Management by the Numbers: A Formal Approach to Deriving Informational and Distributional Properties of "Un-managed" Earnings[1]

Thomas Hemmer
Rice University

Eva Labro
University of North Carolina

May 25 2016

We explore the theoretical relation between earnings and market returns as well as the properties of accounting earnings frequency distributions under the maintained hypothesis that managers use unbiased accounting information benevolently to prudently manage the firms of which they are appointed stewards. We offer this surprisingly uncommon (in the academic literature) perspective to generate untainted benchmarks against which empirically observed earnings-returns relations and aggregate earnings distributions can be evaluated. Our analysis is based on arguably the most parsimonious model of informed managerial decision making and market pricing based on reported earnings possible. It yields the following results: reported losses are less persistent than reported gains, the market response to earnings exhibits an "S-shape" and earnings relate to returns asymmetrically in the way documented by e.g. Basu (1997). Furthermore, the implied frequency distribution of aggregate earnings is neither symmetric nor necessarily unimodal. Instead, it is likely to exhibit a clear discontinuity at zero and looks similar to the plots reported by Burgstahler and Dichev (1997a). However, within our set-up, none of these phenomena are due to reporting noise, bias or some undesirable strategic managerial behavior. They are the natural consequences of using past earnings as the basis for prudent managerial decision making that in turn generates the future earnings on which future decisions will be based.

Keywords: Stewardship, Prudence, Decision Usefulness, Precision, Bias, Persistence, Earnings Management, Earnings Discontinuity
JEL code: M40

[1] Exceptional research assistance by Jim Omartian is much appreciated. We thank Romana Autrey (discussant at AAA meeting), Rich Frankel, Brian Mittendorf (discussant at MAS meeting), Jim Omartian, Richard Saouma, Jack Stecher, Ian Tarrant, Frank Zhou, an anonymous MAS midyear meeting reviewer, workshop participants at Chicago Booth, HEC Lausanne, Penn State, University of Iowa, University of Melbourne, University of New South Wales, Washington University in St. Louis, conference attendees at the 2015 Accounting Research workshop in Zurich, 2015 American Accounting Association Annual meeting in Chicago, AnpCONT 2014 in Brazil, and the 2014 Temple conference on convergence between financial and managerial accounting for comments and suggestions.

1. Introduction

A significant fraction of the academic accounting literature spanning the past 45 plus years has revolved around the peculiar empirical patterns of earnings and around perceived irregularities in how earnings relate to market prices and/or returns. Key examples are seemingly low earnings response coefficients, S-shaped price responses to earnings, a distinct dip at zero in large sample earnings frequency distributions and the more (less) positive relation between returns and earnings for negative (positive) returns. Standard, yet disjoint, explanations offered in the literature include intentional bias, manipulation of reports by managers, leakage or obfuscation of information, and fundamental deficiencies, biases and/or intentional asymmetries in the accounting rules and principles by which accounting measures are constructed.

We do not dispute that all of the above issues may be contributing to persistent, seemingly surprising, empirical patterns. Instead, what we argue (and demonstrate) in this paper is that many of the peculiar empirical patterns are surprising *only* if one assumes that financial reports are produced exclusively for the consumption of external users that are *not* involved in the daily managerial decision making process of the reporting entity. That is, for the type of empirical patterns mentioned above to actually be peculiar, the information on which managers base their ongoing daily decisions to actively manage the operation of the reporting entity must implicitly or explicitly be assumed to be somehow fundamentally different from -and largely uncorrelated with- the information summarized periodically in external financial statements.

More specifically, in this paper we provide some formal insights into the expected empirical properties of earnings, distributional as well as informational, that arise if one drops the extreme and unnatural information assumption that internally and externally reported information are fundamentally independent (Hemmer and Labro 2008). We provide these insights based on a very simple model framework in which firms each need to hire a manager in order to operate. A manager here is an expert agent with the personal ability to select and implement a variety of, on average, (from the perspective of the firm) valuable opportunities available to a particular firm. In direct contrast to much of the accounting literature more generally, and the literature on peculiar informational and distributional properties of earnings specifically, we assume managers are entirely benevolent. They never engage in any strategic behavior, whether it is operations or reporting related: managers always act in the best interest of the outside shareholders of their firm.

At the core of our analysis is the presumption that, key to firm's willingness to maintain highly compensated professional managers, such managers must be able to take managerial actions on an ongoing basis. Moreover, managerial actions must have direct implications for the profitability of the firm by which the manager is employed. Finally, we argue that while largely ignored in the financial accounting literature (e.g. Dechow et al. 2010, 345), those managerial decisions are first-order in determining the properties of reported earnings. In the spirit of parsimony, we focus on a simple binary random walk of real economic earnings over many short periods where a random uninformed selection of an opportunity produces zero terminal value in expectation. Managerial decisions can be "better" or "worse" in that a better decision, if implemented, is assumed to generate favorable earnings over each short period with a higher probability than does a worse decision.

To highlight accounting's central internal decision facilitating role and our premise that the manager does not have perfect knowledge at the outset yet learns from the information generated over time, we assume that managers cannot tell ex-ante the quality of individual choices in their opportunity set but subsequently will be able to make inference about the chosen course of action from observed realized (past) accounting earnings. In particular, a manager will use the internally reported accounting information to update his beliefs in the standard Bayesian way. Then, if at some point in time, based on the accrued evidence, he believes that selecting an alternate opportunity will improve firm profitability, he will do so. The decision to select a new opportunity itself is assumed to be costless to the manager, but may also potentially not be completely effective: past decisions may be somewhat sticky in terms of profitability (for example, due to adjustment costs and the inability to fully undo prior strategy implementations, impairing funds available for future strategy implementations) so even when the manager selects a new strategic direction, it may only result in a new underlying profitability generating process with some probability, and the proportion of better and worse opportunities may differ from the initial. We refer to the process of making such prudent choices decisions as "Firm Management" or FM for brevity.[2]

To avoid contamination of our results by some kind of artificially imposed imperfections, accounting earnings are assumed to measure real economic earnings here without any noise or

[2] "Firm Management" stands in contrast to all other (non-FM) type of actions intended to influence accounting outcomes, which we label as "Accounting Management" or, equivalently, AM.

bias and are reported internally on a timely basis to the manager as the earnings process unfolds and updates thus become available. Also, since we rely on non-strategic and entirely benevolent managers, the less frequent periodic financial reports that are provided to outsiders are simply assumed to be an unperturbed aggregate of the frequent earnings realizations that have occurred over the most recent reporting horizon, and hence not subject to “Accounting Management”. Outsiders, we further assume, use these reports to update their homogeneous and consistent prior beliefs about the value of each firm in a standard Bayesian fashion and price the firm accordingly.

Our Firm Management model provides a unified theory of the following empirically observed informational and distributional properties of earnings: the Earnings Response Coefficient (ERC) is S-shaped, decision making impact diminishes the ERC, earnings relate to returns asymmetrically in the way documented by e.g. Basu (1997), and the frequency distribution of aggregate earnings is neither symmetric nor necessarily unimodal but is likely to exhibit a clear discontinuity at zero and looks similar to the plots reported by Burgstahler and Dichev (1997a). As we will discuss later in more detail, prior work has offered various, yet disjointed, explanations for each separate phenomenon.

First, we demonstrate that the market response to earnings reports is S-shaped in the benchmark case where a firm has no decision making impact (that is, cannot make any decisions to change the course of action). Outside investors’ Baysian updating causes this non-linearity. With our simple structure, reporting higher earnings is the equivalent of more pieces of good news having being obtained during the reporting horizon. The more pieces of good news the investors receive, the higher are their posteriors that the opportunity in place is a better one and thus the higher the valuation they will attach to the reporting firm. But because the posterior is bounded above at one, Bayesian updating is inherently non-linear and the market response to positive earnings will be concave in the absolute level of the earnings. The same bounding of posteriors holds on the negative side.

When the firm has decision making impact, the ERC continues to be S-shaped for the same reason. When the firm’s opportunity set includes strategies of differential earnings generating potential, decision making has value. However, decision making impact will diminish the ERC. Exactly because decision making has value, the expected value of firms with decision making impact at the start of the horizon will be higher, as firms are expected to adapt their decision making to enhance value. Because returns are calculated as the belief revision in expected value of the firm after the release of the earnings report, scaled by the expected value of the firm at the start of the horizon, the returns reaction in decision making impact firms is typically smaller. This may explain a puzzle in the financial accounting literature where researchers have found that observed ERCs are smaller than one would expect (e.g. Lev and Zarowin 1999).

Second, the optimal managerial decision rule here is notable because of its inherent asymmetrical nature. Based on this feature alone, it should be obvious that the market response to periodically reported earnings is not going to be symmetric for gains and losses when the internal decision making role of accounting information is considered. Exactly because of the asymmetric nature of managerial intervention, bad news is going to be less persistent than good. This follows since when the evidence is sufficiently strong that a worse opportunity is currently in place, managerial intervention decreases the probability that it will continue to be so. Hence, the market reaction to low earnings is more muted than the reaction to high earnings as documented in e.g. Burgstahler and Chuk (2013). It also leads directly to the type of kinked return-earnings relation obtained from a Basu (1997) regression, even though there is no deliberate conservative accounting bias in our model.

While the prior argument suggests a simple abandonment option in the spirit of Hayn (1995) (albeit in our case the manager is abandoning an implemented strategic opportunity rather than investors abandoning a firm), the intuition does not stop there. We document that an increase in decision making impact (as modeled by an increase in the differential quality of the opportunities in the opportunity set) results in the opposite asymmetry: a more muted returns reaction to good news than to bad news. The intuition is twofold. While observing negative reported earnings does increase the investor's posterior that a worse opportunity is in place and hence is likely to be replaced, the evidence also indicates firm value forgone by spending time and resources on a worse opportunity. Whether or not the (non-linear Bayesian) updating is more muted or stronger for negative than for positive earnings depends on the tradeoff between the confirmation that value was forgone and the opportunity to create more value in the future. When the differential quality of opportunities in the opportunity set of the firm is small, the latter will be more important. Furthermore, expectations for firms with a high quality opportunity are on average higher, in which case there is little return reaction to a high earnings news report. As a result, returns have a stronger association with negative than with positive earnings for such firms. However, as we will explain in more detail in Section 4, because of the limited weight given to high decision making impact firms in OLS regressions, in a large COMPUSTAT-like sample of all types of firms, researchers will observe the type of kinked return-earnings relation obtained from a Basu (1997) regression with more muted return reactions to bad news than to good news. Therefore, we suggest further empirical research to test our hypothesis of a flipped asymmetry in the returns-earnings relation in subsets of the COMPUSTAT sample.[3]

Our final modeling result explores the implication of earnings based managerial decision making for the distributional properties of reported earnings. This part of our study is motivated by Burgstahler and Dichev (1997a) and the literature on the large sample properties of realized earnings this seminal study initiated. The main feature of the basic distribution of earnings in which we are interested is, of course, the discontinuity at zero: the under-representation of slightly negative earnings realizations relative to what one would expect if the distribution of reported earnings, absent any form of strategic intervention purely aimed at decoupling the reported numbers from fundamentals, were to be bell-shaped, unimodal and reasonably symmetric.

While some studies in this literature have challenged the existence of this discontinuity on econometric grounds, others have raised questions about its very reason for existing. Durtschi and Easton (2005), for example, suggest that the particular scaling and sample selection criteria used by Burgstahler and Dichev (1997a) are the cause of this seeming anomaly. In contrast, others, such as Dechow et al. (2003), suggest that while the discontinuity does appear real, management of discretionary accruals measured using standard (but arguably somewhat ad-hoc) abnormal accrual measurement approaches, appears not to be its cause. The purpose of our paper is fundamentally different from these. We simply aim to provide a better and more formal understanding of the large sample distributional properties of realized earnings frequencies under

[3] Note that, while authors have offered partial explanations, as aptly described by Lipe et al. (1998, , pp. 212), the existing theory of ERCs is incomplete. The empirical regularity is that $ERC_{XL}<ERC_{XH}<ERC_{M,}$ where XL stands for extremely low, XH for extremely high and M for medium earnings. Freeman and Tse (1992) and Subramanyam (1996) offer explanations for the S-shaped earnings-return relation that suggest that both ERC_{XH} and ERC_{XL} are smaller than ERC_M. They suggest that price response to extreme news is lower because of low permanence and low precision of information, respectively. However, they do not have a prediction that $ERC_{XL}<ERC_{XH}$. Both Hayn (1995) and Basu (1997) predict that $ERC_{XL}<ERC_{XH}$, based on the investors' abandonment option and asymmetric timeliness in accounting earnings, respectively. However, neither of them predicts that $ERC_{XH}<ERC_M$. Our decision making theory provides a *complete* prediction of the documented empirical regularity: $ERC_{XL}<ERC_{XH}<ERC_M$.

our null that firms are properly managed based on such earnings, while earnings itself are not managed. We argue that a better comprehension of the distributional implications of this null is critical to advancing our understanding of the implications of the distributional properties of the real world data.

For this part of the study, we construct a sample of firms that each have earnings probability distributions consistent with our prior informational analysis and obtain the predicted frequency distribution of realized earnings reports. Our model structure is designed to create smooth and symmetrically (Normally) distributed earnings if all firms in the sample lack managerial decision making impact. However, we show that when both firms with no decision making impact and firms with high decision making impact (that have a right skewed unimodal earnings frequency distribution because the managerial decision making intervention shifts earnings mass to the right) are combined in a sample, the predicted earnings frequency distribution becomes bimodal with a distinct dip and the higher right-hand mode becomes significantly higher than the lower left-hand one. Indeed, for a large range of parameter values, the frequency pattern generated by our model has a clear resemblance with, for example, figure 3 in Burgstahler and Dichev (1997a). However, this pattern is indicative of firm management rather than earnings management as our model is free from reporting noise, bias and undesirable strategic managerial (operational or disclosure) behavior. While we do not claim that earnings management does not exist, much like we do not claim that conservatism does not exist (it is indeed mandated by accounting rules), we call for a reconsideration of the benchmarks against which to test for these phenomena.

The remainder of the paper is organized as follows. First, we detail the structure and the basic assumptions of our model. In the subsequent section, we derive the optimal managerial decision making rule. Section 4 provides our insights regarding the consequences of prudent earnings based firm management for the informational and cross-sectional properties of the returns reaction to earnings. Section 5 addresses the implications of firm management for the distributional properties of unmanaged earnings. Section 6 discusses connections to the existing empirical literature, and outlines further empirically testable hypotheses. We end with some concluding remarks about the need for reasonable theoretical benchmarks in empirical research and suggestions on how to rethink the direction of accounting research more broadly in the light of our findings.

2. Basic Model Structure

A manager makes n sequential decisions during a n period horizon; an initial at time $t = 0$ and $n - 1$ subsequent indexed by their timing, t = 1 ,..., n-1. No decision is thus made at time $t = n$, where the model concludes. At each decision point the manager is faced with an opportunity set £ with opportunities l, indexed by h=1, …, L (whereby L > 1) opportunities that are different in terms of their expected value. These opportunities can be thought of as any feasible decision the manager can make from strategic to tactical choices w.r.t. to revenue streams and costs. While the manager knows the overall quality of his firm's opportunity set (the proportion of each type l available), he cannot initially distinguish between the different types of opportunities (as defined by their expected value) but will have to pick one. The initial probability that the resulting chosen opportunity will end up being of type $l = 1, \dots, L$ is given by $\gamma_{0,l} \in \Gamma_0$ and we refer to the pair $\{£, \Gamma_0\}$ as the initial decision set. γ is common knowledge.

The particulars of appropriately managing the resources of a firm to maximize firm (or shareholder) value are highly complex and firm-specific undertakings, with intricacies that are (likely) impossible to be fully captured in the form of a formal model; certainly in one as parsimonious as ours. We avoid letting the perfect be the enemy of the good, however, by leaving case specific nuances and idiosyncrasies aside. To make our points as parsimoniously as possible we employ a simple binary representation of the resulting economic earnings process. Specifically, we assume that, at the end of each of the n periods, economic earnings of $v_t \in \{-1,1\}$ are realized. Thus, once a chosen opportunity of type $l \in £$ is implemented, true economic earnings for all subsequent periods where the opportunity remains in place follow a binomial random walk where the probability of high economic earnings, $v_t = 1$, being realized is denoted p_l. The probability of low economic earnings, $v_t = -1$, for a chosen opportunity of type l is thus $1 - p_l$. We denote aggregate earnings up to time t by $V_t \equiv \sum_{j=1}^{t} v_j$. Aggregate earnings for the entire horizon are referred to simply as V.

Rather than *imposing* a particular statistical structure on the initial opportunity set, in our model we restrict attention to what we consider a natural benchmark case. Specifically, since our focus is on the role of accounting information in value creating managerial decision making, we assume that, absent any potential for subsequent informed decisions, the uninformed and thus random initial decision produces zero terminal value in expectation. That is, absent any informed managerial input, the opportunities available to a firm are (in expectation) worthless. An expected firm value at $t = 0$ higher than zero, then, is a consequence of the nature of the opportunities available to the firm *and* the input of the manager.[4] Stated more formally,

A1. *Absent earnings-based decision making intervention subsequent to the initial uninformed decision, the terminal expected aggregate earnings are zero.*

We also assume that in the case where there is no scope for informed managerial decision making intervention, observing aggregate earnings of zero at any given time during the *n*-period horizon is equivalent to receiving no news. To be precise, we assume that absent any potential for informed managerial intervention, observing realized earnings totaling zero simply confirms a Bayesian individual's priors that the firm's terminal expected aggregate earnings will be zero. Formally,

A2. *Absent earnings-based decision making intervention, observing aggregate earnings* $V_t = 0$ *at any given time results in a Bayesian observer's posterior beliefs being identical to his initial at the start of the horizon.*

After making the initial (and thus uninformed) decision at $t = 0$, the subsequent $n - 1$ decisions may potentially be better informed than the initial one due to an internal reporting system that reveals realized earnings privately to the manager every period. The accounting system is here assumed to be perfect in the sense that it measures economic earnings without any bias or noise. However, economic earnings are a noisy measure of the quality of the underlying quality of the opportunity adopted. Each manager is assumed to be Bayesian and to make decisions that maximize the expected future profit of the firm he manages. The $n - 1$ subsequent decisions are confined to either continuing with the most recent chosen opportunity or draw a new opportunity from the available opportunity set.[5]

While we assume that the type of opportunities available to the manager remains the same over the entire horizon, we allow Γ_s, the probability distribution of the available opportunities subsequent to the initial decision, to differ from Γ_0. Specifically, we *allow* for differences that ensure that the initial opportunity set $\{£, \Gamma_0\}$ weakly stochastically dominates the

[4] This is consistent with standard production functions such as the Cobb-Douglas.
[5] If the manager draws a new opportunity, his learning about its quality starts afresh.

subsequent opportunity sets, $\{\pounds, \Gamma_s\}$. We do this for two distinctly different reasons. First, if weak stochastic dominance is not maintained, a manager would be better off abandoning the initial decision in favor of a new opportunity after the first period, even if he receives no new information. Hence, the model would not capture our core interest in managerial decision making informed by accounting information. Second, this models adjustment costs when changing decisions and the inability to fully undo prior strategy implementations, which impairs funds available for future strategy implementations, and hence restricts the subsequent opportunity set. The initially made decision, then, will have lasting consequences and thus be somewhat "sticky".

Finally, to close up the general structure of our model, aggregate earnings are reported in the form of financial statements to external investors for the purpose of valuing the firm every h period(s), where $h \in [2, n]$ and n/h is restricted to be an integer. That is, earnings are reported more frequently internally than externally. Earnings (the change in book value) are reported externally at least once over the n-period model horizon and the publication of the last such (full length) report occurs after exactly n periods. The earnings reported in the financial statement are assumed to be an unbiased, noiseless, and unmanaged aggregate of the earnings provided more frequently internally. Given common knowledge about the proportion of opportunities of different types l in the opportunity set of the firm, outside investors are also assumed to be Bayesian in their updating on the quality of the implemented strategic opportunity. Furthermore, they trade in a frictionless market for the firms' shares and, with the exception of the internal reports, have the exact same information as the managers of the firms. Managers thus differ from outsiders only in their ability to view more frequent information from which to learn about the quality of the implemented opportunity and to implement opportunities available to their firm.[6] In the last period, the liquidating dividend that is paid out is the book value, and outsiders are hence trying to predict terminal date book value. Market value can deviate from book value in every period until the last, where it is the same by construction. Hence returns and market value differ from earnings and book value at any other point in time, other than the last.

[6] While outsiders could more accurately price the firm if the firm made more frequent disclosures, there is no economic value to the firm from making such voluntary disclosures, so with epsilon disclosure cost the firm will not make additional disclosures.

3. Analysis : The Optimal Managerial Decision Rule and its Earnings Implications

While for the model outlined above the manager will necessarily have to make an initial, uninformed choice, afterwards it is optimal for the manager to choose a new opportunity if and only if the available evidence in the form of realized earnings suggest that doing so is better than staying the course. To establish exactly what evidence at what time will result in managerial decision making intervention to select an alternate opportunity, however, one needs to have specific information about the properties of the initial and subsequent decision sets. As it turns out, *A*1 and *A*2 imply a specific and very simple structure for our model. Lemma 1 identifies the specifics.

Lemma 1: For the model outlined in Section 2, assumptions *A*1 and *A*2 imply that *i*) $L = 2$, *ii*) $p_1 = 1 - p_2$ and *iii*) $\gamma_{0,1} = \gamma_{0,2} = 1/2$.

Proof: Consider the $n/2$ nodes of the binomial tree where observed realized earnings are zero. Using Bayes rule and based on our aggregate earnings ($V_t = 0$) neutrality requirement (*A*2), $\forall\ k = 1, \ldots, n/2$ we then must have

$$\Pr(l \mid v_{2\times k} = 0) = \frac{p_l^k (1 - p_l)^k \gamma_l}{\sum_{h=1}^{L} p_h^k (1 - p_h)^k \gamma_h} = \gamma_l,$$

or equivalently

$$p_l^k (1 - p_l)^k = p_l^k (1 - p_l)^k \gamma_l + \sum_{h \neq l} p_h^k (1 - p_h)^k \gamma_h.$$

Because the power transformation is strictly convex for $k > 1$, it is easily verified that for $n \geq 2$, this is satisfied *iff* the last sum only has one term (so $L = 2$), $p_l(1 - p_l) = p_h(1 - p_h)$ and $\gamma_l = (1 - \gamma_h)$. Further, since the available options are different, $p_l \neq p_h$ for $l \neq h$, and $p_l(1 - p_l) = p_h(1 - p_h)$ is thus satisfied if and only if $p_l = 1 - p_h$. Then *iii*) follows directly from *A*1.

Q.E.D.

Hence, Lemma 1 shows that our core assumptions on expected earnings and beliefs absent earnings based decision making (A1 and A2) endogenously lead to a simple structure for our model, where two opportunities are available, that have symmetric probabilities of success. We refer to the $L = 2$ opportunities available to a given firm simply as opportunities that are "worse" (*W*) and opportunities that are "better" (*B*). With probabilities $p_W < ½ < p_B$, then, worse and better choices result in positive earnings in any given future period during which the chosen opportunity remains in place. Also, we hereafter simply use $\gamma_0 \equiv \gamma_{0,B}$ ($= .5$) to represent the fraction of the initially available opportunities that are better and γ_s to represent the fraction of better opportunities available in any subsequent period. The (weak) stochastic dominance requirement then implies that $\gamma_s \in [0, \gamma_0]$.

The manager's decision to stick with the current opportunity or to select an alternate opportunity then involves evaluating, based on the earnings history, whether "sampling" from the subsequently available opportunity set in expectation dominates continuing with the current opportunity. With this, we are ready to characterize the optimal managerial decision rule.

Lemma 2: Let f denote the smallest integer that exceeds $\ln\left(\frac{1-\gamma_s}{\gamma_s}\right)\Big/\ln\left(\frac{1-p_B}{p_B}\right)$. Then, for the model outlined in Section 2, it is optimal for the manager to continue with the initial decision at time t if and only if $\sum_{j=1}^{t} v_j \geq f$. It is optimal for the manager at time t to continue with an opportunity chosen at time $\hat{t}$ if and only if $\sum_{j=\hat{t}}^{t} v_j \geq 0$.

Proof: It is optimal to continue with the initial decision at time t if and only if the probability that we have the better opportunity in place given the observed performance up to time t is higher than the probability of picking a better opportunity when returning to the opportunity set to select an alternate one. Formally,

$$\Pr(B|V_t) = \frac{\gamma_0 p_B^{\frac{t+V_t}{2}}(1-p_B)^{\frac{t-V_t}{2}}}{\gamma_0 p_B^{\frac{t+V_t}{2}}(1-p_B)^{\frac{t-V_t}{2}} + (1-\gamma_0)p_W^{\frac{t+V_t}{2}}(1-p_W)^{\frac{t-V_t}{2}}} \geq \gamma_s,$$

or

$$\frac{p_B^{\frac{t+V_t}{2}}(1-p_B)^{\frac{t-V_t}{2}}}{p_B^{\frac{t+V_t}{2}}(1-p_B)^{\frac{t-V_t}{2}} + p_W^{\frac{t+V_t}{2}}(1-p_W)^{\frac{t-V_t}{2}}} \geq \gamma_s$$

which simplifies to

$$\frac{1}{1+\left(\frac{1-p_B}{p_B}\right)^{V_t}} \geq \gamma_s,$$

or

$$\left(\frac{1-p_B}{p_B}\right)^{V_t} \leq \frac{1-\gamma_s}{\gamma_s},$$

which is satisfied for $V_t \geq \hat{V}_t$, where

$$\hat{V}_t \equiv \ln\left(\frac{1-\gamma_s}{\gamma_s}\right)\Big/\ln\left(\frac{1-p_B}{p_B}\right).$$

Then, for $\gamma_s = \gamma_0\ (= .5)$, $V_t \geq 0$, satisfies the condition for continuing with the initial decision. For $\gamma_s < \gamma_0$, continuing with the initial decision is optimal iff $V_t \geq \hat{V}_t$, whereby $\hat{V}_t$ is a finite negative number. Or, since the feasible levels of earnings are all integers here, continuing with the initial decision is optimal *iff* $V_t \geq f$.

Next, we look at decisions that follow a strategy choice that was triggered by breaching the decision floor f at time $\hat{t}$. It is optimal at time t to continue with that new opportunity the manager decided to implement at time $\hat{t}$ if and only if

$$\Pr(B|V_{\hat{t}}, V_t) = \frac{\gamma_s p_B^{\frac{t-\hat{t}+V_t-V_{\hat{t}}}{2}}(1-p_B)^{\frac{t-\hat{t}-V_t+V_{\hat{t}}}{2}}}{\gamma_s p_B^{\frac{t-\hat{t}+V_t-V_{\hat{t}}}{2}}(1-p_B)^{\frac{t-\hat{t}-V_t+V_{\hat{t}}}{2}} + (1-\gamma_s)p_W^{\frac{t-\hat{t}+V_t-V_{\hat{t}}}{2}}(1-p_W)^{\frac{t-\hat{t}-V_t+V_{\hat{t}}}{2}}} \geq \gamma_s,$$

or,

$$\frac{p_B^{\frac{t-\hat{t}+V_t-V_{\hat{t}}}{2}}(1-p_B)^{\frac{t-\hat{t}-V_t+V_{\hat{t}}}{2}}}{\gamma_s p_B^{\frac{t-\hat{t}+V_t-V_{\hat{t}}}{2}}(1-p_B)^{\frac{t-\hat{t}-V_t+V_{\hat{t}}}{2}} + (1-\gamma_s)(1-p_B)^{\frac{t-\hat{t}+V_t-V_{\hat{t}}}{2}}p_B^{\frac{t-\hat{t}-V_t+V_{\hat{t}}}{2}}} \geq 1,$$

or,

$$\frac{1}{\gamma_s + (1-\gamma_s)\left(\frac{1-p_B}{p_B}\right)^{V_t-V_{\hat{t}}}} \geq 1$$

or,

$$\gamma_s + (1-\gamma_s)\left(\frac{1-p_B}{p_B}\right)^{V_t-V_{\hat{t}}} \leq 1$$

or equivalently,

$$\left(\frac{1-p_B}{p_B}\right)^{V_t-V_{\hat{t}}} \leq 1,$$

which is satisfied iff $V_t - V_{\hat{t}} \geq 0$, or alternatively iff $\sum_{j=\hat{t}}^{t} v_j \geq 0$. *Q.E.D.*

Lemma 2 outlines the optimal decision rule for our managers: as long as aggregate earnings exceed the initial (firm specific) decision floor threshold, the manager sticks with his original selection and thus stays the course. If aggregate earnings move below the threshold, he chooses a different opportunity from the opportunity set that is available to him at that time, and starts to learn afresh from the earnings generated by this newly implemented opportunity. The lower the decision floor, the less likely that the manager will choose to select a new opportunity from the opportunity set and the more likely that he will continue with the opportunity already in place. Note that the decision floor f increases in γ_S: if opportunities in the future are of high(er) quality, it is more likely that the manager will decide to select a new opportunity in the future. Because $f = \left\lceil \ln\left(\frac{1-\gamma_s}{\gamma_s}\right) / \ln\left(\frac{p_W}{1-p_W}\right) \right\rceil$ and $p_w<0.5$, the decision floor f decreases in p_w: selecting an opportunity from the new opportunity set is more attractive because γ_S is more likely to be higher than p_w when p_w decreases. Subsequent to such informed decision to select an alternate opportunity, the optimal strategy becomes to "stay the course" as long as aggregate earnings since the most recent (past) "change of course" are positive. That is, the decision floor is now zero. Otherwise, the manager will select a new opportunity again.

The following example illustrates this decision making process for a tree in which the decision maker takes n=6 sequential decisions. Figure 1A indicates that the terminal node values range from 6 to -6 with an increment of 2. A success at time 1 (V_1=1) is achieved with probability p_B or p_W depending on whether B or W was randomly chosen at the outset. Failure at time 1 (V_1=-1) occurs with probability 1-p_B or 1-p_W. As the bold full line in Figure 1B indicates, only one path in the binomial tree leads to a terminal outcome of 6. In the case where the decision maker randomly starts with picking opportunity B, he will stick (as indicated by the circle at the decision node) with this opportunity throughout since at every node the aggregated earnings indicate that he has the better opportunity in place ($\sum_{j=1}^{t} v_j \geq f$). Because the manager never decides to select an alternate opportunity from the pool and there is only one path that leads to earnings level V=6, it is easy to calculate the probability of realizing this earnings level: $\gamma_0 {p_B}^6 + (1-\gamma_0){p_W}^6$.

In other cases, however, after the initial randomly picked opportunity seems unprofitable the manager will select a new opportunity from the available opportunity set. So while there also is only one path in the tree (indicated by the dashed line in Figure 1B) leading to a terminal value of -6, here all the realized earnings along this path indicate to the manager that he is not doing well. Accordingly, in this case he will continue to select a new opportunity from the opportunity set, as indicated by the triangle at each decision node.

Consider now an intermediate terminal node such as *V*=0. Here twenty different paths lead to a terminal value of 0 with one of those paths indicated in bold in Figure 1C. In this particular example, the manager starts with randomly choosing opportunity B. At time 1, the information in the earnings numbers indicate that this opportunity is unsuccessful (V_1=-1<f), and he decides to select another opportunity from the opportunity set (as indicated by the triangle). However, at time 2, the reported earnings indicate a "worse" opportunity ($\sum_{j=1}^{2} v_j = -1 < 0$). He selects a new opportunity from the pool, and from here on the earnings numbers continue to indicate that he has a "better" opportunity in place. For example, $\sum_{j=2}^{5} v_j = 3 \geq 0$, so he stays the course at time 5.

Insert Figures 1A, B and C about here.

Despite the simplicity of our minimalistic Binomial random-walk representation of the earnings process, the number of potential earnings histories grows rapidly even for a relatively modest number of (sub-)periods, *n*. This presents some technical challenges because the managerial decision making intervention detailed by Lemma 2 inherently makes the probability of observing some level of aggregate earnings path-dependent and because the issues we are interested in can only meaningfully be addressed for a non-trivial number of periods. To overcome these challenges, we next establish two key technical results that enable us to simplify matters significantly and on which the remainder of our analysis therefore is predicated.

Lemma 3. For every path of length *n*, with a decision floor of *f*, ending in node $V<n+2f$ and exactly $d \in [1, \ldots., (n-V+2f)/2]$ informed decisions whereby a new opportunity is selected[7], the probability of realizing a total earnings level of *V* is given as:

$$\begin{aligned}\Pr(V,n,f,d) = {} & (\gamma_0(1-p_B)^{1-f} + (1-\gamma_0)(1-p_W)^{1-f}) \\ & \times \left(\gamma_s(1-p_B) + (1-\gamma_s)(1-p_W)\right)^{d-1} \\ & \times \left(\gamma_s\, p_B^{\frac{n+V}{2}} (1-p_B)^{\frac{n-V-2(d-f)}{2}} \right. \\ & \left. + (1-\gamma_s)\, p_W^{\frac{n+V}{2}} (1-p_W)^{\frac{n-V-2(d-f)}{2}}\right),\end{aligned}$$

independent of the timing of the *d* decisions.

Proof: For some path ending in node *V*, let τ_1 through τ_d denote the times at which the first through the last of the $d \in [1, \ldots., (n-V+2f)/2]$ informed decisions are made. Consider first the case where $d=1$. The probability of such a path is given by

$$\begin{aligned}& \left(\gamma_0 p_B^{\frac{\tau_d+f-1}{2}}(1-p_B)^{\frac{\tau_d-f+1}{2}} + (1-\gamma_0)p_W^{\frac{\tau_d+f-1}{2}}(1-p_W)^{\frac{\tau_d-f+1}{2}}\right) \\ & \times \left(\gamma_s p_B^{\frac{n-\tau_d+(V+d-f)}{2}}(1-p_B)^{\frac{n-\tau_d-(V+d-f)}{2}} + (1-\gamma_s)p_W^{\frac{n-\tau_d+(V+d-f)}{2}}(1-p_W)^{\frac{n-\tau_d-(V+d-f)}{2}}\right).\end{aligned}$$

Because $p_W = (1-p_B)$, it is easily verified that this product does not depend on τ_d. For $d \in [2, \ldots., (n-V+2f)/2]$ informed decisions along a path with end-node *V*, the probability is given by

$$\left(\gamma_0 p_B^{\frac{\tau_1+f-1}{2}}(1-p_B)^{\frac{\tau_1-f+1}{2}} + (1-\gamma_0)p_W^{\frac{\tau_1+f-1}{2}}(1-p_W)^{\frac{\tau_1-f+1}{2}}\right)$$

[7] Note that this excludes the initial decision as that is uninformed.

$$\times \prod_{\omega=1}^{d-1} \left(\gamma_s p_B^{\frac{\tau_{\omega+1}-\tau_\omega-1}{2}} (1-p_B)^{\frac{\tau_{\omega+1}-\tau_\omega+1}{2}} + (1-\gamma_s) p_W^{\frac{\tau_{\omega+1}-\tau_\omega-1}{2}} (1-p_W)^{\frac{\tau_{\omega+1}-\tau_\omega+1}{2}} \right)$$
$$\times \left(\gamma_s p_B^{\frac{n-\tau_d+(V+d-f)}{2}} (1-p_B)^{\frac{n-\tau_d-(V+d-f)}{2}} + (1-\gamma_s) p_W^{\frac{n-\tau_d+(V+d-f)}{2}} (1-p_W)^{\frac{n-\tau_d-(V+d-f)}{2}} \right).$$

As for the case where $d = 1$, it is easily verified that for any two adjacent terms ordered as in the above expression, the earlier of the decision times cancel out. Since the product thus can be written without reference to each τ_1 through τ_d, it can be written without reference to any τ_1 through τ_d. The probability is thus independent of decision timing and any *n*-period path with the same end-node, the same decision floor and the same number of decisions (>0), therefore has the same probability as one where the decision floor is breached after $1 - f$ periods and the subsequent $d - 1$ decisions (if any) occur immediately thereafter over the subsequent $d - 1$ periods. The expression in the lemma is the expression for that sequence of events. *Q.E.D.*

Clearly Lemma 3 simplifies the calculations involved in deriving the properties of earnings distributions since there may be many paths for each terminal node that optimally involve the same *number* of decisions but are made at different *times*. Lemma 3 shows that each of those paths has the same probability of that terminal node. We return to the 6 period example to illustrate Lemma 3. Assume a decision floor *f* of -2. Figure 2 depicts 2 paths that lead to a terminal node of V=-2 and both involve one decision where the course of action is changed. However, for the path depicted by the full bold line, that decision to change is taken at time t=5, whereas for the path indicated with the dotted bold line, this happens at time t=3. The probability of the path depicted by the full bold line is calculated as $[\gamma_0 p_B(1-p_B)^4 + (1-\gamma_0)p_W(1-p_W)^4][\gamma_s p_B + (1-\gamma_s)p_W] = [p_B(1-p_B)][\gamma_0(1-p_B)^3 + (1-\gamma_0)(1-p_W)^3][\gamma_s p_B + (1-\gamma_s)p_W]$, which is equal to the probability of the path depicted by the dotted bold line of $[\gamma_0(1-p_B)^3 + (1-\gamma_0)(1-p_W)^3][\gamma_s p_B^2(1-p_B) + (1-\gamma_s)p_W^2(1-p_W)] = [p_B(1-p_B)][\gamma_0(1-p_B)^3 + (1-\gamma_0)(1-p_W)^3][\gamma_s p_B + (1-\gamma_s)p_W]$. This is also equal to the probability of $\Pr(V,n,f,d) = \Pr(-2,6,-2,1)$ as calculated by Lemma 3: $[\gamma_0(1-p_B)^{1+2} + (1-\gamma_0)(1-p_W)^3][\gamma_s(1-p_B) + (1-\gamma_s)(1-p_W)]^{1-1} \times \left[\gamma_s\, p_B^{\frac{6-2}{2}} (1-p_B)^{\frac{6+2-2(1+2)}{2}} + (1-\gamma_s)p_W{}^2(1-p_W)^1\right] = [\gamma_0(1-p_B)^3 + (1-\gamma_0)(1-p_W)^3][\gamma_s p_B^2(1-p_B) + (1-\gamma_s)p_W^2(1-p_W)]$.

Insert Figure 2 about here.

Because by Lemma 3 the timing of individual decisions is irrelevant, for a given horizon all histories with the same terminal node, the same number of decisions and the same decision floor can be dealt with as a group. The remaining technical problem, then, is to be able to quantify the number of such identical (from a probability standpoint) histories within each such separate group. An application of Lemma 1 in Hemmer et al. (1998) generates the following two (equivalent) expressions for the number of n-period earnings histories terminating in node V with exactly $d \in \{\max[1; -(V+1)], \ldots, \frac{n-V}{2}\}$ decisions:

$$\sum_{\omega=0}^{\eta} {}_{d+2\omega}H_{\omega} \times {}_{1+n-(d+2\omega)}H_{\underline{\frac{n-V-2(\omega+d)}{2}}} \qquad (1a)$$

and

$$\sum_{\omega=0}^{\eta} {}_{V+d+2\omega}H_{\omega} \times {}_{1+n-(V+d+2\omega)}H_{\underline{\frac{n-V-2(\omega+d)}{2}}} \qquad (1b)$$

where
$$\eta = \frac{n-V-2d}{2},$$
$$ {}_{\alpha}H_{\beta} = \frac{\alpha-2\beta}{\alpha-\beta}\, {}_{\alpha-1}C_{\beta}$$

and ${}_{\alpha-1}C_{\beta}$ is the standard Binomial Coefficient for β successes in α-1 trials.

While the calculations involved in (1a) and (1b) certainly are doable, from both a practical and formal vantage point these expressions are somewhat cumbersome and neither is easily compared for different combinations of V and d, in particular as n grows large. Based on (1a) and (1b), however, we are able to derive a much more convenient and transparent way of identifying the number of n-period earnings histories terminating in node V with exactly $d \in \{\max[1; -(V+1)], \ldots, \frac{n-V}{2}\}$ decisions. Lemma 4 provides the specifics.

Lemma 4 For a n-period horizon, let $\mathcal{H}_{n,V,d}$ be the number of histories or paths with terminal node V and $d \geq 1$ informed decisions, where accordingly $V \leq n-2(1-f)$. Then for

$$d \begin{cases} = f + \frac{n-V}{2} \\ \in \left]f - V, f + \frac{n-V}{2}\right[, \\ = f - V \\ = f - V - 1 \end{cases} \qquad \mathcal{H}_{n,V,d} = \begin{cases} 1 \\ {}_{n+1}H_{\eta} \\ {}_{n+1}H_{\eta} - {}_{n}H_{\eta} \\ {}_{n}H_{\eta-1} \end{cases}$$

Proof: To be provided.

Lemma 4 covers the number of histories when at least one decision d to alter the course of action is taken. Of course, the number of histories with d=0 that lead to a terminal node V can now be calculated as the binomial coefficient for node V minus the sum of the number of paths to V with d>0. Continuing with the illustration in Figure 2, the second case of Lemma 4 holds when one decision to change the course of action is made over the six period horizon. Hence, Lemma 4 calculates the number of paths for which the terminal node V equals -2 and d=1 to be ${}_{6+1}H_3 = \frac{1}{4}\binom{6}{3} = 5$. Indeed, other than the two paths indicated in Figure 2, there are a further 3 paths that have the same V and d: a path with a failure, followed by a success, three failures and a success with a decision to change the course of action at t=5, a path with two failures, followed by a success, two failures and a success with a decision at t=5, and a path with three failures, followed by a success, a failure and another success with a decision at t=3.

4. Earnings and Returns

S-shaped "ERC"

To maintain a reasonable level of tractability, for this section we focus on a model where $n = 12$ and the number of periods covered by each financial report $h = 6$. Also, for some firms γ_S equals 0.5, and hence the decision floor *f* equals 0. These firms are nimble decision makers who react quickly to information that suggest they should change their course of action. For other firms, γ_S equals 0, and hence the decision floor *f* equals *–n+1*.[8] These firms are unable to change their course of action, and provide us with a no-decision making benchmark case. First, we calculate the expected future earnings for these benchmark firms for which *f* equals -11 so that no decisions are made throughout the 12 periods. We start by calculating the firm value at $t = 6$ immediately after the only going-concern financial report is made public for each level of reported earnings $V_6 \in \{-6, -4, -2\ , 0\ , 2, 4, 6\}$. Firm value is defined as the expected future earnings generated by the opportunity in place, calculated as the sum of the posterior probability that the better (worse) opportunity is in place times the earnings to be generated over the next (last) 6 periods by the better (worse) opportunity as follows:

[8] Note that this simplifying assumption complies with the weak stochastic dominance requirement. We are currently working on extending our model to allow for a more general structure where γ_S can change in every period.

$$E[V_{12}|V_6 = 6, f = -11]$$
$$= 6 \times (2p_B - 1) \times \frac{1/2\, p_B^6}{1/2\, p_B^6 + 1/2\, p_W^6} - 6 \times (2p_B - 1) \times \frac{1/2\, p_W^6}{1/2\, p_B^6 + 1/2\, p_W^6}$$
$$= (12p_B - 6) \times \frac{p_B^6 - p_W^6}{p_B^6 + p_W^6},$$
$$E[V_{12}|V_6 = 4, f = -11] = (12p_B - 6) \times \frac{p_B^5(1-p_B) - p_W^5(1-p_W)}{p_B^5(1-p_B) + p_W^5(1-p_W)},$$
$$E[V_{12}|V_6 = 2, f = -11] = (12p_B - 6) \times \frac{p_B^4(1-p_B)^2 - p_W^4(1-p_W)^2}{p_B^4(1-p_B)^2 + p_W^4(1-p_W)^2},$$
$$E[V_{12}|V_6 = 0, f = -11] = (12p_B - 6) \times \frac{p_B^3(1-p_B)^3 - p_W^3(1-p_W)^3}{p_B^3(1-p_B)^3 + p_W^3(1-p_W)^3} = 0,$$
$$E[V_{12}|V_6 = -2, f = -11] = (12p_B - 6) \times \frac{p_B^2(1-p_B)^4 - p_W^2(1-p_W)^4}{p_B^2(1-p_B)^4 + p_W^2(1-p_W)^4},$$
$$E[V_{12}|V_6 = -4, f = -11] = (12p_B - 6) \times \frac{p_B(1-p_B)^5 - p_W(1-p_W)^5}{p_B(1-p_B)^5 + p_W(1-p_W)^5},$$
$$E[V_{12}|V_6 = -6, f = -11] = (12p_B - 6) \times \frac{(1-p_B)^6 - (1-p_W)^6}{(1-p_B)^6 + (1-p_W)^6}.$$

Note that, because expected future earnings at time t=0 is zero, when we observe V_6=0, we have not learned anything new about whether or not we have the better or worse opportunity in place, so $E[V_{12}|V_6 = 0, f = -11] = 0$, still. Returns are calculated at t=6 as the belief revision of next period's firm value, given the earnings news, or $r_{6,no\ DM} = \frac{E[V_{12}|V_6] - E[V_0]}{E[V_0]}$. It is easily verified from the above that the relation between expected future earnings and realized past earnings absent any decision making is going to be S-shaped and symmetric around zero.

Empiricists have documented such S-shaped earnings-return relation, and have proposed improved empirical specifications to deal with this non-linearity (Freeman and Tse 1992; Beneish and Harvey 1998).[9] However, Kothari (2001, pp. 135) argues that "a strong economic foundation [for these empirical specifications] is not apparent". As an explanation for this empirical fact, studies typically propose that earnings should be split in two components

[9] While most empirical ERC studies have looked at the response to earnings *surprises*, several studies have also documented a levels result (Lev 1989; Easton and Harris 1991; Biddle and Seow 1991). Das and Lev (1994) show the S-shaped earnings-return relation in a levels analysis. As a robustness check, we modeled an adjustment to earnings levels for the market's earnings expectation. Our conclusions and predictions are qualitatively similar for such earnings surprise model, and hence our paper can also be interpreted as relevant to this literature.

(permanent and transitory) and suggest that the market does not expect extreme earnings (or earnings changes) to be permanent. Hence, the price adjustment is smaller for extreme earnings news (Freeman and Tse 1992; Lipe et al. 1998). However, while transitory items do account for some of the non-linearity, they do not explain it entirely (Das and Lev 1994). An alternate explanation is based on the precision of information. When the market associates a lower precision with extreme news, it attaches less weight to extreme information, and the price response is smaller (Subramanyam 1996; Burgstahler and Chuk 2013).[10]

Our theory explains this empirical fact by the diminishing impact on belief revision of extreme earnings through Bayesian updating. Reporting higher (lower) earnings is the equivalent of more pieces of good (bad) news having being obtained during the reporting horizon. The more pieces of good (bad) news the investors receive, the higher are their (and the manager's) posteriors that the opportunity in place is a better (worse) one and thus the higher (lower) the valuation they will attach to the reporting firm. But because the posterior is bounded above at one, Bayesian updating is inherently non-linear for extreme earnings levels, resulting in the S-shape. Figure 3 plots this relation by the dashed line, and will provide the no decision making benchmark against which to evaluate a case in which the manager does take informed decisions.[11] In the extreme case where the manager has no decision making impact and cannot make any changes to the initial strategy that was randomly chosen, the resulting earnings-return relation will be perfectly symmetric.

-Insert Figure 3 about here-

Decision making impact diminishes ERC

Now turn to firms for which γ_S=0.5, and hence the manager has maximum decision making impact, which implies that $f = 0$[12] and start again with the case where reported realized earnings are $V_6 = 6$. In this case the manager will not choose a new opportunity over the

[10] Other explanations rely on increased competition for firms that are doing well, mean reversion and diminished returns to investment.

[11] Because in the no decision impact case (γ_S=0) E[V_0] equals 0, in figure 3 we assume that the firm has an additional asset worth 1 initially that will be liquidated at time t=12 so that we would have a non-zero denominator.

[12] $\ln\left(\frac{1-\gamma_s}{\gamma_s}\right)/\ln\left(\frac{1-p_B}{p_B}\right)$ is a negative number. In order for $f = \left\lceil \ln\left(\frac{1-\gamma_s}{\gamma_s}\right)/\ln\left(\frac{p_W}{1-p_W}\right)\right\rceil$ to be equal to zero, the case of maximum decision making impact, $\ln\left(\frac{1-\gamma_s}{\gamma_s}\right)/\ln\left(\frac{p_W}{1-p_W}\right)$ needs to be strictly greater than -1. This implies that $\gamma_s > p_W$. Since $p_W < 0.5$, the simplifying assumption that γ_S=0.5 is consistent with a decision floor of zero for any value of p_w.

remaining 6 periods. Also, no decision to choose a different opportunity has been made in the previous 6 periods. The expected future earnings thus are $E[V|V_6 = 6, f = 0] = (12p_B - 6) \times \frac{p_B^6 - p_W^6}{p_B^6 + p_W^6}$, and identical to the "no decision" case above.

Next, consider the histories that lead to earnings of $V_6 = 4$. By Lemma 4, there is 1 history with 1 decision to change opportunities and ${}_6C_1 - 1 = 5$ histories where the manager stuck with the initial decision leading to this point. The posterior for the opportunity in place being of the better "B" (worse "W") type does depend on whether an informed decision to choose a different opportunity was made during the first 6 periods or not. Specifically,

$$Pr[B|V_6 = 4, d = 0] = \frac{p_B^5(1 - p_B)}{p_B^5(1 - p_B) + p_W^5(1 - p_W)}$$

and

$$Pr[B|V_6 = 4, d = 1] = \frac{\gamma_s p_B^5}{\gamma_s p_B^5 + (1 - \gamma_s)p_W^5}.$$

$Pr[W|V_6 = 4, d = 0]$= 1- $Pr[B|V_6 = 4, d = 0]$, and similarly for the case in which d=1. If no decisions to pick a new opportunity were made prior to t=6 (i.e. d=0), then the implied decision floor *f* at the start of the next 6 period time span (period 7 through 12) is -4. However, if instead 1 decision was made to select a new opportunity (i.e. d=1), then the implied floor *f* for the next period is -5.

Using the probabilities that the firm has either an opportunity B or W in place at the start of period 7, we can now calculate the expected earnings at the end of the 12th period, under these implied decision floors *f*. For example, if the implied floor is -4, $E(V_{12}|V_6 = 4, d = 0) = \frac{p_B^5(1-p_B)}{p_B^5(1-p_B)+p_W^5(1-p_W)} \times \Big[6p_B^6 + 4 \times 6p_B^5(1 - p_B) + 2 \times 15p_B^4(1 - p_B)^2 + 0 - 2 \times 15p_B^2(1 - p_B)^4 - 4 \times \Big(5p_B(1 - p_B)^5 + (1 - p_B)^5\big(\gamma_s p_B + (1 - \gamma_s)p_W\big)\Big) - 6(1 - p_B)^5\big(\gamma_s(1 - p_B) + (1 - \gamma_s)(1 - p_W)\big)\Big] + \left(1 - \frac{p_B^5(1-p_B)}{p_B^5(1-p_B)+p_W^5(1-p_W)}\right) \Big[6p_W^6 + 4 \times 6p_W^5(1 - p_W) + 2 \times 15p_W^4(1 - p_W)^2 + 0 - 2 \times 15p_W^2(1 - p_W)^4 - 4 \times \Big(5p_W(1 - p_W)^5 + (1 - p_W)^5\big(\gamma_s p_B + (1 - \gamma_s)p_W\big)\Big) - 6(1 - p_W)^5\big(\gamma_s(1 - p_B) + (1 - \gamma_s)(1 - p_W)\big)\Big]$. A similar calculation for $E(V_{12}|V_6 = 4, d = 1)$ can be made for an implied decision floor of -5. Since the firm only can achieve V_6=4 by making either 0 or 1 decision to alter the opportunity in place, $\Pr(V_6 = 4) =$

$\Pr(\mathrm{d}=0 \text{ and } V_6=4) + \Pr(\mathrm{d}=1 \text{ and } V_6=4) = 5 \times 0.5 \times \left(p_B^5(1-p_B) + p_W^5(1-p_W)\right) + 0.5 \times (\gamma_s p_B^5 + (1-\gamma_s)p_W^5)$. The expected future earnings when observing $V_6=4$ can now be calculated as

$$E[V_{12}|V_6=4, f=0]$$
$$= E(V_{12}|V_6=4, d=0) \times \frac{5 \times 0.5 \times \left(p_B^5(1-p_B) + p_W^5(1-p_W)\right)}{5 \times 0.5 \times \left(p_B^5(1-p_B) + p_W^5(1-p_W)\right) + 0.5 \times (\gamma_s p_B^5 + (1-\gamma_s)p_W^5)}$$
$$+ E(V_{12}|V_6=4, d=1) \times \frac{0.5 \times (\gamma_s p_B^5 + (1-\gamma_s)p_W^5)}{5 \times 0.5 \times \left(p_B^5(1-p_B) + p_W^5(1-p_W)\right) + 0.5 \times (\gamma_s p_B^5 + (1-\gamma_s)p_W^5)}.$$

It is easily verified that $E[V_{12}|V_6=4, f=0] > E[V_{12}|V_6=4, f=-11]$ (as calculated in the no decision impact benchmark case), and that the difference is due to the potential for informed managerial intervention. Even though it becomes a little more involved when more histories with various numbers of decisions to change the opportunity in place are possible, a similar calculation can be made for each observed level of V_6. The expected value of the firm at time 0 is then $E[V_0] = \sum_{i=-6,-4,-2,0,2,4,6} E[V_{12}|V_6=i, f=0] \times Pr(V_6=i)$. It can be verified that this expected value is greater than zero; this positive value is caused in its entirety by the decision making impact of the manager. Returns at t=6 can now be calculated as $r_{6,DM} = \frac{E[V_{12}|V_6]-E[V_0]}{E[V_0]}$.

Figure 3 plots the Earnings Response Coefficient (ERC) for firms with opportunity sets of varying differential quality, as measured by p_B (0.6, 0.75 and 0.9). The higher p_B, the bigger the difference in value of the opportunities in the opportunity set of the firm, and the higher the benefit (cost) of making a good (bad) decision. Increasing the differential quality of the opportunity set depresses the ERC. The intuition for this is twofold. First, the expected value at time zero is larger for firms in which managers have decision making impact than in firms where managers are stuck with the initial randomly selected opportunity. That is, $E\left[V_{0,DM}\right] > E[V_{0,no\ DM}]$. Hence, the denominator in the returns calculation is larger for FM firms. Second, $E[V_{12,DM}|V_6] \geq E[V_{12,no\ DM}|V_6]$ as both outsiders and insiders anticipate that managers will make decisions in the next set of periods to improve value.[13] However, the absolute value of the belief revision numerator $|E[V_{12}|V_6] - E[V_0]|$ is never as big for FM firms as it is for firms without decision making impact for any $V_{12} \neq 0$, because managers already used previously internally reported earnings to adapt their decision making. That is, $\left|E\left[V_{12,DM}|V_6\right] - E\left[V_{0,DM}\right]\right| < \left|E\left[V_{12,no\ DM}|V_6\right] - E\left[V_{0,no\ DM}\right]\right|\ \forall V_{12} \neq 0$.[14] Overall, it follows that ERC is muted for FM firms.

Asymmetry in "ERC"

While Figure 3 depicts unscaled ERCs for individual firms for which decision making impact differs, we also construct various cross-sectional samples to identify the earnings-return relation using firms with different values of p_B (0.6, 0.75 and 0.9). Using the probabilities of the seven levels of earnings calculated earlier ($Pr(V_6)$) as the relative frequencies, we construct three samples of each 100 firms for each of the levels of p_B. Then, we follow the empirical convention to scale current earnings by lagged price (e.g. Basu 1997; Patatoukas and Thomas 2011).[15] Next, we run two OLS regression of scaled earnings on returns in each of these samples, one for the positive returns firms and one for the negative returns firms. The regression results are documented in Panel A of Table 1.

Insert Table 1 about here.

First, consider the first column in Table 1, Panel A, and the full line in Figure 3 for p_B=0.6. These are results for firms for which the differential quality of available opportunities is fairly small (p_w=0.4). The regression coefficient for negative returns is much higher than for positive returns in Table 1, Panel A, and the ERC is asymmetric with less return reaction associated with low earnings in Figure 3. This pattern is consistent with the relation documented by Basu (1997) where, unlike in our setting, the market leads earnings; since conservative accounting incorporates bad news more timely in earnings and hence such news will be less persistent, negative earnings have a greater tendency to reverse. As the outside investors

[13] $E\left[V_{12,DM}|V_6\right] > E\left[V_{12,no\ DM}|V_6\right]$ for all levels of V_6 other than for the highest level (V_6=6), where $E\left[V_{12,DM}|V_6 = 6\right] = E\left[V_{12,no\ DM}|V_6 = 6\right]$ as after observing V_6=6 there is certainty about having the better opportunity in place.

[14] Because by assumption 2 there is no news content when aggregate earnings are zero absent earnings-based decision making intervention, $V_{12}=0$ is the only exception for which this relation is reversed.

[15] Basu (1997) also adjusts for the average earnings-price ratio, but this adjustment is not normally applied in later literature (Dietrich et al. 2007; Patatoukas and Thomas 2011). As Patatoukas and Thomas (2011) note, this adjustment will solely affect the intercept (downwardly) in an earnings returns regression.

rationally do not expect the bad news to be persistent, price revisions are less strong for bad news than for good news.

In our setting, however, this asymmetry is not indicative of a deliberate bias in accounting, yet solely a result of managerial earnings-based decision making. When the evidence is sufficiently strong that a worse opportunity is currently in place, managerial intervention decreases the probability that it will continue to be so.[16] Accordingly, while observing negative reported earnings does increase the investor's posteriors that a worse opportunity is in place, the (non-linear Bayesian) updating is more muted for negative than for positive earnings in this case, as the assessment that more value will be created with the change of course in the next period outweighs.

Moving to the second and the third column in Table 1, Panel A, and to the dotted and small dashed line in Figure 3 for values of p_B=0.75 and 0.9, however, the above pattern is reversed: the OLS coefficient for negative returns is much lower than the coefficient for positive returns in both samples. The intuition for this result is twofold. First, as before, when the evidence is sufficiently strong that a worse opportunity is currently in place, managerial intervention decreases the probability that it will continue to be so. However, because the differential quality of the available opportunities is very big, investors will at the same time also realize that a lot of value was foregone by developing the much worse opportunity for several periods. In this case, the second effect outweighs.[17] Second, expectations for firms with higher quality opportunities are on average higher. Hence, high earnings are expected, resulting in little updating for high earnings. Low earnings, to the contrary, are highly unusual, and hence will attract a stronger returns reaction.

Given the reversed pattern for these two subsamples, one could speculate that a combined sample that encompasses all 3 types of firms in equal proportions would likely be inconsistent with the earnings-return relation documented in Basu (1997) because the slope coefficient for

[16] This is somewhat similar to the argument in Hayn (1995), albeit that in her paper it is the investors that have the option to abandon the firm instead. Hayn (1995) suggests that investors perceive losses (and low earnings) to be temporary, since shareholders can always exercise a put option on the firm, rather than suffer from indefinite losses. Again, market response will be muted. The result in Subramanyam and Wild (1996) that earnings informativeness is inversely related to an entity’s probability of termination is consistent with Hayn (1995)’s explanation.

[17] Since in this case the value forgone in the first reporting period outweighs the benefit of improved decision making for the next period, it is likely that adding more reporting and decision making periods reverses this effect and hence Basu-like asymmetries will also be observed for firms with a large differential quality in their opportunity sets. We are currently working on an extension to the model that includes more reporting periods.

negative returns is much lower than the slope coefficient for positive returns for two-thirds of the combined sample, a relation which is very extreme in the 0.9 subsample. However, the final column in Table 1, Panel A, shows exactly the opposite: for the combined cross-sectional sample, the slope coefficient for negative returns is much higher than for positive returns. Figure 4 illustrates why this happens. The regression lines for each of the reported samples and the combined sample are drawn for their respective negative and positive subsamples separately.

Insert Figure 4 about here.

The North-East quadrant has the regression lines for the positive returns subsamples, while the South-West quadrant depicts the regression lines for the negative returns subsamples. Indeed, the NE quadrant 0.9 full regression line is relatively steep, compared to a relatively shallow regression line in the SW quadrant. The NE quadrant 0.6 dashed regression line is shallower, while the SW quadrant 0.6 dashed regression line is steeper. Note, however, that there exists a substantial range differential between these lines. The 0.9 full regression line range is relatively tight, whereas the 0.6 dashed regression line range is more extensive. This range differential is due to the scaling of earnings by lagged price, since lagged price is lower for a firm that doesn't have a very high quality opportunity than for a firm that does. The third of the sample that has p_B=0.9 hence does not impact the full sample regression estimation much, while the p_B=0.6 part of the sample does impact a lot. Overall, therefore, the combined sample regression lines as indicated by the black full lines are mainly driven by the p_B=0.6 sub-sample and are therefore consistent with the asymmetric earnings-return relation documented by Basu (1997).[18]

So far, we have used a subsample analysis in which we separated the positive returns firms from the negative returns firms. In what follows, we estimate the OLS regression that the empirical literature on earnings-returns relations typically uses following Basu (1997) for our combined sample. We estimate the following OLS regression:

[18] Note that this combined sample equally weighs firms in which there is very little variation in the quality of the strategies in the opportunity set (p_B=0.6 and p_W=0.4) and firms in which there is a lot of variation in the quality of the strategies in the opportunity set (p_B=0.9 and p_W=0.1). In a COMPUSTAT sample, it is likely that the latter firms are less frequently present as managers and employees with a particular skill set are likely to create opportunities of similar quality. Hence, the effect of the 0.9 subsample may be even less pronounced than indicated in Figure 4. Not scaling by lagged price doesn't resolve the conservatism test. Scaling solely leads to firms with low differential quality in their opportunities (and more muted updating for negative than for positive earnings) impacting the regression more, but both the asymmetry and reverse asymmetry can be observed in the unscaled earnings return Figure 3 too.

$$\frac{V_{i6}}{P_{i0}} = \alpha_0 + \alpha_1 Dr_{i6} + \beta_0 r_{i6} + \beta_1 r_{i6} \times Dr_{i6} + \varepsilon_i$$

where Dr_{i6} is a dummy that takes on the value of 1 if returns are negative, 0 otherwise. All other variables are defined as before. The results are reported in the first line of Table 1, Panel B. The interactive slope coefficient β_1 is positive and significant, which implies that earnings is about 1.5 ($= \frac{\beta_1+\beta_0}{\beta_0} = \frac{10.0388+4.8554}{10.0388}$) times more sensitive to negative than to positive returns in this sample, even though there is no bias in the accounting information. Note that all coefficients in this regression have the same sign as documented by Basu (1997, table 1, panel A and B). This equally weighed sample of 300 firms has more firms that have positive returns than firms that have negative returns (ss^+=168 and ss^-=132). Although such imbalance of negative and positive returns is typically also observed in empirical studies that use COMPUSTAT to study the phenomenon, we want to exclude that this imbalance is causing our results. We construct a balanced sample of 600 firms by the including the 200 firms with the highest returns from the 0.6 sample and the 100 firms with the highest returns from the 0.75 sample twice. This sample achieves a balance of negative and positive returns (ss^+=298 and ss^-=302). As the second line of Table 1, Panel B, reports, the interactive slope coefficient β_1 for this sample of 600 firms is even more positive and significant, and implies that earnings is about 1.8 times more sensitive to negative than to positive returns in this sample.

Note that the above result does not entail that accounting conservatism does not lead to asymmetries in the earnings-returns relation. Since (conservative) accounting recognizes bad news quicker than good news, it is highly likely that the market reacts differently to good and bad news. In fact, Basu (1997) documents that earnings is about 4.5 times more sensitive to negative than to positive returns. However, since we document an asymmetric earnings-return reaction by applying a model in which there is no bias in the accounting information on which the manager bases their decisions, we suggest that the benchmark against which to document differential market reaction due to accounting conservatism needs to be reconsidered. Stated differently, while accounting conservatism may well lead to a more asymmetric relation between earnings and returns, finding such a relation empirically is not sufficient to establish that earnings are conservative.

5. Cross-sectional Earnings Frequency Distributions

The last part of our study is motivated by Burgstahler and Dichev (1997a) and the literature on the large sample properties of realized earnings this seminal study initiated. Several studies have challenged the existence of this discontinuity (e.g. Durtschi and Easton 2005, 2009), while others have proposed methodological improvements to document it (e.g. Kernstein and Raj 2007; Jacob and Jorgensen 2007; Jorgensen et al. 2013; Burgstahler and Chuk 2015). Other studies suggest that, while the discontinuity does appear real, management of discretionary accruals appears not to be its cause (Dechow et al. 2010, , pp. 365). For example, Dechow et al. (2003) list several alternative explanations, such as real actions taken to improve firm performance, exchange listing requirements that have a selection bias towards profitable firms, and conservatism and accounting rules precluding the recording of small losses and encouraging the recording of small profits. Jorion and Schwarz (2014) show how incentive fees can mechanistically create a kink at zero in hedge fund net return distributions.

Our objective here is different from all of the above. Heeding Ball (2013)'s call to understand what so-called un-managed earnings look like so that we would understand what the benchmark is against which to document earnings management, in this section we no longer are concerned with the informational qualities of financial reports but seek to establish the expected frequencies of realized earnings in a large sample of diligently managed firms. The distribution of realized earnings frequencies for a large sample is derived through combining the realized earnings probability distributions of individual firms.

We start with studying realized earnings probability distribution for the extreme benchmark case where the decision floor *f*=-*n*, or equivalently, $\gamma_s = 0$. In this case, the manager picks an opportunity randomly in the first period, and after that never makes an informed decision to alter the opportunity in place, yet stays the course. The probability with which every potential level of earnings V at the end of the model's horizon is observed in this case equals $0.5\left(\left(p_B^{\frac{n+V}{2}} \times (1-p_B)^{\frac{n-V}{2}}\right) + \left(p_W^{\frac{n+V}{2}} \times (1-p_W)^{\frac{n-V}{2}}\right)\right) \times \binom{n}{\frac{n+V}{2}}$. This realized earnings probability distribution only leads to a Normal distribution when two conditions are satisfied. First, the manager cannot make any informed choices and is stuck with his initial choice over the entire horizon. Second, his initial random pick is of no consequence in that it does not matter for earnings which opportunity he initially randomly picked. This is the case when p_B=p_W=*0.5*, so technically one opportunity is not better than the other. The resulting distribution is indicated by the dashed line in Figure 5, Panel A, for an n=18 period case.[19]

Insert Figure 5 about here.

For the benchmark case where no informed decisions are made ($\gamma_s = 0$) but $p_B \neq p_W$, the earnings distribution clearly becomes bimodal. The negative mode is centered on the most likely earnings generated by opportunity W, while the positive mode is centered on the most likely earnings generated by opportunity B. Because the initial pick is random, each mode in this bimodal distribution is equally likely. If the quality of the opportunities is different enough, those modes will virtually separate. Because p_B=1-p_W the probability that zero earnings are generated will at that point be very low. Panel A of Figure 5 shows such bimodal distribution for p_B=0.75 with a dotted line. The reason for the non-normality is due to a simple but perhaps subtle distinction. While the sum of normally distributed *variables* (aggregate earnings for all firms) is normally distributed, the sum of normal *distributions* (the aggregate *number* of earnings observations) generally isn't. When different firms end up with either better or worse (normal) earnings distributions, adding these earnings realizations up results in bi-modality when the difference between better and worse firms is sufficiently large.

Now introduce a meaningful role for informed decision making where $\gamma_s = 0.5$, or equivalently, f=0. Because earnings here are allowed to inform real decisions and because such decisions now impact earnings, clearly the final distribution of realized earnings observations with FM, should not be expected to be similar to the benchmark distribution that results absent FM. While it is easily verified in our setting that engaging in FM is value enhancing on average, the shift away from the benchmark distribution caused by FM actually comes in two flavors.

First, if the manager starts out on a bad track, the accounting information is more likely to lead him to change direction and thus to end up with the better opportunity in place. This is of course the desired effect of informed decision making, which shifts the probability mass to the right on more favorable outcomes, and away from the unfavorable outcomes on the left. However, because earnings are noisy signals, by chance the manager could end up concluding that a better opportunity is currently in place when a truly worse opportunity is in place. In this case, he makes the bad decision to stick with the opportunity in place. The effect is unfortunate

[19] Note that in the case where p_B=p_W=0.5, the $\gamma_s = 0$ case is equivalent to the $\gamma_s = 0.5$ case. Adding more periods (than 18) will make the model very hard to trace, but will not offer additional insights.

in the sense that it keeps probability mass on the left. Overall though, there will be a lower weight on the distribution of the worse opportunity and a higher weight on the distribution of the better opportunity. When the quality of the opportunity set is very high ($\gamma_s = 0.5$) and the manager hence has a lot of decision making impact, this will result in a right skewed and here unimodal distribution as depicted by the full line in Figure 5, Panel A, for a value of p_B=0.75.

In the cross-section, both firms in which managers have limited or no decision making impact ($\gamma_s = 0$) and firms in which managers have significant decision making impact ($\gamma_s = 0.5$) are likely to be present. We use the parameter $w_{DM} \in [0,1]$ to represent the proportion of the firms in the sample in which managers have a lot of decision making impact and hence the proportion of firms in the sample that have an earnings probability distribution generated by our model in the case of maximal decision making impact ($\gamma_s = 0.5$). *1-* w_{DM} is the proportion of firms in which managers have no decision making impact and hence follow an earnings probability distribution generated by the $\gamma_s = 0$ model. The earnings frequency distribution in Figure 5, Panel B, is drawn for a sample where the proportion of firms with and without decision making impact is equal (that is, w_{DM} =0.5), with all other parameters as before in this section (p_B=0.75 and n=18). The earnings frequency distribution our approach generates matches the corresponding frequency distribution of individual earnings realizations obtained from COMPUSTAT quite well in that it is distinctly bi-modal and shows a dip around zero.

We do of course acknowledge that given the necessarily simple nature of our set-up, our approach may not be a particularly accurate description of the specific decision process in any given firm. The options available, the ability to respond in a timely manner and the consequences of getting it "right" likely differ across firms and over time. In the bigger sample, however, the implications of such idiosyncrasies are quite likely to wash out without any systematic implications for the overall aggregate frequency distribution of realized earnings. What should not wash out, however, are the fundamental effects of decision making based on past earnings: sometimes the wrong inferences are made and, thus, sometimes bad decisions are made, but on average, making informed decisions is earnings-improving. That is the exact effect our model captures. It also is what leads to the dip in the distribution, and it certainly does not appear to be contradicted by the data.

Moreover we recognize that our analyses and results do not rule out that AM to avoid small losses takes place and that it has distinct distribution-level implications.[20] We do believe, however, that the explanation for the observed irregular earnings frequency distribution that managers are diligently managing their firms is certainly no less plausible than the AM-based explanations. Furthermore, since our predictions about the unmanaged distribution (in the AM sense) are entirely caused by FM, the plausibility of our explanation lends itself to further empirical scrutiny. Arguably, there are systematic differences across industries in terms of how nimbly their member firms can respond to earnings and/or how sticky decisions are. As outlined below, such differences have predictable distributional implications under our FM-perspective.

6. Further empirical connections and testable hypotheses

Decision making impact diminishes ERC

Existing empirical evidence can be interpreted to support the hypothesis that firms with high quality opportunity sets have lower ERCs. In such firms, the impact of decision making is high. Lev and Zarowin (1999) find that firms that increase R&D intensity see an abnormally steep decrease in their ERC. They hypothesize (but do not test) that this effect is due to reported earnings based on immediate expensing of this R&D differing materially from economic earnings that arguably capitalize such intangibles. Using a similar argument, Amir and Lev (1996) find that value relevance of accounting earnings is low in the high-intangible cellular phone industry. Using SGA intensity, R&D intensity and market-to-book ratio as proxies for intangible intensity, Srivastava (2014) demonstrates that the declining relevance of earnings over time can be attributed to the higher intangible intensity of firms added to the population over time.[21] If an increase in R&D intensity or the presence of many intangible assets reflects the increased availability of alternate high quality opportunities, our managerial decision making theory can also explain these empirical results. In line with our predictions, Burgstahler and Dichev (1997b) find that when adaptation value of a firm's resources is high in that they can easily be put to some alternative superior use, earnings (which are a measure of recursion value, the discounted value of the stream of future earnings under the assumption that the firm

[20] Note too that we do not speak to the researcher's ability to document other (non-distributional) forms of AM such as the use of discretionary accruals, restatements, and meeting or beating analyst forecasts.
[21] Note that Collins et al. (1997), using an industry classification to proxy for high intangible intensity, do not find that this contributes to the declining value relevance of earnings over time.

continues to apply its current business technology to its resources) will explain less of the market value of equity.

Furthermore, cross-sectional studies could use proxies of the manager's decision impact to test our theory. For example, the number of bureaucratic layers in the firm and the formality of the budgeting and approval for capital investment processes are both negatively correlated with timely managerial decision making. The founder status of the manager, the frequency of strategic management meetings, compensation of manager (as a proxy for managerial ability), and the slack in debt covenants allowing quick access to funding are all positively correlated with decision making impact in a timely matter.[22] Start-ups typically are more nimble decision makers than mature firms. Also, decision making impact may be correlated with industry. For example, in the pharmaceutical or information technology industries, decisions on which R&D project or technology to pursue are very impactful. In mining, decisions on drilling locations are high stakes. In contrast, in the fast food industry, menu choices may have less impact, while in the utilities sector more decisions are left in the hands of the regulator. Our theory predicts lower ERCs in cross-sectional settings where decision making impact is higher.

Asymmetry in "ERC"

Only limited empirical evidence on the relation between the asymmetric earnings response and the availability of quality opportunities exists.[23] Khan and Watts (2009) document that firms with longer investment cycles, as measured by a smaller ratio of depreciation over lagged assets, have higher accounting conservatism. While they propose that agency issues related to the uncertainty of investment opportunities are resulting in a demand for higher conservatism, our managerial decision making theory also predicts that the "Basu-shape" should be most apparent when high quality opportunities are not immediately available in the firm's opportunity set. Our theory also predicts that firms where managers have at least some decision making impact, ERC will be asymmetric, with more muted returns reactions to low earnings.

[22] We suggest various empirical proxies to develop clear empirical implications. Since some of these suggestions may be interpreted as proxies of other concepts, a thorough empirical study likely will document the results across a set of proxies of decision making impact.

[23] Much of the literature, however, has studied the opposite causality: taking the financial reporting quality as exogenously given, authors show that investment behavior is influenced (Biddle et al. 2009). For example, Bushman et al. (2011) find that timely loss recognition in the financial reporting environment strongly influences investment behavior when firms face declining investment opportunities. In our model, however, asymmetry in the financial report is endogenously derived through the manager's selection of opportunities.

When the available opportunity is of very high quality and managers have a lot of decision making impact, however, the asymmetry in the ERC flips, and returns reactions are muted for high earnings. These predictions can be tested using empirical proxies for the quality of the opportunity set such as R&D intensity (Lev and Zarowin 1999), intangibles intensity (Srivastava 2014) or Tobin's q (Adam and Goyal 2008). Furthermore, the proxies for managerial decision making impact outlined above are other candidates for empirical tests of the impact of decision making on the ERC asymmetry. When ranked over deciles from no decision making impact to major decision making impact, we hypothesize that the ERC should change from symmetric over asymmetric with a more muted returns reaction to low earnings to asymmetric with a more muted returns reaction to high earnings.

While we use the term "earnings" throughout the entire paper, our results hold for any measure of the underlying quality of the opportunity in place that is reported frequently internally in the firm, and of which an aggregate is reported externally less frequently. Hence, we predict that same return responses to aggregate cash flow reports as those derived in this paper for earnings. This is consistent with empirical evidence provided by Dietrich et al. (2007) and Hsu et al. (2012) who find that, when disaggregating earnings, a substantial portion of the Basu coefficient emanates from the cash flow from operations (CFO) component, which should not be affected by accounting rules on timely disclosure. Collins et al. (2014) attribute this asymmetry in CFO to differences in how firms are valued in different life-stages and how CFO reflects good news in growth versus mature firms. They propose to use accruals-only-based measures in studies of asymmetric timeliness to tease out the remaining effect of conservatism on accounting earnings. If accruals in and by themselves are also to be seen as a performance measure that is more frequently reported internally for decision making purposes, and reported in the aggregate externally less frequently, this correction will not fully rule out the impact of managerial decision making.[24]

Earnings frequency distributions

Section 5 and Figure 5, panel B, show that in a generated cross-sectional sample where half of the firms have a lot of decision making impact while the other half does not (w_{DM}=0.5) and where the differential quality of opportunities is medium (at p_B=0.75), the cross-sectional

[24] For example, when bad debt accruals are booked, managers may decide to alter selling practices and approach more credit-worthy customers.

earnings frequency distribution that results looks surprisingly similar to the COMPUSTAT distribution. However, in our setting, the bimodality and dip around zero are caused by the decision making impact of managers (FM), rather than by any form of strategic disclosure (AM).[25] Our model allows us to derive further empirically testable predictions on the impact on the earnings frequency distribution of both the fraction of firms in a sample that have decision making impact and the quality of the opportunities available to firms. Figure 6 documents these comparative statics.

Insert Figure 6 about here.

First, Panel A depicts the impact of changing the fraction of firms that have decision making impact in the sample. Holding p_B constant at 0.75, if we increase the value of w_{DM} from the 0.5 (full line) used in Figure 5, the dashed line in Figure 6, Panel A shows that weight of the earnings frequency distribution shifts away from the negative mode onto the positive mode. As a larger proportion of firms exhibit decision making impact, naturally, the earnings generated on average for the entire sample are higher. As a result, the dip around zero earnings becomes less outspoken, and will even disappear for very large fractions of firms with decision making impact in the sample (untabulated). Reducing the fraction of firms with decision making impact in the sample (the dotted line), to the contrary, shifts mass to the left, as more firms may be stuck with bad opportunities they cannot undo, while other firms are lucky enough to get stuck with a good opportunity. The dip between the two modes becomes much more outspoken. As mentioned earlier, the proportion of firms in which managers have big decision making impact predictably varies across sectors. We expect that earnings frequency distributions for sectors such as retail and utilities (with less decision making impact) exhibit a more outspoken dip at zero and bimodality than for sectors such as technology, pharmaceuticals and mining (with more decision making impact). Within sector, concentration may be a proxy for the proportion of important decision making firms. Other cross-sectional cuts (not limited to sectors) can be derived based on the earlier decision making importance proxies outlined. For example, we expect the earnings frequency distribution of firms in which the CEO is the founder to exhibit a smaller or even no dip at zero, as founders have a big decision making impact.

[25] If our no-decision making impact firm can expect to earn the risk-free interest rate, rather than zero aggregate earnings, the dip ends up close to that risk-free rate, which is close to zero currently.

Panel B in Figure 6 varies the quality of the opportunities available to the firms in the sample, holding w_{DM} constant at 0.5. When p_B decreases (the dashed line), the differential quality of the opportunities available to the firm becomes smaller. The modes for the good and the bad opportunity are hence closer to each other in height, while the decision making firms shift some weight from the negative mode to the positive mode. As a result, the dip around zero becomes much less outspoken, and will disappear if the quality of the available opportunities does not differ much (untabulated). However, when p_B increases (the dotted line), the modes for the good and bad opportunity are pushed farther away from each other, which results in a deeper dip around zero. To our knowledge, no empirical study has looked at the impact of the quality of the opportunities available to the firms on the shape of earnings frequency distributions, and hence future research can test our predictions.[26] We predict that cross-sectional samples in which there is a larger proportion of firms with higher R&D or intangibles intensity and higher Tobin's q show a more clear bimodality and dip at zero than samples in which this proportion is lower. Recent longitudinal work (Gilliam et al. 2015) has shown that the zero-earnings discontinuity existed prior to 2002, but disappeared after that. While Gilliam et al. (2015) attribute this to the introduction of the Sarbanes-Oxley act, future research could look into whether this disappearance is related to increases in the portion of firms with decision making impact and/or to decreases in the quality differential of opportunities in the firms' opportunity sets over time.[27] Note too that our theory applies to all firms and not just to public firms that face capital market pressures. We predict a dip in the earnings frequency distribution for a sample of private firms as well.

7. Conclusion

In this paper, we explore the theoretical relation between earnings and market returns as well as the properties of accounting earnings frequency distributions under the maintained hypothesis that managers use unbiased accounting information benevolently to prudently manage the firms of which they are appointed stewards. Based on arguably the most parsimonious model of informed managerial decision making and market pricing based on reported earnings possible,

[26] Limited empirical evidence is however available that higher quality opportunity sets are positively correlated with high earnings management, using alternate earnings management research designs (Chen et al. 2010; Skinner 1993).
[27] Speculating on the latter, it may well be the case that improvements in information technology which helps transmitting information throughout the firm and the education level of managers ensure more homogenous opportunity sets with similar quality opportunities.

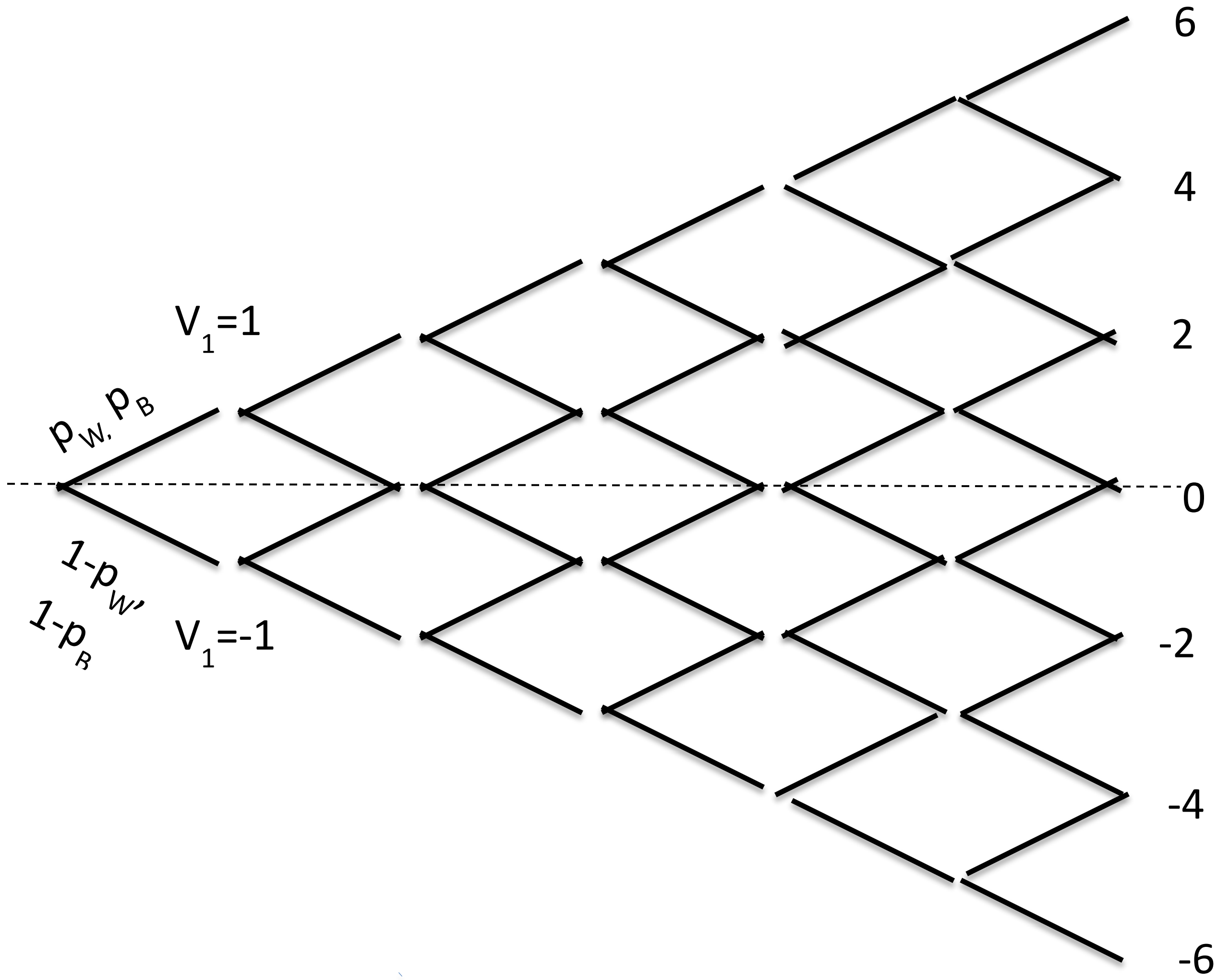
this surprisingly uncommon (in the academic literature) perspective generates untainted benchmarks against which empirically observed earnings-returns relations and aggregate earnings distributions can be evaluated.

Our simple unified managerial decision making theory leads to the following empirical predictions, as observed in the data: reported losses are less persistent than reported gains, ERCs are lower than otherwise expected, the market response to earnings exhibits an "S-shape" and earnings relate to returns asymmetrically in the way documented by Basu (1997). Furthermore, the implied frequency distribution of aggregate earnings is neither symmetric nor necessarily unimodal. Instead, it is likely to exhibit a clear discontinuity and looks remarkably similar to the plots reported by Burgstahler and Dichev (1997a). However, within our set-up, none of these phenomena are due to reporting noise, bias or some undesirable strategic managerial behavior. They are the natural consequences of using past earnings as the basis for prudent managerial decision making that in turn generates the future earnings on which future decisions will be based.

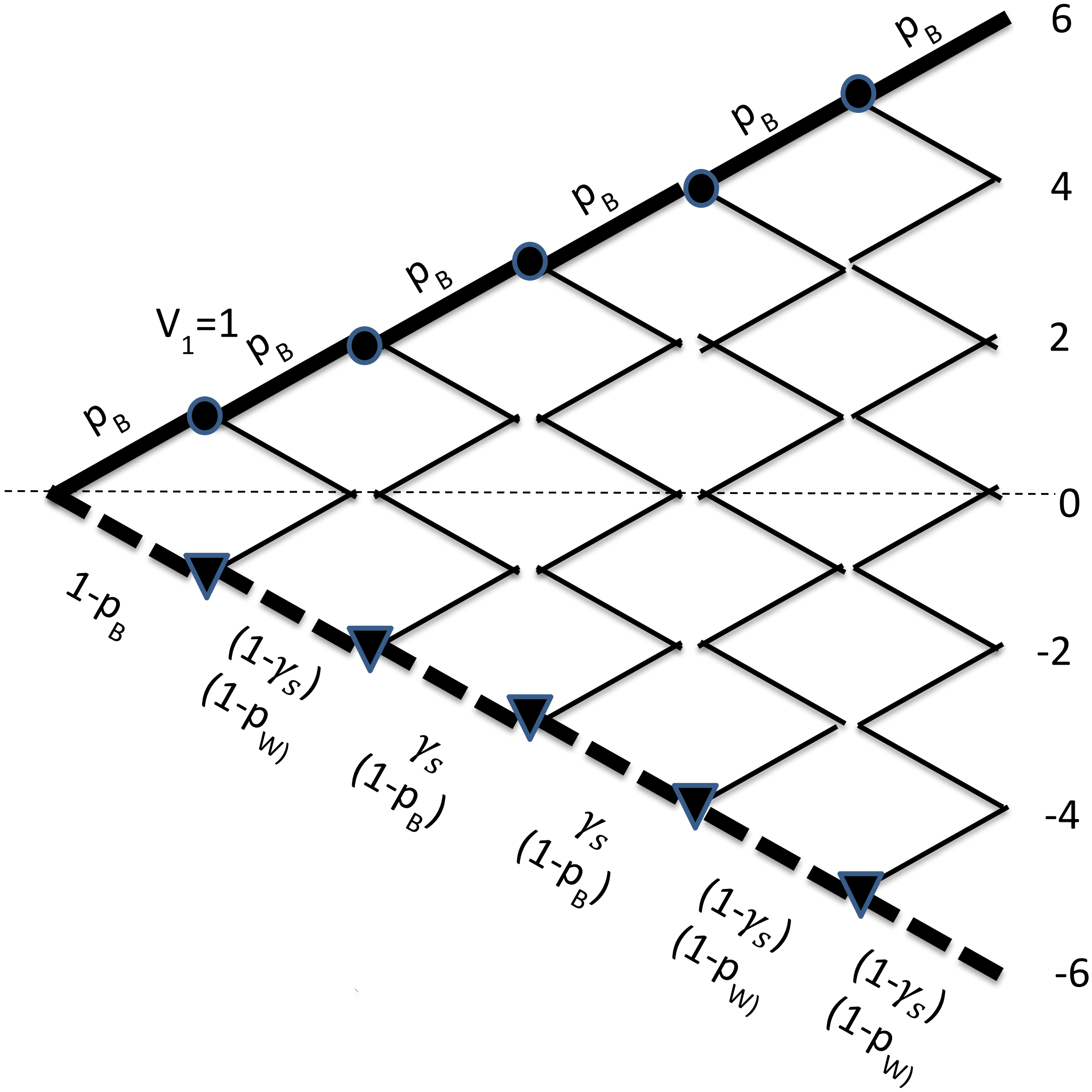
Our maintained FM-only hypothesis may also lend itself well to generating insights into other areas of accounting research not addressed by this study. For example, it would be possible to examine whether compensation structure (incentive intensity as well as pay levels) is based on something as simple as the economic importance of managerial decision making. Our study also speaks to the large literature on disclosure transparency. Our approach suggests that there is also such a thing as inherent operational transparency that differs across firms because of the differential observability of managerial decisions. Further predictions can be made about the fundamentals underlying firms' performance, such as the quality of the opportunity set, decision making impact of managers, and the business cycle (recessions and booms likely see different quality opportunities) on regularities typically thought to belong in the domain of financial reporting. With optimism fueled by the resurgence of academic interest in connecting different areas of accounting research, we look forward to work that addresses the above issues.

Figure 1: Six sub-period tree example

Panel A: Description of 6 sub-period tree example



Panel B: Example: Two paths leading to extreme terminal node values



Panel C: Example: A path leading to a terminal node value of zero

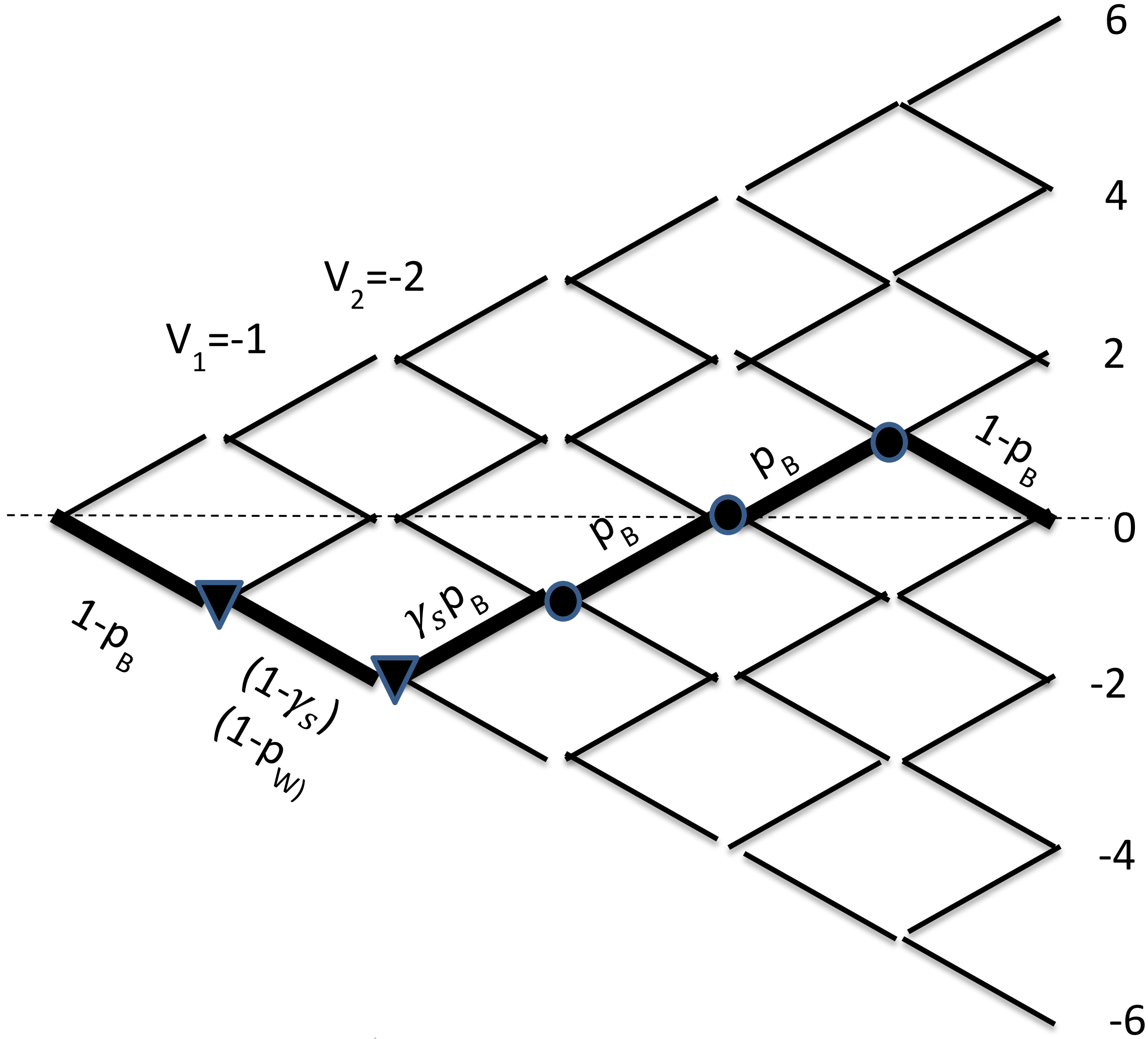


Figure 2: Example of Lemma 3

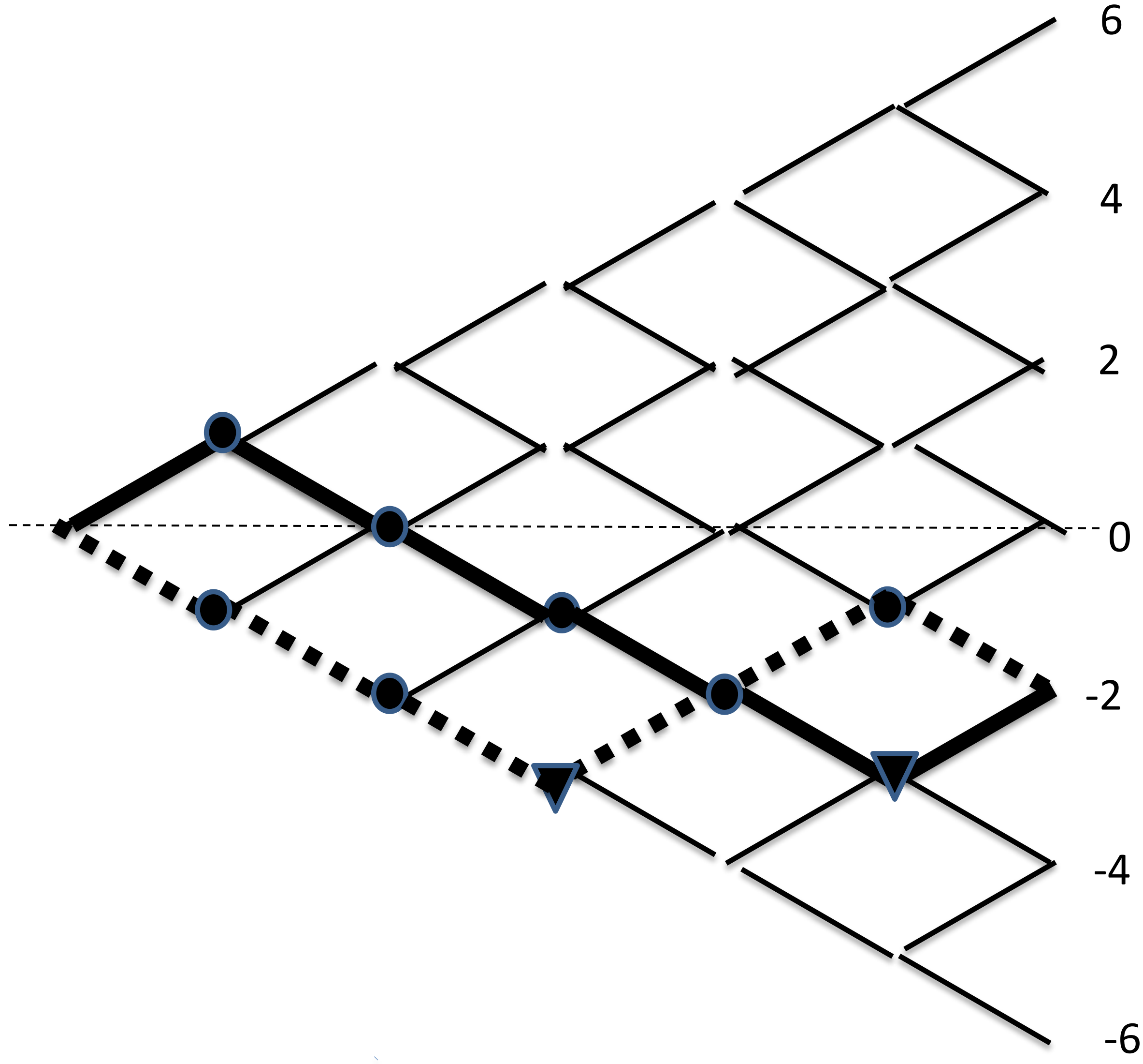


Figure 3: Earnings Response Coefficient (ERC) for individual firms with differing levels of decision making impact caused by alternate available opportunity sets

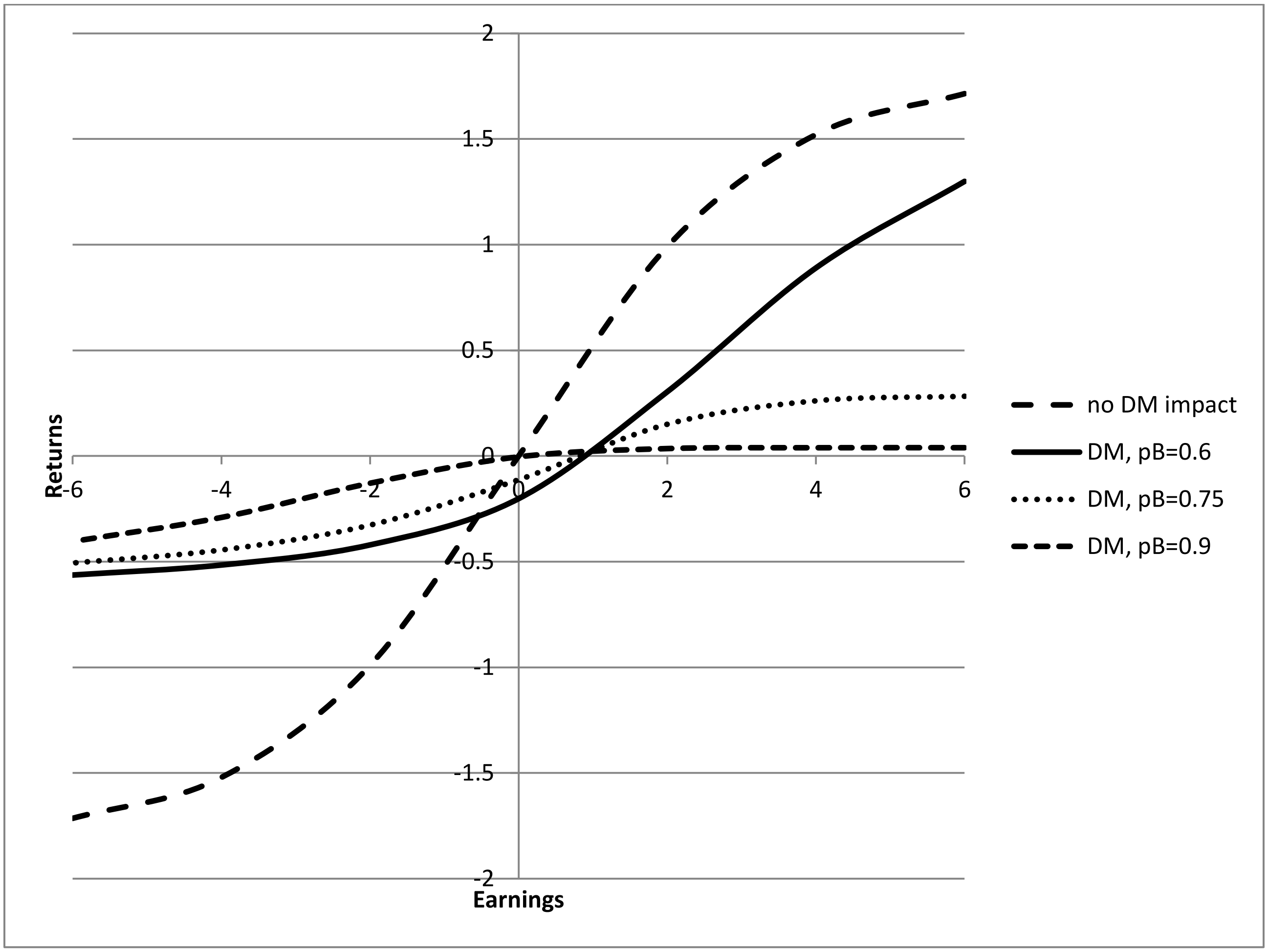


Figure 4: Earnings-return relation for samples of firms with differing levels of decision making impact caused by alternate available opportunity sets

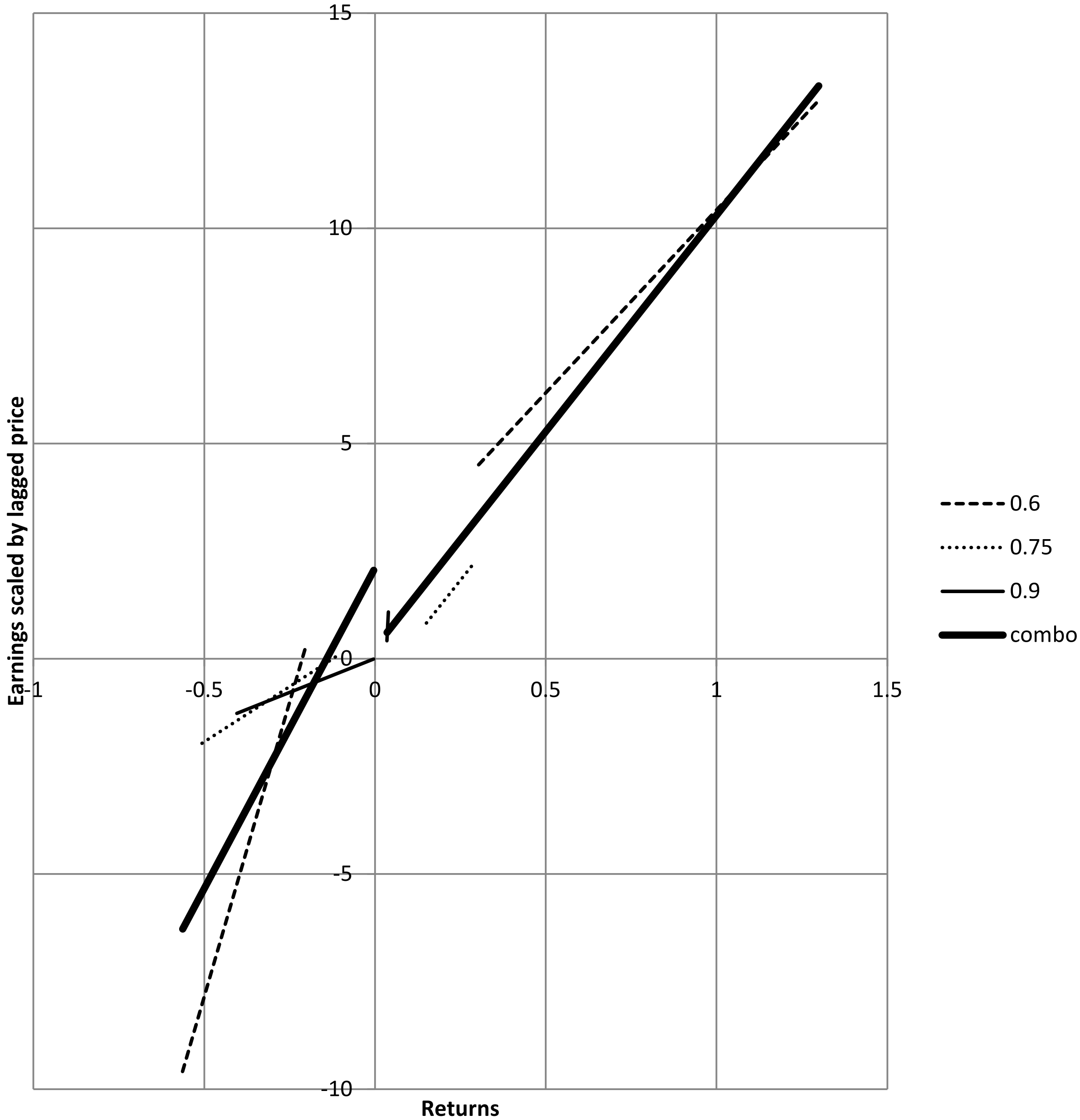
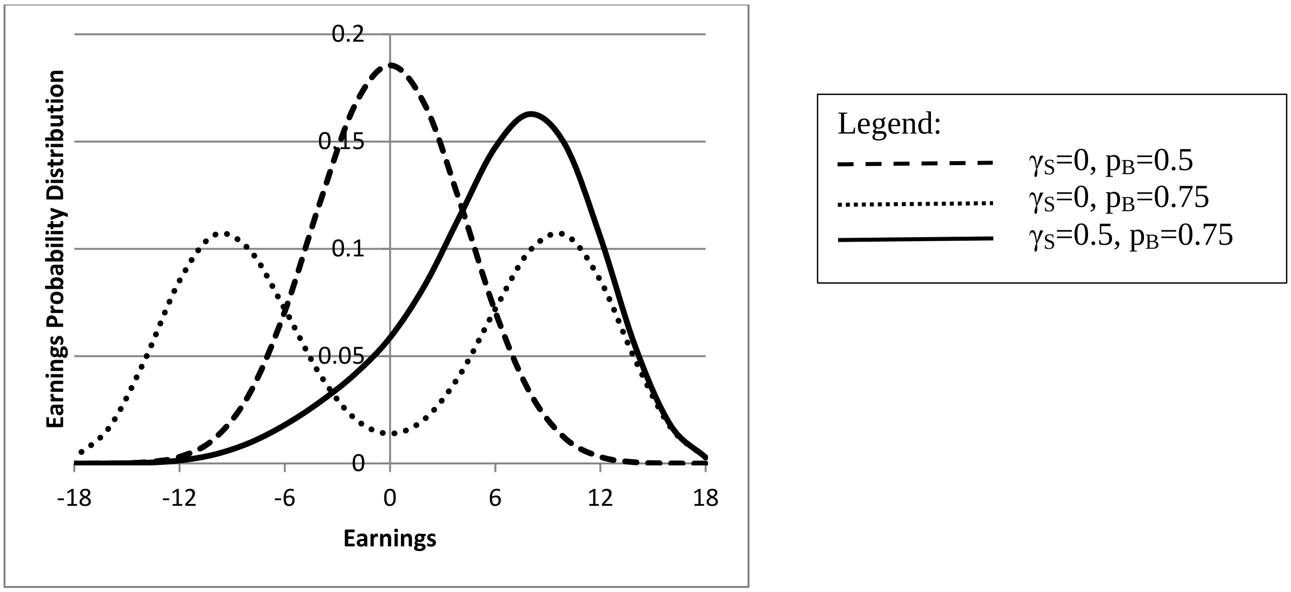
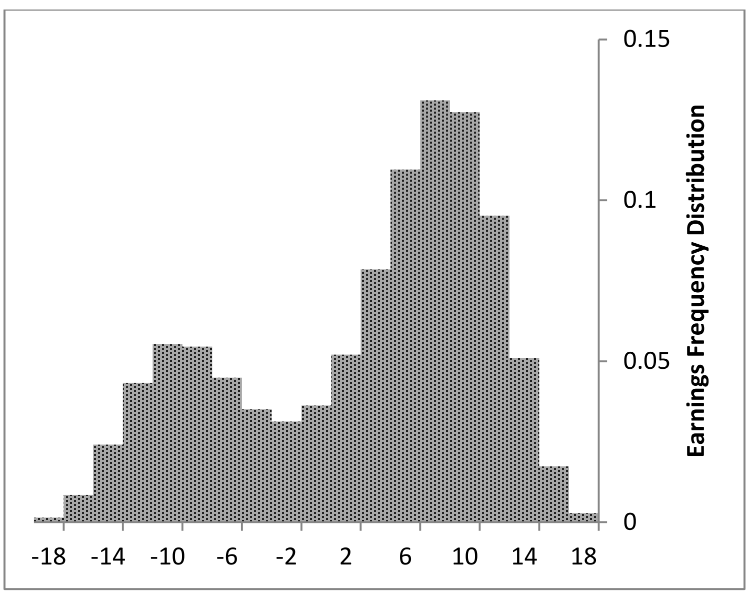


Figure 5: Realized Earnings Probability and Cross-Sectional Earnings Frequency Distributions

Panel A: Realized Earnings Probability Distributions



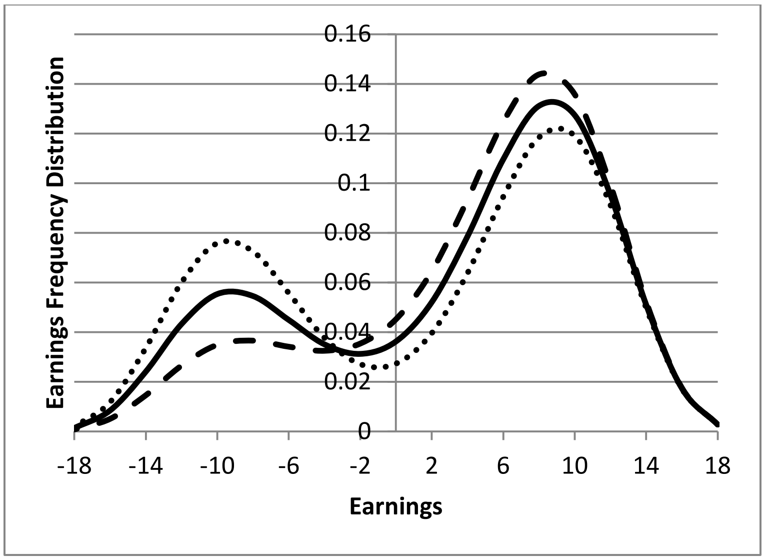
Panel B: Cross-Sectional Earnings Frequency Distribution



Drawn for p_B=0.75, n=18 and w_{DM}=0.5.

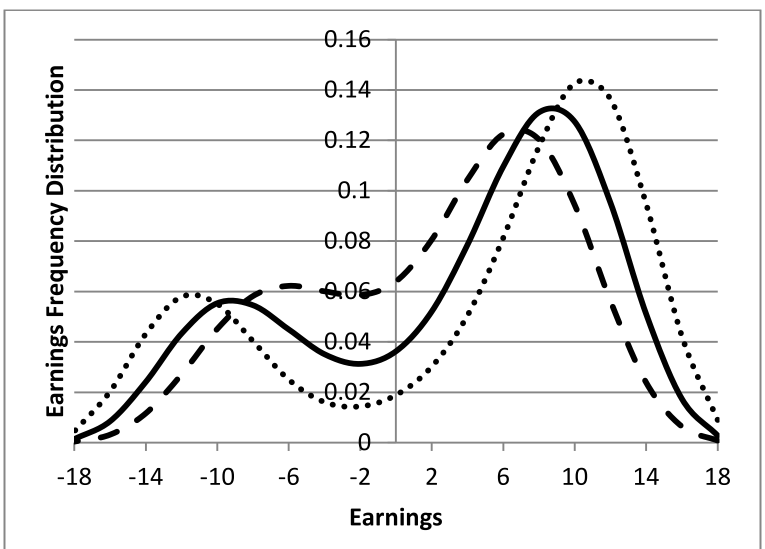
Figure 6: Comparative Statistics for Cross-sectional Earnings Frequency Distributions

Panel A: Cross-Sectional Earnings Frequency Distributions for Samples with Different Proportions of Decision making Firms



Drawn for p_B=0.75 and n=18.

Panel B: Cross-Sectional Earnings Frequency Distributions for Samples with Different Quality Opportunities Available



Drawn for w_{DM}=0.5 and n=18.

Table 1: Earnings-Return Relations

Panel A: OLS regressions in positive and negative return subsamples

p_B	0.6		0.75		0.9		combined	
ss	100		100		100		300	
	Positive returns	Negative returns	Positive returns	Negative returns	Positive returns	Negative returns	Positive returns	Negative returns
Slope (s.e.)	8.500 (0.139)	27.350 (1.047)	9.986 (0.522)	5.139 (0.189)	161.754 (11.654)	3.145 (0.043)	10.039 (0.234)	14.894 (1.045)
Intercept	1.921	5.818	-0.674	0.638	-5.254	0.003	0.264	2.111
R^2	0.99	0.92	0.88	0.94	0.722	0.99	0.92	0.61

Panel B: OLS regressions in overall samples, estimated by

$$\frac{V_{i6}}{P_{i0}} = \alpha_0 + \alpha_1 Dr_{i6} + \beta_0 r_{i6} + \beta_1 r_{i6} \times Dr_{i6} + \varepsilon_i$$

ss	α_0	α_1	β_0	β_1	R^2	ss^+	ss^-
300	0.2612	1.8500 (0.2747)	10.0388 (0.4085)	4.8554 (0.8687)	0.85	168	132
600	0.2262	2.8399 (0.2196)	10.2729 (0.2789)	8.0181 (0.6286)	0.89	298	302

Coefficients are presented with standard errors in parenthesis. All coefficients are significant at the 1% level. V_{i6} is the earnings realized at time 6, P_{i0} is the lagged price, r_{i6} are the returns at time 6, and Dr_{i6} is a dummy that is one (zero) when returns are negative (positive). ss^+ (ss^-) is the number of firms with positive (negative) returns in the sample. ss is the total sample size.

References

Adam, T., and V. K. Goyal. 2008. The Investment Opportunity Set and Its Proxy Variables. *The Journal of Financial Research* 31 (1):41-63.

Amir, E., and B. Lev. 1996. Value-relevance of nonfinancial information: The wireless communications industry. *Journal of Accounting and Economics* 22:3-30.

Ball, R. 2013. Accounting Informs Investors and Earnings Management is Rife: Two Questionable Beliefs. *Accounting Horizons* 27 (4):847-853.

Basu, S. 1997. The Conservatism Principle and the Asymmetric Timeliness of Earnings. *Journal of Accounting and Economics* 24:3-37.

Beneish, M. D., and C. R. Harvey. 1998. Measurement Error and Nonlinearity in the Earnings-Returns Relation. *Review of Quantitative Finance and Accounting* 11:219-247.

Biddle, G. C., G. Hilary, and R. S. Verdi. 2009. How does financial reporting quality relate to investment efficiency? *Journal of Accounting and Economics* 48:112-131.

Biddle, G. C., and G. S. Seow. 1991. The Estimation and Determinants of Associations Between Returns and Earnings: Evidence from Cross-industry Comparisons. *Journal of Accounting, Auditing and Finance*:183-232.

Burgstahler, D., and E. Chuk. 2013. Earnings Precision and the Relations Between Earnings and Returns. In *Working Paper*. University of Washington.

———. 2015. Do Scaling and Selection Explain Earnings Discontinuities? *Journal of Accounting and Economics* 60 (1):168-186.

Burgstahler, D., and I. D. Dichev. 1997a. Earnings Management to Avoid Earnings Decreases and Losses. *Journal of Accounting and Economics* 24:99-126.

———. 1997b. Earnings, Adaptation and Equity Value. *The Accounting Review* 72 (2):187-215.

Bushman, R. M., J. D. Piotroski, and A. Smith. 2011. Capital Allocation and Timely Accounting Recognition of Economic Losses. *Journal of Business Finance & Accounting* 38 (1&2):1-33.

Chen, K. Y., R. J. Elder, and S. Hung. 2010. The Investment Opportunity Set and Earnings Management: Evidence from the Role of Controlling Shareholders. *Corporate Governance: An International Review* 18 (3):193-211.

Collins, D. W., P. Hribar, and S. Tian. 2014. Cash Flow Asymmetry: Causes and Implications for Conditional Conservatism Research. *Journal of Accounting and Economics* 58 (2-3):173-200.

Collins, D. W., E. L. Maydew, and I. S. Weiss. 1997. Changes in the value-relevance of earnings and book values over the past forty years. *Journal of Accounting and Economics* 24:39-67.

Das, S., and B. Lev. 1994. Nonlinearity in the Returns-Earnings Relation: Tests of Altemative Specifications and Explanations. *Contemporary Accounting Research* 11 (I-II):353-379.

Dechow, P., S. Richardson, and I. Tuna. 2003. Why Are Earnings Kinky? An Examination of the Earnings Management Explanation. *Review of Accounting Studies* 8:355–384,.

Dechow, P., G. Weili, and C. M. Schrand. 2010. Understanding Earnings Quality: A review of the Proxies, their Determinants and their Consequences. *Journal of Accounting and Economics* 50:344-401.

Dietrich, J. R., K. A. Muller III, and E. J. Riedl. 2007. Asymmetric timeliness tests of accounting conservatism. *Review of Accounting Studies* 12:95-124.

Durtschi, C., and P. D. Easton. 2005. Earnings Management? The Shapes of the Frequency Distributions of Earnings Metrics Are Not Evidence Ipso Facto. *Journal of Accounting Research* 43 (4):557-592.

———. 2009. Earnings Management? Erroneous Inferences Based on Earnings Frequency Distributions. *Journal of Accounting Research* 47 (5):1249-1281.

Easton, P. D., and T. S. Harris. 1991. Earnings as an Explanatory Variable for Returns. *Journal of Accounting Research* 29 (1):19-36.

Freeman, R. N., and S. Y. Tse. 1992. A Nonlinear Model of Security Price Responses to Unexpected Earnings. *Journal of Accounting Research* 30 (2):185-209.

Gilliam, T., F. Heflin, and J. Paterson. 2015. Evidence that the Zero-Earnings Discontinuity has Disappeared. *Journal of Accounting and Economics* 60 (1):117-132.

Hayn, C. 1995. The Information Content of Losses. *Journal of Accounting and Economics* 20:125-153.

Hemmer, T., and E. Labro. 2008. On the Optimal Relation between the Properties of Managerial and Financial Reporting Systems. *Journal of Accounting Research* 46 (5):1209-1240.

Hemmer, T., S. R. Matsunaga, and T. Shevlin. 1998. Optimal Exercise and the Cost of Granting Employee Stock Options with a Reload Provision. *Journal of Accounting Research* 36 (2):231-255.

Hsu, A., J. O'Hanlon, and K. Peasnell. 2012. The Basu Measure as an Indicator of Conditional Conservatism: Evidence from UK Earnings Components. *European Accounting Review* 21 (1):87-113.

Jacob, J., and B. N. Jorgensen. 2007. Earnings Management and Accounting Income Aggregation. *Journal of Accounting and Economics* 43:369–390.

Jorgensen, B. N., Y. G. Lee, and S. Rock. 2013. The Shapes of Scaled Earnings Histograms Are Not Due to Scaling and Sample Selection: Evidence from Distributions of Reported Earnings per Share. *Contemporary Accounting Research*.

Jorion, P., and C. Schwarz. 2014. Are Hedge Fund Managers Systematically Misreporting? Or Not? *Journal of Financial Economics* 111 (2):311-327.

Kernstein, J., and A. Raj. 2007. Intra-year shifts in the earnings distribution and their implications for earnings management. *Journal of Accounting and Economics* 44 (399–419).

Khan, M., and R. L. Watts. 2009. Estimation and empirical properties of a firm-year measure of accounting conservatism. *Journal of Accounting and Economics* 48 (132-150).

Kothari, S. P. 2001. Capital Markets Research in Accounting. *Journal of Accounting and Economics* 31:105-231.

Lev, B. 1989. On the Usefulness of Earnings and Earnings Research: Lessons and Directions from Two Decades of Empirical Research. *Journal of Accounting Research* 27 supplement:153-192.

Lev, B., and P. Zarowin. 1999. The Boundaries of Financial Feporting and How to Extend Them. *Journal of Accounting Research* 37 (353–385).

Lipe, R. C., L. Bryant, and S. K. Widener. 1998. Do Nonlinearity, Firm-Specific Coeffcients, and Losses Represent Distinct Factors in the Relation between Stock Returns and Accounting Earnings? *Journal of Accounting and Economics* 25:195-214.

Patatoukas, P. N., and J. K. Thomas. 2011. More Evidence of Bias in the Differential Timeliness Measure of Conditional Conservatism. *The Accounting Review* 86 (5):1765–1793.

Skinner, D. J. 1993. The Investment Opportunity Set and Accounting Procedure Choice. *Journal of Accounting and Economics* 16:407-445.

Srivastava, A. 2014. Why have measures of earnings quality changed over time? *Journal of Accounting and Economics* 57 (2-3):196-217.

Subramanyam, K. R. 1996. Uncertain Precision and Price Reactions to Information. *The Accounting Review* 71 (2):207-220.

Subramanyam, K. R., and J. J. Wild. 1996. Going-Concem Status, Eamings Persistence, and Informativeness of Eamings. *Contemporary Accounting Research* 13 (1):251-273.