Abstract

This paper evaluates both efficiency increasing and efficiency decreasing mergers in a procurement setting in which firms differ in the likelihood that they have high production costs. Profitable efficiency increasing mergers often decrease the expected price, but profitable efficiency decreasing mergers always increase it. For a particular pair of firms, there may be no profitable mergers. If there are, then the most profitable merger may decrease efficiency. Consequently, merging firms that are able to choose their postmerger level of efficiency may profitably elect to decrease it.
The Competitive Effects of Mergers Between Asymmetric Firms

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1 Introduction

The existence of asymmetries between firms provides a strong motive for horizontal mergers, because mergers may allow combinations of assets that make the merging firms more efficient. For example, merging firms may provide complementary products or may operate in non-overlapping distribution areas. In these instances, a merger that profitably lowers the parties’ costs may increase the intensity of competition by creating a more aggressive firm. Weighing against this possible consumer benefit of a merger is the potential for reduced competition resulting from the removal of a close rival. Contemporary horizontal merger analysis attempts to balance these conflicting forces in order to allow competition-enhancing mergers and prevent competition-dampening ones.

This paper characterizes the profitability and price effect of mergers in a homogeneous product procurement auction setting in which firms potentially have different cost structures. The model admits several interpretations in which “costs” relate to a firm’s ability to competitively serve the market. For example, firms of different size might with different frequencies face binding capacity constraints caused by random fluctuations in productive capabilities, or firms might differ in their advertising coverage. I discuss these interpretations in Section 2.

Auction models recently have been used by competition authorities to examine markets in which asymmetric information and bidding play a key role. One reason that auction models have become popular is because many mergers take place in intermediate goods markets, which often have features similar to theoretical auction models. For example, if the intermediate good is a small element of the final product’s total cost, then the input’s derived demand likely is quite inelastic. Moreover, firms in procurement settings, such as defense contracting or the provision of accounting services, often do not know their rivals’ production and opportunity costs, though particular firms might consistently have high or low costs. Both private cost information and differences in firms’ likely costs can be incorporated in the auction framework.

Given the relevance of auction models to actual transactions, from both public policy and litigation perspectives it is important to understand how mergers affect behavior in auction markets. Several recent papers have addressed this topic by using models in which firms’ costs are random draws from continuous distributions. General results about competition and mergers in asymmetric

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1The Federal Trade Commission recently has begun to exploit auction models in merger analyses. For example, auction results were used in evaluating the recent merger between Rite-Aid and Revco. See Baker [1996]. See also Section 2.21 of the Horizontal Merger Guidelines [1997].

2This claim is made in Scheffman [1993], while Dalkir, Logan, and Masson [2000] estimate that auction models are potentially applicable to between one-third and one-half of the mergers that receive close scrutiny from U.S. antitrust agencies.
first-price auction markets are lacking in this setting, because analytic solutions cannot be derived
in asymmetric models with arbitrary cost distributions.\textsuperscript{3}

In this paper I take an alternative approach by examining an asymmetric first-price auction
model in which firms’ costs are random draws from discrete distributions. This assumption on the
cost structure makes it possible to explicitly calculate the firms’ expected profits and the expected
price, and therefore to provide analytic conditions describing a merger’s effect.

I also depart from the previous literature by using a more general formulation of a merger’s
effect on the merged firm’s cost distribution. In particular, I permit mergers to either increase or
dercrease efficiency, in contrast to the typical approach that assumes mergers always weakly increase
efficiency. The more general approach reveals a large class of profitable mergers that normally
would have been overlooked.

The following are the paper’s main results. First, both efficiency decreasing and efficiency
increasing mergers can be profitable. Although decreasing efficiency directly harms the merged
firm by increasing its expected cost relative to that of the merging firms, it also can indirectly benefit
the merged firm by softening price competition. Second, the most profitable merger for a pair of
firms may be an efficiency decreasing one, which suggests that merging firms may profitably elect
to decrease their efficiency. Unfortunately for consumers, profitable efficiency decreasing mergers
strictly increase the expected price. Third, there exist profitable yet price-reducing mergers between
firms with relatively high expected costs. Consequently, mergers between these weaker firms can
be procompetitive despite the resulting increase in concentration.

To provide a background against which to evaluate the discrete-type approach, the following four
papers represent what has been learned about mergers when firms’ costs are drawn from continuous
distributions. Waehrer [1999] examines per-member expected profits for coalitions in both first-
price and second-price auction settings. In the part of the analysis specifically examining mergers,
firms are modeled as taking different numbers of draws from a common cost distribution, with each
firm’s realized production cost being the minimum of its draws. The author finds that profitable
mergers in first-price auction markets increase the expected profits of nonmerging firms and all
potential entrants. In contrast, profitable mergers in second-price auction markets increase only
the expected profits of the merging firms. From these results he concludes that entry potentially
can counteract the effect of price-increasing mergers in first-price auction markets, but not in
second-price auction markets.

\textsuperscript{3}\textsuperscript{See Maskin and Riley [2000], Lebrun [1996], and Waehrer [1999].}
Waehrer and Perry [2003] analyze second-price auctions in which firms’ costs are drawn from the same type of distributions used in Waehrer [1999]. Because firms in second-price auctions have dominant strategies, equilibrium prices and profits can be analytically determined. The authors find that mergers are profitable, increase industry concentration, and increase the expected price. However, they also find that the buyer can partially or fully offset a merger’s effect, or in some instances prevent the merger entirely, if it optimally selects the reserve price.

Tschantz, Crooke, and Froeb [2000] numerically analyze first-price and second-price auctions in which bidders’ values are drawn from Extreme Value distributions with the same variance and different means. Using a three-firm example in which two of the firms draw their values from the same distribution, the authors show that a merger of the identical bidders has a larger price effect when those two firms are larger than the third firm rather than smaller.

Dalkir, Logan and Masson [2000] numerically analyze first-price auctions in which firms are one of two types, taking the minimum of either a high number or a low number of draws from a Uniform distribution. The authors show that using symmetric auction models to simulate a merger in this setting may overstate the acquisition’s price effect.

2 The One-Shot Procurement Model

Consider a market with \( N \geq 2 \) firms that compete for the unit demand of a buyer. The firms produce homogeneous goods, and each firm is privately informed about its marginal production cost. A firm’s marginal cost is an independent draw from its commonly known cost distribution, and is either \( c_L \) or \( c_H \), with \( 0 = c_L < c_H \). Firm \( i \) is low-cost \((c_L)\) with probability \( \alpha_i \in (0,1] \), the firms are labeled such that \( \alpha_N \leq \alpha_{N-1} \leq \cdots \leq \alpha_1 \), and \( \alpha = \{\alpha_1, \ldots, \alpha_N\} \) is the vector of efficiency probabilities. I refer to a firm \( i \) such that \( \alpha_i \geq \alpha_j \) for all \( j \in \{1, \ldots, N\} \) as a best firm. Note that there may be several best firms. The buyer solicits simultaneous and secret price offers from the firms after each has learned its production cost, then buys from the firm offering the lowest price at the price that it offered. In the auction literature this setting is referred to as a first-price auction in an independent private values environment. When \( \alpha_i \neq \alpha_j \) for some firms \( i \) and \( j \), the environment is asymmetric, in the sense that some firms differ in their likelihood of being low-cost.

Versions of the model have been used to describe behavior in a variety of strategic settings. Athey and Bagwell [2001] and McAfee and McMillan [1992] use the symmetric version to examine collusion in auctions, while Lang and Rosenthal [1991] and Janssen and Rasmusen [2002] use it to examine entry and pricing. Maskin and Riley [1985] use the asymmetric version to examine the price performance of different auction formats, while Thomas [2002] uses it to extend and refine
results from Lang and Rosenthal [1991]. This model also is formally equivalent to the model of advertising in McAfee [1994], in which the \( \alpha_i \)'s represent the coverage rate of sellers' advertising to initially uninformed consumers.

The model has other interpretations in which all firms have access to the same underlying technology and are subject to random shocks. In the capacity interpretation, larger firms are better able to smooth out production or demand shocks, and so are less likely to face binding capacity constraints. In the model, capacity constrained firms have cost \( c_H \) and cannot serve the market as efficiently as can firms with excess capacity, who have cost \( c_L \). In the geographic interpretation, larger firms have broader geographic coverage and are more likely to have facilities close to customers who are randomly located in space. In the consumer taste interpretation, some firms have products that are more likely to be acceptable to customers who are randomly located in the space of consumer tastes.

Because there are only two possible cost levels, the unique equilibrium involves mixed strategy pricing by low-cost firms (unless \( \alpha_1 = \alpha_2 = 1 \), which yields marginal cost pricing). I assume that the buyer can produce the item in-house for cost \( c_H \), so it credibly can commit to a reserve price of \( c_H \). This convention allows the simplifying assumption that high-cost firms do not submit price offers, which still yields a Nash equilibrium but avoids an uninteresting equilibrium existence problem in the asymmetric setting that arises from the possibility of ties between high-cost and low-cost firms at \( c_H \).

Because low-cost firms use mixed strategies, each price in the support of a low-cost firm \( i \)'s equilibrium price-setting distribution yields the same interim expected profit, \( \pi_i(\alpha) \). Therefore, firm \( i \)'s ex ante expected profit is \( \alpha_i \pi_i(\alpha) \). Result 1 provides formulas for a low-cost firm's interim expected profit, the expected price, \( P(\alpha) \), and expected social welfare, \( W(\alpha) \).

**Result 1 (McAfee [1994])** The following hold for a given vector of efficiency probabilities, \( \alpha \):  
1. \( \pi_i(\alpha) = \pi(\alpha) = \left( \prod_{k=2}^{N} (1 - \alpha_k) \right) c_H \) for all \( i \)  
2. \( P(\alpha) = (1 + \alpha_2 + \cdots + \alpha_N) \pi(\alpha) \)  
3. \( W(\alpha) = \left[ 1 - \left( \prod_{k=1}^{N} (1 - \alpha_k) \right) \right] c_H \)

For readers unfamiliar with McAfee [1994], some features of Result 1 are worth highlighting. First, the best firm’s efficiency probability does not influence a low-cost firm’s interim expected profit or the expected price. The best firm faces the weakest competition, in the sense that

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\( ^4 \text{See Maskin and Riley [1985] and McAfee and McMillan [1992] for two other means of dealing with this problem.} \)
it is more likely to face no rivals than is any other firm. Of all firms, the best firm therefore
has the highest minimum payoff from setting a price of $c_H$, which implies that it also has the
highest minimum price that it is willing to set. One can show that all low-cost firms’ price-setting
distributions have the same minimum, and that minimum is the highest minimum that the best
firm is willing to set. Hence the minimum equilibrium price, which directly affects both expected
profits and the expected price, is influenced only by the probability that the best firm faces no
competition, and not by the best firm’s efficiency probability. Because the minimum price wins
with probability one and is common to all firms’ equilibrium price-setting distributions, each low-
cost firm has the same interim expected profit. Second, expected social welfare is determined
entirely by the probability that at least one firm is low-cost, because demand is inelastic and the
product is always provided by a firm with the lowest realized cost. Therefore, any merger that
does not change the firms’ overall productive efficiency has no effect on social welfare, and any
changes in outcomes are simply transfers between the firms and the buyer.

3 The Effect of Mergers

Result 1 provides a straightforward means to evaluate any merger’s profitability and price effect.
Although with a large number of firms there are many potential mergers to be analyzed, all mergers
in this model may be usefully partitioned into four classes, depending on whether or not the merger
involves the best firm and whether or not it yields a new best firm. This partition stems from the
best firm’s efficiency probability not influencing interim expected profits or the expected price.

A merger between firms $i$ and $j$ is modeled by considering a new efficiency probability $\tilde{\alpha}_{ij} \in (0, 1]$ for
the merged firm, with the vectors of premerger and postmerger efficiency probabilities denoted
by $\alpha$ and $\tilde{\alpha} = \{\alpha_1, \ldots, \tilde{\alpha}_{ij}, \ldots, \alpha_N\}$, respectively. I put no restrictions on $\tilde{\alpha}_{ij}$, in contrast to
the existing literature’s typical assumption that the merged firm’s cost distribution is at least as
favorable as letting the merged firm take the minimum of two draws, one from each merger partner’s
cost distribution.\footnote{For example, see Waehrer [1999], Dalkir, Logan, and Masson [2000], Tschantz, Crooke, and Froeb [2000], and Waehrer and Perry [2003].} In general, such an assumption is represented by a postmerger cost distribution $\tilde{G}_{ij}(c)$ for which $\tilde{G}_{ij}(c) \geq 1 - (1 - G_i(c)) [1 - G_j(c)]$, where $G_i(c)$ and $G_j(c)$ denote the merging firms’ premerger cost distributions. In the discrete-type model used here, such an assumption requires that $\tilde{\alpha}_{ij} \geq 1 - (1 - \alpha_i)(1 - \alpha_j)$. While researchers presumably do not contemplate that the merged firm literally keeps all production facilities postmerger, and for a given contract produces from the facility with the lowest realized cost, this lower bound provides a natural benchmark in
the sense that one might think that the merged firm would want to do at least this well. However, it will be shown that a merger may be more profitable with an even lower efficiency probability.

I say that a merger strictly increases efficiency if \( \bar{\alpha}_{ij} > 1 - (1 - \alpha_i)(1 - \alpha_j) \) and strictly decreases efficiency if \( \bar{\alpha}_{ij} < 1 - (1 - \alpha_i)(1 - \alpha_j) \). Applying Result 1 shows that efficiency increasing mergers increase total welfare, while efficiency decreasing ones decrease it. To see that this definition of efficiencies is sensible from the merging firms’ perspective, consider the model’s geographic interpretation. If the merging firms can reposition their production assets to provide broader geographic coverage than they independently provided premerger, then that is equivalent to the merged firm’s being more likely to be low-cost than either firm premerger.

Merger profitability requires that \( \bar{\alpha}_{ij} \pi(\bar{\alpha}) \geq (\alpha_i + \alpha_j) \pi(\alpha) \), in order for there to exist a split of the profits such that each merging firm is at least as well off as it was premerger. This condition reveals that merger profitability is influenced by the interplay between two forces. For a given vector of premerger efficiency probabilities, a large value of \( \bar{\alpha}_{ij} \) directly benefits the merged firm by making it likely that the firm is low-cost, and hence earns the interim expected profit of a low-cost firm. However, a large value of \( \bar{\alpha}_{ij} \) also can indirectly harm the merged firm by causing aggressive price-setting that lowers its interim expected profits.

Using Result 1 to examine the interaction between the direct and indirect effects of \( \bar{\alpha}_{ij} \) yields some general principles regarding merger profitability that influence all of the formal results to follow. First, it is possible that market conditions are such that a merger between two firms is not profitable for any \( \bar{\alpha}_{ij} \). This will occur if premerger profits are high relative to the highest possible postmerger profits. Second, if the merged firm is not the best firm, then its profits are highest when \( \bar{\alpha}_{ij} = 0.5 \). This level of \( \bar{\alpha}_{ij} \) just balances the opposing forces just described, and identical increases and decreases of \( \bar{\alpha}_{ij} \) from 0.5 identically influence profitability. Hence, merger profitability in this instance requires that \( \bar{\alpha}_{ij} \) be neither too high nor too low, but instead be sufficiently intermediate. To achieve this, firms with high expected costs want to increase efficiency, while firms with low expected costs want to decrease efficiency. Third, if the merged firm is the best firm, then its profits increase as \( \bar{\alpha}_{ij} \) increases, because increasing \( \bar{\alpha}_{ij} \) lowers expected costs but does not increase the intensity of competition. Merger profitability in this instance requires that \( \bar{\alpha}_{ij} \) be sufficiently high. To achieve this, firms with high expected costs want to increase efficiency. Firms with low expected costs also tend to want to increase efficiency, although they may be able to profitably merge while decreasing efficiency.

The following four propositions use the intuition just described to determine conditions under
which values of $\alpha_{ij}$ exist such that a given merger is profitable. The first two propositions examine mergers that do not involve the best firm premerger, and the second two examine mergers that do. All proofs are in the Appendix.

**Proposition 1** Consider mergers between two non-best firms $i$ and $j$ such that the merged firm is not uniquely the best firm. If $\alpha_i$ and $\alpha_j$ are both sufficiently low, then there exist only profitable mergers that increase efficiency, each of which strictly decreases the expected price. If at least one of $\alpha_i$ and $\alpha_j$ is sufficiently high, then there exist only profitable mergers that decrease efficiency, each of which strictly increases the expected price.

Figures 1 and 2 illustrate the conditions underlying Proposition 1. Applying Result 1 shows that the merger is profitable if and only if $\bar{\alpha}_{ij} (1 - \bar{\alpha}_{ij}) - (\alpha_i + \alpha_j) (1 - \alpha_i)(1 - \alpha_j) \geq 0$. For given $\alpha_i$ and $\alpha_j$, the line in Figure 1 represents this quantity as a function of $\bar{\alpha}_{ij}$, and it crosses the $\bar{\alpha}_{ij}$-axis at the values of $\bar{\alpha}_{ij}$ for which premerger and postmerger expected profits are equal. For all $\bar{\alpha}_{ij}$ between the two roots, the merger strictly increases profits, which corresponds to the intuition that $\bar{\alpha}_{ij}$ must be sufficiently intermediate for the merger to be profitable. The location of the line is obviously influenced by $\alpha_i$ and $\alpha_j$, and Figure 2 illustrates values of $\alpha_i$ and $\alpha_j$ for which there do and do not exist $\bar{\alpha}_{ij}$ such that the merger is profitable.

**Figures 1 and 2 here**

Proposition 1 has several interesting features. First, there may not exist any profitable mergers. The $(\alpha_i, \alpha_j)$ pairs in Region B in Figure 2 are those for which the quadratic equation $\bar{\alpha}_{ij} (1 - \bar{\alpha}_{ij}) - (\alpha_i + \alpha_j) (1 - \alpha_i)(1 - \alpha_j) = 0$ has no real roots, and hence the region in Figure 1 is empty. There do not exist profitable mergers in Region B because the intermediate values of $\alpha_i$ and $\alpha_j$ are such that premerger expected profits are so high that even the most profitable merger leads to lower joint profits for the merging firms.

Second, there may exist profitable efficiency decreasing mergers, but not profitable efficiency increasing ones. In Region C in Figure 2, at least one of $\alpha_i$ and $\alpha_j$ is sufficiently high that the only profitable mergers are efficiency decreasing ones that blunt the severity of competition caused by the high value of $\alpha_i$ or $\alpha_j$. Efficiency increasing mergers are unprofitable because the extent to which they intensify competition outweighs the direct benefit of reducing the merged firm’s expected cost. Because efficiency decreasing mergers increase the expected price, the profitable merger of a strong firm with either a strong or a weak firm strictly increases the expected price by diminishing competition.
Third, profitable efficiency increasing mergers always strictly decrease the expected price. In Region A in Figure 2, both $\alpha_i$ and $\alpha_j$ are sufficiently low that the only profitable mergers are efficiency increasing ones. While increasing efficiency causes more severe competition, the merging firms can offset that effect through their increased likelihood of being low-cost. Interestingly, if $\tilde{\alpha}_{ij}$ is high enough to make the merger profitable, then it leads to sufficiently intense competition that the expected price decreases postmerger. Therefore, the profitable merger of two relatively weak competitors creates a stronger competitor that causes the expected price to fall despite the increase in concentration.

**Proposition 2** Consider mergers between two non-best firms $i$ and $j$ such that the merged firm is uniquely the best firm. If either $\alpha_i$ and $\alpha_j$ are both sufficiently low, or at least one is sufficiently high, and if $\alpha_1$ is sufficiently low, then for sufficiently high $\tilde{\alpha}_{ij}$ there exist profitable mergers. If $\alpha_i$ and $\alpha_j$ are sufficiently low, then every profitable merger increases efficiency and strictly decreases the expected price. Otherwise, profitable mergers may either increase or decrease efficiency, and the expected price increases if $\alpha_i$, $\alpha_j$, and $\sum_{k \neq 1, i, j} \alpha_k$ are sufficiently high.

Because there is a limit on the level of postmerger profits, profitably merging to become the best firm requires that premerger profits be sufficiently low. The merging firms’ efficiency probabilities influence this in one of two ways. If both $\alpha_i$ and $\alpha_j$ are low, then the merging firms’ ex ante expected profits are low premerger because they are unlikely to be low-cost. If at least one of $\alpha_i$ and $\alpha_j$ is high, then the merging firms’ ex ante expected profits are low premerger because competition is so severe. However, these restrictions on $\alpha_i$ and $\alpha_j$ by themselves are insufficient to guarantee that the merger is profitable. If $\alpha_1$ is too high, then postmerger expected profits will be too low to make the merger profitable, due to the severe competition from firm 1 that is introduced by its no longer being the best firm.

To provide some perspective on the implications of Propositions 1 and 2, consider mergers between firms 2 and 3 in a triopoly example in which $\alpha_1 = 0.9$ and $\alpha_2 = \alpha_3 = 0.8$. The most profitable merger that creates a new best firm is an efficiency increasing one that has $\tilde{\alpha}_{23} = 1$, which increases the two firms’ profits by $0.036c_H$. The most profitable merger that does not create a new best firm is an efficiency decreasing one that has $\tilde{\alpha}_{23} = 0.5$, which increases the two firms’ profits by $0.186c_H$. Therefore, if firms 2 and 3 can select their postmerger expected cost, then they strictly prefer decreasing efficiency and remaining a non-best firm to increasing efficiency and becoming the best firm.
The next two propositions examine mergers that involve the best firm premerger, which without loss of generality can be assumed to be firm 1. These results are influenced by a firm that is the best of the nonmerging firms, and for discussion purposes it is useful to denote by $m \neq 1$ a firm for which $\alpha_m \geq \alpha_k$ for all $k \in \{2, ..., N\}\{j\}$, where $j$ is firm 1’s merger partner. Note that firm $m$ may be a best firm premerger.

**Proposition 3** Consider mergers between the best firm and any other firm $j$ such that the merged firm is not the best firm. If $\alpha_1$, $\alpha_j$, and $\alpha_m$ each are sufficiently low, or if at least one of $\alpha_j$ and $\alpha_m$ is sufficiently high, then for sufficiently intermediate $\bar{\alpha}_{1j}$ there exist profitable mergers. Each of these mergers decreases efficiency and strictly increases the expected price.

While it may seem strange that the best firm would consider a merger that reduces its efficiency probability by so much that the firm is no longer the best firm, consider a situation in which the best firm and its closest rival both are very likely to be low-cost. This corresponds to the case in which at least one of $\alpha_j$ and $\alpha_m$ is high. It may make sense for the best firm to consummate a merger that reduces its own probability of being low-cost, with either its closest rival or some other firm, in order to soften price competition and increase its interim expected profits in those cases in which it actually is low-cost. Of course, the merger will not be profitable if the merged firm’s efficiency probability is too low. This tension reflects the fact that, as a non-best firm, the merged firm’s profits are maximized when $\bar{\alpha}_{1j} = 0.5$.

**Proposition 4** Consider mergers between the best firm and any other firm $j$ such that the merged firm is the best firm. There exist mergers that weakly increase efficiency, and each of them is profitable and strictly increases the expected price. There also exist profitable mergers that strictly decrease efficiency, and each of them strictly increases the expected price.

The merger of the best firm and some other firm $j$, such that the merged firm is the best firm, is equivalent to eliminating firm $j$ and changing firm 1’s efficiency probability. Applying Result 1 reveals the intuitively appealing finding that removing firm $j$ strictly increases the remaining firms’ ex ante expected profits and strictly increases the expected price. Moreover, changing firm 1’s efficiency probability has no countervailing effect on the expected price, because firm 1 is the best firm. Therefore, profitable mergers of this sort increase the expected price.

Continuing with the example following Proposition 2 reveals additional insights regarding the comparison between various profitable mergers. With $\alpha_1 = 0.9$ and $\alpha_2 = \alpha_3 = 0.8$, the most
profitable merger between firms 1 and 2 for which the merged firm remains the best firm is an efficiency increasing one that has \( \tilde{\alpha}_{12} = 1 \), which increases the two firms’ profits by 0.132\( c_H \). The most profitable merger that makes firm 3 the new best firm is an efficiency decreasing one that has \( \tilde{\alpha}_{12} = 0.5 \), which increases the two firms’ profits by 0.182\( c_H \). Therefore, if firms 1 and 2 can select their postmerger expected cost, then they strictly prefer decreasing efficiency and becoming a non-best firm to increasing efficiency and remaining the best firm. Moreover, the most profitable merger among any two firms in this setting is the efficiency decreasing one between firms 2 and 3 that does not create a new best firm. This example suggests that there may exist strong incentives for firms to reduce their efficiency in order to soften competition, even in cases in which they could increase efficiency.

4 Conclusion

This paper characterizes the profitability of both efficiency increasing and decreasing mergers between firms that compete in first-price procurement markets, allowing for across-firm asymmetries both premerger and postmerger. The general formulation of efficiency changes illustrates a tension that influences a merger’s profitability: Increasing efficiency directly benefits the merged firm by reducing its expected costs, but it can indirectly harm the merged firm by increasing the severity of competition. The resolution of this conflict reveals that there exist profitable mergers of both kinds. In fact, the indirect effect can be so strong that in some instances the most profitable merger available to a pair of firms decreases efficiency. In other instances there may be no profitable mergers.

The analysis also reveals that profitable mergers that weakly decrease efficiency always strictly increase the expected price. This occurs because a profitable efficiency decreasing merger must strictly increase a low-cost firm’s interim expected profit, which implies that all firms’ ex ante expected profits increase. Because efficiency decreasing mergers also decrease total welfare, the increase in all firms’ ex ante expected profits implies that the expected price must increase.

However, despite the fact that increasing efficiency tends to create incentives for aggressive pricing, I find that profitable mergers that weakly increase efficiency may either increase or decrease the expected price. For example, a profitable efficiency increasing merger of two relatively small firms creates a stronger competitor that can cause the expected price to fall, despite the resulting increase in market concentration. In contrast, a profitable merger of the firm with the lowest expected cost and any other firm increases the expected price, regardless of any efficiency gains.

Appendix
Proof of Proposition 1: Using Result 1 and the fact that the merged firm is not the best firm, merger profitability requires $\bar{\alpha}_{ij} (1 - \bar{\alpha}_{ij}) \geq (\alpha_i + \alpha_j) (1 - \alpha_i) (1 - \alpha_j)$. Therefore, the merger is profitable for all $\bar{\alpha}_{ij}$ between the roots of the preceding quadratic equation in $\bar{\alpha}_{ij}$, which amounts to requiring

$$\bar{\alpha}_{ij} \in \left[ \frac{1 - \sqrt{1 - 4(\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j)}}{2}, \frac{1 + \sqrt{1 - 4(\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j)}}{2} \right].$$

It must be the case that $1 - 4(\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j) \geq 0$, in order for the preceding set to be nonempty.

One can now determine whether or not the merger increases efficiency. Consider the case $1 - (1 - \alpha_i)(1 - \alpha_j) > 0.5$. Suppose that there exist profitable mergers that increase efficiency. Then it must be the case that the upper end of the range of permissible $\bar{\alpha}_{ij}$ exceeds $1 - (1 - \alpha_i)(1 - \alpha_j)$. Upon simplification, this implies

$$\sqrt{1 - 4(\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j)} \geq 1 - 2(1 - \alpha_i)(1 - \alpha_j).$$

Because $1 - (1 - \alpha_i)(1 - \alpha_j) > 0.5$, the right hand side of the preceding expression is positive. Squaring both sides yields

$$1 - 4(\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j) \geq 1 - 4(1 - \alpha_i)(1 - \alpha_j) + 4(1 - \alpha_i)^2(1 - \alpha_j)^2,$$

which can be simplified to $0 \geq \alpha_i \alpha_j$, which cannot hold. Therefore, if $1 - (1 - \alpha_i)(1 - \alpha_j) > 0.5$, then every profitable merger decreases efficiency. A similar argument establishes that if $1 - (1 - \alpha_i)(1 - \alpha_j) < 0.5$, then every profitable merger increases efficiency.

In terms of price effects, profitable efficiency decreasing mergers increase the expected price. To see this, recall that expected social welfare is the sum of expected consumer and producer surplus. Therefore,

$$W(\bar{\alpha}) - W(\alpha) = \left\{ [c_H - P(\bar{\alpha})] + \bar{\alpha}_{ij} \pi(\bar{\alpha}) + \sum_{k \neq i,j} \alpha_k \pi(\bar{\alpha}) \right\} - \left\{ [c_H - P(\alpha)] + (\alpha_i + \alpha_j) \pi(\alpha) + \sum_{k \neq i,j} \alpha_k \pi(\alpha) \right\}.$$
\[
= \{ P(\alpha) - P(\bar{\alpha}) \} + \{ \tilde{\alpha}_{ij} \pi(\bar{\alpha}) - (\alpha_i + \alpha_j) \pi(\alpha) \} + \left\{ \left[ \pi(\bar{\alpha}) - \pi(\alpha) \right] \sum_{k \neq i,j} \alpha_k \right\}.
\]

Merger profitability implies that the second term above is weakly positive, but the merger’s efficiency reduction additionally implies that interim expected profits strictly increase. Therefore, the third term above is strictly positive. As the merger is assumed to weakly decrease efficiency, expected social welfare weakly decreases, which implies that the expected price must strictly increase. To see how profitable efficiency increasing mergers affect the price, consider the case \(1 - (1 - \alpha_i)(1 - \alpha_j) < 0.5\), in which all profitable mergers increase efficiency. The sign of the difference between the postmerger price and the premerger price is identical to the sign of

\[
\left(1 + \sum_{k \neq 1,i,j} \alpha_k + \tilde{\alpha}_{ij}\right)(1 - \tilde{\alpha}_{ij}) - \left(1 + \sum_{k \neq 1,i,j} \alpha_k + \alpha_i + \alpha_j\right)(1 - \alpha_i)(1 - \alpha_j).
\]

For the profitable merger with the lowest possible \(\tilde{\alpha}_{ij}\), this condition can be simplified to show that the sign of the price change is identical to the sign of \(1 - (1 - \alpha_i)(1 - \alpha_j) - \tilde{\alpha}_{ij}\). Because the merger increases efficiency, it decreases the expected price. Using Result 1, further increases in \(\tilde{\alpha}_{ij}\) reduce the postmerger expected price. Therefore, all profitable efficiency increasing mergers decrease the expected price.

**Proof of Proposition 2:** Using Result 1 and the fact that the merged firm is uniquely the best firm, merger profitability requires \(\tilde{\alpha}_{ij}(1 - \alpha_1) \geq (\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j)\). Therefore, for given \(\alpha_1, \alpha_i, \) and \(\alpha_j\), \(\tilde{\alpha}_{ij}\) must be such that

\[
\tilde{\alpha}_{ij} \geq \frac{(\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j)}{(1 - \alpha_1)}.
\]

\(\tilde{\alpha}_{ij}\) is also constrained to be weakly less than 1. Therefore, for there to exist \(\tilde{\alpha}_{ij}\) such that the merger is profitable, it must be the case that

\[
\frac{(\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j)}{(1 - \alpha_1)} \leq 1,
\]

which occurs if and only if \(\alpha_1 \leq 1 - (\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j)\). Note that \(\alpha_i < 1 - (\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j)\) and \(\alpha_j < 1 - (\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j)\). Therefore, there always exists \(\alpha_1, \alpha_i, \) and \(\alpha_j\) such that a suitable \(\tilde{\alpha}_{ij}\) exists. That is, for some premerger efficiency probabilities there exist profitable mergers creating a new best firm. However, for particular \(\alpha_1, \alpha_i, \) and \(\alpha_j\) there
may not exist profitable mergers of this sort. The sign of the difference between the postmerger price and the premerger price is identical to the sign of

\[ \left( 1 + \sum_{k \neq 1, i, j} \alpha_k + \alpha_1 \right) (1 - \alpha_1) - \left( 1 + \sum_{k \neq 1, i, j} \alpha_k + \alpha_i + \alpha_j \right) (1 - \alpha_i) (1 - \alpha_j). \]

This can be simplified to show that the sign of the price change is identical to the sign of

\[ [1 - (1 - \alpha_i)(1 - \alpha_j) - \alpha_1] \left( \alpha_1 + \alpha_i + \alpha_j + \sum_{k \neq 1, i, j} \alpha_k \right) - \alpha_i \alpha_j (1 - \alpha_1). \]

If \( \alpha_1 \geq 1 - (1 - \alpha_i)(1 - \alpha_j) \), then every profitable merger increases efficiency (because \( \tilde{\alpha}_{ij} > \alpha_1 \)) and strictly decreases the expected price. If \( \alpha_1 < 1 - (1 - \alpha_i)(1 - \alpha_j) \), then every profitable merger may or may not increase efficiency, depending on the level of \( \tilde{\alpha}_{ij} \) relative to \( 1 - (1 - \alpha_i)(1 - \alpha_j) \). Such a merger also may or may not increase the expected price, depending on \( \alpha_1, \alpha_i, \alpha_j \), and \( \sum_{k \neq 1, i, j} \alpha_k \). The merger is more likely to increase the price in this instance the larger are \( \alpha_i, \alpha_j \), and \( \sum_{k \neq 1, i, j} \alpha_k \), and the smaller is \( \alpha_1 \). Of course, profitable efficiency decreasing mergers always increase the expected price, as shown in the proof of Proposition 1.

**Proof of Proposition 3:** The merged firm’s not being the best requires \( \tilde{\alpha}_{1j} < \alpha_m \), where \( \alpha_m \geq \alpha_k \ \forall \ k \in \{2, \ldots, N\} \setminus \{j\} \), for \( m \neq 1, j \). Using Result 1 and the fact that the merged firm is not the best firm, merger profitability requires \( \tilde{\alpha}_{1j} (1 - \tilde{\alpha}_{1j}) \geq (\alpha_1 + \alpha_j) (1 - \alpha_j) (1 - \alpha_m) \).

Therefore, the merger is profitable for all \( \tilde{\alpha}_{1j} \) between the roots of the preceding quadratic equation in \( \tilde{\alpha}_{1j} \), which amounts to requiring

\[ \tilde{\alpha}_{1j} \in \left[ \frac{1 - \sqrt{1 - 4 (\alpha_1 + \alpha_j) (1 - \alpha_j) (1 - \alpha_m)}}{2}, \frac{1 + \sqrt{1 - 4 (\alpha_1 + \alpha_j) (1 - \alpha_j) (1 - \alpha_m)}}{2} \right]. \]

It must be the case that \( 1 - 4 (\alpha_1 + \alpha_j) (1 - \alpha_j) (1 - \alpha_m) \geq 0 \), in order for the preceding set to be nonempty. To ensure there exist \( \tilde{\alpha}_{1j} \) such that \( \tilde{\alpha}_{1j} < \alpha_m \), it suffices to look at the lower end of the interval of permissible \( \tilde{\alpha}_{1j} \). That is, it must be the case that

\[ \frac{1 - \sqrt{1 - 4 (\alpha_1 + \alpha_j) (1 - \alpha_j) (1 - \alpha_m)}}{2} < \alpha_m, \]

which is equivalent to \( 1 - 2 \alpha_m < \sqrt{1 - 4 (\alpha_1 + \alpha_j) (1 - \alpha_j) (1 - \alpha_m)} \). If \( \alpha_m > 0.5 \), then the preceding inequality holds. If \( \alpha_m \leq 0.5 \), then both sides of the preceding inequality can be
squared and simplified to yield \((\alpha_1 + \alpha_j)(1 - \alpha_j) < \alpha_m\). Consequently, for the merged firm not to be the best firm and for there to exist profitable mergers, it must be the case that \(\alpha_m > \text{Min}[0.5, (\alpha_1 + \alpha_j)(1 - \alpha_j)]\). Because \(\bar{\alpha}_{1j} < \alpha_1 < 1 - (1 - \alpha_1)(1 - \alpha_j)\), all profitable mergers in this case decrease efficiency. They therefore increase the expected price, as shown in the proof of Proposition 1.

**Proof of Proposition 4:** Using Result 1 and the fact that the merged firm is the best firm, merger profitability requires \(\bar{\alpha}_{1j} \geq 1 - (1 - \alpha_1)(1 - \alpha_j) - \alpha_j^2\). Because the merged firm’s profit increases as \(\bar{\alpha}_{1j}\) increases, the merger is profitable for \(\bar{\alpha}_{1j} \in [1 - (1 - \alpha_1)(1 - \alpha_j) - \alpha_j^2, 1]\). A similar argument establishes \(P(\bar{\alpha}) > P(\alpha)\). Because \(\alpha_m \leq \alpha_1 < 1 - (1 - \alpha_1)(1 - \alpha_j)\), the profitability condition encompasses all efficiency increasing mergers and a set of efficiency decreasing ones.

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