a_lb^2\right)}{q+m_l+(1-a_i)n_l}}{1-\frac{n_l^2}{[q+m_l+(1-a_l)n_l][q+m_l+(1-a_i)n_l]}a_la_ib^2}a_i\right),\\
\omega_i^* &= \gamma_l^{*D}\left(1+\frac{a_ia_l}{(1-a_i)(1-a_l)}b^2\right)\left(1+\frac{\frac{n_l\left[1-a_l+\left[1+(1-a_i)\beta_l^{*D}\right]a_lb^2\right]}{(1-a_i)[q+m_l+(1-a_i)n_l]}+\frac{1}{1-a_i}}{1-\frac{n_l^2}{[q+m_l+(1-a_l)n_l][q+m_l+(1-a_i)n_l]}a_la_ib^2}a_i\right)\\
h_i^* &= \left(\frac{\gamma_i^*}{m_i}-\frac{b\omega_i^*}{m_l}\right)q\bar{\theta}.
\end{aligned}
$$

The other case when the shadow bank is deterred from entry can be derived similarly.

I now follow the higher-order-belief approach outlined in Morris and Shin (2002) and show that this linear equilibrium is indeed the unique equilibrium. I first show that the k-th order expectation of the fundamentals θ_i, by the group of investors, takes the following functional form, where $i \in \{1,2\}$,

$$\bar{E}^k[\theta_i] = \delta_{ik}\, z_i + l_{ik}\,\theta_i + r_{ik}\,\bar{\theta}, \qquad (58)$$

where

$$
\begin{aligned}
\delta_{ik} &= \frac{m_i}{m_i+q}\left[1-\left(\frac{n_i}{m_i+n_i+q}\right)^k\right], \qquad (59)\\
l_{ik} &= \left(\frac{n_i}{m_i+n_i+q}\right)^k,\\
r_{ik} &= \frac{q}{m_i+q}\left[1-\left(\frac{n_i}{m_i+n_i+q}\right)^k\right].
\end{aligned}
$$

This can be shown by induction. At $k=1$, I know an individual j's expectation of θ_i is:

$$E_j(\theta_i) = \frac{m_iz_i+n_ix_{ij}+q\bar{\theta}}{m_i+n_i+q}, \qquad (60)$$

[17] With some abuse of notation, I let $\beta_2^{*D} = \frac{n_2}{q+m_2+(1-a_2)n_2}$ and $\gamma_2^{*D} = \frac{m_2}{(1-a_2)[q+m_2+(1-a_2)n_2]}$.

therefore, the expectation of θ_i by the group of investors becomes

$$\bar{E}[\theta_i] = \int_0^1 E_j(\theta_i)di = \frac{m_i z_i + n_i \theta_i + q\bar{\theta}}{m_i + n_i + q}. \tag{61}$$

Now suppose (58) holds for $k-1$. Then

$$\begin{aligned} E_j[\bar{E}^{k-1}[\theta_i]] &= E_j[\delta_{ik-1}\, z_i + l_{ik-1}\, \theta_i + r_{ik-1}\, \bar{\theta}] \\ &= \delta_{ik-1}\, z_i + r_{ik-1}\, \bar{\theta} + l_{ik-1}\, E_j[\theta_i] \\ &= \delta_{ik-1}\, z_i + r_{ik-1}\, \bar{\theta} + \frac{m_i z_i + n_i x_{ij} + q\bar{\theta}}{m_i + n_i + q}, \end{aligned} \tag{62}$$

therefore,

$$\bar{E}^k[\theta_i] = \delta_{ik-1}\, z_i + r_{ik-1}\, \bar{\theta} + \frac{m_i z_i + n_i \theta_i + q\bar{\theta}}{m_i + n_i + q}, \tag{63}$$

and after a few simplifying steps, this gives

$$\bar{E}^k[\theta_i] = \delta_{ik}\, z_i + l_{ik}\, \theta_i + r_{ik}\, \bar{\theta}, \tag{64}$$

which concludes the proof on the linear form of $\bar{E}^k[\theta_i]$. In addition, notice also the individual optimal investments can be rewritten as:

$$\boldsymbol{k}_j = \boldsymbol{A}\, \boldsymbol{E}_j[\boldsymbol{\theta}] + \boldsymbol{B}\, \boldsymbol{E}_j[\boldsymbol{K}], \tag{65}$$

where

$$\begin{aligned} \boldsymbol{k}_j &= \begin{bmatrix} k_{1j} \\ k_{2j} \end{bmatrix}, \\ \boldsymbol{E}_j[\boldsymbol{\theta}] &= \begin{bmatrix} E_j[\theta_1] \\ E_j[\theta_2] \end{bmatrix}, \\ \boldsymbol{E}_j[\boldsymbol{K}] &= \begin{bmatrix} E_j[K_1] \\ E_j[K_2] \end{bmatrix}, \\ \boldsymbol{A} &= \begin{bmatrix} 1 & -b \\ -b & 1 \end{bmatrix}, \\ \boldsymbol{B} &= \begin{bmatrix} a_1 & -ba_2 \\ -ba_1 & a_2 \end{bmatrix}, \end{aligned} \tag{66}$$

and the aggregate investment can be similarly written as:

$$\boldsymbol{K} = \boldsymbol{A}\, \bar{\boldsymbol{E}}[\boldsymbol{\theta}] + \boldsymbol{B}\, \bar{\boldsymbol{E}}[\boldsymbol{K}]. \tag{67}$$

Substituting the aggregate investment into the individual investment gives

$$\begin{aligned}\boldsymbol{k}_j &= \boldsymbol{A}\,\boldsymbol{E}_j[\boldsymbol{\theta}] + \boldsymbol{B}\,\boldsymbol{A}\,\boldsymbol{E}_j\bar{\boldsymbol{E}}[[\boldsymbol{\theta}]] + \boldsymbol{B}^2\,\boldsymbol{A}\,\boldsymbol{E}_j\,[\bar{\boldsymbol{E}}^{\boldsymbol{2}}[\boldsymbol{\theta}]] + \,... \\ &= \sum_{k=0}^{\infty} \boldsymbol{B}^k\,\boldsymbol{A}\,\boldsymbol{E}_j[\bar{\boldsymbol{E}}^{\boldsymbol{k}}[\boldsymbol{\theta}]],\end{aligned} \tag{68}$$

where I have shown that

$$\bar{\boldsymbol{E}}^k\,[\boldsymbol{\theta}] = \begin{bmatrix}\delta_{1k}\,z_1 + l_{1k}\,\theta_1 + r_{1k}\,\bar{\theta} \\ \delta_{2k}\,z_2 + l_{2k}\,\theta_2 + r_{2k}\,\bar{\theta}\end{bmatrix}. \tag{69}$$

It can also be verified that given $0 < a_1 < \frac{1}{2}, 0 < a_2 < \frac{1}{2}$, and $0 < b < 1$, the eigenvalues of $\boldsymbol{B}$ are all between 0 and 1. Therefore, the sum $\sum_{k=0}^{\infty} \boldsymbol{B}^k\,\boldsymbol{A}\,\boldsymbol{E}_j[\bar{\boldsymbol{E}}^{\boldsymbol{k}}[\boldsymbol{\theta}]]$ is converging. After a few simplifying steps, this sum reduces to the exact linear forms as in Proposition 3 and hence I have verified the uniqueness of the linear equilibrium. Equation (68) shows well that the importance of the higher-order beliefs in determining the investments is dictated by the convergence rate of the matrix, $\boldsymbol{B}$. ■

Proof of Corollary 2

Proof. These can be shown by directly computing the derivatives. ■

Proof of Proposition 4

Proof. In equilibrium, the first-order conditions for the shadow bank are given as follows:

$$\begin{aligned}\frac{\partial \Pi_2}{\partial m_2} &= 0, \\ \frac{\partial \Pi_2}{\partial a_2} &= 0.\end{aligned} \tag{70}$$

By the implicit function theorem, the following holds:

$$\begin{aligned}\frac{\partial^2 \Pi_2}{\partial m_2 \partial m_1} + \frac{\partial^2 \Pi_2}{\partial m_2^2}\frac{\partial m_2^*}{\partial m_1} + \frac{\partial^2 \Pi_2}{\partial m_2 \partial a_2}\frac{\partial a_2^*}{\partial m_1} &= 0, \\ \frac{\partial^2 \Pi_2}{\partial a_2 \partial m_1} + \frac{\partial^2 \Pi_2}{\partial a_2^2}\frac{\partial a_2^*}{\partial m_1} + \frac{\partial^2 \Pi_2}{\partial a_2 \partial m_2}\frac{\partial m_2^*}{\partial m_1} &= 0.\end{aligned} \tag{71}$$

It can be verified that $\frac{\partial^2 \Pi_2}{\partial m_2 \partial m_1} = 0$. Hence,

$$\begin{aligned} \frac{\partial a_2^*}{\partial m_1} &= -\frac{\frac{\partial^2 \Pi_2}{\partial a_2 \partial m_1}\frac{\partial^2 \Pi_2}{\partial m_2^2}}{\frac{\partial^2 \Pi_2}{\partial a_2^2}\frac{\partial^2 \Pi_2}{\partial m_2^2} - \frac{\partial^2 \Pi_2}{\partial a_2 \partial m_2}\frac{\partial^2 \Pi_2}{\partial m_2 \partial a_2}}, \\ \frac{\partial m_2^*}{\partial m_1} &= \frac{\frac{\partial^2 \Pi_2}{\partial a_2 \partial m_1}\frac{\partial^2 \Pi_2}{\partial m_2 \partial a_2}}{\frac{\partial^2 \Pi_2}{\partial a_2^2}\frac{\partial^2 \Pi_2}{\partial m_2^2} - \frac{\partial^2 \Pi_2}{\partial a_2 \partial m_2}\frac{\partial^2 \Pi_2}{\partial m_2 \partial a_2}}. \end{aligned} \tag{72}$$

Notice first that since (a_2^*, m_2^*) maximizes Π_2, the associated second-order condition requires that

$$\begin{aligned} \frac{\partial^2 \Pi_2}{\partial a_2^2}\frac{\partial^2 \Pi_2}{\partial m_2^2} - \frac{\partial^2 \Pi_2}{\partial a_2 \partial m_2}\frac{\partial^2 \Pi_2}{\partial m_2 \partial a_2} &> 0, \\ \frac{\partial^2 \Pi_2}{\partial a_2^2} &< 0, \\ \frac{\partial^2 \Pi_2}{\partial m_2^2} &< 0. \end{aligned} \tag{73}$$

In addition, substituting $\Pi_2 = Cov(\theta_2 + a_2 s_2', s_2') - \frac{1}{2}Var(s_2') + \bar{\Pi}_2 - \frac{1-2a_2}{2}Var(s_1')$, gives,

$$\begin{aligned} \frac{\partial^2 \Pi_2}{\partial a_2 \partial m_1} &= -\frac{\partial^2}{\partial a_2 \partial m_1}\left(\frac{1-2a_2}{2}Var(s_1')\right) \\ &= \frac{\partial}{\partial a_2}\left(-\frac{1-2a_2}{2}\right)\frac{\partial Var(s_1')}{\partial m_1} - \frac{1-2a_2}{2}\frac{\partial}{\partial a_2}\left(\frac{\partial Var(s_1')}{\partial m_1}\right) \\ &= \frac{\partial Var(s_1')}{\partial m_1} - \frac{1}{2}(1-2a_2)\frac{\partial^2 Var(s_1')}{\partial a_2 \partial m_1}, \end{aligned} \tag{74}$$

where the first term $\frac{\partial Var(s_1')}{\partial m_1}$ corresponds to the *weighting effect* and the second term $\frac{1}{2}(1 - 2a_2)\frac{\partial^2 Var(s_1')}{\partial a_2 \partial m_1}$ corresponds to the *overreaction effect.* It can be verified that when n_1 is sufficiently large, there exists a threshold $\hat{b}_1$ such that $b > \hat{b}_1$, $\frac{1}{2}(1-2a_2)\frac{\partial^2 Var(s_1')}{\partial a_2 \partial m_1} > \frac{\partial Var(s_1')}{\partial m_1}$, which makes $\frac{\partial^2 \Pi_2}{\partial a_2 \partial m_1} < 0$ and $\frac{\partial a_2^*}{\partial m_1} < 0$. Similarly, I also verify that when n_2 is sufficiently large, there exists a threshold $\hat{b}_2$ such that $b > \hat{b}_2$, $\frac{\partial^2 \Pi_2}{\partial a_2 \partial m_2} < 0$ and $\frac{\partial^2 \Pi_2}{\partial a_2 \partial m_1} < 0$, which makes $\frac{\partial m_2^*}{\partial m_1} > 0$. ■

Proof of Proposition 5

Proof. By the envelope theorem, I have

$$\frac{d\Pi_2\left(m_2^*\left(m_1\right), a_2^*\left(m_1\right), m_1\right)}{dm_1} = \frac{\partial \Pi_2(m_2, a_2, m_1)}{\partial m_1}\Big|_{m_2 = m_2^*(m_1), a_2 = a_2^*(m_1)} < 0, \tag{75}$$

where the last inequality can be verified by directly computing the derivative. That is, $\Pi_2(m_2^*(m_1), a_2^*(m_1), m_1)$ is strictly decreasing in m_1. Since the entry is not blockaded by the monopoly choice of m_1^{*M} by the traditional bank, I have

$$\Pi_2\left(m_2^*\left(m_1^{*M}\right), a_2^*\left(m_1^{*M}\right), m_1^{*M}\right) > \overline{U}. \tag{76}$$

I also assume that $\overline{U}$ is not too small such that there exists some choice of m_1 that deters entry. In particular, since $\frac{d\Pi_2(m_2^*(m_1), a_2^*(m_1), m_1)}{dm_1} < 0$, entry must be deterred at least at $m_1 = \infty$. That is,

$$\lim_{m_1 \to \infty} \Pi_2(m_2^*(m_1), a_2^*(m_1), m_1) < \overline{U}. \tag{77}$$

By the intermediate value theorem, there exists a unique $m_1^D > m_1^{*M}$ that solves $\Pi_2(m_2^*(m_1), a_2^*(m_1), m_1) = 0$. In addition, the shadow bank decides to enter if and only if $m_1 < m_1^D$. ■

Proof of Proposition 6

Proof. I first characterize the disclosure decision of the traditional bank when it accommodates entry. The equilibrium m_1^* is given by solving the following optimization program,

$$\begin{aligned} &\max_{m_1} \Pi_1\left(m_1, m_2^*\left(m_1\right), a_2^*\left(m_1\right)\right), \\ &s.t.\, m_1 < m_1^D. \end{aligned} \tag{78}$$

In Appendix II, I show that there exists a unique interior solution m_1^* to $\frac{d\Pi_1}{dm_1} = \frac{\partial \Pi_1}{\partial m_1} + \frac{\partial \Pi_1}{\partial m_2}\frac{\partial m_2^*}{\partial m_1} + \frac{\partial \Pi_1}{\partial a_2}\frac{\partial a_2^*}{\partial m_1} = 0$. In addition, from Proposition 7, $m_1^* < m_1^{*M}$ and from Proposition 5, $m_1^{*M} < m_1^D$. Thus $m_1^* < m_1^D$. Therefore, m_1^* is indeed the equilibrium. Now consider the case when the traditional bank deters entry. The equilibrium m_1^{*D} is given by solving

$$\begin{aligned} &\max_{m_1} \Pi_1^M\left(m_1\right), \\ &s.t.\, m_1 \geq m_1^D. \end{aligned} \tag{79}$$

Given the entry is not blockaded, $m_1^D > m_1^{*M}$. Since the payoff of the traditional bank, Π_1^M, is concave in m_1 and when the entry is deterred, Π_1 is maximized at the monopoly choice, m_1^{*M}, it is optimal for the traditional bank to set $m_1^{*D} = m_1^D$ since given $m_1^D > m_1^{*M}$, a further deviation from m_1^{*M} would impair the traditional bank's payoff. Therefore, $m_1^{*D} = m_1^D$.

In equilibrium, the traditional bank chooses between m_1^* and m_1^{*D} by comparing the payoff from accommodating entry $\Pi_1(m_1^*, m_2^*(m_1^*), a_2^*(m_1^*))$ with that from deterring entry $\Pi_1^M\left(m_1^{*D}\right)$. Before comparing $\Pi_1(m_1^*, m_2^*(m_1^*), a_2^*(m_1^*))$ and $\Pi_1^M\left(m_1^{*D}\right)$, I first compare $\Pi_1(m_1, m_2, a_2)$ and $\Pi_1\left(m_1\right)$ for any set of $\{m_1, m_2, a_2\}$. Recall that

$$\begin{aligned}
\Pi_1(m_1, m_2, a_2) &= \frac{1}{q}\left[(\beta_1^* + \gamma_1^*) - \frac{1-2a_1}{2}(\beta_1^* + \gamma_1^*)^2\right] \qquad (80)\\
&\quad - \frac{1-2a_1}{2}\left[\frac{(\gamma_1^*)^2}{m_1} + \frac{(\omega_1^*)^2}{m_2} + \frac{(\lambda_1^* + \omega_1^*)^2}{q}\right] + \bar{\Pi}_1 \\
&= \frac{1}{2q(1-2a_1)} - \frac{1-2a_1}{2q}\left[(\beta_1^* + \gamma_1^*) - \frac{1}{1-2a_1}\right]^2 \\
&\quad - \frac{1-2a_1}{2}\left[\frac{(\gamma_1^*)^2}{m_1} + \frac{(\omega_1^*)^2}{m_2} + \frac{(\lambda_1^* + \omega_1^*)^2}{q}\right] + \bar{\Pi}_1,
\end{aligned}$$

Similarly,

$$\Pi_1^M(m_1) = \frac{1}{2q(1-2a_1)} - \frac{1-2a_1}{2q}\left[(\beta_1^{*D} + \gamma_1^{*D}) - \frac{1}{1-2a_1}\right]^2 - \frac{1-2a_1}{2}\frac{\left(\gamma_1^{*D}\right)^2}{m_1} + \bar{\Pi}_1^M, \qquad (81)$$

By observing the expressions of Π_1 and Π_1^M, it is straightforward to verify their continuity in a_1. For any set of $\{m_1, m_2, a_2\}$, $\Pi_1(m_1, m_2, a_2|a_1) > \Pi_1^M(m_1|a_1)$ if and only if

$$\begin{aligned}
&\left[(\beta_1^* + \gamma_1^*) - (\beta_1^{*D} + \gamma_1^{*D})\right]\left[1 - \frac{(\beta_1^{*D} + \gamma_1^{*D}) + (\beta_1^* + \gamma_1^*)}{2}(1-2a_1)\right] \qquad (82)\\
> \; & q\left[\bar{\Pi}_1^M - \bar{\Pi}_1 + \frac{1-2a_1}{2}\left(\frac{(\gamma_1^*)^2}{m_1} - \frac{\left(\gamma_1^{*D}\right)^2}{m_1} + \frac{(\omega_1^*)^2}{m_2} + \frac{(\lambda_1^* + \omega_1^*)^2}{q}\right)\right].
\end{aligned}$$

The LHS of condition (82) can be viewed as the benefit of accommodating entry resulted from the multiplier effect shown in Corollary 2. Notice that because of the multiplier effect, $\beta_1^* + \gamma_1^* > \beta_1^{*D} + \gamma_1^{*D}$. The expression suggests that this multiplier effect benefits the bank only when the investor inertia is sufficiently large, $\frac{(\beta_1^{*D} + \gamma_1^{*D}) + (\beta_1^* + \gamma_1^*)}{2} < \frac{1}{1-2a_1}$. On the other hand, the RHS represents the cost of accommodating the entry. The expression suggests two costs related to the entry. First, the difference $\bar{\Pi}_1^M - \bar{\Pi}_1 > 0$ describes the usual competition effect. The entry of the shadow bank erodes the traditional bank's expected profit. Second, the rest terms capture the increase in the volatility of K_1 caused by the competition. Because of the competition, K_1 also varies with the signals of the shadow bank and becomes more volatile. Since $a_1 < \frac{1}{2}$, the traditional bank's payoff is concave in K_1 and thus this increase in the volatility is detrimental. Now consider two extreme cases $a_1 = \frac{1}{2}$ and $a_1 = 0$. At $a_1 = \frac{1}{2}$, the LHS of the condition becomes $(\beta_1^* + \gamma_1^*) - (\beta_1^{*D} + \gamma_1^{*D}) > 0$, while the RHS becomes $q\left(\bar{\Pi}_1^M - \bar{\Pi}_1\right)$. It can be shown that for q sufficiently small, $(\beta_1^* + \gamma_1^*) - (\beta_1^{*D} + \gamma_1^{*D}) > q\left(\bar{\Pi}_1^D - \bar{\Pi}_1\right)$ and thus $\Pi_1(\cdot|a_1 = \frac{1}{2}) > \Pi_1^M(\cdot|a_1 = \frac{1}{2})$. At $a_1 = 0$, on the LHS of condition (82), for q sufficiently small, I have $\beta_1^{*D} + \gamma_1^{*D}$ close to $\frac{1}{1-a_1} = 1$

and $\beta_1^* + \gamma_1^* > \beta_1^{*D} + \gamma_1^{*D}$. Therefore, $\frac{1}{1-2a_1} < \frac{(\beta_1^{*D}+\gamma_1^{*D})+(\beta_1^*+\gamma_1^*)}{2}$, which makes the LHS negative. On the RHS, since $\bar{\Pi}_1^M > \bar{\Pi}_1$ (the usual competition effect) and $\gamma_1^* > \gamma_1^{*D}$ (the multiplier effect), the RHS is positive. As a result, $\Pi_1(\cdot|a_1 = 0) < \Pi_1^M(\cdot|a_1 = 0)$.

Since $\Pi_1(\cdot|) - \Pi_1^M(\cdot|)$ is continuous in a_1, by the intermediate value theorem, in the compact set $a_1 \in [0, \frac{1}{2}]$, there exists at least one a_x that makes $\Pi_1(\cdot|a_1 = a_x) - \Pi_1^M(\cdot|a_1 = a_x) = 0$. Define

$$f(a_1) = \Pi_1(\cdot) - \Pi_1^M(\cdot), \tag{83}$$

I now show for any set of $\{m_1, m_2, a_2\}$, there is a unique root $\hat{a}(m_1, m_2, a_2, b)$ that makes $f(\hat{a}) = 0$. The idea is that, the profit difference for the traditional bank between accommodation and deterrence is either strictly increasing in its liquidity exposure a_1 or U-shaped (first decreasing and then increasing). In either case, the profit difference can be zero only at a unique intermediate value of $a_1 = \hat{a}$. Specifically, it can be verified that for q sufficiently small, there exists a unique $a_1' \leq \frac{1}{2}$ such that $\frac{\partial f}{\partial a_1} > 0$ if and only if $a_1 > a_1'$. Consider two cases. Suppose first that $a_1' \leq 0$. Thus for $a_1 > 0 \geq a_1'$, $\frac{\partial f}{\partial a_1} > 0$. That is, the profit difference $f(a_1)$ is strictly increasing in a_1 and has a unique root $\hat{a}$ that makes $f(\hat{a}) = 0$. Second, suppose that $a_1' > 0$. Thus for $0 < a_1 < a_1'$, $\frac{\partial f}{\partial a_1} < 0$ while for $a_1 \geq a_1'$, $\frac{\partial f}{\partial a_1} \geq 0$. That is, the profit difference $f(a_1)$ is U-shaped and turns at $a_1 = a_1'$. Recall that at $a_1 = 0$, $f(0) = \Pi_1(\cdot|a_1 = 0) - \Pi_1^M(\cdot|a_1 = 0) < 0$. Therefore, at any $a_1 \in [0, a_1']$, $f(a_1') < f(a_1) < f(0) < 0$. Thus there is no root for $f(a_1) = 0$ in $[0, a_1']$. For $a_1 > a_1'$, since $f(a_1') < f(0) < 0$ and $f(\frac{1}{2}) = \Pi_1(\cdot|a_1 = \frac{1}{2}) - \Pi_1^M(\cdot|a_1 = \frac{1}{2}) > 0$, by the intermediate value theorem, there exists a root $\hat{a}$ that makes $f(\hat{a}) = 0$. This root is also unique given $f(a_1)$ is monotonically increasing in a_1 for $a_1 > a_1'$. I thereby have verified that for any set of $\{m_1, m_2, a_2\}$, there exists a unique root $\hat{a}$ that makes $f(\hat{a}) = 0$. In sum, $\Pi_1(m_1, m_2, a_2) > \Pi_1^M(m_1)$ if and only $a_1 > \hat{a}(m_1, m_2, a_2)$.

Now I compare $\Pi_1(m_1^*, m_2^*(m_1^*), a_2^*(m_1^*))$ and $\Pi_1^M(m_1^{*D})$. For $a_1 > \hat{a}(m_1^{*D}, m_2^*(m_1^{*D}), a_2^*(m_1^{*D}))$,

$$\Pi_1(m_1^{*D}, m_2^*(m_1^{*D}), a_2^*(m_1^{*D})) > \Pi_1^M(m_1^{*D}). \tag{84}$$

In addition, notice that $m_1^{*D} = m_1^D$ is also within the feasible set of the accommodation case marginally (i.e., $m_1 < m_1^D$). Since m_1^* is the optimal decision by the traditional bank in the accommodation case, I have

$$\Pi_1(m_1^*, m_2^*(m_1^*), a_2^*(m_1^*)) > \Pi_1(m_1^{*D}, m_2^*(m_1^{*D}), a_2^*(m_1^{*D})), \tag{85}$$

where the inequality is strict since $m_1^* \neq m_1^{*D}$. Thus

$$\Pi_1(m_1^*, m_2^*(m_1^*), a_2^*(m_1^*)) > \Pi_1(m_1^{*D}, m_2^*(m_1^{*D}), a_2^*(m_1^{*D})) > \Pi_1^M(m_1^{*D}), \tag{86}$$

which indicates that the traditional bank always accommodates the entry of the shadow bank when $a_1 > \hat{a}(m_1^{*D}, m_2^*(m_1^{*D}), a_2^*(m_1^{*D}))$. On the other hand, when $a_1 < \hat{a}(m_1^{*D}, m_2^*(m_1^{*D}), a_2^*(m_1^{*D}))$, I have,

$$\Pi_1(m_1^{*D}, m_2^*(m_1^{*D}), a_2^*(m_1^{*D})) < \Pi_1^M(m_1^{*D}). \tag{87}$$

In order to deter the shadow bank from entry, the traditional bank needs to move its disclosure precision upward and chooses m_1^{*D} instead of m_1^*, where $m_1^{*D} > m_1^*$. Therefore, if the traditional bank accommodates entry, it would choose m_1^* instead of m_1^{*D}, which saves the additional disclosure cost, measured by $\frac{c_m}{2}\left(m_1^{*D}\right)^2 - \frac{c_m}{2}\left(m_1^*\right)^2$. However, I find that as long as the disclosure cost c_m is not too high, I have[18]

$$\begin{aligned} & \Pi_1(m_1^*, m_2^*(m_1^*), a_2^*(m_1^*)) - \Pi_1(m_1^{*D}, m_2^*(m_1^{*D}), a_2^*(m_1^{*D})) \\ < \ & \frac{c_m}{2}\left(m_1^{*D}\right)^2 - \frac{c_m}{2}\left(m_1^*\right)^2 < \Pi_1^M(m_1^{*D}) - \Pi_1(m_1^{*D}, m_2^*(m_1^{*D}), a_2^*(m_1^{*D})), \end{aligned} \tag{88}$$

which gives,

$$\Pi_1^M(m_1^{*D}) > \Pi_1(m_1^*, m_2^*(m_1^*), a_2^*(m_1^*)), \tag{89}$$

that is, the traditional bank prefers to deter the shadow bank from entry. ■

Proof of Corollary 3

Proof. Since entry is not blockaded by the monopoly choice m_1^{*M} of the traditional bank, $m_1^{*M} < m_1^D$. That is, m_1^{*M} is within the feasible set of the optimization program for the accommodation case. Since m_1^* maximizes the traditional bank's payoff in this case, I have

$$\Pi_1(m_1^*, m_2^*(m_1^*), a_2^*(m_1^*)) > \Pi_1(m_1^{*M}, m_2^*(m_1^{*M}), a_2^*(m_1^{*M})), \tag{90}$$

where the inequality is strictly since $m_1^* < m_1^{*M}$ as shown in Proposition 7. In addition, from Proposition 6, when a_1 is sufficiently large $(a_1 > \hat{a}(m_1^{*M}, m_2^*(m_1^{*M}), a_2^*(m_1^{*M})))$,

$$\Pi_1(m_1^{*M}, m_2^*(m_1^{*M}), a_2^*(m_1^{*M})) > \Pi_1^M(m_1^{*M}), \tag{91}$$

[18] To see why the first inequality holds, denote the payoff to the traditional bank without the disclosure cost as $B_1\left(m_1, m_2^*\left(m_1\right), a_2^*\left(m_1\right)\right)$. Thus $\Pi_1 = B_1 - \frac{c_m}{2}m_1^2$. It can be verified that for c_m sufficiently small, $B_1|m_1^* < B_1|m_1^{*D}$. Thus $\Pi_1|m_1^* - \Pi_1|m_1^{*D} = B_1|m_1^* - B_1|m_1^{*D} + \frac{c_m}{2}\left(m_1^{*D}\right)^2 - \frac{c_m}{2}\left(m_1^*\right)^2 < \frac{c_m}{2}\left(m_1^{*D}\right)^2 - \frac{c_m}{2}\left(m_1^*\right)^2$.

and hence

$$\Pi_1(m_1^*, m_2^*(m_1^*), a_2^*(m_1^*)) > \Pi_1^M(m_1^{*M}). \tag{92}$$

■

Proof of Corollary 4

Proof. $\frac{\partial \Pi_1}{\partial m_2} < 0$ can be show by directly computing the derivative. In addition, when q is close to zero, it can be shown that $\frac{\partial \Pi_1}{\partial a_2} > 0$ when $a_1 > \hat{a}(b)$, where

$$\hat{a}(b) = \frac{7 - b^2 - \sqrt{17b^4 + 2b^2 + 17}}{8 - 8b^2}. \tag{93}$$

■

Proof of Proposition 7

Proof. I first prove that $m_1^* < m_1^c$. Observe first when the traditional bank's decision of m_1 is observable to the shadow bank, its equilibrium decision, m_1^* is characterized instead by the following first-order condition:

$$\frac{d\Pi_1(m_1^*, m_2^*(m_1^*), a_2^*(m_1^*))}{dm_1} = \frac{\partial \Pi_1}{\partial m_1} + \frac{\partial \Pi_1}{\partial m_2}\frac{\partial m_2^*}{\partial m_1} + \frac{\partial \Pi_1}{\partial a_2}\frac{\partial a_2^*}{\partial m_1} = 0. \tag{94}$$

However, in the benchmark case in which the traditional bank's decision is not observable to the shadow bank, its equilibrium decision, m_1^c, is characterized by the first-order condition:

$$\frac{\partial \Pi_1(m_1^c, m_2^*(m_1^c), a_2^*(m_1^c))}{\partial m_1} = 0, \tag{95}$$

where the shadow bank's decisions $(m_2^*(m_1), a_2^*(m_1))$ are identical with those in the main setting in which the traditional bank's decision is observable. This is because in a rational expectation equilibrium, the shadow bank correctly conjectures the decision by the traditional bank even without observing the actual decision (Bagwell, 1995). Recall that by Lemma 4, $\frac{\partial \Pi_1}{\partial m_2} < 0$ and $\frac{\partial \Pi_1}{\partial a_2} > 0$. In addition, Proposition 4 also suggests that $\frac{\partial m_2^*}{\partial m_1} > 0$ and $\frac{\partial a_2^*}{\partial m_1} < 0$. Thus, in the unobservable case,

$$\frac{d\Pi_1(m_1^c, m_2^*(m_1^c), a_2^*(m_1^c))}{dm_1} = \frac{\partial \Pi_1}{\partial m_1} + \frac{\partial \Pi_1}{\partial m_2}\frac{\partial m_2^*}{\partial m_1} + \frac{\partial \Pi_1}{\partial a_2}\frac{\partial a_2^*}{\partial m_1} < 0, \tag{96}$$

this is because the last two terms in the first-order condition on m_1 are both negative and the first term is zero. Given the concavity of Π_1 in m_1, this implies that:

$$m_1^* < m_1^c. \tag{97}$$

I now show that $m_1^c < m_1^{*M}$. When the traditional bank is the monopoly, its disclosure decision is given by

$$\frac{\partial \Pi_1^M}{\partial m_1} = \frac{1}{q}\left[1-(1-2a_1)\left(\beta_1^{*D}+\gamma_1^{*D}\right)\right]\frac{\partial(\beta_1^{*D}+\gamma_1^{*D})}{\partial m_1} - \frac{1-2a_1}{2}\left[\frac{2\gamma_1^{*D}}{m_1}\frac{\partial \gamma_1^{*D}}{\partial m_1} - \frac{\left(\gamma_1^{*D}\right)^2}{m_1^2}\right] - c_m m_1 = 0, \tag{98}$$

while m_1^c is given by,

$$\frac{\partial \Pi_1}{\partial m_1} = \frac{1}{q}\left[1-(1-2a_1)\left(\beta_1^{*}+\gamma_1^{*}\right)\right]\frac{\partial(\beta_1^{*}+\gamma_1^{*})}{\partial m_1} - \frac{1-2a_1}{2}\left[\frac{2\gamma_1^{*}}{m_1}\frac{\partial \gamma_1^{*}}{\partial m_1} - \frac{\left(\gamma_1^{*}\right)^2}{m_1^2}\right] - c_m m_1 = 0. \tag{99}$$

Therefore, $\frac{\partial \Pi_1}{\partial m_1} < \frac{\partial \Pi_1^M}{\partial m_1}$ if and only if

$$\begin{aligned} &\left[1-(1-2a_1)\left(\beta_1^{*}+\gamma_1^{*}\right)\right]\frac{\partial(\beta_1^{*}+\gamma_1^{*})}{\partial m_1} - \left[1-(1-2a_1)\left(\beta_1^{*D}+\gamma_1^{*D}\right)\right]\frac{\partial(\beta_1^{*D}+\gamma_1^{*D})}{\partial m_1} \\ < \; &\frac{q(1-2a_1)}{2}\left[\frac{2\gamma_1^{*D}}{m_1}\frac{\partial \gamma_1^{*D}}{\partial m_1} - \frac{\left(\gamma_1^{*D}\right)^2}{m_1^2} - \frac{2\gamma_1^{*}}{m_1}\frac{\partial \gamma_1^{*}}{\partial m_1} + \frac{\left(\gamma_1^{*}\right)^2}{m_1^2}\right], \end{aligned} \tag{100}$$

It can be verified that $\frac{\partial^2}{\partial b \partial m_1}\left(\frac{\left(\gamma_1^*\right)^2}{m_1}\right) > 0$ and thus $\frac{\partial}{\partial m_1}\left(\frac{\left(\gamma_1^*\right)^2}{m_1}\right) = \frac{2\gamma_1^*}{m_1}\frac{\partial \gamma_1^*}{\partial m_1} - \frac{\left(\gamma_1^*\right)^2}{m_1^2} > \frac{\partial}{\partial m_1}\left(\frac{\left(\gamma_1^{*D}\right)^2}{m_1}\right) = \frac{2\gamma_1^{*D}}{m_1}\frac{\partial \gamma_1^{*D}}{\partial m_1} - \frac{\left(\gamma_1^{*D}\right)^2}{m_1^2}$. Therefore, given the RHS of (100) is positive, a sufficient condition for $\frac{\partial \Pi_1}{\partial m_1} < \frac{\partial \Pi_1^M}{\partial m_1}$ is that its LHS is negative, i.e.,

$$\left[1-(1-2a_1)\left(\beta_1^{*}+\gamma_1^{*}\right)\right]\frac{\partial(\beta_1^{*}+\gamma_1^{*})}{\partial m_1} - \left[1-(1-2a_1)\left(\beta_1^{*D}+\gamma_1^{*D}\right)\right]\frac{\partial(\beta_1^{*D}+\gamma_1^{*D})}{\partial m_1} < 0. \tag{101}$$

From Appendix II, when b is sufficiently large (e.g., b^2 close to $\frac{a_1(1-a_2^*)}{a_2^*(1-a_1)}$), $\beta_1^* + \gamma_1^*$ becomes close to $\frac{1}{1-2a_1}$, which makes $1-(1-2a_1)\left(\beta_1^*+\gamma_1^*\right)$ close to zero while $1-(1-2a_1)\left(\beta_1^{*D}+\gamma_1^{*D}\right) > \frac{a_1}{1-a_1} > 0$. In addition, since $\frac{\partial(\beta_1^{*D}+\gamma_1^{*D})}{\partial m_1}, \frac{\partial(\beta_1^{*}+\gamma_1^{*})}{\partial m_1} > 0$, I have $\left[1-(1-2a_1)\left(\beta_1^{*D}+\gamma_1^{*D}\right)\right]\frac{\partial(\beta_1^{*D}+\gamma_1^{*D})}{\partial m_1} > \left[1-(1-2a_1)\left(\beta_1^{*}+\gamma_1^{*}\right)\right]\frac{\partial(\beta_1^{*}+\gamma_1^{*})}{\partial m_1}$ and hence $\frac{\partial \Pi_1}{\partial m_1} < \frac{\partial \Pi_1^M}{\partial m_1}$. Therefore, $m_1^{*M} > m_1^c$.

Third, from Proposition 5, since entry is not blockaded by the monopoly choice, $m_1^{*M} < m_1^{*D} = m_1^D$.

Lastly, since $\frac{\partial m_2^*}{\partial m_1} > 0$ and $\frac{\partial a_2^*}{\partial m_1} < 0$, thus,

$$m_2^*\left(m_1^*\right) < m_2^*\left(m_1^c\right) < m_2^*\left(m_1^{*M}\right) < m_2^*\left(m_1^{*D}\right), \tag{102}$$

and

$$a_2^*\left(m_1^*\right) > a_2^*\left(m_1^c\right) > a_2^*\left(m_1^{*M}\right) > a_2^*\left(m_1^{*D}\right). \tag{103}$$

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