Wages and Productivity Growth in a Dynamic Monopoly *

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Abstract

This paper studies the intertemporal problem of a monopolistic firm that engages in productivity enhancing innovations to reduce labor costs. The optimal innovation policy is not monotone and the rate of productivity growth is the highest when the firm’s size is in some intermediate range. As long as its initial productivity is not too low, the firm eventually reaches a steady state where the rate of productivity growth is identical to the rate of wage growth. Productivity dependent wage differentials do not affect productivity growth in the steady state; they increase, however, the firm’s long–run equilibrium cost level.

Keywords: innovation, labor productivity, wages, wage differentials, dynamic programming, monopoly

JEL Classification No.: D24, D42, D92, J30, J51

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1 Introduction

This paper studies the impact of labor costs on the incentives for process innovations. It considers the intertemporal problem of a firm that engages in productivity enhancing innovations to reduce its labor costs. While the firm acts as a monopoly in the output market, it takes the current competitive wage in the labor market as given. At each point in time, the wage rate and the firm’s productivity determine its unit labor cost. Through investments in process innovations it can raise productivity and thus reduce its cost at subsequent dates. This generates a dynamic optimization problem because innovation affects future labor costs and the incentives for innovation depend on the evolution of these costs.

We formulate this problem as an infinite horizon optimization programme. In this way we can study the long-run evolution of productivity growth. At each point in time, the optimal innovation policy is determined by the firm’s current unit labor cost. We show that this policy is not monotone but has an inverted U–shape: The optimal innovation rate is higher for intermediate values of the firm’s unit labor cost than for low or high values. Since the firm’s output and employment decrease with its cost, the empirical implication is that also the relation between innovation and the firm’s size, as measured by its output or employment, is not monotone. In fact, on an optimal path with output increasing over time, there may be an initial phase of accelerating productivity growth which is followed by a phase of decelerating productivity growth.

The long-run prediction of our analysis is that productivity growth converges towards the rate of wage growth in all firms that survive in the long-run. Indeed, we show that increasing wages will not drive the firm out of the market as long as the initial level of its labor cost is not too high. In this case the rate of productivity growth approaches the rate of wage growth under the optimal innovation policy. Eventually, the firm reaches a steady state where these two rates coincide so that its unit labor cost remains constant over time. Interestingly, this steady state is independent of the level of wages; it only depends on their growth rate. The level of wages determines only whether the optimal policy tends towards the steady state or whether the firm will go extinct in the long-run.

In our model it is the evolution of wages that stimulates productivity enhancing innovations at the firm level. Our cost push argument addresses the interaction between labor market conditions and the innovative performance of industries and countries. This interaction is the subject of a number of
empirical studies both at the macroeconomic (see e.g. Gordon (1987)) and the microeconomic level (see e.g. Doms, Dunne and Troske (1997), Chen-nells and Van Reenen (1997), Mohnen et.al. (1986) and Van Reenen (1996)). An important issue in this context is the relation between unionization and firms’ R&D investment and innovation activities, which has been the subject of a relatively small number of theoretical and empirical studies. Theoretical studies by Baldwin (1983), Grout (1984) and van der Ploeg (1987) show that unionization is associated with underinvestment. In the absence of legally binding contracts, once a firm has incurred the sunk costs of investment, its union pushes for higher wages in order to capture a share of the quasi-rents created by the firm’s innovation. Due to the union’s hold-up behavior, the firm’s incentives to innovate are decreasing with the union’s bargaining power. This underinvestment result may, however, not hold if unionized firms are competing in the market and invest strategically to increase their market shares and profits: Tauman and Weiss (1987) show that a unionized firm, competing with a non-unionized rival, may overinvest in R&D under plausible conditions. Ulph and Ulph (1994, 1998, 2001) show that overinvestment may be observed in a unionized duopoly where firms are involved in a patent race for a single cost-reducing innovation.

We address the issue of rent sharing in Section 5, where we endogenize the firm’s wage rate by introducing productivity dependent wage differentials. These may reflect the employees’ bargaining power within the firm. In contrast with the static hold-up problem our dynamic analysis reveals that rent sharing has some quite different long-run effects. Indeed, the firm’s steady state rate of innovation is independent of the degree of rent sharing. This is so because in the steady state the rate of productivity growth equals the rate of wage growth and the ratio of the firm’s wage to the competitive wage remains constant over time. Our model thus indicates that unionization does not influence the firm’s long-run innovation behavior. The higher the degree of rent sharing, however, the higher is the firm’s long-run equilibrium cost level and the lower is its output. Yet, employment at each date after the steady state has been reached is independent of the degree of rent sharing. This is so because rent sharing has two exactly counterbalancing effects on employment: On the one hand, it reduces the firm’s steady state output; on the other hand it also lowers the long-run level of labor productivity. While the output effect tends to decrease employment, this is offset by the increase in employment due to the productivity effect.

\[1\] For an excellent discussion of the hold-up problem in labor markets see Malcomson (1997)
The empirical evidence of the impact of unionization on productivity enhancing activities is mixed and, to a large extent, inconclusive. Studies for USA show that unionization is associated with significantly less investment in physical capital (Bronars and Deere (1993); Hirsch, 1990); significantly less innovation (Acs and Audretsch (1987, 1988); Hirsch and Link (1987)); significantly less investment in R&D (Connolly et al. (1986); Addison and Hirsch (1989; Bronars et al., (1994)); but significantly more investment in employer-related training (Tan et al. (1992)). The empirical evidence from UK is more inconclusive. A few studies identify a positive, but not necessarily significant, impact of unions on investment (Machin and Wadhwani (1991); Latreille (1992)), while others identify a negative impact (Denny and Nickell (1992)). Unionism may however be related with a higher probability of receiving formal training (Booth (1992); Tan et al. (1992)). More recent studies also report mixed evidence on the relation between union power and productivity enhancing activities (Addison and Wagner (1994); Menezes-Filho et al. (1997, 1998)).

This paper complements our study in Bester and Petrakis (1998) of the relation between wages and productivity growth in a competitive industry with free entry and exit where the last period’s best technology is freely available to any firm. Also in the competitive framework it is the growth rate of wages that determines the industry’s long-run behavior. There are, however, some important differences between the competitive case and the present model. First, under perfect competition the individual firm does not face a truly dynamic optimization problem to determine its optimal innovation policy. This is so because free entry and exit, and free availability of last period’s best technology imply that a firm’s future profits are zero, independently of its innovation decision. Second, and more importantly, under perfect competition and free entry and exit the individual firm does not have to consider the impact of its production cost on the output price. Therefore, in Bester and Petrakis (1998) there is a positive relation between the incentive for innovation and the current cost level. This is no longer the case in the present monopoly model or, more generally, under imperfect competition. Instead, under the optimal policy the relation between $R&D$ investment and current cost has an inverted $U$–shape: When the monopoly’s cost are relatively low, there is little incentive to reduce these costs even further. Also

\footnote{For a recent survey on how the unions affect firms’ performance, innovation and labor productivity see Flanagan (1999). The author concludes by stressing that this issue is highly controversial.}

\footnote{An extension of our study towards a general equilibrium model is presented in Hellwig and Irmen (2001).}
relatively high costs, however, reduce the gains from cost reduction because of the low output associated with a high monopoly price. As a result, the monopoly’s highest innovation effort occurs for some intermediate level of labor costs and its adjustment path towards the steady state may not be monotone.

In our model the firm’s output and employment are both negatively related to its unit labor cost. Thus, if we view output or employment as a measure of firm size, an empirical implication of our analysis is that the relation between innovation and firm size is not monotone. Instead this relation has an inverse U–shape as the rate of productivity growth is highest when the firm’s size is in some intermediate range. These findings are in line with Machin and Wadhwani (1991), who report that “initially, an increase in size is associated with higher investment, though, eventually, size appears to be a disadvantage.” In contrast, most of the literature considers the R&D–firm size relationship as monotonic. For example, Cohen and Klepper (1992, 1996) provide an explanation for an inverse relationship between R&D productivity and firm size. Klette and Griliches (2000) present a model of firm growth in which R&D investment is proportional to sales. Some empirical studies give evidence for a positive relation between R&D and firm size in selected industries (e.g. Grabowski (1968) and Mansfield (1964)). Others find that firm size barely matters for R&D or that the relation may even be negative (e.g. Acs and Audretsch (1988), Pavitt, Robson and Townsend (1987), Scherer (1980)). Moreover, Acs and Audretsch (1987) indicate that a large or a small firm may have the relative innovative advantage, depending on a number of industry features, such as the degree of concentration and unionization, the capital intensity, the proportion of skilled labor etc. This kind of studies, however, typically measures the firms’ innovative activity without distinguishing between process and product innovations. Therefore, it is worth emphasizing that our theoretical findings only refer to productivity enhancing innovations.

This paper is organized as follows. Section 2 describes the firm’s infinite horizon optimization problem. The steady state solutions of this problem are derived in Section 3. Section 4 analyses the adjustment dynamics of the optimal innovation policy. The impact of productivity dependent wage differentials is studied in Section 5. Section 6 presents conclusions. The proofs of Lemma 1 and Propositions 1 and 2 are relegated to the Appendix in Section 7.
2 The Model

Our model depicts the evolution of labor productivity in a dynamic monopoly. For mathematical convenience we take the time horizon to be infinite; yet this should be interpreted as an approximation of a situation where the firm expects to operate for a large number of periods so that it does not take into account that its lifetime is finite. At each date \( t = 0, 1, 2 \ldots \), the monopolist employs labor in combination with other inputs to produce a single good. Also, he engages in process innovation to increase labor productivity. His innovation behavior at date \( t \) determines his technology at the subsequent date. To focus on the impact of labor costs on productivity growth, we abstract from activities such as product innovations or advertising, by which the firm is able to increase demand. We first analyse the evolution of productivity under the assumption that the wage rate is exogenous and that the monopolist takes the going rate as given. Later, in Section 5 we consider the possibility of productivity dependent wage differentials, which may reflect the employees’ bargaining power within the firm.

More formally, the model is specified as follows. At each date the monopolist faces the inverse demand function \( P(x) \) and so his revenue is \( R(x) \equiv P(x)x \). To produce \( x \) units of output at date \( t \), he has to invest the amount \( C(x) \) in inputs other than labor. The required labor input is \( x/a_t \), where \( a_t \) is the firm’s labor productivity at date \( t \).

The firm can engage in labor productivity enhancing process innovation. We assume that it can increase current productivity by the factor \((1 + q_t)\) by spending the amount \( K(q_t) \). To keep the problem tractable, we take the innovation cost \( K(q) \) to be independent of the actual level of labor productivity. Thus, if \( a_t \) describes the technology available at date \( t \), labor productivity at \( t + 1 \) becomes

\[
a_{t+1} = (1 + q_t)a_t
\]

if the amount \( K(q_t) \) is spent on innovations. Let \( K(0) = 0, K'(0) = 0, K'(^\infty) = \infty \) and \( K'(q) > 0, K''(q) > 0 \) for all \( q > 0 \). The initial level of productivity \( a_0 = \bar{a}_0 \) is exogenous.

The exogenous wage rate \( w_t \) grows at the rate \( \gamma > 0 \) so that \( w_{t+1} = (1 + \gamma)w_t \), with \( w_0 = \bar{w}_0 > 0 \). One possible interpretation is that \( \gamma \) represents the average growth rate of labor productivity in the entire economy. Therefore, also wages grow at the rate \( \gamma \) in the equilibrium of the economy - wide labor market. Since the firm under consideration constitutes only a small part of the whole economy, its impact on the equilibrium wage rate is negligible. Note that the growth of wages and income in the economy does not affect...
the firm’s demand. For simplicity, we abstract from such wealth effects by assuming that demand is generated by consumers with quasi-linear utility functions.

Let \( c_t \equiv w_t/a_t \). Then, after investing \( K(q) \) at date \( t \), the firm’s labor cost per unit of output at the next date is

\[
  c_{t+1} = \frac{1 + \gamma}{1 + q} c_t. 
\]  

The firm’s sales profit \( \Pi \) depends on its unit labor cost according to

\[
  \Pi(c) \equiv \max_x R(x) - C(x) - cx. 
\]  

We assume that \( R(x) - C(x) \) is strictly concave and \( R(0) \geq C(0), R'(0) > C'(0) \) and \( R'(\infty) < C'(\infty) \). Therefore, the profit maximizing output \( x^*(c) \) is uniquely determined and continuous in \( c \). Moreover, \( x^*(c) \) is positive and strictly decreasing in \( c \) as long as \( c \) is not too large. Notice that, by the envelope theorem, we have \( \Pi'(c) = -x^*(c) \).

For our analysis the shape of the function \( -\Pi'(c)c \) is crucial. Since \( -\Pi'(c)c = x^*(c)c \) this function describes how the firm’s total wage bill depends on its unit labor cost. Clearly, the firm’s wage bill tends to zero in the limit \( c \to 0 \). But also for \( c \) sufficiently large one has \( x^*(c)c = 0 \), because then \( x^*(c) = 0 \). Thus the firm’s total wage cost attains an interior maximum for some positive \( c \). As Lemma 1 below shows, the following assumption ensures that \( -\Pi'(c)c \) is well-behaved in the sense that it is single-peaked.

**Assumption 1** \( [R'(x) - C'(x)]x \) is strictly quasi-concave whenever \( P(x) > 0 \). Moreover, \( [R'(x) - C'(x)]x \to 0 \) as \( x \to 0 \).

This assumption is satisfied for standard cost and demand functions: For instance, \( R'(x) x \) is strictly concave if \( P(x) \) is linear or if it is iso-elastic with a price elasticity of demand larger than unity. In addition, \( -C'(x)x \) is concave if \( C(x) = k x^\omega \) with \( k > 0 \) and \( \omega \geq 1 \). Furthermore, also the second part of Assumption 1 is satisfied by these demand and cost functions.

**Lemma 1** There is a \( c^m > 0 \) such that \( -\Pi'(c)c \) is increasing in \( c \) if \( c < c^m \). If \( c > c^m \) and \( \Pi(c) > 0 \), then \( -\Pi'(c)c \) is decreasing in \( c \). Moreover, \( -\Pi'(0) < \infty \) and \( -\Pi'(c)c \to 0 \) for \( c \to \infty \).

The monopolist discounts future payoffs by the factor \( 0 < \delta < 1 \). We assume that

\[
  -\delta \Pi'(c^m(1 + \gamma)) \geq K'(\gamma). 
\]  


For a given discount factor \( \delta \), this condition is satisfied if the growth rate \( \gamma \) of wages is sufficiently small. By Lemma 1 the firm’s wage bill \(-\Pi'(c) c\) attains a maximum at \( c = c^m \). Since the incentive for innovation is positively related to the firm’s total labor cost, condition (4) ensures that for \( c = c^m \) the marginal benefit from innovation exceeds the marginal cost of raising labor productivity at the rate \( \gamma \). Indeed, as we show in the proof of Proposition 1 below, condition (4) implies that for values of \( c \) in the neighborhood of \( c^m \) the monopolist chooses an innovation rate \( q > \gamma \). \(^4\) If this were not the case, his innovation effort would always lag behind the growth of wages; thus his unit labor cost would increase over time and ultimately he would be forced to leave the market.

We can now state the monopolist’s intertemporal problem of choosing an optimal innovation strategy. He solves

\[
\max_{c_t, q_t} \sum_{t=0}^{\infty} \delta^t [\Pi(c_t) - K(q_t)]
\]

subject to

\[
c_{t+1} = \frac{1 + \gamma}{1 + q_t} c_t, \quad c_0 = \bar{c}_0.
\]

The solution of this problem determines an optimal trajectory \( \{c^*_t, q^*_t\}_{t=0}^{\infty} \). We can view (5) as an optimal control problem. This has a single state variable \( c \), a single control \( q \), an infinite horizon and is autonomous. Therefore, the optimal innovation policy may also be described by a function \( q^*(c) \) with the interpretation that the monopolist optimally chooses \( q^*_t = q^*(c_t) \) when his unit labor cost at \( t \) equals \( c_t \).

### 3 Steady States

To derive the optimal innovation policy \( q^*(\cdot) \) we employ the technique of dynamic programming. Let \( V(c) \) denote the value function associated with problem (5). Thus, at date \( t \) the monopolist’s present value of profits under the optimal policy is \( V(c_t) \) when his current unit cost is \( c_t \). The Bellman equation for problem (5) is

\[
V(c) = \Pi(c) - K(q^*(c)) + \delta V \left( \frac{c}{1 + q^*(c)} \right)
\]

\[
= \max_q \Pi(c) - K(q) + \delta V \left( \frac{1 + \gamma}{1 + q} \right).
\]

\(^4\)Note that (4) is not a necessary but only a sufficient condition to guarantee that \( q > \gamma \) for \( c \) close to \( c^m \).
We assume that \( V(\cdot) \) is continuously differentiable. The optimal innovation policy \( q^*(\cdot) \) then necessarily satisfies the first order condition

\[
-\delta V' \left( c \frac{1 + \gamma}{1 + q^*(c)} \right) \frac{(1 + \gamma)c}{1 + q^*(c)} = (1 + q^*(c))K'(q^*(c)).
\]  

(8)

By the envelope theorem we obtain from (7) that

\[
V'(c) = \Pi'(c) + \delta V' \left( c \frac{1 + \gamma}{1 + q^*(c)} \right) \frac{1 + \gamma}{1 + q^*(c)}.
\]  

(9)

In what follows we characterize the optimal policy \( q^*(\cdot) \) by examining the implications of conditions (8) and (9).

The optimal innovation policy determines the dynamic path of the monopolist’s wage cost according to

\[
c^*_{t+1} = \frac{1 + \gamma}{1 + q^*(c^*_t)}c^*_t.
\]  

(10)

We first look at the possible steady state values of \( c^*_t \). These values are the candidates for the long-run level of the firm’s wage–productivity ratio. In a steady state, \( c^*_t \) is stationary over time and so \( c^*_t = \hat{c} \) for all \( t \). Obviously, (10) implies that \( c^*_t = \hat{c} \) always constitutes a steady state for the evolution of \( c^*_t \). In this steady state the firm requires no labor input and so its optimal policy trivially satisfies \( q^*(\hat{c}) = 0 \). More relevant are the steady states in which \( \hat{c} \) is positive. By (10) this requires \( q^*(\hat{c}) = \gamma \). In such a steady state, the firm matches the growth rate of wages by the rate of productivity growth so that its labor cost per unit of output remains constant. Because of our assumption of stationary demand, also the steady state output \( x^*(\hat{c}) \) does not change over time. If the economy is growing at the rate \( \gamma \), this means that the firm’s share of total output and employment in the economy is gradually declining. Indeed, this may be viewed as a characteristic feature of a single industry in an growing economy in which the variety of available products expands over time. In such an environment it is the increase in the number of industries rather than the growth of the individual firm that accompanies the growth of income.

For \( q^*(\hat{c}) = \gamma \), condition (9) becomes \( V'(\hat{c}) = \Pi'(\hat{c})/(1 - \delta) \) so that by (8)

\[
-\Pi'(\hat{c}) \hat{c} = K'(\gamma)(1 + \gamma)(1 - \delta)/\delta.
\]  

(11)

By Lemma 1 and (4) this equation has exactly two solutions, \( \hat{c}_{II} \) and \( \hat{c}_{III} \). Moreover, \( 0 < \hat{c}_{II} < c^m < \hat{c}_{III} \).
Figure 1: The Determination of the Steady States

Figure 1 illustrates the determination of the steady state values $\hat{c}_{II}$ and $\hat{c}_{III}$ by equation (11). It also shows how these values change with the parameters of the model: An increase in $\gamma$ shifts the line $K'(\gamma)(1+\gamma)(1-\delta)/\delta$ upwards. As a result, $\hat{c}_{II}$ increases while $\hat{c}_{III}$ decreases with growth rate $\gamma$ of wages. In contrast, an increase in the monopolist’s discount factor $\delta$ lowers $\hat{c}_{II}$ and raises $\hat{c}_{III}$.

We now prove that $c \in \{\hat{c}_{II}, \hat{c}_{III}\}$ implies $q^*(c) = \gamma$ to establish the following result:

**Proposition 1** The optimal policy satisfies $q^*(c) = \gamma$ if and only if $c \in \{\hat{c}_{II}, \hat{c}_{III}\}$.

Interestingly, there are two steady state values for the firm’s wage–productivity ratio where the monopolist has the same incentive to spend on labor productivity enhancing innovation. The intuition is as follows. Obviously, the firm has a stronger *direct* incentive to invest in productivity enhancing activities at the steady state $\hat{c}_{III}$ where the unit labor costs are higher. Note,
however, that at $\hat{c}_{III}$ the monopolist produces a lower output than at the low wage–productivity steady state $\hat{c}_{II}$, i.e. $x^*(\hat{c}_{II}) > x^*(\hat{c}_{III})$. Hence, any given unit labor cost savings apply to a larger output at the low wage–productivity steady state. Therefore, the monopolist has a stronger indirect incentive to invest in productivity enhancing activities at the steady state $\hat{c}_{II}$. This latter output effect counterbalances the direct effect, and the monopolist has the same incentive for innovation in the two steady states $\hat{c}_{II}$ and $\hat{c}_{III}$.

4 Innovation Dynamics

In the foregoing section we have shown that the trajectory $\{c_t^*, q_t^*\}_{i=0}^\infty$ has three steady states, in which $c_t^*$ and $q_t^*$ remain constant over time. Typically, however, the initial value $\bar{c}_0$ of the monopolist’s wage–productivity ratio will only accidentally coincide with one of the three steady state values, $\hat{c}_{I}, \hat{c}_{II}$ and $\hat{c}_{III}$. In general $c_t^*$ will evolve over time until it possibly reaches one of the steady states. From the theory of dynamic optimization it is well-known that the optimal path of $c_t^*$ is monotone, i.e. the sign of $c_t^* - c_t^*$ is the same for all $t$ (see e.g. Kamien and Schwartz (1991, p. 179)).

The following Proposition characterizes the behavior of the optimal policy $q^*(c)$ for values of $c$ other than its steady state values:

**Proposition 2** The optimal policy satisfies $q^*(c) > \gamma$ for all $c \in (\hat{c}_{II}, \hat{c}_{III})$. For all $c \notin [\hat{c}_{II}, \hat{c}_{III}]$ the optimal policy satisfies $q^*(c) < \gamma$.

It turns out that for intermediate values of $c$ the monopolist has a strong incentive for cost reductions: Within the range $c \in (\hat{c}_{II}, \hat{c}_{III})$ he chooses an innovation rate that exceeds the growth rate of wages. This means that $c_t^*$ decreases monotonically over time when the initial value $\bar{c}_0$ happens to lie between $\hat{c}_{II}$ and $\hat{c}_{III}$. For intermediate values of wage–productivity ratio, the monopolist has a strong incentive to invest in labor productivity enhancing activities. Not only its unit labor cost is not too low so that the direct innovation incentive is strong. But also the firm’s output is high enough to induce a strong indirect incentive to invest in cost reduction. In summary, both the direct effect and the output effect are rather strong for intermediate values of $c$. This is not the case for values of $c$ outside the range $[\hat{c}_{II}, \hat{c}_{III}]$, where wages grow faster than productivity under the optimal policy.

Figure 2 illustrates the evolution of $c_t$ as determined by equation (10). Within the range $c_t \in (\hat{c}_{II}, \hat{c}_{III})$, the innovation rate exceeds the growth rate

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6 We are grateful to an anonymous referee for suggesting this diagram.
of wages and so we have \( c_{t+1} < c_t \). For values of \( c_t \) outside the range \([\hat{c}_{II}, \hat{c}_{III}]\), we have \( c_{t+1} > c_t \) because \( q^*(c_t) < \gamma \). This has the following implications for the long–run equilibrium: For any starting point \( \bar{c}_0 \) in the range \((0, \hat{c}_{II})\), the firm’s wage–productivity ratio \( c^*_t \) ultimately approaches \( \hat{c}_{II} \). On the optimal path \( c^*_t \) increases monotonically over time because for each \( t \) one has \( q^*(c^*_t) < \gamma \). Also when \( \bar{c}_0 \) is in the interval \((\hat{c}_{II}, \hat{c}_{III})\), \( c^*_t \) approaches \( \hat{c}_{II} \) in the long–run. On such a path, however, \( c^*_t \) decreases over time because \( q^*(c^*_t) > \gamma \). If \( \bar{c}_0 \) exceeds the critical value \( \hat{c}_{III} \), the firm will become eventually extinct under the optimal policy, which satisfies \( q^*(c^*_t) < \gamma \). On this path, the monopoly’s labor cost increases over time and so its output decreases until it ultimately equals zero.

Unless \( \bar{c}_0 > \hat{c}_{III} \), the rate of productivity growth converges to the rate of wage growth. The long–run behavior of the monopoly’s innovation strategy is thus independent of its initial productivity \( \bar{a}_0 \) and the level of the wage rate \( \bar{w}_0 \). It is not the level of wages but the growth rate of wages that eventually determines the monopoly’s innovation behavior and its wage–productivity ratio in the steady state \( \hat{c}_{II} \). In this steady state, also the firm’s output \( x^*(\hat{c}_{II}) \) and its price \( P(x^*(\hat{c}_{II})) \) are constant and do not depend on the wage level. This level, however, has a profound effect on the long–run level of

Figure 2: The Dynamics of Innovation
employment: As $\hat{c}_{II}$ is constant, a one percent increase in $\hat{w}_0$ raises the long-run level of labor productivity by one percent. Since the steady state output $x^*(\hat{c}_{II})$ is not affected, employment eventually falls by one percent. In other words, the monopoly’s long-run elasticity of labor demand with respect to the wage level equals minus unity.

Recall that the higher the exogenous growth rate of wages $\gamma$ is, the higher is the steady state value of wage–productivity ratio $\hat{c}_{II}$ and the lower is the steady state value $\hat{c}_{III}$. Therefore, as $\gamma$ increases, the range of parameters for which the firm will become eventually extinct becomes larger. Moreover, for the rest of the parameter values, the higher the growth rate of wages $\gamma$ is, the higher is the monopolist’s long-run wage–productivity ratio $\hat{c}_{II}$. The monopolist will thus face a higher unit cost of labor in the long-run in economies where wages grow faster. The monopolist’s output will then be lower and his price higher in the long-run. Further, as labor productivity will increase faster in the steady state, the monopolist’s demand for labor will decrease faster in the long-run. Similarly, the firm’s adjustment behavior depends on its discount factor $\delta$ and the size of the demand:7 If demand is relatively weak or the discount factor relatively high, it is more likely that the firm eventually becomes extinct. Also, it faces a higher wage–productivity ratio in the long–run if it is optimal to stay in the market forever.

## 5 Productivity Dependent Wages

Our analysis has so far considered a monopoly that pays the competitive wage rate to its labor force. It is often argued, however, that there is some sharing of the rents from innovation between workers and firms. For instance, if wage determination is the result of bargaining between a firm and a union, the wage will be positively related to the firm’s productivity. The idea is that the available rents increase with productivity and that this feeds through into higher wages as in the wage bargaining process the workers appropriate a share of the additional surplus.8

To investigate the impact of productivity dependent wages on the monopoly’s optimal innovation strategy, we now consider the case where the

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7 We may describe changes in demand by shifts in the demand function $P(x)$. A reduction in “market size” is then represented by a downward shift of $P(x)$. As a result, $\hat{c}_{II}$ increases and $\hat{c}_{III}$ decreases because the $-\Pi'(c)c$ schedule in Figure 1 is shifted downwards.

8 For details see the Introduction. For a survey of wage bargaining models with trade unions see e.g. Booth (1995), Ulph and Ulph (1990), Layard et.al. (1991). An empirical analysis can be found in Van Reenen (1996).
firm specific wage \( w_t \) depends on the competitive wage \( \tilde{w}_t \) and current labor productivity \( a_t \) according to

\[
w_t = \alpha \tilde{w}_t + \beta a_t.
\]  
(12)

We assume \( \alpha > 0, \beta > 0 \) and \( \beta/(1-\alpha) \geq \tilde{w}_t/a_t \) so that \( w_t \geq \tilde{w}_t \). The factor \( \beta \) reflects the extent to which the employees benefit from the firm’s productivity. In a wage bargaining model, it is positively related to the union’s bargaining power. Implicitly equation (12) assumes that neither innovations nor the level of employment are contractible and that wages are determined by a sequence of short–term bargaining agreements ex post after the innovation. Neither the firm nor its employees can commit to a long–term contract that would in advance specify the wage in subsequent periods. This rules out, for instance, that the firm could get concessions from its workers by promising higher wages in the future.

This can be illustrated in the right–to–manage model of wage bargaining where the firm chooses its output \( x^*(c_t) \) after the wage rate \( w_t \) has been determined. Therefore its employment equals \( x^*(w_t/a_t)/a_t \). The union’s payoff at date \( t \) may be specified as \( U(w_t) = (w_t - \tilde{w}_t) x^*(w_t/a_t)/a_t \). Suppose that at each date the firm and the union bargain about the current wage so that \( w_t \) is determined by the generalized Nash bargaining solution. Then \( w_t \) maximizes

\[
\Pi \left( \frac{w_t}{a_t} \right)^{1-r} U \left( w_t \right)^r,
\]  
(13)

where \( r \in (0,1] \) indicates the union’s bargaining power. If the monopolist’s demand and costs are specified as \( P(x) = A - x \) and \( C(x) = x^2 \), we have from (3) that, for \( A > c \), \( \Pi(c) = (A - c)^2/8 \). The solution of the above maximization problem yields

\[
w_t = \frac{2 - r}{2} \tilde{w}_t + \frac{Ar}{2a_t},
\]  
(14)

which is of the same form as equation (12).

The monopoly’s wage–productivity ratio at date \( t \) equals

\[
c_t = \alpha \frac{\tilde{w}_t}{a_t} + \beta.
\]  
(15)

As before, we assume that the competitive wage grows at the rate \( \gamma \) so that \( \tilde{w}_{t+1} = (1+\gamma)\tilde{w}_t \). If at date \( t \) the firm increases future labor productivity by the factor \( q_t \), its labor cost per unit of output at the subsequent date is

\[
c_{t+1} = \alpha \frac{\tilde{w}_t(1+\gamma)}{a_t(1+q_t)} + \beta,
\]  
(16)
By combining (15) and (16) it becomes apparent that the factor $\beta$ affects the evolution of $c_t$ according to

$$c_{t+1} = [c_t - \beta] \frac{1 + \gamma}{1 + q_t} + \beta. \quad (17)$$

Ceteris paribus, $c_{t+1}$ increases with $\beta$ if $q_t > \gamma$. This is so because the firm’s workforce captures some share of the gains from cost reduction. Conversely, the available rents decrease if $q_t < \gamma$ because then $c_{t+1} > c_t$. In this case, $c_{t+1}$ and $\beta$ are negatively related.

When wages depend on the firm’s productivity, the monopoly’s innovation behavior takes into account the effect of productivity changes on its wage cost. Thus equation (17) replaces constraint (6) in the monopoly’s problem (5). As in the previous analysis of this problem, the critical steady states of the state variable $c$ are those where $q'(\bar{c}) = \gamma$ so that $c_{t+1}^* = c_t^*$ if $c_t^* = \bar{c}$. Analogously to equations (8) and (9), it is straightforward to show that such a $\bar{c}$ must satisfy

$$-\delta V'(\bar{c})[\bar{c} - \beta] = (1 + \gamma)K'(\gamma), \quad V'(\bar{c}) = \Pi'\bar{c}) + \delta V'(\bar{c}). \quad (18)$$

Combining these conditions yields

$$-\Pi'\bar{c})[\bar{c} - \beta] = K'(\gamma)(1 + \gamma)(1 - \delta)/\delta. \quad (19)$$

Similarly to (11), for $\beta$ not too large and $\gamma$ sufficiently small this equation has exactly two solutions, $\bar{c}_{II}(\beta)$ and $\bar{c}_{III}(\beta)$.\footnote{Using the arguments in the proof of Lemma 1, it is easy to show that $-\Pi'(c)[c - \beta]$ attains a unique maximum at some $c^*_\beta$. Propositions 1 and 2 are then easily extended to the case where $c_t$ evolves according to (17) if condition (4) is modified to $-\delta \Pi'(c^*_\beta (1 + \gamma) - \gamma \beta)(c^*_\beta - \beta) > K'\gamma).$}

Proposition 3 states that the steady state value $\bar{c}_{II}(\beta)$ is increasing in $\beta$, and the steady state value $\bar{c}_{III}(\beta)$ is decreasing in $\beta$.

Proposition 3 describes the impact of $\beta$ on the firm’s long-run behavior. The properties of the optimal policy $q^*(c)$ are analogous to the result in Proposition 2: In the interval $(\bar{c}_{II}(\beta), \bar{c}_{III}(\beta))$ it is the case that $q^*(c) > \gamma$.
while \( q^*(c) < \gamma \) for all values of \( c \notin [\bar{c}_{II}(\beta), \bar{c}_{III}(\beta)] \). On the adjustment path, the dependence of wages on productivity tends to reduce the firm’s investment in cost reduction: Since the length of the interval \((\bar{c}_{II}(\beta), \bar{c}_{II}(\beta))\) decreases with \( \beta \), this factor is negatively related to the range of \( c \)-values where \( q^*(c) > \gamma \). The intuition is simply that sharing the productivity gains with its employees reduces the firm’s innovation incentive. By the same argument, \( \bar{c}_{III}(\beta) < \bar{c}_{III} \) so that productivity dependent wages reduce the range of initial values \( \bar{c}_0 \) that allow the firm to survive in the long–run.

The long–run implications of our dynamic model, however, are remarkably different from the usual static hold–up problem: As long as \( \bar{c}_0 < \bar{c}_{III}(\beta) \), the optimal trajectory \( \{c^*_t, q^*_t\}_{t=0}^{\infty} \) converges towards \((\bar{c}_{II}(\beta), \gamma)\). This means that rent sharing between the monopoly and its labor force does not affect the long–run rate of productivity growth and the long–run innovation effort \( K(\gamma) \). On the optimal path, the firm specific wage \( w \) eventually grows at the same rate as the competitive wage \( \bar{w} \) and so the relative wage differential \( w/\bar{w} \) remains constant over time. Because of this differential, the monopoly’s long–run labor cost \( \bar{c}_{II}(\beta) \) is positively related to the factor \( \beta \). Since \( \Pi'(c) < 0 \), its profit in the steady state \( \bar{c}_{II}(\beta) \) depends negatively on \( \beta \).

Consider, for instance, a unionized firm where bargaining between the monopolist and the union determines the wage rate. Our analysis then predicts the following: First, long–run productivity growth does not depend on the union’s bargaining power; it is simply determined by the growth rate of the competitive wage.\(^\text{10}\) Second, as the union’s claims over productivity rents increase, the monopolist’s steady state unit labor cost increases. Hence, due to the presence of the union, output and profits decrease.\(^\text{11}\)

Interestingly, the long–run employment level is independent of \( \beta \). Since the monopolist’s steady state output decreases with the degree of rent sharing, employment also tends to decrease with \( \beta \). This output effect, however, is offset by a labor productivity effect: The stronger the union’s claims over productivity rents are, the lower turns out to be the productivity of labor at each date after the steady state has been reached. In fact, by (15) the

\(^{10}\)For the USA, Hirsch and Link (1984) find that union density and productivity growth are negatively related, while Freeman and Medoff (1984) report an inconclusive evidence. For the UK, Nickell et.al. (1992) and Gregg et.al. (1993) obtain mixed evidence for the impact of union presence on productivity growth.

\(^{11}\)The empirical evidence from the USA and the UK on the union impact on productivity suggests that it is predominantly negative, although it could also be positive or insignificant, depending on the particular industry (see e.g. Booth (1995) and references therein).
long-run level of labor productivity at a date $t$ is given by,

$$a_t = \frac{\alpha \bar{w}_t}{\hat{c}_{II}(\beta) - \beta}.$$  

(20)

Observe from (19) that $\hat{c}_{II}(\beta) - \beta$ is inversely related to the steady state output level $x^*(\hat{c}_{II}(\beta)) = -\Pi'(\hat{c}_{II}(\beta))$. Therefore, for a given level of competitive wage $\bar{w}_t$, the higher $\beta$ is, the lower is the firm’s output and the lower is its level of labor productivity at each date $t$. More interestingly, since employment at date $t$ is the ratio of the steady state output to labor productivity at date $t$, i.e. $L_t = x^*(\hat{c}_{II}(\beta))/a_t$, we obtain by (19) that employment does not depend on $\beta$:

$$L_t = \frac{K'(\gamma)(1 + \gamma)(1 - \delta)}{\alpha \bar{w}_t \delta}.$$  

(21)

This proves the following Proposition:

**Proposition 4** Employment at each date $t$ after the steady state has been reached is independent of $\beta$.

While the degree of rent–sharing, $\beta$, does not affect steady state employment, it follows from (21) that the latter is inversely related to the parameter $\alpha$ in the wage equation (12). In equation (14) of our example, the parameter $\alpha = (2 - r)/2$ in turn depends negatively on the union’s bargaining power $r$. Thus, at least within the framework of our wage bargaining example, steady state employment increases with union bargaining power.

### 6 Conclusions

Modern industrialized countries are characterized by rapid technical progress accompanied with substantial increases in real wages. We have shown that, in a dynamic monopolistic industry, the firm optimally invests each period in productivity enhancing innovations to counterbalance increasing wages. Our analysis presents a dynamic cost–push argument of productivity growth. This argument is in the same spirit as Kleinknecht (1998) who emphasizes that labor market rigidities may have positive dynamic efficiency effects because they create stronger incentives to increase labor productivity. In our model higher current labor costs create stronger incentives for process innovations. Unless the wage level is too high, the monopolist’s rate of productivity growth monotonically approaches the growth rate of wages and eventually the firm reaches a steady state where its unit cost of labor remains constant.
over time. Interestingly, long–run productivity growth only depends on the growth rate of wages and is thus independent of the initial level of wages. While the monopolist’s unit labor cost as well as its output in the long–run depend only on the growth rate of wages, the long–run levels of labor productivity and employment depend on the level of wages.

We have also analyzed the case where the rents stemming from productivity enhancing innovations are shared between the monopolist and its workers. Surprisingly, also in this case the long–run productivity growth only depends on the growth rate of wages and is independent of the share of profits over which the workers have claims (measured e.g. by the union’s power). Hence, unionization does not influence long–run productivity growth, despite the fact that it depresses the short–run incentives for innovation. The union’s bargaining power, however, determines the monopolist’s long–run unit cost of labor and thus its output level: In the steady state its unit cost of labor is the higher and its output is the lower, the higher the union’s power is. Nonetheless, long–run employment may actually be positively related to the union’s bargaining power. This is so because unionization has a negative impact on labor productivity and reduces the sensitivity of the firm–specific wage towards changes in the market wage rate.

Our model could be extended to consider dynamic imperfectly competitive markets. In an intertemporal model where at each date firms compete in prices or quantities it will be interesting to analyze how strategic interactions between the firms affect productivity growth in the short–run and in the long–run. Stimulated by the work of Schumpeter (1947), a large part of the literature on R&D relates the pace of innovative activity to market structure. An imperfect competition version of our model could combine this approach with our cost-push argument.
7 Appendix

This appendix contains the proof of Lemma 1 and Propositions 1 and 2.

**Proof of Lemma 1:** When the optimal output \(x^*(c)\) is positive, the first order condition is \(R'(x^*) - C'(x^*) = c\). Therefore

\[-\Pi'(c) c = x^*(c) c = [R'(x^*(c)) - C'(x^*(c))]x^*(c). \tag{22}\]

Since \(x^*(c) < \infty\), one has \(-\Pi'(0) = x^*(0) < \infty\). For \(c \to \infty\) one has \(x^*(c) \to 0\). Therefore, the second part of Assumption 1 and (22) imply that \(-\Pi'(c) c \to 0\) for \(c \to \infty\).

For \(c \) sufficiently small \(x^*(c) > 0\) and so \(-\Pi'(c) c > 0\). Since \(-\Pi'(c) c \to 0\) for \(c \to 0\) and for \(c \to \infty\), the function \(-\Pi'(c) c\) attains a maximum for some \(0 < c^m < \infty\). Since \(x^*(c)\) is strictly decreasing, Assumption 1 and (22) imply that \(c^m\) is unique and that \(-\Pi'(c) c\) is increasing in \(c\) for \(c < c^m\) and decreasing in \(c\) for \(c > c^m\) as long as \(\Pi(c) > 0\). Q.E.D.

**Proof of Proposition 1:** Since we have already shown that \(q^*(c) = \gamma\) implies \(c \in \{c_{II}, \tilde{c}_{III}\}\), it remains to show that the equation \(q^*(c) = \gamma\) has two solutions \(\tilde{c}_{II}\) and \(\tilde{c}_{III}\). Notice that combining (8) and (9) yields

\[V'(c) c = \Pi'(c) c - K'(q^*(c))(1 + q^*(c)) \tag{23}\]

for all \(c\). Since \(K^m > 0\), this equation has a unique solution \(q^*(c)\) for all \(c\). Moreover, by continuity of \(V'()\) and \(\Pi'()\) the optimal policy \(q^*(\cdot)\) is continuous.

We first show that \(q^*(c^m) > \gamma\). Note that by (23) we have \(-V'(c)c \geq -\Pi'(c)c\) for all \(c\). Suppose \(q^*(c^m) \leq \gamma\). Then this together with (8) implies

\[-\delta \Pi' \left( c^m \frac{1 + \gamma}{1 + q^*(c^m)} \right) \frac{(1 + \gamma)c^m}{1 + q^*(c^m)} \leq 0 \tag{24} \]

\[-\delta V' \left( c^m \frac{1 + \gamma}{1 + q^*(c^m)} \right) \frac{(1 + \gamma)c^m}{1 + q^*(c^m)} = \]

\[ (1 + q^*(c^m))K'(q^*(c^m)) \leq (1 + \gamma)K'(\gamma). \]

As \(q^*(c^m)\) increases from 0 to \(\gamma\), the term \([1 + \gamma)c^m]/[1 + q^*(c^m)]\) decreases from \(c^m(1 + \gamma)\) to \(c^m\). By Lemma 1, \(-\Pi'(c)c\) is decreasing in \(c\) over the interval \([c^m, c^m(1 + \gamma)]\). Accordingly, the first term (24) is increasing in \(q^*(c^m)\) over the interval \([0, \gamma]\) and so it follows from \(q^*(c^m) \leq \gamma\) that

\[-\Pi' \left( c^m \frac{1 + \gamma}{1 + q^*(c^m)} \right) \frac{(1 + \gamma)c^m}{1 + q^*(c^m)} \geq -\Pi' (c^m(1 + \gamma))(1 + \gamma)c^m. \tag{25}\]
This in combination with (24) yields

\[-\delta \Pi'(c^m(1 + \gamma))(1 + \gamma)c^m \leq (1 + \gamma)K'(\gamma),\]

a contradiction to condition (4). This proves that $q^*(c^m) > \gamma$.

Next we show that $q^*(\bar{c}) < \gamma$ for some $\bar{c} > c^m$. Suppose the contrary. Then $q^*(c) \geq \gamma$ and so by (7)

\[V(c) \leq \Pi(c) - K(\gamma) + \delta V\left(c\frac{1 + \gamma}{1 + q^*(c)}\right)\]

for all $c$ sufficiently large. Note that $\Pi(c) \to 0$ as $c \to \infty$. Moreover, we have $q^*(c) < \infty$ for all $c$, because $K(\infty) = \infty$. Therefore, (26) implies

\[\lim_{c \to \infty} V(c) \leq -\frac{K(\gamma)}{1 - \delta} < 0,\]

a contradiction. This proves that $q^*(\bar{c}) < \gamma$ for some $\bar{c} > c^m$.

Since $q^*(0) = 0$, $q^*(c^m) > \gamma$ and $q^*(\bar{c}) < \gamma$ for some $\bar{c} > c^m$, continuity of $q^*(\cdot)$ implies that there is a $\tilde{c}_{II} \in (0, c^m)$ such that $q^*(\tilde{c}_{II}) = \gamma$ and a $\tilde{c}_{III} > c^m$ such that $q^*(\tilde{c}_{III}) = \gamma$.

**Proof of Proposition 2:** We first prove that the optimal policy satisfies $q^*(c) > \gamma$ for all $c \in (\tilde{c}_{II}, \tilde{c}_{III})$. Note that the sign of $q^*(c) - \gamma$ cannot change over some interval $[c', c'']$ if this interval does not contain a steady state. Otherwise, one would obtain a contradiction to the fact that the optimal trajectory $c^*_t$ is monotone. By Proposition 1, therefore, the sign of $q^*(c) - \gamma$ is constant over the interval $c \in (\tilde{c}_{II}, \tilde{c}_{III})$. The proof of Proposition 1 shows that $q^*(c^m) > \gamma$. Since $c^m \in (\tilde{c}_{II}, \tilde{c}_{III})$, this proves that $q^*(c) > \gamma$ for all $c \in (\tilde{c}_{II}, \tilde{c}_{III})$.

We now prove that for all $c \notin [\tilde{c}_{II}, \tilde{c}_{III}]$ the optimal policy satisfies $q^*(c) < \gamma$. By argument above, the sign of $q^*(c) - \gamma$ cannot change over the interval $c \in [0, \tilde{c}_{II})$ or the interval $(\tilde{c}_{III}, \infty)$. Therefore it is sufficient to show that each interval contains a $c'$ such that $q^*(c') < \gamma$. Clearly, this is the case for the interval $[0, \tilde{c}_{II})$ because $q^*(0) = 0$.

Now consider the interval $(\tilde{c}_{III}, \infty)$. The proof of Proposition 1 shows that $q^*(\bar{c}) < \gamma$ for some $\bar{c} > c^m$. Since we have shown that $q^*(c) > \gamma$ for all $c \in (\tilde{c}_{II}, \tilde{c}_{III})$, it must be the case that $\bar{c} > \tilde{c}_{III}$. Therefore also the interval $(\tilde{c}_{III}, \infty)$ contains a $c'$ such that $q^*(c') < \gamma$.

Q.E.D.
8 References


