Preemption and Rent Dissipation under Price Competition

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Abstract

We study a simple duopoly model of preemption with multiple investments and instantaneous price competition on a market of finite size driven by stochastic taste shocks. Different patterns of equilibria may arise, depending on the importance of the real option effect. If the average growth rate of the market is close to the risk-free rate, or if the volatility of demand shocks is high, no dissipation of rents occurs in equilibrium, despite instantaneous price competition. If these conditions do not hold, the equilibrium investment timing is suboptimal, and the firms' long-run capacities may depend on the initial market conditions. Our conclusions contrast sharply with standard rent dissipation results.

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1. Introduction

Since the early work of Tullock (1967) and Posner (1975), the idea that rent-seeking behavior by firms leads to dissipation of monopolistic rents has become a recurrent theme in the industrial organization literature. Posner's argument is that firms compete for monopoly power, and that monopoly profits are offset by the total expenditures made by firms to win this contest, thereby adding to the usual deadweight loss. By modelling the contest for monopoly power as a timing game, the game-theoretic literature on preemption in oligopoly has provided a useful framework to assess this argument.[1] The basic intuition is well captured by Fudenberg and Tirole's (1985) analysis of new technology adoption in a duopoly. In their model, two ex-ante identical firms can invest in a cost reducing technology. Early adoption by one firm delays or prevents adoption by the other. If information lags are negligible, the investment game always has a subgame-perfect equilibrium in which any first-mover advantage is completely dissipated by preemptive adoption. The intuition is akin to static Bertrand competition: if one firm enjoyed a strict first-mover advantage in equilibrium, its rival could strictly increase its equilibrium payoff by preempting its investment by an infinitesimal amount of time.[2]

Most analyses of the preemption phenomenon have focused on environments in which each firm can make at most one indivisible investment. This seems to be an important restriction, since it is a priori reasonable to believe that rent dissipation is alleviated in the presence of long-term relationships between firms. However, as shown by Gilbert and Harris (1984), the full rent dissipation outcome still obtains at equilibrium when firms can accumulate capacity by building new production plants. In a preemption equilibrium, firms threaten to invest whenever a new plant is profitable, so that each new plant built by any firm earns zero profit. Provided marginal revenue is everywhere positive, this rule can be credibly followed by any firm since it depends only on the total capacity installed so far in the industry, and not on a firm's own existing capacity. Thus long-term competition per se does not rule out full rent dissipation on new investments.

The objective of this paper is to assess the robustness of this result. To do so, we consider

[1] Seminal contributions along these lines include Eaton and Lipsey (1979), Gilbert and Newbery (1982), and Fudenberg and Tirole (1985, 1987), among others.

[2] Whether this is the unique equilibrium depends on the specification of payoffs. Basically, if innovation deters imitation for a long period, the preemption equilibrium is the unique subgame-perfect equilibrium. By contrast, if adoption by a firm triggers a fast response by its rival, there exists a continuum of other equilibria. In particular, the optimal joint-adoption date is sustainable as a subgame-perfect equilibrium.

the following stylized model of dynamic Bertrand-Edgeworth competition. There are two firms. Investment by each firm takes the form of irreversible acquisition of indivisible units of capacity at a constant fixed cost. Market demand is driven by consumers' taste for the output. Specifically, while the total size of the market is finite and fixed, the consumers' willingness to pay for any given quantity of the good fluctuates randomly, and tends to increase on average. At each instant, firms compete in prices given their existing capacity. Our main focus is on the timing of investments in a Markov perfect equilibrium of the game, for which one of the natural state variables is the current position of the demand curve.

The trade-off faced by firms when taking their investment decisions is simple (for an early analysis, see Smets, 1991). On the one hand, uncertainty and irreversibility entail an option value of delaying investment. On the other hand, the threat of preemption is likely to accelerate investment, thereby mitigating this real option effect. When one unit of capacity suffices to cover the market, full rent dissipation occurs in equilibrium: the real option value of waiting is totally offset by the threat of preemption. However, when multiple investments are required to serve all consumers, the possibility of segmenting capacity prevents the existence of permanent barriers to entry, much as in the literature on two-stage Bertrand-Edgeworth competition.[3] Indeed, even if a leader firm has already enough capacity to serve all consumers, a laggard firm without capacity can always install a smaller amount of capacity without driving industry profits to zero.

This in turn affects the incentives to accumulate capacity for preempting. Indeed, as the size of the market is finite, the type of investment rule considered by Gilbert and Harris—invest as soon as a new plant can earn profits—cannot be indefinitely applied by an incumbent, as its marginal revenue from new investments drops to zero after a finite number of investments. (For the price inelastic demand function that we will consider, this happens as soon as the incumbent has enough capacity to serve all consumers in the market.) In the same circumstances, however, marginal revenue from new investments is not necessarily zero for a laggard firm. Thus, contrary to what happen if the market size is infinite and the marginal revenue generated by any investment becomes eventually positive, as in Gilbert and Harris (1984), firms with different capacities have in general different incentives to invest. The outcome of the game depends then on whether the laggard or the leader has more

[3]See, e.g., Gelman and Salop (1983), or Allen, Deneckere, Faith and Kovenock (2000). Note however that, although some competition effects in our model are the same as those at work in a standard two-stage game framework, a key feature of our approach is that we explicitly rule out commitment to capacity.

incentives to preempt its rival.

To illustrate this simple intuition, we solve the model analytically under two additional assumptions. First, we suppose that demand is price inelastic at any date. While restrictive, this assumption has the advantage of keeping the model as close as possible to the unit market case: with constant investment cost per unit of capacity, a monopolist would like to perform all his investments at once. Second, to cut down on the complexity of the model, we assume that two units of capacity are needed to serve all consumers. However, we do not require investments to take place sequentially. In particular, each firm has the option of covering the whole market at any date.

Under these assumptions, our results are the following. If the average growth rate of the market is large enough relative to the risk-free rate, or if the volatility of demand changes is high enough, there is a unique Markov perfect equilibrium outcome, namely that firms simultaneously invest in one unit of capacity at the random time that maximizes industry profits. Since the duopoly exactly duplicates the monopoly outcome, no rent dissipation occurs in equilibrium. The interpretation is that it is never worthwhile for a firm to invest in two units of capacity, because the stochastic delay induced for the entry of the second firm is not long enough on average to allow the first firm to recover the fixed cost of a second unit of capacity. While this intuition is close to the cooperative equilibria found by Fudenberg and Tirole (1985), an important difference with their results is that, under the above assumptions, the efficient joint-entry outcome is the *unique* equilibrium outcome in our model. A striking feature of the equilibrium for this constellation of parameters is that despite great indivisibilities—as two units of capacity are enough to serve all consumers—it is not worthwhile for the incumbent to erect temporary barriers to entry by accumulating capacity.

If on the other hand, the risk-free rate is high relative to the average growth rate of the market, and if the volatility of demand is low enough, then multiple Markov perfect equilibria exist. However, if the initial state of demand is low enough, all these equilibria generate the same outcome: firms will simultaneously invest in one unit of capacity at the first date at which it becomes worthwhile to preempt one's rival with two units of capacity. This occurs earlier than the optimal monopolistic investment date, implying that some monopolistic rents are dissipated in equilibrium. Out of equilibrium, or for a high enough initial state of demand, different patterns of investment may be compatible with equilibrium. In particular, there exist equilibria in which firms try to preempt each other with two units of capacity, resulting in over-accumulation of capital compared to the total size of the market.

The paper is organized as follows. Section 2 describes the model. In Section 3, we derive two useful benchmarks: first, the optimal investment policy of a monopolist, and second, the duopoly outcome when the market can be covered with a single unit of capacity. The Markov perfect equilibria of the multiple investments model are described in Section 4 in the special case where two units are needed to cover the market. Section 5 concludes by discussing the robustness of our results. All proofs are in the appendices.

2. The Model

Consider the following strategic investment problem:

Market Demand. Time is continuous, and indexed by $t \geq 0$. At any date t, the demand side of the market is described by a price inelastic unit demand:

$$D_t(P) = \begin{cases} 0 & \text{if} \quad P > P_t \\ [0,1] & \text{if} \quad P = P_t \\ 1 & \text{if} \quad P < P_t \end{cases}, \tag{1}$$

where the total willingness to pay P_t for the commodity produced by the firms is subject to aggregate demand shocks described by a geometric Brownian motion:

$$\mathrm{d}P_t = \alpha P_t \,\mathrm{d}t + \sigma P_t \,\mathrm{d}Z_t. \tag{2}$$

Here P_0, α, and σ are positive constants and $\{Z_t\}_{t\geq 0}$ is a standard Brownian motion.

Firms. There are two firms, labelled by $i, j \in \{1, 2\}$. Both are risk-neutral and discount future revenues and costs at a constant risk-free rate $r > \alpha$.[4] Variable costs are normalized to zero. Investment is irreversible and takes place in a lumpy way. Each unit of capacity allows a firm to cover at most a fraction $1/N$ of the market, for some positive integer N. The cost of each unit of capacity is constant and equal to $I > 0$. Capital has no resale value and does not depreciate.

Competition. Within each instant $[t, t + \mathrm{d}t)$, the timing of the game is the following: (i) First, each firm chooses how many units of capacity to invest in, given the realization of P_t

[4] If $\alpha \geq r$, delaying investment is always better for the firms, and no equilibrium exists.

and the existing levels of capacity; (ii) Next, each firm quotes a price given its new level of capacity and that of its rival; (iii) Last, consumers choose from which firm to purchase, and production and transfers take place.

There are several points of departure between this model and the environment considered by Gilbert and Harris (1984). First, we allow for stochastic fluctuations of demand. This is because we want to assess the impact of demand uncertainty on firms' investment decisions, and how real options may act as a counter force to the preemption motive in strategic contexts. Second, instantaneous competition takes place in prices, and not in quantities.[5] The purpose of this assumption is to investigate whether making market competition more aggressive increases the incentives for preemption, and to contrast the no-dissipation result to the outcome of the game when capacity cannot be segmented, which entails full rent dissipation. The third and crucial difference between our model and Gilbert and Harris' is that the size of the market is finite and constant through time. That is, aggregate demand shocks are of the kind considered in the real option literature (see, e.g., Dixit and Pindyck, 1994): growth of demand arises only through taste shocks that affect the willingness to pay of the consumers, and not, for instance, through mere demographic pressure. It is this feature of the model that rules out full rent dissipation when more than one investment is required to cover the market. Last, we focus on a price inelastic demand curve rather than a downward sloping one. Admittedly, by doing so, we abstract from one important source of preemption, which is to invest early in order to extract rents from consumers with a high willingness to pay for the output. However, this does not eliminate a second source of preemption, which consists in investing early in order to modify the ex-post distribution of profits across firms, thus generating temporary barriers to entry. By focusing on this second source of preemption, we can more conveniently analyze the firms' incentive to build capacity over time, an important objective of this paper.

Markov Strategies. We focus on Markov perfect equilibria (MPE), in which firms' investment and pricing decisions depend only on the current value of the consumers' reserve price p and the firms' capital stock measured in units of capacity, (n^i, n^j). This rules out any kind of implicit collusion between the firms when making their pricing decisions: in any period, firms play an equilibrium of the static Bertrand-Edgeworth pricing game given their current capacities. This assumption is consistent with the main purpose of our paper, which is to

[5]See Boyer, Lasserre, Mariotti and Moreaux (2002) for a related preemption model with instantaneous quantity competition.

study the dynamics of capital accumulation.

Our definition of Markov strategies and payoffs generalizes Fudenberg and Tirole's (1985) concept of preemption strategies to a stochastic environment with multiple investments. To obtain an adequate continuous time representation of limits of discrete time mixed strategy equilibria, we define a strategy for firm i as an *intensity function* s^i, with the interpretation that $s^i_\nu(n^i, n^j, p) \in [0, 1]$ is the intensity with which i invests in ν additional units of capacity given the capital stocks (n^i, n^j) and the state of demand p. The details of the construction are gathered in Appendix A. We denote by $U^i_s(n^i, n^j, p)$ firm i's expected discounted value in state (n^i, n^j, p) given the strategy profile $s = (s^1, s^2)$.

3. Benchmarks

As benchmarks, we study two polar situations. First, we analyze the optimal investment policy of a monopolist. Next, we characterize the MPEs of the investment game when the market can be covered with a single unit of capacity.

3.1. Optimal Investment Policy in a Monopoly

Since demand is price inelastic and variable costs are nil, a risk-neutral monopolist will optimally invest in N units of capacity simultaneously. Once in place, it extracts all the consumer surplus by charging a price P_t at any date t. The expected discounted value of a monopolist when the current state of demand is p is thus given by:

$$V^M_N(p) = \sup_\tau \left\{ \mathrm{E}_p \left[\int_\tau^\infty \mathrm{e}^{-rt} P_t \, \mathrm{d}t - \mathrm{e}^{-r\tau} NI \right] \right\}, \tag{3}$$

where the supremum is taken with respect to all stopping times adapted to $\{P_t\}_{t \geq 0}$, and $\mathrm{E}_p[\cdot]$ is the expectation operator conditional on $P_0 = p$. An optimal stopping time for (3) is to invest when the state of demand reaches the investment trigger:

$$p^M_N = \frac{\beta}{\beta - 1} (r - \alpha) NI, \tag{4}$$

where:

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}\,. \tag{5}$$

(See, e.g., Dixit and Pindyck, 1994, Chapter 5). Since $\beta > 1$, the investment trigger p^M_N is above the Marshallian trigger $p^m_N = (r - \alpha) NI$ at which the value of the firm is equal to zero. Note also that since the monopolist can extract all the social surplus, his investment strategy coincides with that of a benevolent social planner, provided that the risk-free rate r is equal to the social discount rate.

3.2. The Investment Game in the Case $N = 1$

If $N = 1$, Bertrand competition acts as a permanent barrier to entry. Indeed, if an incumbent firm already holds one capacity unit, price competition following the entry by the challenger would drive the industry profits to zero and therefore prevent it from recovering its investment cost I. Similarly, if both firms enter simultaneously, subsequent profits are zero. It is also obvious that no firm wants to acquire more than one unit of capacity in a MPE. In other terms, in any MPE s,

$$\begin{aligned} U_s^i(0,1,p) &= 0, \\ U_s^i(1,0,p) &= \mathrm{E}_p\left[\int_0^\infty \mathrm{e}^{-rt} P_t \,\mathrm{d}t\right] = \frac{p}{r-\alpha}, \qquad (6) \\ U_s^i(1,1,p) &= 0, \end{aligned}$$

for any firm i and state of demand p. We now determine the firms' equilibrium strategies before any firm has invested yet, i.e., in states of the form $(0,0,p)$. First, if $p < p_1^m$, it is clearly a strictly dominant strategy for both firms not to invest. Next, if $p > p_1^M$, both firms have no incentive to delay investment, so that any subgame starting at $(0,0,p)$ is essentially static. As a result, one can construct many continuation MPEs on (p_1^M, ∞). An obvious candidate is that firm i invests in one unit with intensity one, and firm j optimally reacts by not investing. Alternatively, using equation (17) in Appendix A, it is easy to check that if firm j invests with intensity $(p - p_1^m)/p$, all choices of firm i yield a zero payoff. By symmetry, it follows that there exists a mixed strategy continuation equilibrium yielding both firms a zero payoff. (In fact, this turns out to be the unique MPE strategy profile such that both firms are active on (p_1^m, ∞).) More generally, we have the following proposition.

Proposition 1 *If $P_0 \in (0, p_1^M)$ then, in any MPE of the investment game, the expected payoff of each firm at date zero is equal to zero.*

The intuition is straightforward. Suppose that $P_0 < p_1^m$. The maximal payoff any firm can achieve is obtained by investing at the first time τ_1^M where the state of demand reaches the monopolistic trigger p_1^M. But if firm i anticipates that j will invest at τ_1^M, it is better off avoiding being preempted by j by investing at the first date where the state of demand reaches $p_1^M - \varepsilon$ for some small $\varepsilon > 0$. Reasoning backwards, at any $p \in (p_1^m, p_1^M)$, each firm wants to preempt to avoid being preempted later on. At the Marshallian trigger p_1^m, the rents of the leader and the follower are both equal to zero, so that both firms are indifferent between preempting or staying out of the market. Hence, on any MPE path one firm (possibly chosen randomly) will invest at the first time τ_1^m where the state of demand reaches the Marshallian trigger p_1^m, and the other firm never invests afterwards.

Proposition 1 illustrates in a spectacular way the rent dissipation phenomenon described by Posner (1975) and Fudenberg and Tirole (1987), among others. Here, the threat of Bertrand competition implies that in any MPE, the first firm to invest enjoys a permanent monopoly position. In equilibrium, the real option effect is completely offset by the threat of preemption, so that the monopoly rents—hence the social surplus, since demand is price inelastic—are totally dissipated at the investment date.

4. Multiple Investments: The Case $N = 2$

In this section, we contrast the previous rent dissipation result with the outcome of the game if multiple units of capacity are needed to serve all consumers. For simplicity, we focus on the case where the market can be covered with two units of capacity.

4.1. Short-Term Competition

If $N = 2$, a firm never wants to hold more than two capacity units in equilibrium, so that we may assume that $n^i, n^j \in \{0, 1, 2\}$. Let $\pi(n^i, n^j, p)$ denote firm i's instantaneous expected profits in a Nash equilibrium of the instantaneous Bertrand-Edgeworth game when the current state of the game is (n^i, n^j, p). We have:

Lemma 1 *For each* $(i, n^i, p) \in \{1, 2\} \times \{0, 1, 2\} \times [0, \infty)$:

(i) $\pi(n^i, 0, p) = n^i p/2$;

(ii) $\pi(1, 1, p) = p/2$;

(iii) $\pi(2, 2, p) = 0$;

(iv) $\pi(2, 1, p) = p/2$ *and* $\pi(1, 2, p) = p/4$.

The proof of items (i), (ii), (iii) is immediate. In case (iv), i.e., when one firm holds one unit of capacity while the other holds two units, the instantaneous Bertrand-Edgeworth game has no pure strategy equilibrium. In Appendix B, we show that there exists a unique mixed strategy equilibrium with the given payoffs (see also Cripps and Ireland, 1988, for a similar result). In this equilibrium, the firm with one unit of capacity prices more aggressively than its rival. As in Ghemawat (1997), one can interpret this phenomenon as a "stochastic price umbrella" held by the smallest firm.

As is obvious from Lemma 1, an important consequence of the perfect elasticity of demand is that if a firm already holds one unit of capacity while its rival has not invested yet, the rentability of the existing unit, hence the instantaneous profits of the incumbent, do not change following the investment of one unit by its challenger. Similarly, if a firm's capacity is of one unit, the expected profit of its rival are the same whether its own capacity is of one or two units. This property greatly simplifies the analysis of the firms' long-term interactions, to which we now turn.

4.2. Long-Term Competition: Preliminary Results

To characterize the MPEs of the investment game, we proceed by backward induction on the firms' capacity profiles (n^1, n^2). Four cases must be distinguished.

Case 1: $(n^1, n^2) \in \{(1,1),(1,2),(2,1),(2,2)\}$. It is clear from Lemma 1 that when both firms already hold at least one unit of capacity, acquiring additional units of capacity is a strictly dominated strategy from each firm's perspective. Since capital does not depreciate, the investment game is over as soon as one of these positions is reached. Thus, in any MPE s, firm i's continuation payoff is then given by:

$$U_s(n^i, n^j, p) = \mathrm{E}_p\left[\int_0^\infty \mathrm{e}^{-rt}\pi(n^i, n^j, P_t)\,\mathrm{d}t\right] = \frac{\pi(n^i, n^j, p)}{r-\alpha}. \tag{7}$$

(From now on, we drop the firm index in the expressions of equilibrium value functions when the MPE outcome is unique.)

Case 2: $(n^1, n^2) \in \{(0,2),(2,0)\}$. Suppose now that firm i already holds two units of capacity, while its rival j has not invested yet. It is clear that making further investments has no value for the incumbent i. Moreover, it is not in the interest of the challenger j to invest in more than one unit, since doing so would drive industry profits to zero. The challenger's continuation payoff is thus given by:

$$U_s(0,2,p) = \sup_{\tau} \left\{ \mathrm{E}_p \left[\int_{\tau}^{\infty} \mathrm{e}^{-rt} \pi(1,2,P_t)\,\mathrm{d}t - \mathrm{e}^{-r\tau} I \right] \right\}. \tag{8}$$

Replacing $\pi(1,2,P_t)$ by its value given by Lemma 1, and proceeding as for (3), we get that an optimal stopping time for (8) is to invest when the state of demand reaches the investment trigger:

$$p^C = 4\,\frac{\beta}{\beta-1}\,(r-\alpha)\,I. \tag{9}$$

Let τ^C denote the corresponding stopping time. Then:

$$U_s(0,2,p) = \mathrm{E}_p \left[\int_{\tau^C}^{\infty} \mathrm{e}^{-rt} \pi(1,2,P_t)\,\mathrm{d}t - \mathrm{e}^{-r\tau^C} I \right] = \left(\frac{p}{p^C} \right)^{\beta} \left[\frac{\pi(1,2,p^C)}{r-\alpha} - I \right], \tag{10}$$

for any $p \in (0, p^C)$. For any such p, the value of the incumbent is given by:

$$\begin{aligned} U_s(2,0,p) &= \mathrm{E}_p \left[\int_0^{\tau^C} \mathrm{e}^{-rt} \pi(2,0,P_t)\,\mathrm{d}t + \int_{\tau^C}^{\infty} \mathrm{e}^{-rt} \pi(2,1,P_t)\,\mathrm{d}t \right] \\ &= \frac{\pi(2,0,p)}{r-\alpha} + \left(\frac{p}{p^C} \right)^{\beta} \frac{\pi(2,1,p^C) - \pi(2,0,p^C)}{r-\alpha}. \end{aligned} \tag{11}$$

In contrast with Cases 1 and 2, where the MPE outcome of the game is uniquely determined, the remaining cases are more problematic to handle.

Case 3: $(n^1, n^2) \in \{(0,1),(1,0)\}$. Suppose now that firm i has already installed one unit of capacity, while its rival j has not invested yet. By investing in one additional unit, the incumbent can temporarily increase its profits from $\pi(1,0,P_t)$ to $\pi(2,0,P_t)$ and delay the challenger's entry until τ^C. However, it is not a priori clear whether the incumbent can gain from doing so. The only case in which the conclusion is straightforward is when $p \geq p^C$. Indeed, from Case 2, it is then a strictly dominant strategy for the challenger to invest immediately in one unit. The incumbent has thus no incentive to make further investments.

Case 4: $(n^1, n^2) = (0,0)$. If both firms have not invested yet, they may enter with one or two capacity units. As in Case 3, the only situation in which the MPE strategies are straightforward to determine is when the state of demand is $p \geq p^C$. Indeed, the same argument implies that investing in two units is a strictly dominated strategy, and both firms enter simultaneously with one unit.

It turns out that the equilibrium dynamics in Cases 3 and 4 depends crucially on whether $r < 2\alpha + \sigma^2$. To simplify the exposition, we shall first consider the case where this relation holds. We then tackle the analysis in the opposite case.

4.3. *Optimal MPE Trajectories in the Case $r < 2\alpha + \sigma^2$*

The purpose of this section is to show that, in contrast to the benchmark case of Bertrand competition with $N = 1$, the investment choices in a MPE may now match the socially optimal trajectory for certain values of the parameters r, σ and α. The following lemma plays a crucial role in establishing this result.

Lemma 2 *If $r < 2\alpha + \sigma^2$ then, in any MPE,*

$$U_s(2, 0, \cdot) - I < U_s(1, 1, \cdot). \tag{12}$$

Under the assumption of Lemma 2, the incumbent in Case 3 prefers to be preempted by one unit of capacity rather than preempting itself with one unit. Therefore, the only possible rationale for the incumbent to preempt the challenger would be to prevent it from investing in two units at once. However, the challenger never benefits from doing so, since $\pi(2, 1, \cdot) = \pi(1, 1, \cdot)$. Thus, under the assumption of Lemma 2, the preemption incentive of an incumbent is always negative. Since the challenger is not threatened by new investments from its rival, its best response is to invest in one unit at the date τ_2^M where demand reaches the optimal investment trigger p_2^M.

Of course, the assumption $r < 2\alpha + \sigma^2$ is crucial for this result. In the limit deterministic case, $\sigma = 0$, this means that the growth rate α of the consumers' valuation must not be too small compared to the discount rate r. This may be interpreted as follows. When $\sigma = 0$, the optimal investment policy of a challenger is to invest when $p^C = 4rI$ is reached, at date $T^C = \ln(4rI/p_0)/\alpha$, if we suppose that the incumbent holds two units of capacity at time zero. Clearly, the larger α, the shorter the time interval where the incumbent can enjoy full monopoly profits, and thus the weaker the incentive to preempt with two units of capacity. A similar intuition pertains to the stochastic case, $\sigma > 0$. Since $\alpha > 0$, $r < 2\alpha + \sigma^2$ is now equivalent to $\beta \in (1, 2)$. This implies that $p^C = 4(r + \sigma^2\beta/2)I$ is relatively small, and that the stochastic discount factor $\mathrm{E}_t[\mathrm{e}^{-r(\tau^C - t)}] = (p_t/p^C)^\beta$ between any date t and τ^C is relatively large. Again, the reason why it is not worthwhile for a firm to invest in two units of capacity is that doing so does not delay sufficiently the entry of the challenger. Moreover, for a given expected growth rate of demand α, an increase in the volatility σ reduces the preemption incentive. This is a standard real option effect: ceteris paribus, an increase in uncertainty makes firms more willing to delay their investment.

Consider now what happens in Case 4 under the assumption of Lemma 2. From Lemma 1, $\pi(2,1,\cdot) = \pi(1,1,\cdot)$ and $\pi(1,2,\cdot) \geq \pi(2,2,\cdot)$. Therefore,

$$\begin{aligned} U_s(1,1,\cdot) - I &> U_s(2,1,\cdot) - 2I, \\ U_s(1,2,\cdot) - I &> U_s(2,2,\cdot) - 2I. \end{aligned} \tag{13}$$

Hence, if with probability one both firms invest simultaneously in state $(0,0,p)$, each firm is better off investing in one rather than two units. Suppose now that one firm preempts its rival with one unit. Lemma 2 implies that it will never reinvest on any MPE path. Taking advantage from $\pi(1,0,\cdot) = \pi(1,1,\cdot)$, it follows immediately that $U_s(1,0,\cdot) = U_s(1,1,\cdot)$. We may therefore rewrite (12) as:

$$U_s(2,0,\cdot) - 2I < U_s(1,0,\cdot) - I. \tag{14}$$

Intuitively, if $r < 2\alpha + \sigma^2$ a firm prefers to preempt its rival with one rather than two units. Together with (13), this implies that given the unique continuation MPE outcome previously constructed in Cases 1-3, investing in two units of capacity with positive intensity is a strictly dominated strategy in Case 4. It follows immediately that on any MPE path, each firm will invest at most in one unit of capacity. To do so, they are both better off waiting until the state of demand reaches the optimal trigger p_2^M. Hence, we have proved the following result:

Proposition 2 *If $r < 2\alpha + \sigma^2$, there exists an unique MPE outcome. On any MPE path both firms simultaneously invest in one unit of capacity at the optimal investment date τ_2^M and never reinvest thereafter.*

Under the assumption of Proposition 2, the unique MPE outcome exactly duplicates the socially efficient outcome, and no dissipation of rents arises in equilibrium: everything happens as if the two firms were two divisions of a monopolistic firm. This is in sharp contrast with the benchmark case, where each firm could cover the whole market with a single unit of capacity. The main differences are as follows: (i) Firms may now segment their capacity to serve only a fraction of consumers, so that, even if they cannot implicitly collude on prices, there is no permanent barrier to entry; (ii) Covering the whole market may not delay sufficiently the challenger's entry.

If we adopt a reasonable parametrization of $r = 4\%$, the assumption of Proposition 2 holds for any $\alpha \in (0,r)$ as soon as σ is larger than 20%, which is a good approximation of the average standard deviation of the expected rate of return on the New York Stock Exchange.

Of course, the latter is based on a diversified portfolio of assets. However, one may argue that it represents a reasonable lower bound for the volatility on the rate of return on the introduction of a new product, or an R&D venture, two typical applications of our model. Overall, the conclusion of Proposition 2 is likely to hold for a broad set of realistic parameter values.

4.4. MPE Trajectories in the Case $r > 2\alpha + \sigma^2$

To assess the robustness of Proposition 2, it is nevertheless instructive to analyze the firm's MPE investment strategies if $r > 2\alpha + \sigma^2$. The following lemma compares the incumbent's and the challenger's preemption incentives in Case 3.

Lemma 3 *In Case 3, the preemption incentive of the incumbent is always less than the preemption incentive of the challenger:*

$$U_s(2, 0, \cdot) - U_s(1, 1, \cdot) < U_s(1, 1, \cdot) - U_s(0, 2, \cdot). \tag{15}$$

Moreover, if $r > 2\alpha + \sigma^2$, there exist two numbers $p^- \in (0, p_2^M)$ and $p^+ \in (p_2^M, p^C)$ such that for each $p \in [0, \infty)$, $U_s(2, 0, p) - I > U_s(1, 1, p)$ if and only if $p \in (p^-, p^+)$.

The intuition is straightforward. Suppose that the present state of the game is $(1, 0, p)$. From the incumbent's perspective, the gain of preempting relative to being preempted is to reap the additional profits $\pi(2, 0, P_t) - \pi(1, 1, P_t) = P_t/2$ until τ^C. By contrast, the relative gain of preemption for the challenger is $\pi(1, 1, P_t) = P_t/2$ until τ^C, to which must be added the relative gain of a better competitive position in the long-run, $\pi(1, 1, P_t) - \pi(1, 2, P_t) = P_t/4$, on the whole time interval $[\tau^C, \infty)$. Hence in Case 3, the preemption incentive of the challenger is always larger than the incumbent's. However, this does not mean that the challenger will always win the investment race. In particular, if $r > 2\alpha + \sigma^2$, there is a whole interval (p^-, p^+) of current states of demand on which both firms have a positive preemption incentive.

To characterize the MPE outcomes in Case 3, it is helpful to determine each firm's stand-alone optimal investment date, i.e., the random time at which it would invest if it were not threatened by its rival's entry (see Katz and Shapiro, 1987). For the challenger, this is obviously equal to τ_2^M. For the leader, it is the solution of:

$$\sup_{\tau} \left\{ \mathrm{E}_p \left[\int_0^{\tau} \mathrm{e}^{-rt} \pi(1, 0, P_t) \, \mathrm{d}t + \mathrm{e}^{-r\tau} [U_s(2, 0, P_\tau) - I] \right] \right\}. \tag{16}$$

Using Lemma 3, a standard computation shows that the solution to (16) is to invest if the current state of demand p lies in $[p_2^M, p^+]$. The lower bound of this interval coincides with the optimal investment strategy for a myopic incumbent which does not take into account the future entry of the challenger. The intuition is as in Leahy (1993): when computing its stand-alone date, a myopic incumbent overstates by the same amount the value of the investment option and the marginal benefit from investing, leaving the investment rule unaffected. Since demand is price inelastic, the stand-alone investment dates of the challenger and of a myopic incumbent coincide. Of course, if the state of demand is $p > p^+$, a non myopic incumbent does not want to invest since its preemption incentive is negative.

As in the benchmark case, it immediately follows that if $p \in (p_2^M, p^+)$, both the challenger and the incumbent in Case 3 have no incentive to delay investment, so that any subgame starting at $(1, 0, p)$ is essentially static. As a consequence, one may construct many continuation MPEs on (p_2^M, p^+). Next, on (p^+, ∞), there is clearly a unique MPE outcome, namely that the incumbent abstains from investing and the challenger invests immediately in one unit. The following lemma shows how to construct a MPE continuation strategy profile on $[0, p_2^M)$.

Lemma 4 *Suppose Case 3 holds and $r > 2\alpha + \sigma^2$. Then, there exists a MPE such that:*

(i) If $p \in (p^-, p_2^M)$, both firms invest in one unit with positive intensity, and the equilibrium payoffs are $U_s(0, 2, p)$ for the challenger and $U_s(1, 1, p)$ for the incumbent;

(ii) If $p \in [0, p^-)$, neither the incumbent nor the challenger invests. At $p = p^-$, the challenger invests with probability one and the incumbent abstains.

This MPE involves mixed strategies on (p^-, p_2^M), and the positive probability of simultaneous entry drives each firm's profit to its follower level, as in Fudenberg and Tirole (1985). Interestingly, since the preemption incentive of the challenger is larger than that of the incumbent, in this mixed strategy equilibrum the incumbent must invest with a greater intensity than its rival. Note finally that to avoid an openness problem at p^- in the challenger's optimization problem on $[0, p^-)$, it is necessary that the challenger enters with probability one at p^- and that the incumbent optimally abstains to invest. It follows that the challenger's equilibrium payoff exhibits a downward jump from $U_s(1, 1, p^-) - I$ at p^- to $U_s(0, 2, p)$ in a small right neighborhood of p^-. This discontinuity obviously arises because of the firms' asymmetric capacities.

To complete the construction of a MPE, we must describe the firms' equilibrium strategies in Case 4, i.e., when none has invested yet. It is intuitively clear that, for any current value $p \in (0, p^-) \cup (p^+, \infty)$ of P_t, investing in two units of capacity at once is strictly dominated by investing in only one unit of capacity. However, if $p \in (p^-, p^+)$, it is not a priori obvious whether firms will attempt to invest in one or two units of capacity. In fact, both scenarios can occur in equilibrium. The following proposition describes the first type of equilibrium.

Proposition 3 *If $r > 2\alpha + \sigma^2$, there exists a MPE such that, on the equilibrium path, both firms simultaneously invest in one unit of capacity at the first date τ_{p^-} where the stochastic process of demand reaches p^- and never reinvest thereafter.*

The MPE of Proposition 3 generalizes to the case $r > 2\alpha + \sigma^2$ the unique MPE obtained when $r < 2\alpha + \sigma^2$. If the current state of demand is $p \geq p_2^M$, the intuition is similar: each firm is ready to invest exactly in one unit if it anticipates that its rival will do the same. Things are different if $p < p_2^M$. In the case $r < 2\alpha + \sigma^2$, each firm was ready to wait until τ_2^M before investing because it anticipated that its rival would at most invest in one unit, with no impact on its own incentives. By contrast, if $r > 2\alpha + \sigma^2$, being preempted by one unit on (p^-, p_2^M) yields a continuation payoff of $U_s(0, 2, p)$ since, by Lemma 4, an incumbent firm holding one unit has then an incentive to immediately reinvest in a further unit. In other terms, it makes no difference in terms of payoffs whether one is preempted with one or two units on (p^-, p_2^M). It follows that at each $p \in (p^-, p_2^M)$, each firm wants to invest in one unit if its rival does the same. It should be noted that $\lim_{r \downarrow 2\alpha + \sigma^2} p^- = \lim_{r \downarrow 2\alpha + \sigma^2} p^+ = p_2^M$, so that as $r \downarrow 2\alpha + \sigma^2$, the above equilibrium outcome converges to the efficient MPE outcome derived in Proposition 2.

There are other equilibria, however. For instance, it is easy to construct an equilibrium such that: (i) No firm invests on $[0, p^-)$; (ii) At p^-, both firms invest in one unit with probability one; (iii) On (p^-, p_2^M), both firms invest in two units of capacity with positive intensity:

$$s_2(0, 0, p) = \frac{U_s(2, 0, p) - 2I - U_s(0, 2, p)}{U_s(2, 0, p)},$$

and abstain from investing with intensity $s_0(0, 0, p) = 1 - s_2(0, 0, p)$; (iv) On (p_2^M, ∞), both firms invest in one unit with probability one. In this latter equilibrium, the pattern of capacity accumulation depends on the initial state of the economy. If P_0 is low enough ($P_0 \in [0, p^-]$) or high enough ($P_0 \in [p_2^M, \infty)$), then each firm will acquire only one unit of capacity on the equilibrium path. If on the other hand P_0 lies in the intermediary range

(p^-, p_2^M), there will be over-accumulation of capacity along the equilibrium path, since one firm will enter immediately with two units, and the other firm will follow with one unit at time τ^C.

5. Concluding Remarks

In a preemption model with multiple investments, a firm has two motives for accumulating capacity. An obvious reason is to take advantage of its current market power. However, it may also invest only to postpone the next investment of its competitor. The interplay between these two motives may give rise to highly complex investment sequences in equilibrium. In our model, different patterns of equilibria may arise, depending on the importance of the real option effect. If the average growth rate of the market is close to the risk-free rate, or if the volatility of demand changes is high, then the unique equilibrium acquisition process involves joint investment at the socially optimal date. If these conditions do not hold, the equilibrium investment timing is suboptimal, and the firms' long-run capacities depend on the initial market conditions. However, under a broad set of empirically relevant parameters, no dissipation of rents occur in equilibrium, despite instantaneous price competition. This is in sharp contrast with the outcome of the standard winner-take-all model of preemption under Bertrand competition. One implication of our results is that even if capacity can be only very crudely segmented, investment timing per se can be used by firms to reach a co-operative outcome in the absence of any implicit collusion on prices. This suggests that the implications of one-shot models of preemption are likely to prove very fragile and sensitive to both modelling choices and initial conditions.

As mentioned in the introduction, our conclusions differ also sharply from the full rent dissipation result of Gilbert and Harris (1984). The source of this discrepancy lies in how demand evolves in the two environments. When the market size grows in an unbounded way, firms can threaten to preempt each other on each new investment, leading to complete dissipation of rents. However, if the market size is bounded, and the evolution of demand is driven by consumers' tastes, the extent to which rents are dissipated is limited. Admittedly, the no-dissipation result holds in a very specific setting, characterized by a price inelastic demand curve and a very coarse grid of investment opportunities. Moreover, as shown in Subsection 4.4, some rent dissipation may occur even in this setting over a range of the parameters. Due to the dynamic nature of investment decisions, and the absence of commitment to capacity, it would be extremely difficult to analyze the model for an arbitrary grid of investment opportunities. It goes without saying that the no-dissipation result, taken literally, is not likely to survive in more general specifications of the model. The key question, however, is to determine whether the full rent dissipation result of Gilbert and Harris (1984) would hold in such a setting. In that respect, the logic of our argument is actually quite general. Indeed, when the size of the market is bounded, the worst that can happen to a firm is that its rival has already enough capacity to serve all consumers. However, if capacity can be segmented, the follower's rents from its investments will be bounded away from zero even in these circumstances, and this, independently of the form of short-term competition or the size of the market.[6] Thus, even if *some* rent dissipation is likely to occur as a result of preemption, the extent to which this happens has been perhaps overestimated.

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[6]Consider for instance the case where $2N$ units of capacity are needed to serve all consumers in our model, and each unit has a fixed cost of I/N. Suppose that the leader has already $2N$ units. Then, by investing in N units of capacity at τ^C, the challenger can guarantee itself the same payoff as in (10).

Appendix A

Formally, a Markov strategy for firm i is a mapping s^i that associates, to each capacity vector $(n^i, n^j) \in \{0, \ldots, N\} \times \{0, \ldots, N\}$ and state of demand $p \in (0, \infty)$, a probability distribution $s^i(n^i, n^j, p)$ on $\{0, \ldots, N - n^i\}$. (Since each unit of capacity can cover a fraction $1/N$ of the market, it is obvious that firms never wish to acquire more than N units of capacity.) For each $\nu \in \{0, \ldots, N - n^i\}$, the quantity $s^i_\nu(n^i, n^j, p) = s^i(n^i, n^j, p)(\nu)$ will be interpreted as the intensity with which firm i invests in ν units in the state (n^i, n^j, p). The following regularity conditions are imposed for technical convenience:

(R_1) *For each $i \in \{1, 2\}$ and $(n^1, n^2) \in \{0, \ldots, N\} \times \{0, \ldots, N\}$, the mapping $s^i(n^i, n^j, \cdot)$ is piecewise continuous and admits everywhere a right limit.*

(R_2) *For each $p \in (0, \infty)$, if $s^i_0(n^i, n^j, p) = 1$, then $s^i_0(n^i, n^j, \cdot) = 1$ in a left neighborhood of p. If furthermore $p = \inf\{p' > p \,|\, s^i_0(n^i, n^j, p') \neq 1\}$, then for each $\nu \in \{1, \ldots, N - n^i\}$, the right partial derivative $\partial^+_p s^i_\nu(n^i, n^j, p)$ exists and at least one of them is strictly positive.*

For each strategy profile $s = (s^1, s^2)$, let $U^i_s(n^i, n^j, p)$ denote the value of firm i when s is played and the current state of the game is (n^i, n^j, p). Since investment is irreversible and capital does not depreciate, we will be able to compute these value functions recursively from the equilibria of the static pricing game. To do so, we distinguish three cases.

Case 1. First, if $p \in A_s(n^i, n^j) = \{p' \in [0, \infty) \,|\, s^i_0(n^i, n^j, p')\, s^j_0(n^j, n^i, p') \neq 1\}$, then at least one firm is active in the state (n^i, n^j, p). $U^i_s(n^i, n^j, p)$ is then defined as:

$$\frac{\sum_{(\nu^i,\nu^j)\neq(0,0)} s^i_{\nu^i}(n^i, n^j, p)\, s^j_{\nu^j}(n^j, n^i, p)\left[U^i_s(n^i + \nu^i, n^j + \nu^j, p) - \nu^i I\right]}{1 - s^i_0(n^i, n^j, p)\, s^j_0(n^j, n^i, p)}. \tag{17}$$

The intuition underlying (17) is that, if the firms' intensity of investment is not identically zero in state (n^i, n^j, p), with probability one some new investments will take place in this state. With probability $s^i_{\nu^i}(n^i, n^j, p)\, s^j_{\nu^j}(n^j, n^i, p) \,/ [1 - s^i_0(n^i, n^j, p)\, s^j_0(n^j, n^i, p)]$, (ν^i, ν^j) additional units are invested in.

Case 2. Next, if $p \in \partial A_s(n^i, n^j) \setminus A_s(n^i, n^j)$, the intensity of entry of both firms is zero. However, since p is on the boundary of $A_s(n^i, n^j)$, with probability one the first random time $\tau_s(n^i, n^j, p)$ at which the process of demand shocks reaches $A_s(n^i, n^j)$ starting from p is equal to zero. (This is a special case of the 0-1 law for diffusions, see Øksendal, 1995, Corollary

9.2). Using (R_1) and (R_2), $U_s^i(n^i, n^j, p)$ can be computed as the limit of $U_s^i(n^i, n^j, p + \varepsilon)$ when $\varepsilon \downarrow 0$, if necessary using a first-order Taylor expansion of (17). As in Fudenberg and Tirole (1985), the interpretation is that there is an "interval of atoms" following p in at least one firm's strategy, so with probability one some investments will occur at p.

Case 3. Last, if $p \notin A_s(n^i, n^j) \cup \partial A_s(n^i, n^j)$, both firms' intensity of investment is zero in a neighborhood of p. Firm i's continuation value is then standardly computed as:

$$\mathrm{E}_p \left[\int_0^{\tau_s(n^i,n^j,p)} \mathrm{e}^{-rt} \pi(n^i, n^j, P_t)\,\mathrm{d}t + \mathrm{e}^{-r\tau_s(n^i,n^j,p)}\, U_s^i(n^i, n^j, P_{\tau_s(n^i,n^j,p)}) \right], \tag{18}$$

where $\pi(n^i, n^j, P_t)$ is firm i's payoff in a Nash equilibrium of the static pricing game in state (n^i, n^j, P_t), and $\tau_s(n^i, n^j, p)$ is defined as in Case 2.

Appendix B

Proof of Proposition 1. Using (17), it is straightforward to verify that both firms investing with intensity $(p - p_1^m)/p$ forms a MPE strategy profile on (p_1^m, p_1^M). Moreover, it is easy to see that, for any p in this interval, if $s_1^i(0, 0, p) > (p - p_1^m)/p$, then the unique best response of j at $(0, 0, p)$ is $s_0^j(0, 0, p) = 1$. Hence this is the unique Markov strategy profile on (p_1^m, p_1^M) in which both firms are active on this whole interval. Suppose now that there exists a MPE strategy profile s such that $s_0^j(0, 0, p) = s_0^j(0, 0, p) = 1$ for all p in an interval of the form $[p_1^m, \underline{p})$, where $\underline{p} = \inf\{p' > p_1^m \mid s_0^j(0, 0, p') s_0^j(0, 0, p') \neq 1\}$. Hence, starting from $P_0 \in (0, p_1^m)$, the first time at which an investment takes place is strictly larger than the first time τ_1^m at which the state of demand reaches p_1^m. It is obvious from (6) that at least one of the continuation values $U_s^i(0, 0, \underline{p})$, $U_s^j(0, 0, \underline{p})$ must be strictly less than $U_s^i(1, 0, \underline{p}) - I = U_s^j(1, 0, \underline{p}) - I = \underline{p}/(r - \alpha) - I$. Suppose for instance that $U_s^i(0, 0, \underline{p}) < U_s^i(1, 0, \underline{p}) - I$, so that for some $\eta > 0$, it is the case that $U_s^i(0, 0, \underline{p}) = U_s^i(1, 0, \underline{p}) - I - \eta$. Then by (18),

$$U_s^i(0, 0, p) = \left(\frac{p}{\underline{p}} \right)^{\beta} \left[U_s^i(1, 0, \underline{p}) - I - \eta \right] \quad \forall p \in [p_1^m, \underline{p}). \tag{19}$$

Moreover, by continuity of the mapping $U_s^i(1, 0, \cdot)$:

$$\lim_{p \uparrow \underline{p}} \left\{ \left(\frac{p}{\underline{p}} \right)^{\beta} \frac{U_s^i(1, 0, \underline{p}) - I}{U_s^i(1, 0, p) - I} \right\} = 1. \tag{20}$$

It follows then from (19) and (20) that by deviating and investing with positive intensity at p in a small left neighborhood of $\underline{p}$, firm i can get a strictly higher payoff, namely $U_s^i(1, 0, p) -$

I, than its equilibrium payoff $U_s^i(0,0,p)$, a contradiction. It follows that in any MPE, investment occurs as soon as the state of demand reaches p_1^m, which implies the result. $\square$

Proof of Lemma 1. Let us assume that firm i has two units of capacity, while firm j has only one unit. Without loss of generality, we may assume that $p = 1$. A strategy profile is then a vector of prices $(p^i, p^j) \in [0,1] \times [0,1]$. For any strategy profile (p^i, p^j), firms' payoffs are given by the following matrix:

$$\pi^i(p^i, p^j) = \begin{cases} p^i & \text{if} \quad p^i < p^j \\ \\ p^i/2 & \text{if} \quad p^i \geq p^j \end{cases}, \quad \pi^j(p^j, p^i) = \begin{cases} p^j/2 & \text{if} \quad p^j \leq p^i \\ \\ 0 & \text{if} \quad p^j > p^i \end{cases}. \tag{21}$$

It is easy to see that the game (21) has no pure strategy equilibrium. We look for a mixed strategy equilibrium, described by a pair of cumulative distribution functions (F^i, F^j) over $[0,1]$. Note first that any $p^i < 1/2$ is a strictly dominated strategy for i, since i can secure a payoff of $1/2$ by charging $p^i = 1$. Hence necessarily $\text{supp}(F^i) \subset [1/2, 1]$, so that one must have $\text{supp}(F^j) \subset [1/2, 1]$ as well. Define:

$$F^i(p^i) = \begin{cases} 0 & \text{if} \quad p^i \in [0, 1/2] \\ \\ 1 - 1/(2p^i) & \text{if} \quad p^i \in (1/2, 1) \\ \\ 1 & \text{if} \quad p^i = 1 \end{cases}, \quad F^j(p^j) = \begin{cases} 0 & \text{if} \quad p^j \in [0, 1/2] \\ \\ 2 - 1/p^j & \text{if} \quad p^j \in (1/2, 1] \end{cases}. \tag{22}$$

Now, from (22), we have $\pi^j(p^j, F^i) = p^j(1 - F^i(p^j))/2 = 1/4$ for all $p^j \in [1/2, 1)$. Moreover, for $p^j = 1$, we have $\pi^j(1, F^i) = \Pr_{F^i}(p^i = 1)/2 = 1/4$. Hence, given the strategy F^i of firm i, firm j is indifferent between all possible prices in $\text{supp}(F^j) = [1/2, 1]$. Similarly, for all $p^i \in [1/2, 1]$, we have $\pi^i(p^i, F^j) = p^i(1 - F^j(p^i)) + p^i F^j(p^i)/2 = 1/2$. Hence, given the strategy F^j of firm j, firm i is indifferent between all possible prices in $\text{supp}(F^i) = [1/2, 1]$. It follows that (F^i, F^j) is a mixed strategy equilibrium of the static pricing game, with corresponding profits $(1/2, 1/4)$. The proof that this equilibrium is unique is standard (see, e.g., Cripps and Ireland, 1988, or Ghemawat, 1997), and is therefore omitted. $\square$

Proof of Lemma 2. From (7) and (11), we have, after substitution from Lemma 1:

$$U_s(2,0,p) - I - U_s(1,1,p) = \frac{1}{2(r-\alpha)} \left[p - \left(\frac{p}{p^C} \right)^{\beta} p^C \right] - I \quad \forall p \in [0, \infty). \tag{23}$$

Since $\beta > 1$, this quantity attains its global maximum at $p^* = \beta^{1/(1-\beta)} p^C < p^C$. From (23), straightforward manipulations yield:

$$U_s(2,0,p^*) - I - U_s(1,1,p^*) = \left(2\beta^{\frac{1}{(1-\beta)}} - 1 \right) I. \tag{24}$$

It is immediate to check that the expression in (24) is negative if and only if $\beta < 2$, or equivalently $r < 2\alpha + \sigma^2$ since $\alpha \geq 0$. This concludes the proof. $\square$

Proof of Lemma 3. From (7), (10), (11) and the fact that $\pi(2, 0, \cdot) = 2\pi(2, 1, \cdot) = 2\pi(1, 1, \cdot) = 4\pi(1, 2, \cdot)$, it follows that for each $p \in [0, \infty)$:

$$2U_s(1, 1, p) - U_s(0, 2, p) - U_s(2, 0, p) = \mathrm{E}_p \left[\int_{\tau^C}^{\infty} \mathrm{e}^{-rt} \pi(1, 2, P_t) \, \mathrm{d}t + \mathrm{e}^{-r\tau^C} \right]. \tag{25}$$

The expression in (25) is obviously positive, which proves (15). Next, if $r > 2\alpha + \sigma^2$, then from the proof of Lemma 2, $U_s(2, 0, p^*) - I > U_s(1, 1, p^*)$. Moreover, it is straightforward to see that the mapping $U_s(2, 0, \cdot) - U_s(1, 1, \cdot)$ is strictly quasiconcave, and is equal to zero at 0 and p^C. Hence, by the intermediate value theorem, there exist $p^-, p^+ \in (0, p^C)$ such that for all $p \in [0, \infty)$, $U_s(2, 0, p) - I > U_s(1, 1, p)$ if and only if $p \in (p^-, p^+)$. Last, from (23), straightforward manipulations yield:

$$U_s(2, 0, p_2^M) - I - U_s(1, 1, p_2^M) = \left[\left(1 - 2^{1-\beta}\right) \frac{\beta}{\beta - 1} - 1 \right] I. \tag{26}$$

It is immediate to check that the expression in (26) is positive if and only if $\beta > 2$, or equivalently $r > 2\alpha + \sigma^2$ since $\alpha \geq 0$. It follows that $p_2^M \in (p^-, p^+)$, as claimed. $\square$

Proof of Lemma 4. (i) If the current state of demand is $p \in (p^-, p_2^M)$, then $U_s(2, 0, p) - I > U_s(1, 1, p)$ and $U_s(1, 1, p) - I > U_s(0, 2, p)$, so both the incumbent and the challenger have a positive incentive to preempt each other. Using (17), it is straightforward to check that a MPE strategy profile is for both firms to invest in one unit of capacity with intensities:

$$s_1(1, 0, p) = \frac{U_s(1, 1, p) - I - U_s(0, 2, p)}{U_s(1, 1, p) - U_s(1, 2, p)} \tag{27}$$

for the incumbent, and:

$$s_1(0, 1, p) = \frac{U_s(2, 0, p) - I - U_s(1, 1, p)}{U_s(2, 0, p) - U_s(2, 1, p)} \tag{28}$$

for the challenger. From (27) and (28), the associated MPE payoffs are $U_s(1, 1, p)$ for the incumbent and $U_s(0, 2, p)$ for the challenger.

(ii) If the current value of demand is $p \in [0, p^-)$, an incumbent with one unit of capacity cannot gain from investing in a second unit at p since $U_s(2, 0, p) - I < U_s(1, 1, p)$. It follows that the challenger is not threatened by the incumbent on this interval. However, since $p^- < p_2^M$, it is suboptimal for the challenger to invest, and its best response is therefore to wait until p^- is reached. At p^-, the incumbent is indifferent between preempting and being preempted. Suppose that $s_1(1,0,p^-) > 0$ in equilibrium. Then the challenger's payoff at p^- is $U_s(1,1,p^-) - I - \eta$ for some $\eta > 0$. Hence, it follows from (18) that:

$$U_s(0,1,p) = \left(\frac{p}{p^-}\right)^\beta \left[U_s(1,1,p^-) - I - \eta\right] \quad \forall p \in [0,p^-). \tag{29}$$

Moreover, by continuity of the mapping $U_s(1,1,\cdot)$:

$$\lim_{p \uparrow p^-} \left\{ \left(\frac{p}{p^-}\right)^\beta \frac{U_s(1,1,p^-) - I}{U_s(1,1,p) - I} \right\} = 1. \tag{30}$$

Furthermore, $s_0(1,0,p) = 1$ for all $p \in [0,p^-)$. It follows then from (29) and (30) that by deviating and investing with positive intensity at p in a small left neighborhood of p^-, the challenger can get a strictly higher payoff, namely $U_s(1,1,p) - I$, than its equilibrium payoff $U_s(0,0,p)$. However, as noted previously, investing at $p^- - \varepsilon$ is never a best response for any $\varepsilon > 0$ given that the incumbent does not invest on $[0,p^-)$. Therefore, if $s_1(1,0,p^-) > 0$, there is an openness problem in the determination of the challenger's best response in a left neighborhood of p^-. Thus it must be that the incumbent never invests at p^-, and the challenger invests with positive intensity at p^-. In particular, $s_1(0,1,p^-) > 0 = \lim_{p \downarrow p^-} s_1(0,1,p)$ and $s_1(1,0,p^-) = 0 < \lim_{p \downarrow p^-} s_1(1,0,p)$. □

Proof of Proposition 3. Cases 1-3 have already been analyzed. Suppose that Case 4 holds. Let us prove that there is a MPE in which, for any current state of demand $p \geq p^-$, both firms invest in one unit of capacity with intensity $s_1(0,0,p) = 1$. Note first that this is obvious if $p > p_2^M$ since then no firm has an incentive to delay investment. Consider now what happens if $p \in (p^-, p_2^M)$. From the discussion of Case 1, if both firms invest simultaneously in one unit at p, their continuation payoff at p is $U_s(1,1,p) - I$. Suppose now that firm i deviates, and invests in one unit with intensity $s_1^i \in [0,1)$. (It is clear that investing in two units cannot be a best response if firm j invests in one unit.) Then by Lemma 4, firm i's continuation payoff at p is $s_1^i\,(U_s(1,1,p) - I) + (1 - s_1^i)\,U_s(0,2,p)$. However, from Lemma 3, one has $U_s(1,1,p) - I - U_s(0,2,p) > U_s(2,0,p) - I - U_s(1,1,p) > 0$ for any $p \in (p^-, p_2^M)$ so this deviation cannot be profitable. Last, suppose that firm j never invests before p^- is reached. Clearly, investing in two units is never a best response for firm i. Moreover, as $p^- < p_2^M$, investing in one unit at some $p \in [0,p^-)$ is clearly dominated for firm i given the continuation equilibrium just constructed for Case 4 on $[p^-, \infty)$ and a best response is to wait until τ_{p^-} to invest. □

References

Allen, B., Deneckere, R., Faith, T., Kovenock, D., 2000. Capacity precommitment as a barrier to entry: A Bertrand-Edgeworth approach. Economic Theory, 15, 501-530.

Boyer, M., Lasserre, P., Mariotti, T., Moreaux, M., 2002. Lumpy investment, real options, and the dynamics of industry development. Working paper, Université de Toulouse I.

Cripps, M., Ireland, N., 1988. Equilibrium and capacities in a market of fixed size. Working paper, University of Warwick.

Dixit, A.K., Pindyck, R.S., 1994. Investment under uncertainty. Princeton University Press, Princeton.

Eaton, C., Lipsey, R.G., 1981. Capital, commitment, and entry equilibrium. Bell Journal of Economics, 12, 593-604.

Fudenberg, D., Tirole, J., 1985. Preemption and rent equalization in the adoption of new technology. Review of Economic Studies, 52, 383-401.

Fudenberg, D., Tirole, J., 1987. Understanding rent dissipation: On the use of game theory in industrial organization. American Economic Review, 77, 176-183.

Gelman, J.R., Salop, S.C., 1983. Judo economics: Capacity limitation and coupon competition. Bell Journal of Economics, 14, 315-325.

Ghemawat, P., 1997. Games businesses play. MIT Press, Cambridge.

Gilbert, R.J., Harris, R.G., 1984. Competition with lumpy investment. Rand Journal of Economics, 15, 197-212.

Gilbert, R.J., Newbery, D., 1982. Preemptive patenting and the persistence of monopoly. American Economic Review, 72, 514-526.

Katz, M., Shapiro, C., 1987. R&D rivalry with licensing or imitation. American Economic Review, 77, 402-420.

Leahy, J.V., 1993. Investment in competitive equilibrium: The optimality of myopic behavior. Quarterly Journal of Economics, 104, 1105-1133.

Øksendal, B., 1995. Stochastic differential equations: An introduction with applications. Springer-Verlag, Berlin.

Posner, R.A., 1975. The social costs of monopoly and regulation. Journal of Political Economy, 83, 807-827.

Smets, F., 1991. Exporting versus FDI: The effect of uncertainty, irreversibilities and strategic interactions. Working paper, Yale University.

Tullock, G., 1967. The welfare costs of tariffs, monopolies and theft. Western Economic Journal, 5, 224-232.