The Incumbency Protection Power of Network Effects: 
Hype or Reality?

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Abstract

Many IT enabled networks have attained a large user base induced by strong network effects, which are thought to create an economic moat by increasing switching costs, thus offering protection against new entrants. The underlying assumption behind this result involves single-homing, where users choose only one network at any given time. Is the incumbency protection power of the moat as strong in the case of multi-homing, where users with resource constraints can co-exist on multiple networks and incrementally adopt a new entrant? We develop a multi-period analytical model of endogenous adoption decisions in a setting where a new network arrives with a superior capability, and where users derive value from technological capability as well as network effects. We demonstrate that the moat created by network effects with incremental adoption is weaker than that in the case of binary adoption. This enables the new entrant to cause a gradual migration of users through a slow but steady growth of its capabilities. However, with improvements in its capabilities, the incumbent may be able to regain users more easily than in the case of single-homing, leading to higher levels of competitive dynamics than that predicted by the network effects literature. Thus our study suggests that the protection power of network effects and the resulting competitive intensity may be overrated and underplayed respectively in many modern technology settings relative to their levels implied by binary adoption decisions.

Keywords: Network effects, switching cost, incumbency protection, new entry, multi-homing, adoption dynamics
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1. INTRODUCTION

Many IT-enabled networks like Facebook, LinkedIn and Twitter have grown large by building strong network effects (Stross 2010). Such positive externalities increase switching costs (Farrell, Shapiro 1988; Klemperer 1987), and are believed to create an economic moat1 that protects incumbents against new entrants, thereby lowering the intensity of competition. Does this result still hold in modern technology settings where users can incrementally adopt a new platform without completely abandoning the incumbent? The network effects literature would suggest that due to large installed user bases, early online social networks such as MySpace and Orkut would witness minimal migration of their users to late entrants like Facebook. Yet it is well documented that most adopters of early social media significantly reduced the time spent on incumbent networks like MySpace to embrace newer networks like Facebook, even though the latter did not arrive with revolutionary capabilities or features2. Figure 1 shows that during 2007-09, the average time spent on MySpace dropped from thirty to ten minutes, while that spent on Facebook increased steadily. By January 2011, the average time on MySpace had declined to just five minutes, while that on Facebook increased to thirty four minutes (Source: www.alexa.com). If network effects helped MySpace grow in the first place,

1 In describing his investment philosophy, Warren Buffet once stated: “In business, I look for economic castles protected by unbreachable moats.”

Source: http://tech.fortune.cnn.com/tag/warren-buffet/

how can we explain the decline in MySpace activity over time and a corresponding increase on Facebook? In other words, did network effects not offer sufficient protection to the incumbent?

<Insert Figure 1 here>

A closer scrutiny of the early network effects literature (e.g., Katz and Shapiro 1985, 1994) reveals that the key results involving incumbency protection implicitly assume that a user can be a member of only one network at a point in time. For example, in the case of Beta and VHS standards battle or QWERTY and DVORAK keyboard adoption (Liebowitz and Margolis 1994), users are assumed to completely adopt only one of the standards. Traditionally, the focus of the network effects literature has been on such binary adoption decisions, which is referred to as single homing (SH) (Rochet and Tirole 2003). However, in many instances today, users do not have to choose one network or technology over another. Rather, they may choose to co-adopt multiple networks simultaneously, which is referred to as multi-homing (MH) (Rochet and Tirole 2003; Gabszewicz and Wauthy, 2004). For example, the Super Audio Compact Disc (SACD) and Digital Versatile Disc Audio (DVD-A) are competing formats for multi-channel audio. In spite of predictions of a standards war driving out one format (e.g., Shapiro and Varian 1998), both continue to co-exist a decade after their introduction. Such co-adoption is made possible by the availability of universal players, which allow consumers to buy their favorite albums on DVD-A or SACD based on availability, and rely on CD or MP3 for other albums (Mock 2004). Along similar lines, applications like Move2Picasa³ help users transfer photos from Facebook to Google+, while work-arounds exist to export friend lists from Facebook to

other social networks. With switching costs due to format differences being reduced, consumers are able to choose multiple formats simultaneously.

As shown in Table 1, the extant literature on MH (e.g., Rochet and Tirole 2003; Gabszewicz and Wauthy 2004; Armstrong and Wright 2005; Parker and Van Alstyne 2005; Armstrong 2006; Doganoglu and Wright 2006; Eisenmann et al. 2006) has not studied the extent and dynamics of adoption of multiple platforms through the allocation of limited resources. For example, in Gabszewicz and Wauthy (2004), a visitor can buy passes to more than one exhibition center, while an exhibitor can choose to display in multiple centers. SH users choose only one center to visit or display products, while MH users choose multiple centers. Gabszewicz and Wauthy (2004) do not investigate the case where a visitor and an exhibitor may have a time and budget constraint respectively. Under such resource constraints in MH, visitors and exhibitors would behave strategically and divide their resources (time and money respectively) across venues in order to maximize their benefits. In many MH scenarios such as social networks or traditional CD versus DVD-A/SACD, users or adopters have a limited amount of resources, which can be allocated toward the adoption of networks, products or platforms. That is, users have a choice of the amount of resource to allocate to a platform (and hence the extent of adoption), which can vary from period to period in a temporal model. One of the two key research issues in this study involves whether network effects offer strong protection to incumbents in MH settings where users can incrementally adopt a new network by dividing limited resources between the incumbent and a new entrant, and then gradually increase the resource(s) allocated to the new entrant under certain conditions. The dynamics of migration in such cases remain an open question in the literature. For example, how large a technological capability does the new entrant need to enter the market? Should it initially target users with

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high affinity to technological capability or network effects? We also investigate the incumbent’s reaction which determines the competitive dynamics of this market. How long can the incumbent afford to wait before offering improved capabilities in order to maintain its market share in the presence of a new entrant with superior capabilities? If the new entrant is able to breach the moat created by network effects in incremental MH, can the incumbent also regain lost users more easily in such an environment?

<Insert Table 1 here>

We assume that a user derives benefits from the capability of a network or technology, and from network effects (Katz and Shapiro 1985; Liebowitz and Margolis 1999). Network capability may be attributed to proprietary technology, functionality, control over information/privacy, interface, customizability and applications. Network effect is the benefit that each user of a good or service realizes as more people use the same good or service. For instance, both Skype and GoogleTalk have instant messaging and voice-over-IP features, though Skype is believed to be superior in voice quality and PC-to-phone calls. However, GoogleTalk is integrated with Gmail, Google Docs and other applications, thus enabling a user to add new contacts from her mailing list and manage all interactions through email or chat. Thus a user may choose Skype for superior voice quality and PC-to-phone calls, but also use GoogleTalk to interact with her contacts already using the latter option.

We demonstrate through a multi-period analytical model that in the MH case where a user can coexist on the incumbent and the new entrant networks by dividing her total time spent on networking, even a marginally superior technological capability of the new network will start a slow bleeding of the incumbent network, thus triggering a decline in the total time spent on the incumbent. Eventually, the network effects on the new entrant may become strong at the cost of that in the incumbent network. This may create an avalanche, whereby the remaining users on the incumbent network allocate increasing amounts of time to the new network,
leading to a major shrinkage of time spent on the former. By contrast, in the SH case considered in the early network effects literature, a quantum leap in capability is necessary to make users migrate at once to the new network. Thus the incumbent may be more vulnerable than previously considered on the basis of the extant literature. Further, the highest technological capability required to cause migration in SH network must be provided earlier than in the case of MH. Therefore, in the latter setting, the new entrant does not have to deliver the highest required capability upfront; since the required capability increases gradually in MH, the new entrant has more time relative to SH to improve its technology to attract users.

We also demonstrate that if the incumbent responds to the new entrant in a timely manner by developing new capabilities, it may be able regain the time lost to the new entrant more easily relative to the case of SH. Thus the competitive intensity and dynamics in modern technology settings may be stronger than that suggested by the strong protective power of network effects advocated in the extant literature. Since the extant MH literature has not focused on partial and incremental adoption of multiple platforms, the required capability of the new entrant and the dynamics of the migration and potential reverse migration have received little attention.

The key contribution of our study is to demonstrate that the incumbency protection power of network effects described in the classical network effects literature may be overrated in modern technology settings where other types of switching costs have been lowered to enable co-adoption of multiple technologies or networks. Our results offer an explanation as to why it has been possible in recent times to breach the moat created by incumbents with large installed

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5 While this phenomenon may appear to be similar to the diffusion processes described by Rogers (1962), the S-curve proposed by Rogers is exogenous in nature, while the adoption levels are endogenously chosen by users in our model. Further differences between our model and the key results in the diffusion studies are discussed in the literature review section.
bases and underscores an increased vulnerability to new entrants with less-than-revolutionary technology offerings but with gradual improvements over time. The study also demonstrates an increase in competitive intensity where the incumbent may regain lost share more easily than in the case of SH or complete adoption. Our work also contributes to the diffusion literature by treating migration or diffusion across networks as an endogenous and incremental choice by users rather than an exogenously specified model parameter.

2. PAST RESEARCH

The ‘Information Economy’ has traditionally been believed to be propelled by strong network effects (Shapiro and Varian 1998; Katz and Shapiro 1985; Katz and Shapiro 1994). For example, a Facebook user who uses the network to keep in touch and/or play multi-player games like Farmville with friends would value the network more as more of her friends join the network. It is suggested that such positive externalities would make a strong network stronger and a weak network weaker (Shapiro and Varian 1998). However, this result is based on the fact that the incumbent network induces a large switching cost for users through network effects, thus shielding the incumbent from a new entrant with superior capabilities. In the context of online social networks as with many other technologies, there is no direct membership cost, while the learning curve is minimal owing to similarity in the basic functionality and usability across networks. Thus, network effects are the primary source of switching cost in online social networks.

Prior theories of competition in technology networks accentuate the path dependence (Liebowitz and Margolis 1995) of outcomes, and suggest that inferior products that have entered the market early and developed large networks may drive out superior products and “win the market” (Arthur 1989, 1994; Farrell and Saloner 1986). However, our thesis is that in many modern technology networks where users can multi-home incrementally, and where network
effects is the major component of switching cost, such lock-ins with inferior products may not be feasible. In spite of the presence of network effects, history shows that new technology products and services have been launched successfully in competitive markets and that market segments have been penetrated into and captured. This is often made possible due to innovation in terms of revolutionary technology (e.g., the Apple iPhone), path breaking business models (e.g., Apple iTunes) or an evolutionary strategy, whereby the new player enters the market by being compatible with the incumbent. The revolutionary strategy is marked by a large improvement or “discontinuity” in technological capability, whereas the evolutionary strategy follows a smoother transition (Shapiro and Varian 1998).

The SH literature assumes that users choose only one of the networks. Katz and Shapiro (1985) develop a single period model where a consumer’s decision to adopt a technology or network depends on the price and the rational expectations about the final network size. When new products are introduced, the market may display a bias towards existing product, resulting in excess inertia or a rush to the new entrant due to insufficient friction (Katz and Shapiro, 1985). In a two-period SH model of dynamic competition with new adopters entering the market, Farrell and Shapiro (1988) show that an inferior product can enter a market where there are economies of scale and switching cost for consumers. However, they note that it will be difficult for such a new entrant to lure the installed base, and that it may be more successful in attracting new adopters. Beggs and Klemperer (1992) propose an infinite period model where new consumers arrive in the market every period and a fraction of old consumers leaves. Switching costs make the market more attractive to a new entrant in spite of an installed base. Farrell & Saloner (1985) model sequential decisions to completely adopt one amongst multiple platforms (i.e., single homing) where the timing of adoption is endogenous. However, in our model, the participants sequentially decide on the extent of adoption of the incumbent and the new entrant in a multi-homing context.
Another relevant stream of literature involves the diffusion of innovation. Diffusion is a process of communicating ideas about an innovation amongst users who are potential adopters (Rogers, 1962). Rogers proposed that the adoption curve follows a normal distribution. He classified adopters into innovators, early adopters, early majority, late majority and laggards. Bass (1963, 1969) presented an analytical model describing this phenomenon, popularly known as the 'Bass diffusion model', which is a special case of the Gompertz distribution. The Bass diffusion model considers the adoption of new innovations as a result of interactions between existing user base and potential adopters. In this model, the rate of change of adoption depends on the cumulative adopters at a given time, the 'coefficient of external influence' and the 'coefficient of internal influence'. The model aids practitioners in predicting sales based on historic information or derived from analogous product sales in the past. However, in the Bass model, the adoption pattern is exogenously specified, whereas we treat the adoption process as endogenous. The Bass diffusion model does not explicitly specify if new adopters enter the market. Thus it is not clear if the adopters renounce a product to adopt one whose diffusion pattern is being predicted. In our model, there are no new adopters in order to isolate the dynamics of network effects and technological capabilities and their impact on diffusion. Thus, in order to adopt the new entrant, current users of the incumbent network have to leave the incumbent either completely (in SH) or partially (in MH).

3. THE MODEL

We consider an incumbent (network 1) with technological capability $c_1$. A new entrant (network 2) arrives with a better capability $c_2 (c_2 > c_1)$. The capability of a network may depend on the features provided by specific technologies. For instance, let the incumbent and the new entrant networks serve only two applications, namely music streaming and gaming. Let the
capabilities of the incumbent and the new entrant for music streaming be \( m_1 \) and \( m_2 \), and those for gaming be \( g_1 \) and \( g_2 \) respectively, such that \( c_1 = f(m_1, g_1) \) and \( c_2 = f(m_2, g_2) \). We consider the case where each network is superior in one dimension than the other, although network 2 is superior from an overall standpoint\(^6\). We assume that each user is in all other users’ contact list in the incumbent network. To assess the impact of innovation in technology on migration patterns of existing users, we isolate the model from new adopters\(^7\). In SH environment, the benefit derived by a user \( i \) from network \( j \) in period \( \rho \) is given by \( B^{SH}_{ij \rho} \) which has two parts: i) A private benefit derived from using the technology provided by the network (e.g., single user applications providing information, entertainment, etc.), and ii) social utility derived from interactions with other users of the technology. Thus:

\[
B^{SH}_{ij \rho} = c_j \alpha_i U_{ip}(T) + c_j \gamma_i S_{ip} \left( T, \sum_{d=1}^{D} \omega_d \tilde{m}_{ijd \rho} \right)
\]

\(^6\) If the new network’s music streaming and gaming technology capabilities are both better than those of the incumbent, i.e., if \( m_2 > m_1 \) and \( g_2 > g_1 \) then a user would have no reason to spend time on the incumbent network from a capability standpoint. In other words, the user would have no incentive to split time across two networks to derive benefits from both networks’ capabilities.

\(^7\) If new users are considered, they are more likely to join the new entrant than the incumbent because of the former’s superior technological capability. The fact that they had not already joined the incumbent implies that they may be more affine to technological capabilities than network effects. Thus new users would only help strengthen our results involving the vulnerability of the incumbent.
where \( c_j \alpha_i U_{ip}(T) \) is the private benefit derived by user \( i \) from network \( j \), and where \( U_{ip}(T) \) is the private utility derived by user \( i \) in period \( \rho \) by allocating time \( T \) to network \( j \) with capability \( c_j \). The parameter \( \alpha_i \) \((0 < \alpha_i \leq 1)\) represents the affinity of user \( i \) to utility derived from single user applications, and \( T \) represents the total time allocated to network \( j \).

The second part of the benefit function, \( S_{ip}\left(T, \sum_{d=i}^{D} \omega_d \bar{m}_{-i,d,\rho}\right) \), is the social utility derived by user \( i \) from network effects in period \( \rho \) by allocating time \( T \) to network \( j \), where \( \gamma_i \) represents the affinity of user \( i \) to network effects. \( \bar{m}_{-i,d,\rho} \) represents the mean time allocation by other users (represented by \(-i\) ) in network \( j \) whose influence on user \( i \) is ranked \( d \). Network effect related benefits for a user increase with the time allocated to a network by (i) other users and (ii) herself. The sum of weights associated with other users is normalized to 1. The above mean is calculated with respect to the total number of users of similar rank in network 1. Thus, the mean network effect is 0 or \( T \) for SH networks and lies between 0 and \( T \) for MH networks. The influence depends upon the similarity in users' affinities to network effects. For instance, a network effects affine user shall assign a higher weight (and therefore a higher rank) to users with higher affinity to network effects than her because such users have a higher influence on others' decisions regarding resource allocation in online platforms. Similarly, the weight assigned to users with similar affinity will be higher than the weight for users with lower affinities. \( \omega_d \) denotes the weight associated with the rank \( d \) of the influence. A higher ranked (denoted by a higher value of \( d \) ) contact will receive a larger weight. We assume that for a given influence based on relative affinity to network effects, the user has the ability to rank them and that the weight and ranking associated with a user remains same across the two networks. \( D \) is the highest rank of influence for user \( i \). Thus, even though all users are connected to each other facilitating the information flow, the time allotment decision of a user depends on the influence
that other users have on her. If the diffusion process is such that the users with a lower affinity to network effects make decision before a user with higher affinity to network effects, then user $\rho$ will assign weights to other decision making users such that $\omega_1 < \omega_2 < \ldots < \omega_{p-1}$. User $D$ with the highest affinity to network effects is assigned the highest weight $\omega_D$. The weights are normalized such that $\sum_{d=1}^{D} \omega_d = 1$.

The benefit derived by user $i$ in a MH environment is given by $B_{i\rho}^{MH}$. In MH, user $i$ may choose to co-exist in both the networks by allocating time $t_{ij\rho}$ to network $j$ in period $\rho$ and derive benefits from splitting time across both networks. Thus:

$$B_{i\rho}^{MH} = \sum_{j=1}^{2} c_j \left[ \alpha_i U_{i\rho}(t_{ij\rho}) + \gamma_i S_{i\rho}(t_{ij\rho}, \sum_{d=1}^{D} \omega_d \bar{M}_{-ijd\rho}) \right]$$

where the total time $T$ is fixed, i.e., $t_{1\rho} + t_{2\rho} = T$.

### 3.1 Model Assumptions

Diffusion of information about the new entrant is such that users in network 1 are progressively informed about the details of capability of network 2 to be able to make a decision to allocate time between the two networks. This could be the reason why a new network does not witness instant and complete migration of all users. For example, when Google+ was launched, invitations were sent to select users, who, in turn, could send invitations to other users in their network. Thus, though most users on other competing networks like Facebook knew about the existence of Google+, only users with invitations were in a position to make a migration decision and invite other users from Facebook to experience Google+. Since the number of invitations is limited, users present on Google+ would invite users with similar affinity for technology capabilities or network effects. Users with high affinity to technology (or network effects) will be
more likely to take a decision to allocate time to the new entrant if they receive an invitation from a user with similar affinity to technology (or network effects). Along similar lines, one user or a group of users with the same affinities for technology and network effects decide to allocate time \((\geq 0)\) to the new network in each period.

When users know the types of other users who will make a decision to allocate time between the two networks, they form expectations about the mean time allotment of other users\(^8\). We consider the temporal dynamics of decision making in which expectation formations are myopic (Sobel 1981; Farrell and Shapiro 1988; Beggs and Klemperer 1992; Brock and Durlauf 2001). That is, users take the last period’s actual mean time allotment to make forecasts about the current period.

**Parametric assumptions**

a. The private utility of user \(i\) is concave: \(\frac{\partial^2 U_i(t_{ij})}{\partial (t_{ij})^2} < 0\). This form provides the user the motivation to split time across two networks in MH. A function of the form \(U_i(t_{ij}) = (t_{ij})^\beta\) where \(0 < \beta < 1\) satisfies the parametric assumption. The intuition behind \(\beta\) is that it determines the value of a user’s time. All users are assumed to have the same valuation of time.

\(^8\) In period \(\rho\), users or user groups \(1, 2, ..., \rho - 1\) may have already allocated some time to network 2. The \(\rho\)th user or group, who has spent the full time \(T\) on network 1 until period \(\rho - 1\), will decide for the first time whether to allocate any time to network 2 based on its capability and the total time spent on network 2 by users \(1, 2, ..., \rho - 1\). Users \(1, 2, ..., \rho - 1\) will also reallocate their time to network 2 based on their expectation of user \(\rho\)’s decision.
b. We consider social utility functions that demonstrate constant strategic complementarity between the time allotted by the user and the expected mean time allotment by other users.

\[
\frac{\partial^2}{\partial t_{ij,p} \partial \bar{m}_{-ij,p}} S_{ij,p} \bigg( t_{ij,p}, \bar{m}_{-ij,p} \bigg) = \xi > 0
\]

A function of the form \( S_{ij,p} \big( t_{ij,p}, \bar{m}_{-ij,p} \big) = \xi \left( 1 + t_{ij,p} \right) \sum_{d=1}^{D} \omega_d \bar{m}^r_{-ijd,p} \) satisfies the above requirement\(^9\) and represents the social utility derived per unit capability from time spent by others. A given network size (measured by the total time spent on the network) may generate disparate levels of network effects depending on level of interactions supported by the platform. E.g., social media platforms provide collaboration tools enabling interactive creation and diffusion of content, while professional networks like LinkedIn and Glassdoor offer features to conduct interviews or check crowd-sourced salary information. A platform supporting such interactive experiences will be successful in creating a higher network effect, which is captured by \( \xi \), a network characteristic which has a higher value for a platform that allows higher levels of interaction between users. In contrast to the multi-user functionality represented by \( \xi \), the capability of the network, \( c \), refers to features or functionality that is valuable to a single user regardless of her interactions with others. For the social utility to play a significant role in MH, the network must satisfy a minimum level of interactions between users. For example, a network where a user can only provide content individually will have a lower level of network effects.

\(^9\)The discussion of social utility function uses the notation \( \bar{m} \) for mean network effects from other homogeneous users. It holds true for heterogeneous users as well where the expected mean network effects from other users is denoted by \( \bar{M} \).
interaction than a platform where users can collaboratively generate and filter content. This constraint on the level of interactions that the platform must support is discussed in the next section.

We assume that users preserve their social influence and affinities to technology and network effects when they migrate from network 1 to network 2 in both SH and MH cases. However, it is possible that social influence, affinities to technology and network effects vary with time or platform. We do not analyze such variation in user characteristics in our model. Thus, for an exogenous value of time \( T \), user \( i ' \) s characteristics are defined by \( \{ \alpha_i, \beta, \gamma_i \} \) and the network \( j ' \)'s characteristics are defined by \( \{ c_j, \xi \} \).

### 3.2 Homogeneous Users

Homogeneous users, the starting point of our analysis, have the same affinity to technology and network effects. Let \( \alpha_i = \alpha \) and \( \gamma_i = \gamma \) for all \( i \). The diffusion of information about the capabilities of network 2 occurs in a way such that in the first period, one user, who currently spends the entire time \( T \) on network 1, gets the opportunity to make a decision regarding time allocation to network 2. In each subsequent period, a different user gets the opportunity to make a first-time decision regarding time allocation between the two networks. In each of these periods, the capability required by the new entrant must be sufficient to overcome benefits from both the technology and network effects provided by the incumbent.

In SH, users forego the benefits from capability and network effects on network 1 to adopt network 2 completely. Thus it is difficult to migrate the initial users because they do not have significant network effects in network 2 in the early stages. In MH, the capability provided by network 2 should be sufficient to only partially migrate users to network 2. Therefore, the network effect on network 2 in MH builds more slowly than in SH. However, because of complete adoption in SH, network 2 must deliver a higher technology upfront in SH as
compared to MH. As stated before, the extent of interaction between users must be higher than a minimum threshold, which ensures that the social utility is affected sufficiently by the strategic interactions between the user and her network.

**Proposition 1a:** For homogeneous users, the capability required by the new entrant to initiate migration in SH network is highest in the first period. In MH networks, the capability required to initiate incremental migration is less than that in SH networks.

All proofs are provided in the Appendix.

Even though the network 2 in MH requires a lower capability in the initial periods to initiate migration, the capability requirement may grow as more users are induced to increase their level of adoption of network 2. However, it is of interest to know whether a complete migration in MH is possible by slowly incrementing the technology but keeping the highest technology requirement strictly lower than that needed upfront in SH. This has significant implications for the technology development process for MH networks. Moreover, it also raises concerns for the incumbent network because of the possibility of complete migration in MH with a lower technology than in SH.

Technology networks provide features to enable interaction amongst users and to leverage benefits derived from social utility (as embodied in $\xi$). However, users also derive higher benefits from social utility if they have higher affinity to network effects. Thus, the minimum required level of interactions increases with the affinity to technology and decreases with affinity to network effects. In networks with the minimum required level of interactions, the new entrant in MH may completely migrate homogeneous users with a capability lower than the highest capability required in SH. Further, such migration is easier if the user has a lower value for the time spent on the network. As the users’ valuation of time ($\beta$) gets closer to 1, the time allocation of users in MH network becomes similar to SH. Thus, networks where users have
lower valuation of time are more likely to witness migration to newer networks since it is easier for the new entrant to pull such users compared to networks where users have high valuation of time. For example, Facebook may have been able to gradually attract users away from MySpace because of the potentially lower valuation of time by college students relative to users of professional networks such as LinkedIn. Our model indicates that it may be relatively more difficult for a professional network competing with LinkedIn to migrate users away because professionals, who form the majority in such networks, may have a relatively higher valuation of time than college students.\footnote{http://www.forbes.com/sites/ciocentral/2011/02/16/why-linkedin-is-more-valuable-than-facebook/}

**Proposition 1b:** In MH, homogenous users will completely migrate to a new entrant with a technology capability lower than the highest capability required in SH, if in every period,

\[
\frac{\text{Net loss in benefits from the incumbent by choosing MH over SH.}}{\text{Benefit from new entrant by choosing MH.}} < 2^{2-\beta} - 1
\]

This threshold \(2^{2-\beta} - 1\) is higher when the valuation of time by homogeneous users (denoted by \(\beta\)) is lower and vice versa.

Proposition 1b specifies the conditions necessary for a complete migration of homogeneous users under MH within the same time frame as SH. If the new entrant network increases technology slowly over a longer period of time (beyond the number of periods
considered in the analysis) without any response from the incumbent, full migration could take place under less stringent conditions\(^{11}\).

In case it is not feasible to keep the required capability on network 2 lower than that required in SH throughout the migration process, the new entrant can still buy more time to develop such capability in MH rather than SH, where network 2 must deliver the highest technology upfront. This extra time must be supported by regular release of usable parts (modules) so that a steady migration in MH is sustained.

Shapiro and Varian (1998) describe a “revolutionary” or “10x” improvement strategy for new entrants to be able to penetrate a market characterized by an incumbent enjoying strong network effects. Proposition 1a and 1b suggest that a new entrant may not need a giant leap in technology to eventually capture the entire market; rather, in MH networks with certain characteristics, the new entrant can introduce small increments in technology at regular intervals and still manage to gain market share over time\(^{12}\). For example, MySpace and Facebook were launched in 2003 and 2004 respectively with very similar social networking tools like the ability to add friends by searching existing profiles, and posting messages and photos. Facebook continuously improved the social networking experience by adding

\(^{11}\) In the absence of any limitation of the number of periods within which migration must be completed, a homogenous user in MH may increment her time by \(T/\lambda\) where \(\lambda(\geq D)\) is the period by which migration will be completed and \(D\) is the number of periods (= number of users or groups of similar users) in which the migration is completed in SH.

\(^{12}\) Shapiro and Varian (1998) suggest an incremental strategy as an alternative to revolutionary improvement; however, they define such a strategy as providing backward compatibility with the status quo. Further, their prescription implicitly involves single homing, such as the adoption of MS Word by WordPerfect users through conversion help features.
capabilities gradually, while MySpace began losing its main demographics of teenagers due to poor design and privacy issues. As Facebook incrementally added features like Share, News Feeds, Mini-Feed, Marketplace, Translation tools, Facebook Chat, Like, Virtual Gift Shop, etc., and created a platform for Facebook application developers maintaining transparency in design and privacy control, MySpace struggled with its design or privacy related issues. Thus, MySpace’s decline was not a result of a revolutionary technology shift induced by Facebook, but can be attributed to the latter’s gradual but steady stream of improvements. Similarly, Google+ entered the market in 2011 with a different design of social networking and has gained a sizable membership in a short time period, and is steadily adding features to its platform. For example, since inception, Google+ has developed new features such as hangouts (which enable group video chats), customization of news feeds, and circles (which provide users more granular control over privacy than Facebook). If the incumbent (Facebook) does not innovate continuously, it may slowly lose market share to Google+ in spite of the network effects it has built. The next section numerically illustrates the above migration patterns.

3.2.1 Numerical illustrations for homogeneous users

For a SH user, the decision is whether to adopt the new entrant network completely (by choosing $T$) or stay with the incumbent. We investigate the MH case where a user can allocate less than $T$ to the new network. Since the information received about the new entrant is reliable in our model, users have no incentive to reduce their time on the new network in a later period.

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13 http://newsroom.fb.com/content/default.aspx?NewsAreaId=20

14 http://www.msnbc.msn.com/id/19717700/

15 http://www.huffingtonpost.com/2011/07/15/google-plus-vs-facebook_n_899922.html#s300815&title=Hangouts_Video_Chat
In examples below, we use $c_i = 50$, $T = 40$, $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 0.5$ and $D = 200$. We analyze the following two cases.

**MH case 1:** The first decision maker increments her time on the new network by either 0 or $T/D$ in a network where everyone is connected to everyone, where $D$ is the total number of decision making periods (and also the highest rank of measure of influence). In every period, all homogeneous users allocate the same time to network 2.

**MH case 2:** The first decision maker increments her time on the new network by either 0 or $xT/D$, where $D$ is the total number of decision making periods and $x$ is an integer ($x > 1$). In every period, all homogeneous users allocate the same time to network 2. In this case, all decision making users migrate completely in period $D/x$. Thereafter, all new decision makers adopt network 2 completely when they make their first decision regarding time allocation. Table 2 shows the time allocation by various users over different periods. MH case 1 is represented by $x = 1$, where $D$ is the period in which migration is completed.

Table 3 shows that when homogeneous users are more capability affine, network 2 requires the least capability to ensure complete migration in SH, MH case 1 and 2. For more network affine homogeneous users, network 2 requires the highest capability to motivate all users to migrate. As users become more affine to social utility and network effects rather than capability, it becomes more difficult for the new entrant to ensure complete migration.

Figure 2 shows the capability requirement of network 2 in homogeneous SH, MH cases 1 and 2 for users with equal affinity to both capability and network effects.
Figure 3 shows that network 2 ensures complete migration in the final period (200\textsuperscript{th} period) in MH case 2, while in MH case 1, the MH network replicates the behavior of a SH network from the 20\textsuperscript{th} period onwards, owing to faster build-up of network effects and social utility in case 1. The above examples illustrate the vulnerability of the incumbent. For example, case 1 indicates that when users are highly capability affine, a new entrant can cause a complete migration in MH with a capability that is almost thirteen times smaller than that required in SH. Even when users are highly affine to network effects, the new entrant only needs about one third the capability required for the case of SH. So far we have assumed that users are homogeneous in the affinity towards capability and network effects, i.e., $\alpha$ and $\gamma$ are constant for all users. In the next section, we relax this assumption and analyze how heterogeneous users affect our earlier propositions.

3.3 Heterogeneous Users

Heterogeneous users have different affinities to single-user and multi-user applications offered by the network. Let us assume that there are three types of users (as shown in Table 4): Type 1 users, who have high affinity to capability but low affinity to network effects, Type 2 users, who have moderate levels of affinity to both capability and network effects, and Type 3 users, who have high affinity to network effect but low affinity to capability. As in previous section, we still assume that all users are connected to each other.

The diffusion of innovation in the real world begins with innovators, then spreads to early and late majority, and is finally adopted by laggards (Rogers, 1962). In the context of our model and social networks, an innovation in capability is most likely to attract users who are more affine to technology (e.g., single-player option in gaming networks) than users who are
more network affine (e.g., multi-player option in gaming networks) in the early periods when the new entrant has not built sufficient network effects. For example, single-player games may be ideal to attract users who derive more benefits from their private utility of a technology in a social gaming network. Over time the technology to support multi-player options may be developed, thus targeting users who are more network effects affine. The diffusion pattern has strategic implications for network 2. Our model suggests that targeting the capability affine users first may help the new entrant in developing its technology over time and build network effects in order to attract network affine users in later periods. According to our model, a new entrant is better off focusing on attracting the Type 1 users followed by Type 2 and 3 in order to slowly build up the network effects in network 2 and in the process not requiring delivery of a path-breaking technology in the initial periods.

**Proposition 2:** For heterogeneous users, in both SH and MH, the new entrant network will prefer delivering capability to attract users in the following order: Type 1, Type 2 and Type 3.

Proposition 2 may be extended to a continuum of $\alpha$ and $\gamma$ whereby the new entrant would prefer users with highest affinity to capability to make migration decisions. From Proposition 2, we note that the new entrant in SH may gradually increase the capability required to be delivered by targeting users of Type 1 first followed by other types. Thus, a new entrant in SH may not have to deliver the highest technology in the first period as in the case of homogeneous users (Proposition 1).

**Proposition 3a:** For heterogeneous user in SH, the capability requirement to enable complete migration first increases, reaches a peak and then decreases.

Comparing the capability required for heterogeneous users in SH and MH, we note that the network effect builds faster in SH due to complete adoption. However, to enable this complete adoption, SH requires a higher capability in the initial periods. This required
capability rises as the affinity to individual utility from capability reduces and affinity to social utility increases with network effects across periods. However, at some point in migration in SH, the capability requirement starts decreasing when the network effects in network 2 has been built sufficiently, and the remaining decision making users are more affine to network effects. This will make it easier for the network 2 in SH to migrate users completely in later periods.

Similarly, in MH, the capability requirement rises in the initial periods and then declines in later periods. Since network effect builds slowly in MH as compared to SH, the highest capability required by MH may exceed SH if sufficient network effects have not built up. However, if there is sufficient demand for the technology delivered by network 2 in MH, then migration on MH may continue with a lower technology than in SH.

Similar to the case of homogeneous users, the minimum required level of interactions for heterogeneous increases with the highest affinity to technology and decrease with the lowest affinity to network effects. In such networks, the new entrant in MH may completely migrate heterogeneous users with a capability lower than the highest capability required in SH. As in the case of homogeneous users, migration in MH with a lower capability than SH is easier if the user has a lower value for the time spent on a network.

**Proposition 3b:** In MH, heterogeneous user will completely migrate to a new entrant with a technology capability lower than the highest capability required in SH, if in every period,

\[
\text{Net loss in benefits from the incumbent by choosing MH over SH.} < 2^{2^{-\beta}} - 1
\]

This threshold \(2^{2^{-\beta}} - 1\) is higher when the valuation of time by heterogeneous users (denoted by \(\beta\)) is low and vice versa.
3.3.1 Numerical examples for heterogeneous users

We numerically analyze the case where heterogeneous users can choose increments of time (up to $T$) to spend on the new network and where the affinities to technological capability and network effects are uniformly distributed. We use the following learning model:

$$\tilde{M}_{-i,j,d,\rho} = \tilde{M}_{-i,j,d,\rho-1} + \lambda \left( M^a_{-i,j,d,\rho-1} - \tilde{M}_{-i,j,d,\rho-1} \right)$$

where $\tilde{M}_{-i,j,d,\rho}$ is the expectation that user $i$ forms about the mean time allotment of all other users with social influence $d$ in network 1 in period $\rho$. $M^a_{-i,j,d,\rho-1}$ is the actual time allotment by other users in the previous period. $\lambda$ is a parameter which corrects for biases in expectation formation. When $\lambda = 0$, $\tilde{M}_{-i,j,d,\rho} = \tilde{M}_{-i,j,d,\rho-1}$. When $\lambda = 1$, $\tilde{M}_{-i,j,d,\rho} = M^a_{-i,j,d,\rho-1}$. While this analysis may not encompass all learning strategies, it helps us illustrate the dynamics in migration patterns in SH and MH platforms.

In the examples below, $c_i = 50$, $T = 40$, $\beta = 0.5$, $D = 200$, $\lambda = 1$, $\alpha \in [0,1]$ and $\gamma \in [0,1]$. $\alpha$ and $\gamma$ follow a uniform distribution and reflects the heterogeneity of users. The diffusion of information is such that users with high (low) affinity to technology make a migration decision in the early (later) periods. We use tools in Matlab 7.9 to perform nonlinear optimization of time allotment by heterogeneous users to the two platforms. The definition of optimality is based on Karush-Kuhn-Tucker (KKT) conditions, which ensure that the gradient is zero at the minima but are modified by the total time constraint. Active Set algorithms are used to compute optimal time allocation. The algorithm starts from a feasible point, approximates the solution to the problem defined by active constraints, computes a Quasi-Newton approximation of the Hessian of the Lagrangian, removes constraints with negative Lagrange multiplier and searches for infeasible constraints in order to compute an optimal solution\(^{16}\).

\(^{16}\) http://www.mathworks.com/help/toolbox/optim/
Figure 4 illustrates the growth of the new entrant in SH and MH where it offers the same capability for both SH and MH \( (c_2^{SH} = c_2^{MH} = 80, \bar{\xi} = 1.262) \). This capability is sufficient for MH to initiate migration from network 1 in the initial periods leading to a large scale move to network 2 in the 125th period; however, the SH network stalls with the same capability, whereby network 2 may become a niche provider to a few users with high affinity to technology. For instance, starting in 1995, eBay has been the dominant online auction site in the U.S. for a wide variety of products. By contrast, audiogon.com, a much smaller online marketplace which started business operations several years later, has been successful in creating a niche marketplace for high end audio products.

If the required capability is delivered to continue the migration process, network growth may be slow initially in the case of MH, but the total time spent on the new network catches up to the level of SH in later periods (Figure 6). Only innovators and early adopters may choose to migrate to a new entrant when only SH is permitted; however, the same group of users may act as a trigger for slow bleeding of the incumbent network in MH. It should be noted that in order to enable complete migration in MH, the capability required may not always be below SH. Thus, it possible that initially the capability required is low in MH, however, it exceeds that of SH in later periods. This suggests that while new entrants in SH have to deliver a higher technology faster, a MH new entrant has more time deliver a better technology. In Figure 5, in order to enable complete migration, SH must deliver its highest technology by period 4. In contrast, the MH new entrant may deliver a technology lower than SH till the 96th period and still continue migration. While SH network had only four periods to deliver this highest capability, MH network can take twenty four times longer to deliver the highest required capability. Similarly, in Figure 6, MH network 2 must deliver its highest technology in the 147th period in order to enable complete migration.
Since the *bleeding* in the incumbent network is generally slow when users make unconstrained optimal decisions, it may initially go undetected or not raise an alarm even when detected. Yet it is critical for the incumbent to take corrective action in order to pull back the users slowly migrating away from the incumbent, before sufficient network effect is reached on the new entrant to cause large scale migration in later periods. However, there are cases where the incumbent may not be in a position to react to capability improvements by the new entrant. For example, MySpace lost a majority of its user base to Facebook, but could not match the capabilities being delivered steadily by the latter. Thus, we suggest that the economic moat of network effects is susceptible to breach in the case where users do not have to give up on the incumbent in a single period to incrementally adopt a new entrant.

To demonstrate how an incremental technological progress may breach the incumbency protection provided by network effects, we isolated the model from the incumbent’s reaction to the new entrant’s arrival. In the following section, we discuss the effect of the incumbent’s delivery of an improved capability as a reaction to the new network’s superior technology.

### 3.4 Incumbent’s Reaction to the New Entrant: Reverse Migration

If the incumbent does not react\(^\text{17}\) to the new entrant’s capability with a steady stream of developments in its own platform, the network effect on the new network becomes progressively

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\(^{17}\) In our analysis, the new entrant does not anticipate the incumbent’s reaction to choose its capability as in a Stackelberg leadership model. Further, many online social networks like MySpace failed to respond quickly to a new entrant’s moves due to technological and other
stronger. Because of the pattern of user decisions to migrate, an incumbent with a large user base will enjoy a strong network effect for a longer period of time. Since the new entrant will take more time to build up stronger network effects, the incumbent has the opportunity to deliver an improved capability and pull back the time lost to the competitor. Nault and Vandenbosch (1996) suggest that incumbent firms should “eat their own lunch” by leapfrogging in innovation to face competition from a new entrant. Any improved capability delivered by the incumbent after the arrival of the new entrant has to overcome the pull created by the network effects developing on the new network. Thus, the incumbent should strive to deliver the desired improvement in capability as early as possible. As the affinity to technology increases, the user derives more private benefit from a given capability level. Thus, the capability improvement required to ensure that the user has an incentive to migrate back to the incumbent is lower for a user with higher affinity to private utility. If the incumbent delivers an improvement soon enough, then it may cause the reversal of migration with a capability lower than that of the new entrant owing to its large user base. Thus a continuous development of the platform is a necessity for survival for both the incumbent and the new entrant.

In the initial periods, when the required capability of MH is below that of SH, the incumbent must deliver a high technology to motivate users to increase the time spent on network 1. Further, network effects have not developed sufficiently on network 2. Therefore, in the initial periods it is relatively easier than later periods for both SH and MH network 1 to regain lost time from users who have partially migrated to network 2. However, for initial users in SH, who have migrated to the new entrant, the incumbent finds reverse migration more difficult in the initial periods. When users have a lower value of their time on a network, the incumbent may be able to regain its share of the total time more easily in the case of MH. For example, relative to a constraints. Further, even if the new entrant expects a reaction from the incumbent, there will be an increase in the former’s required capability in both SH and MH cases.
competitor of LinkedIn, Facebook may find it easier to attract back members who have partially adopted Google+ in the early periods because many users of such social networks may not value their time as much as busy professionals in LinkedIn.

**Proposition 4:** For homogeneous users, it becomes increasingly easier for the MH network (compared to SH) to cause reverse migration as the valuation of time by users decreases.

If there is no uncertainty about the quality of the capability offered (as in our model), the incumbent’s timely reaction may cause a ‘ping-pong’ behavior, where users spend more time with the network offering the best combination of capability and network effects in absence of other switching costs. It is however possible that a high reputation of the network in improving its capabilities may undermine reactive efforts of its competitor preventing such behavior.

In the first part of the paper, we have shown how a new entrant can induce migration in MH for both homogeneous and heterogeneous users with capability smaller than the incumbent and slowly develop its capabilities over time to cause a large scale migration if the incumbent does not react with innovation. We have also shown that a new entrant in MH can cause complete migration by keeping its technology lower than SH, if the networks facilitate the minimum level of interactions. In the latter part we see how the incumbent can combat such a strategy by the new entrant and innovate in order to cause a reverse migration. We find that in the initial periods, it is relatively easier to cause reverse migration in MH compared to SH. This suggests that, in the modern era where MH platforms are common, both the incumbent and the new entrant should strive for continuous innovation and not rely only on network effects. The demise of online networks like MySpace and ensuing competition between Facebook and Google+ underscore the importance of continuous innovation.
4. LIMITATIONS AND FUTURE WORK

In this study, we assumed that the ranks or weights a user assigns to others do not change across the two networks. However, it is plausible that a user may assign a higher or lower weight to a contact on the new network relative to that in the incumbent. For example, if the new entrant has superior multi-user games, a user may choose to interact more closely on the new network with contacts that enjoy such activities. While such a scenario will change the capabilities that need to be delivered by the new network to cause migration as well as the incumbent to regain lost time, both SH and MH settings will be affected in the same manner, thus not affecting the key results.

For analytical tractability we assumed that the total time spent by a user, T, does not change due to the new network. However, it is possible that T may increase due to better capabilities of network 2 for all users; alternatively users who are more affine to technological capabilities may increase the time spent on networking. Thus relaxing the assumption of T being a constant will change the migration pattern. For example, we may not obtain a linear increase in the total time in SH. However, the main result of the ease in enabling migration in MH relative to SH will not be affected. If T increases due to the new network, it will help the latter grow even faster, thus supporting our thesis.

We have not used any switching cost in the model other than that caused by network effects. This is plausible in a social network setting without membership fees. The ability of users to realize benefits by splitting time among multiple social networks is driven by the capability of the networks. Thus the level of adoption of the new network is determined in part by the total time available rather than by a switching cost (other than that due to network effects). If there are additional switching costs, the required capability for the new entrant will increase in both SH and MH settings, while the key results should remain unaffected.
In complex networks (Newman, 2003), network effects may be localized. Sundarajan (2007) studies the influence of local network effects on decisions to adopt a new product. Though we considered a fully connected network in our analysis, not everybody contributes equally to network effects for a user in our model, and are assigned different weights based on their similarity with the focal user.

We assumed that the diffusion of information about the new entrant takes place in a fully connected network through users with similar affinities to technology and network effects. This is similar to the notion of homophily (McPherson et al. 2001) in networks which suggests that users with similar characteristics tend to be associated with each other. If the network is not fully connected, groups of similar users in the network may not be directly connected to each other. Diffusion of information in such network topologies may be made easier by targeting users who connect disparate subnetworks of users (Granovetter 1973). Future research can delve into how complex network topologies impact the ease of migrating users in the multi-homing context.

5. CONCLUSION

The information economy has been characterized as a setting with strong network effects, and prior research has indicated that such forces favor the incumbent and can make it difficult for superior new technologies or networks to be adopted. We have noted that in the increasingly common scenario of multi-homing, an incumbent may be more vulnerable to a new entrant’s threats than previously believed in the network effects literature. Our contribution to the multi-homing literature is the introduction of dynamics of incremental adoption with resource constraints, a widely observed phenomenon that has received surprisingly little attention in the extant literature.

We also demonstrated that in a setting where users may incrementally adopt a new network by increasing the level of adoption over time, a new entrant need not provide the maximum
technological capability at the outset as in the case of single homing; it has the opportunity to arrive with a marginally superior capability and to improve such capability steadily over time. In other words, the incumbency protection power of network effects appear to be overrated in the modern context of simultaneous and incremental adoption of multiple technologies. Moreover, we show that a multi-homing network can witness complete migration if the new entrant initially targets more capability affine users. Given the public nature of profiles, preferences and actual behavior of users on various social media, such targeting has become increasingly easier to implement.

While our results underscore the vulnerability of the incumbent, we have also demonstrated that in the absence of switching costs other than that created by network effects, the incumbent can regain lost market share by delivering an improved capability before the network effects have built up sufficiently on the new entrant’s network. We find that it is relatively easier for the incumbent to cause reverse migration in the initial periods in multi homing networks compared to single homing networks, especially when users have a lower value for the time spent on a network. Thus, continuous innovation is critical for both the incumbent and the new entrant. These results indicate that network effects may contribute less to the competitive advantage of an incumbent than previously believed; further, the competitive intensity may increase considerably due to the opportunity for new entrants to compete with a powerful incumbent with just a marginally superior offering followed by incremental but steady improvements, and for the incumbent to react quickly to regain market share more easily than in the case of single homing.

REFERENCES


FIGURES AND TABLES

**Figure1:** Time spent on MySpace and Facebook (www.compete.com)

**Table 1:** Extant literature on multi-homing

<table>
<thead>
<tr>
<th>Study</th>
<th>Focus</th>
<th>Extent of adoption of platforms studied?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rochet and Tirole (2003)</td>
<td>Role of governance structure in pricing in the presence of MH. Incremental adoption issues not considered.</td>
<td>No</td>
</tr>
<tr>
<td>Gabszewicz and Wauthy</td>
<td>Two-sided markets and price competition in the presence of MH. A visitor can buy</td>
<td>No</td>
</tr>
<tr>
<td>(2004)</td>
<td>passes to two exhibition centers, while an exhibitor can also exhibit in both centers.</td>
<td></td>
</tr>
<tr>
<td>--------------------------------------------</td>
<td>----------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Armstrong and Wright (2005)</td>
<td>Pricing dynamics of two sided markets in the presence of MH and externalities. No</td>
<td></td>
</tr>
<tr>
<td>Parker and Van Alstyne (2005)</td>
<td>Analyzes when a firm can offer a free good in two-sided markets. No</td>
<td></td>
</tr>
<tr>
<td>Armstrong (2006)</td>
<td>Equilibrium prices to be paid by platform users with cross side externalities in 2-sided markets. Extent and timing of adoption of multiple platforms not considered. No</td>
<td></td>
</tr>
<tr>
<td>Doganoglu and Wright (2006)</td>
<td>Effect of compatibility on private and social incentives in multi-homing, which involves complete adoption of multiple platforms. No</td>
<td></td>
</tr>
<tr>
<td>Eisenmann et al. (2006)</td>
<td>Strategies for two-sided markets -- “winner-takes-all” scenarios are possible for high MH costs and cross-side network effects. The dynamics of incremental adoption of multiple networks are not considered. No</td>
<td></td>
</tr>
<tr>
<td>Landsman and Stremersch (2011)</td>
<td>Effect of level of MH on sales. Platform level MH hurts sales when platforms have low market share. However, mature platforms with large market shares witness more seller level multi-homing. No. This paper focuses on whether platform or seller level MH is beneficial, and not on how sellers may choose to divide their investment in developing games across multiple platforms.</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2:** Time allocation in MH by homogeneous users for cases 1 and 2

<table>
<thead>
<tr>
<th>User #</th>
<th>Period 1</th>
<th>Period 2</th>
<th>....</th>
<th>Period D/x</th>
<th>....</th>
<th>Period D</th>
</tr>
</thead>
<tbody>
<tr>
<td>User 1</td>
<td>$xT/D$</td>
<td>$2xT/D$</td>
<td>....</td>
<td>$T$</td>
<td>....</td>
<td>$T$</td>
</tr>
<tr>
<td>User 2</td>
<td>0</td>
<td>$2xT/D$</td>
<td>....</td>
<td>$T$</td>
<td>....</td>
<td>$T$</td>
</tr>
<tr>
<td>....</td>
<td>....</td>
<td>....</td>
<td>....</td>
<td>....</td>
<td>....</td>
<td>....</td>
</tr>
<tr>
<td>User D/x</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$T$</td>
<td>....</td>
<td>....</td>
</tr>
<tr>
<td>....</td>
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<td>....</td>
<td>....</td>
<td>....</td>
<td>....</td>
<td>....</td>
</tr>
<tr>
<td>User D</td>
<td>0</td>
<td>0</td>
<td>....</td>
<td>....</td>
<td>....</td>
<td>$T$</td>
</tr>
</tbody>
</table>

**Table 3:** Highest capabilities required for complete migration in SH and MH networks of homogeneous users.
<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>Highest capability required in SH</th>
<th>Highest capability required in MH case 1</th>
<th>Highest capability required in MH case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>673.7</td>
<td>53.3</td>
<td>313.3</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>2853</td>
<td>452.3</td>
<td>1746</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>10800</td>
<td>3972</td>
<td>4630</td>
</tr>
</tbody>
</table>

**Figure 2:** Capability requirements of the new entrant to cause migration in SH and MH case 1 and 2.

**Table 4:** Types of heterogeneous users in network 1 in period 0.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>User type</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>Low</td>
<td>Type 1</td>
</tr>
<tr>
<td>Moderate</td>
<td>Moderate</td>
<td>Type 2</td>
</tr>
<tr>
<td>Low</td>
<td>High</td>
<td>Type 3</td>
</tr>
</tbody>
</table>
Figure 3: Growth of the new entrant network in SH and MH cases 1 and 2.

Figure 4: Growth of new entrant network in SH and MH when users are heterogeneous and the new entrant offers the same capability to both SH and MH.
Figure 5: Capability required by the new entrant to continue migration in SH and MH networks.

Figure 6: Growth of the new entrant in SH and MH network corresponding to the capability delivered in Figure 5.
APPENDIX: Analytical Proofs

Proof of Propositions 1a and b

The benefit derived by user \( i \) in period \( \rho \) from network 1 in the SH case is given by

\[
B_{1,i,\rho}^{SH} = c_1 \left[ \alpha U_{i,p} (T) + \gamma S_{i,p} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{i,d,\rho} \right) \right]
\]

Similarly, the benefit derived by user \( i \) in period \( \rho \) from network 2 in SH is

\[
B_{2,i,\rho}^{SH} = c_2 \left[ \alpha U_{i,p} (T) + \gamma S_{i,p} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{i,d,\rho} \right) \right]
\]

The benefit derived by user \( i \) from splitting time between networks 1 and 2 in MH is given by

\[
B_{i,\rho}^{MH} = \sum_{j=1}^{2} c_j \left[ \alpha U_{i,p} \left( t_{ij,p} \right) + \gamma S_{i,p} \left( t_{ij,p}, \sum_{d=1}^{D} \omega_d \bar{M}_{i,d,\rho} \right) \right]
\]

The capability required in period \( \rho \) for SH in order to migrate user \( i \) and other users who are socially influenced by user \( i \)'s decision is given by the minimum value of \( c_2^S \) that is derived from

\[
B_{2,i,\rho}^{SH} \geq B_{1,i,\rho}^{SH}
\]

\[
c_2^{SH} \geq c_1 \frac{\alpha U_{i,p} (T) + \gamma S_{i,p} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{i,d,\rho} \right)}{\alpha U_{i,p} (T) + \gamma S_{i,p} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{i,d,\rho} \right)}
\]

From the equality, the first order condition reveals
\[ \frac{\partial c_{2p}^{SH}}{\partial \rho} = k_1 \left[ \frac{\partial S_{ip} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{-1d} \right)}{\partial \rho} - k_2 \frac{\partial S_{ip} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{-2d} \right)}{\partial \rho} \right]. \]

\[ k_1 = \gamma c_i \left[ \alpha U_{iip} (T) + \gamma S_{ip} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{-1d} \right) \right]^2, \quad k_2 = \alpha U_{iip} (T) + \gamma S_{ip} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{-2d} \right), \quad \text{and} \]

\[ k_3 = \alpha U_{iip} (T) + \gamma S_{ip} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{-1d} \right). \]

It can be shown that if the incumbent does not retaliate, i.e., \( \frac{\partial c_i}{\partial \rho} = 0 \), the mean estimated time allotment of other users (and therefore social utility) on network 1 decreases due to appropriate supply of technological innovation: \( \frac{\partial \bar{m}_{-1d}}{\partial \rho} \leq 0 \). Thus the mean estimated time allotment (and therefore social utility) on network 2 increases, i.e., \( \frac{\partial \bar{m}_{-2d}}{\partial \rho} \geq 0 \). Therefore \( c_{2p}^{SH} \) must be highest in the first period (\( \frac{\partial c_{2p}^{SH}}{\partial \rho} < 0 \)).

The capability required in period \( \rho \) for MH in order to migrate user \( i \) and other users who are influenced by user \( i \)'s decision is given by \( c_{21}^{MH} \) that is derived from \( B_{ip}^{MH} = \max \left( B_{1i, \rho}^{SH}, B_{1i, \rho}^{SH} \right) \) where

\[ B_{ip}^{MH} = B_{1i, \rho}^{SH} + B_{1i, \rho}^{SH}. \]

In MH, the capability required in the first period to allow multi-homing is given by \( c_{21}^{MH} \) such that

\[ c_{21}^{MH} \left\{ \alpha U_{iip} (t_{i1}) + \gamma S_{i1} \left( t_{i1}, \sum_{d=1}^{D} \omega_d \bar{M}_{-1d1} \right) \right\} + c_1 \left\{ \alpha U_{iip} (t_{i1}) + \gamma S_{i1} \left( t_{i1}, \sum_{d=1}^{D} \omega_d \bar{M}_{-1d1} \right) \right\} \geq \]

\[ c_{21}^{SH} \left\{ \alpha U_{iip} (T) + \gamma S_{i1} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{-1d1} \right) \right\}. \]
However, in SH, users completely migrate to the new entrant. This is possible only if $c_{21}^{SH} > c_{21}^{MH}$.

In MH, the capability delivered in period $\rho$ must satisfy the condition $c_{2,\rho}^{MH} > c_1$. The highest capability required to enable partial migration in MH is given by

$$c_{2,\rho}^{MH} \geq c_1 - \frac{\alpha \{U_{ip}(T) - U_{ip}(t_{i,\rho})\} + \gamma \left\{S_{ip}\left(T, \sum_{d=1}^{D} \omega_d \bar{M}_{-iid,\rho}\right) - S_{ip}\left(t_{i,\rho}, \sum_{d=1}^{D} \omega_d \bar{M}_{-iid,\rho}\right)\right\}}{\alpha U_{ip}(t_{i,\rho}) + \gamma S_{ip}(t_{i,\rho}, \sum_{d=1}^{D} \omega_d \bar{M}_{-i2d,\rho})}$$

The above inequality may be substituted by its parametric form (Section 3.1) and rewritten as

$$\xi > \frac{\alpha \left(t_{i,p}^\beta + t_{i,22}^\beta - T^\beta\right)}{\gamma \left(T^2 - t_{i,1}^2 - t_{i,22}^2\right) \sum_{d=1}^{D} \omega_d} \frac{1}{1}$$

Thus the minimum value of level of interactions that must be satisfied for a MH network is given as

$$\xi_{\text{min}} = 2 \left(2^{1-\beta} - 1\right) \frac{\alpha}{\gamma} T^{\beta-2}$$

Note that $\xi_{\text{min}}$ increases with $\alpha$ and decreases with $\gamma$.

For MH to enable migration with a lower capability then the highest capability needed for SH (in period 1), i.e. for $c_{21}^{SH} > c_{2,\rho}^{MH}$, we have the following necessary condition:

$$\frac{\alpha \{U_{ip}(T) - U_{ip}(t_{i,\rho})\} + \gamma \left\{S_{ip}\left(T, \sum_{d=1}^{D} \omega_d \bar{M}_{-iid,\rho}\right) - S_{ip}\left(t_{i,\rho}, \sum_{d=1}^{D} \omega_d \bar{M}_{-iid,\rho}\right)\right\}}{\alpha U_{ip}(t_{i,\rho}) + \gamma S_{ip}(t_{i,\rho}, \sum_{d=1}^{D} \omega_d \bar{M}_{-i2d,\rho})} < 1 + \frac{\gamma S_{1,1}(T, \sum_{d=1}^{D} \omega_d T)}{\alpha U_{11}(T)}$$

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Say, \( \eta = 1 + \frac{\gamma}{\alpha} \frac{S_{11}(T, \sum_{d=1}^{D} \omega_d T)}{U_{11}(T)} = 2^{2^2} - 1 \)

The left hand side (lhs) of this inequality is positive for \( \xi > \xi_{\min} \). Differentiating the lhs of the above inequality with respect to \( \beta \) reveals that the lhs increases with \( \beta \). We also know that \( \eta = 2^{2^2} - 1 \). Thus, the threshold \( \eta \) decreases with an increase in \( \beta \). Since the left and right hand sides of the above inequality are moving in opposite directions with an increase in \( \beta \) and may have a crossover point, this suggests that it is relatively easier to satisfy the above inequality for a lower \( \beta \).

**Proof of Proposition 2**

We first analyze the SH case. Heterogeneity is created by three types of users: Type \( \tau \) with affinities \( \alpha_{\tau} \) and \( \gamma_{\tau} \) where \( \tau \in \{1, 2, 3\} \). The capability required by the SH network 2 for type \( \tau \) as given by \( c_{2,\tau}^{SH} \) is shown below.

\[
c_{2,\tau}^{SH} \geq c_{1,\tau} \frac{U_{1p}(T) + \frac{\gamma_{\tau}}{\alpha_{\tau}} S_{1p}(T, \sum_{d=1}^{D} \omega_d \bar{m}_{-i1d,\tau})}{U_{1p}(T) + \frac{\gamma_{\tau}}{\alpha_{\tau}} S_{1p}(T, \sum_{d=1}^{D} \omega_d \bar{m}_{-i2d,\tau})}
\]

We know that in the initial periods, the network effect in 1 is higher than that in network 2. Thus, in order to provide a lower capability, network 2 will prefer a lower value of \( \gamma_{\tau} \) and a higher value of \( \alpha_{\tau} \). Thus Type 1 user should be the first to make a decision. After Type 1 users have
migrated, the network effect will reduce in network 1 and increase in network 2. Thus, the required technology to pull Type 2 users will be lower than that required for Type 3, given Type 2’s relatively higher affinity to capability. Finally, Type 3 users with highest affinity to network effects should be targeted. The Type 3 users have lost benefits from Type 1 and Type 2 users because they have already migrated to the network 2. Thus, this category of users will find it beneficial to migrate completely at once to make up for lost network effects.

In MH, the capability required by network 2 to partially migrate is given by

$$c_{2,\rho}^{MH} \geq c_{1,\rho}$$

$$\alpha_{i} \left( U_{ip}(T) - U_{ip}(t_{1,\rho}) \right) + \gamma_{r} \left( S_{ip} \left( T, \sum_{d=1}^{D} \omega_{d} \bar{M}_{-1ld,\rho} \right) - S_{ip} \left( t_{1,\rho}, \sum_{d=1}^{D} \omega_{d} \bar{M}_{-1ld,\rho} \right) \right)$$

We know that

$$\left( U_{ip}(T) - U_{ip}(t_{1,\rho}) \right) < U_{ip}(t_{2,\rho})$$

$$\left( S_{ip} \left( T, \sum_{d=1}^{D} \omega_{d} \bar{M}_{-1ld,\rho} \right) - S_{ip} \left( t_{1,\rho}, \sum_{d=1}^{D} \omega_{d} \bar{M}_{-1ld,\rho} \right) \right) > S_{ip} \left( t_{2,\rho}, \sum_{d=1}^{D} \omega_{d} \bar{M}_{-1ld,\rho} \right)$$

Therefore, similar to SH, network 2 in MH will prefer to attract users with high affinity to private utility first and low affinity to network effect or social utility later.

**Proof of Proposition 3a**

From Proposition 2, when users are heterogeneous, the new entrant network targets users with decreasing affinity to private utility and increasing affinity to network effects or social utility.
Thus, \( \frac{\partial (\gamma_\rho / \alpha_\rho)}{\partial \rho} = \mu \), where \( \mu \) is positive. Since the new entrant delivers the appropriate capability required for migration, if \( \frac{\partial \hat{c}_{c, \rho}}{\partial \rho} = 0 \) (i.e., if the incumbent does not react), the following is true:

\[
\frac{\partial S_{ip}}{\partial \rho} \left( T, \sum_{d=1}^{D} \omega_d \bar{M}_{\text{idle}} \right) = -\varphi \quad \text{and} \quad \frac{\partial S_{ip}}{\partial \rho} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{\text{idle}} \right) = \varphi \quad \text{where} \ \varphi \ \text{is positive.}
\]

\[
\frac{\partial \hat{c}_{c, \rho \rho}}{\partial \rho} = c_{i, \rho} \frac{\partial}{\partial \rho} \left[ \frac{\alpha_\rho U_{ip} (T) + \gamma_\rho S_{ip} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{\text{idle}} \right)}{\alpha_\rho U_{ip} (T) + \gamma_\rho S_{ip} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{\text{idle}} \right)} \right]
\]

The condition for \( \frac{\partial \hat{c}_{c, \rho \rho}}{\partial \rho} > 0 \) can be expanded and written as

\[
S_1 - S_2 > \frac{1}{U_{ip} (T) \alpha_\rho \mu} \left\{ \gamma_\rho \varphi \left( 2 + \frac{\gamma_\rho}{\alpha_\rho} S_T \right) \right\}
\]

where \( S_1 = S_{ip} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{\text{idle}} \right) \), \( S_2 = S_{ip} \left( T, \sum_{d=1}^{D} \omega_d \bar{m}_{\text{idle}} \right) \) and \( S_T = S_1 + S_2 \).

Thus, when the difference in social utility is greater than the above threshold, the capability requirement for a new entrant increases in \( S_T \). However, when the difference in social utilities equals the threshold, the SH network reaches its highest required capability, i.e. \( \frac{\partial \hat{c}_{c, \rho \rho}}{\partial \rho} = 0 \), and when \( S_2 \) increases further, it results in lowering the requirement for capability required for migration, i.e., \( \frac{\partial \hat{c}_{c, \rho \rho}}{\partial \rho} < 0 \).

The threshold

\[
\frac{1}{U_{ip} (T) \alpha_\rho \mu} \left\{ \gamma_\rho \varphi \left( 2 + \frac{\gamma_\rho}{\alpha_\rho} S_T \right) \right\}
\]

increases with gamma and decreases with alpha.
Proof of Proposition 3b:

Similar to Proposition 1, the minimum level of interactions that must be satisfied for a MH network for heterogeneous users is given as

$$\xi_{\text{min}} = 2 \left(2^{1-\beta} - 1\right) \frac{\alpha_{\text{max}}}{\gamma_{\text{min}}} T^{\beta-2}$$

The required level of interactions increases with $\alpha$ and decreases with an increase in $\gamma$. Thus, in a network with more technology affine users, the network should implement functionalities to motivate users to gain from social aspects of the network.

For MH to have a lower capability than that for SH, we find a condition such that in any period $\rho$ MH requires a lower technology than the highest technology required by SH, i.e., $c_{21}^{SH} > c_{2\rho}^{MH}$.

From Proposition 1 (for homogeneous users), the condition can be rewritten as,

$$\frac{\alpha \left(U_{i\rho} (T) - U_{i\rho} \left(t_{1\rho}\right)\right) + \gamma \left(S_{i\rho} \left(T, \sum_{d=1}^{D} \omega_d \bar{m}_{-1ld\rho}\right) - S_{i\rho} \left(t_{1\rho}, \sum_{d=1}^{D} \omega_d \bar{M}_{-1ld\rho}\right)\right)}{\alpha U_{i\rho} \left(t_{2\rho}\right) + \gamma S_{i\rho} \left(t_{2\rho}, \sum_{d=1}^{D} \omega_d \bar{M}_{-2ld\rho}\right)} < 1 + \frac{\gamma_{11}}{\alpha_{i}} \frac{S_{11} \left(T, \sum_{d=1}^{D} \omega_d T\right)}{U_{11} (T)}$$

Say, $\eta = 1 + \frac{\gamma_{11}}{\alpha_{i}} \frac{S_{11} \left(T, \sum_{d=1}^{D} \omega_d T\right)}{U_{11} (T)} = 2^{2-\beta} - 1$
The left hand side (lhs) of this inequality is positive for $\xi > \xi_{\text{min}}$. Differentiating the lhs of the above inequality with respect to $\beta$ reveals that the lhs increases with $\beta$. We also know that $\eta = 2^{2-\beta} - 1$. Thus, the threshold $\eta$ decreases with an increase in $\beta$. Since the left and right hand sides of the above inequality are moving in opposite directions with an increase in $\beta$ and may have a crossover point, this suggests that it is relatively easier to satisfy the above inequality for a lower $\beta$.

**Proof of Proposition 4**

We analyze the capability requirements for homogeneous users from the perspective of network 1, which now reacts to network 2’s improvement in technology. Given the capability, $c_{2,\rho}^{\text{SH}}$, of the new network, in period $\rho$, the incumbent must provide a new capability given by

$$c_{1,\rho}^{\text{SH}} \geq c_{2,\rho}^{\text{SH}} \frac{\alpha U_{ip}(T) + \gamma S_{ip}(T, \sum_{d=1}^{D} \omega_d \bar{m}_{-1,2,\rho})}{\alpha U_{ip}(T) + \gamma S_{ip}(T, \sum_{d=1}^{D} \omega_d \bar{m}_{-1,\rho})}$$

Similarly, for MH the requirement by the incumbent to cause reverse migration is given by

$$c_{1,\rho}^{\text{MH}} \geq c_{2,\rho}^{\text{MH}} \frac{\alpha U_{ip}(t_{1,\rho}) + \gamma S_{ip}(t_{1,2,\rho}, \sum_{d=1}^{D} \omega_d \bar{M}_{-1,2,\rho})}{\alpha \{U_{ip}(t_{1,\rho}) - U_{ip}(t_{1,\rho})\} + \gamma \left[S_{ip}(T, \sum_{d=1}^{D} \omega_d \bar{m}_{-1,\rho}) - S_{ip}(t_{1,\rho}, \sum_{d=1}^{D} \omega_d \bar{M}_{-1,\rho})\right]}$$

We know that for homogeneous users, the SH network new entrant has to provide the highest capability upfront. To compare the minimum capabilities required for reverse migration, we can
rewrite the above requirements in terms of benefit functions as follows: \( c^S_{11} > c^M_{p} \) if

\[
\frac{c^S_{21}}{c^M_{2p}} \geq \frac{\alpha \left( U_{i,p}(T) - U_{i,p}(t_{i,p}) \right) + \gamma S_{i,p} \left( T, \sum_{d=1}^{D} \omega_d \bar{M}_{-1d_{i,p}} \right) - S_{i,p} \left( t_{i,p}, \sum_{d=1}^{D} \omega_d \bar{M}_{-1d_{i,p}} \right)}{\gamma \left( T, \sum_{d=1}^{D} \omega_d T \right)} > 1 + \frac{\gamma}{\alpha} U_{11}(T)
\]

From Proposition 1, \( \bar{\xi} \geq \bar{\xi}_{\text{min}} \) ensures that \( c^S_{21} > c^M_{2p} \)

\[
\alpha \left( U_{i,p}(T) - U_{i,p}(t_{i,p}) \right) + \gamma S_{i,p} \left( t_{i,p}, \sum_{d=1}^{D} \omega_d \bar{M}_{-1d_{i,p}} \right) > \alpha U_{i,p}(t_{i,p}) + \gamma S_{i,p} \left( t_{i,p}, \sum_{d=1}^{D} \omega_d \bar{M}_{-1d_{i,p}} \right)
\]

We also know that \( \bar{\xi}_{\text{min}} = 2 \left( 2^{1-\beta} - 1 \right) \frac{\alpha}{\gamma} T^{\beta-2} \). Therefore, the condition under which the incumbent’s capability required in MH for reverse migration is lower than in SH is given by

\[
\frac{c^S_{21}}{c^M_{2p}} \geq \frac{\alpha \left( U_{i,p}(T) - U_{i,p}(t_{i,p}) \right) + \gamma S_{i,p} \left( T, \sum_{d=1}^{D} \omega_d \bar{M}_{-1d_{i,p}} \right) - S_{i,p} \left( t_{i,p}, \sum_{d=1}^{D} \omega_d \bar{M}_{-1d_{i,p}} \right)}{\gamma \left( T, \sum_{d=1}^{D} \omega_d T \right)} > 2^{2-\beta} - 1
\]

Since \( \beta \in (0,1) \), the required threshold (i.e., the right hand side of the above equation) decreases as users place a higher value on their time spent on a network. Moreover, in the initial periods when sufficient network effects have not developed on the new entrant, the left hand side of the equation is higher than in later periods. Therefore it is easier for MH to cause reverse migration.