An Alternating Move Price-Setting Duopoly Model with Stochastic Costs

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Abstract

This paper examines an alternating move price-setting duopoly model in which marginal costs are stochastic without persistence. It is shown that, provided marginal costs do not fluctuate excessively, equilibria in which firms match the current monopoly price do not exist. By contrast, equilibria in which firms always match their rival along the equilibrium path do exist. In these equilibria, deviations off of the equilibrium path result in extended price wars, in which firms repeatedly undercut each other. Finally, examples in which such price wars are observed in equilibrium are constructed.

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1 Introduction

In 1964, Stigler argued that a difficulty a cartel faces is ensuring that member firms will not undercut the cartel's price, and that price wars signal a collapse of collusion. Since then, theories have been developed to understand price wars and their role in maintaining collusion. In the first stage of research, the credible threat of a price war is used to ensure that no firm in the collusive arrangement will cheat; because in these models price wars are not observed in equilibrium, they are thought of as "off-equilibrium" price war theories. The next stage identified reasonable models in which price wars are observable in equilibrium. Such models include that of Green and Porter (1984), in which price wars occur because firms cannot distinguish between market demand shocks and deviations by rivals, and Slade (1989), in which firms use price wars to learn about the demand curve after a demand shock.

While many theories predict price wars, few predict wars that resemble those observed in many industries. A common feature of observed price wars is the wide range of prices set through the course of the war, due to repeated undercutting. Price wars featuring an increased degree of price volatility have been documented for example in the prices of retail gasoline (Slade,1992), bromine (Levenstein, 1996), and cigarettes (Boudreaux *et al*,1995). However, the literature has provided few explanations for the wide range of prices sometimes observed in price wars, even considering wars that occur off of the equilibrium path.[1]

[1] Exceptions include Slade(1989) and Staiger and Wolak (1992), in which decreases in demand cause capacity-constrained firms to switch from collusive prices to mixed strategies, in which the lower priced firm serves the entire market. Extended price fluctuations in which prices may decline slowly over time have also been generated in models of sales (Conlisk, Gerstner and Sobel, 1984), and models with customer loyalties (Rosenthal and Chen, 1996).

The purpose of this paper is to consider a model in which the threat of price wars covering a wide range of prices permits the firms to maintain a collusive price level, and to discuss extensions in which such price wars are observed in equilibrium. This paper extends the infinite-horizon, alternating-move, price setting duopoly model of Maskin and Tirole (1988).[2] In their model, two firms set price in an alternating fashion, each taking the other's price as fixed when setting its own. Equilibrium pricing in which each firm matches its rival at the monopoly price can be supported using simple strategies, in which a firm's price depends only on its rival's most recent price. In the event of a deviation below the monopoly price, a firm responds by dropping its price to the point that induces its rival to restore the monopoly price. Deviations above the monopoly price are ignored. Thus, this model has been viewed as a formalization of the 'kinked demand' explanation of price rigidities.[3]

This paper extends Maskin and Tirole (1988) by examining price-matching strategies when marginal costs vary over time.[4] This extension is motivated for several reasons. First, the robustness of the price matching equilibria discussed above to the introduction of time-varying costs can be explored, providing insights as to the appropriateness of such a model in explaining price rigidities. Second, testable implications of such a model regarding the relationship between retail prices and marginal costs can be identified. Finally, the type of

[2]This model has been extended in other directions. Eaton and Engers (1990) examine a discrete form of product differentiation. Wallner (1999) considers the implications of the infinite horizon assumption, and Eckert(2003) examines the role of the tie-breaking rule.

[3]See for example Blanchard and Fischer (1989) for discussion.

[4]Maskin and Tirole (1988) also generate equilibria in which prices are not rigid, but instead exhibit regular cycles. Using numerical examples, Noel (2002) describes such equilibrium price cycles when marginal costs vary from period to period.

price war that can be used to support collusion in such a setting can be explored.

In this paper I wish to gain intuition about the effect time-varying costs have on the incentives of firms to match or undercut the prices of rivals, and on whether tacitly collusive price levels can be supported in the face of time varying costs. Therefore, most of this paper considers a stylized representation of marginal costs, in which marginal costs can be either high or low with equal probability. I also discuss the expected effects of introducing persistence in marginal costs through a Markov process. While the marginal cost process studied is clearly an abstraction, it provides a tractable setting from which to obtain intuition regarding time-varying marginal costs.

My analysis yields the following results. Provided the difference between high and low marginal costs is not too large, when there is no persistence in marginal cost equilibria do *not* exist in which firms always match the current monopoly price when that price is set by the firm's rival. By contrast, equilibria do exist in which the firms match each other at an equilibrium price level that does not vary with marginal costs. Firms are deterred from deviating from the tacitly-collusive price level by the threat of an extended price war of stochastic length involving repeated undercutting. Intuitively, when costs are low, a firm would prefer to respond to a deviation from the collusive price level with a higher price than the price that would force its rival to restore the collusive price. This incentive, to delay forcing a restoration of the collusive price in order to serve the entire market at low cost and high prices, results in off equilibrium path behavior in which firms repeatedly undercut each other. Finally, I demonstrate that by adding the possibility of demand or cost shocks that occur with low probability and that are sufficiently persistent, it is possible to observe such price wars in equilibrium.

This paper proceeds as follows. Section 2 presents the model. Section 3 discusses equilibria under high persistence in the marginal cost process. Section 4 supposes marginal costs are high or low each period with equal probability, and Section 5 discusses examples in which price wars occur in equilibrium. Section 6 concludes.

2 The Model

This section extends the Maskin and Tirole (1988) alternating move duopoly model by supposing that marginal costs in each period are stochastic. There are two firms, indexed by $i = 1, 2$. The infinite horizon discrete-time model has periods indexed by $t = 0, 1, 2, ...$ Firm 1 chooses a price in every even period, committing to that price for two periods. Firm 2 sets its price in odd periods. This timing reflects short run commitments to price.

The firms sell homogeneous products. The demand is the same in each period, and is given by $D(p)$ where p is the lowest price charged by either firm in the market. The prices that each firm can choose are restricted to a finite grid, with the distance between available prices equal to k. That is, if firm i sets a price p, and firm j wants to undercut p, the highest price it can set is $p - k$. This ensures that an optimal undercut exists.[5]

Marginal costs are assumed to be constant over output, exogenous, and identical for each firm. There are no fixed costs or capacity constraints. To gain intuition regarding the role of time varying marginal costs in the existence and characterization of equilibria, I extend Maskin and Tirole (1988) by supposing that marginal costs follow a Markov process.

[5] The grid size k may also be the result of physical restrictions. In gasoline retailing, the signs used by retailers restrict them to prices ending in tenths of a cent.

Marginal cost in any period can be either low (c_L) or high (c_H). The probability that in any period marginal cost will remain unchanged from the previous period is $\alpha \in [0, 1]$, and the probability that costs switch states is $1 - \alpha$. I assume that each firm observes the current marginal cost before setting its price.

The parameter α can be viewed as measuring the rigidity of marginal cost, and describes both the likelihood of a cost shock and the persistence of a shock should one occur. For example, if $\alpha = 0.99$ then a change to marginal cost is essentially unexpected, but any such shock would have almost permanent implications. Alternatively, if $\alpha = 0.5$, then costs are high or low with equal probability in a given period, and are as likely next period to change as stay the same. Throughout most of this paper, I will assume that $\alpha = 0.5$, but will also consider the implications of other values.

Define the industry operating profit function, assumed to be strictly concave with respect to p, as

$$\Pi(p, c) := (p - c)D(p). \tag{1}$$

Further define p_L^M and p_H^M as the monopoly prices under low and high marginal costs.

Let p^i denote firm i's price in period t. Then firm i's operating profits in period t can be expressed as follows:

$$\pi^i(p^i, p^j, c) = \begin{cases} \Pi(p^i, c) & \text{if } p^i < p^j \\ \frac{1}{2}\Pi(p^i, c) & \text{if } p^j = p^i \\ 0 & \text{if } p^i > p^j, \end{cases}$$

This equation states that firm i serves all of the market if its price is less than firm j's price, one half of the market if its price is equal to firm j's price, and none of the market otherwise.

In principle, a firm's choice in any period in which it makes a pricing decision could depend upon the entire history of play up to that period. I follow Maskin and Tirole (1988) by restricting a firm's choice to depending only upon payoff relevant variables: those variables that directly enter its payoff function.[6] If as in Maskin and Tirole (1988) marginal cost is constant over time, the only payoff-relevant variable is the price set by the rival in the previous period. In the current model, however, there are *two* payoff relevant variables: the price set by the firm's rival in the previous period, and the current marginal cost. Therefore, firm i's strategy is a (possibly random) dynamic reaction function, $R_i(p, c)$, which depends on the price set previously by i's rival, and on the current marginal cost.[7] This reaction function specifies a price response for every possible rival price and for each possible marginal cost. Each firm chooses its strategy to maximize the present discounted value of profits. The discount factor δ is the same for both firms and is assumed to be close to one.[8]

The solution concept is Markov Perfect equilibrium(MPE). Given a pair of strategies $R_1(p, c)$ and $R_2(p, c)$, define $V_i(p, c)$, $i = 1, 2$, as the expected present discounted profit of

[6] Maskin and Tirole (1997) provide a thorough discussion of the advantages of working with Markov strategies. In particular, any equilibrium within the space of Markov strategies is also an equilibrium in the space of history dependent strategies. As well, Markov strategies can be viewed as representing the simplest behavior consistent with rationality.

[7] Note that the Markov chain assumption is an important simplification; with more sophisticated processes, the number of payoff relevant variables would increase, as past values of marginal cost are relevant for predicting future marginal cost.

[8] While some results in this paper are independent of δ (Proposition 1 and related results), other results are derived by fixing α and supposing that δ is sufficiently near 1. In general, lowering δ is expected to reduce the attractiveness of raising price above rivals in order to increase market prices in future periods, thus leading to lower prices before such restorations occur. In a related model, Noel(2002) confirms this intuition with numerical examples.

firm i in any period in which it has to choose a price, firm j's price is p, current marginal cost is c and all subsequent play is determined by (R_1, R_2). Similarly, define $W_i(p, c)$ as firm i's expected present discounted profit, prior to knowing current costs, in any period in which firm j chooses price, i's price is p and the marginal cost *in the previous period* (in which i had chosen its price) was c. Then a pair of strategies $(R_1(p, c), R_2(p, c))$ is an MPE if for all prices $p*$, and $i = 1, 2$,

$$V_i(p*, c) = \max_p [\pi^i(p, p*, c) + \delta W_i(p, c)], \tag{2}$$

and

$$W_i(p*, \tilde{c}) = E[\pi^i(p*, R_i(p*, c), c) + \delta V_i(R_i(p*, c), c)|\tilde{c}], \tag{3}$$

where $R^i(p*, c)$ is the optimal choice of p, the expectation is taken with respect to the distributions of $R^j(p*, c)$ and c, and the notation $E[\quad|\tilde{c}]$ indicates that the expectation is conditional on the previous period's costs being $\tilde{c}$.

3 Low Probability Cost Shocks

To introduce the intuition behind the role of stochastic marginal costs in the existence and characterization of equilibria, I first suppose that α is sufficiently near 1 that the strategies employed by the firms are the same as would be employed if $\alpha = 1$. That is, I suppose that once cost changes it is very unlikely to change back, and that the probability of a shock is sufficiently low as to not affect the equilibrium strategies.[9] I then discuss how the incentives to deviate from the equilibrium strategies change as the persistence of costs is decreased.

[9] The intuition for the effect of low-probability shocks is discussed in Maskin and Tirole (1988).

Throughout this paper, I will focus on symmetric equilibria, in which $R^1(p,c) = R^2(p,c)$ for all p and c. Therefore, the superscripts i and j can be dropped from all reaction functions and value functions. As well, I focus on two classes of equilibria. The first consists of equilibria in which the focal price depends on current costs, whereas in the second class, the focal price does not depend on current costs. Formally, define a *single focal price equilibrium* as one in which, for some price p^f, $R(p^f, c_H) = R(p^f, c_L) = p^f$. That is, in a single focal price equilibrium, there exists a price that, if set by a firm's rival in the previous period, the firm will match regardless of current marginal costs. Alternatively, in an *alternating focal price equilibrium*, for some p_L^f and p_H^f, $R(p_L^f, c_L) = p_L^f, R(p_H^f, c_H) = p_H^f$, but $R(p_L^f, c_H) \neq p_L^f$ and $R(p_H^f, c_L) \neq p_H^f$. That is, a firm matches p_L^f when costs are low but not when costs are high, and matches p_H^f when costs are high but not when they are low. Note that I have not placed any restrictions on the prices set by a firm in response to prices other than the focal prices.

For α near 1, it is possible to construct both single focal price and alternating focal price equilibria. Suppose for example, that demand is given by $D(p) = 1 - p$, k is equal to 1/20, $c_L = 0$, $c_H = 1/10$, and $\delta = 0.99$.[10] Prices are indexed $p_0 = 0, p_1 = 1/20, p_2 = 2/20, ...p_{20} = 1$. The first three columns of Table 1 give the price and corresponding period profits when cost is high or low (to avoid decimals, profits are multiplied by 400).

The remaining columns of Table 1 contain dynamic reaction functions supporting an MPE for $\alpha > 0.84$, based on strategies constructed by Maskin and Tirole(1988) to support the monopoly price when marginal costs are constant over time. In the alternating focal

[10] While the points of this section could be made in a simpler example with a larger grid size, a fine grid will be required to illustrate results later in the paper.

price equilibrium described in Table 1, firms match the low-cost monopoly price p_{10} when marginal cost is low, and the high-cost monopoly price p_{11} when marginal cost is high.

Suppose first that marginal costs are low, and that α is equal to 1. If a firm's rival has set the focal price p_{10}, the firm will match that price. If a firm's rival has set a price just below the focal price, the firm prefers to respond with a price sufficiently low (in this case p_2) to force its rival to restore the focal price p_{10} with some probability, rather than undercutting to a price such as p_8 at which it would serve the entire market for a period but would delay the restoration of the focal price. In this example, firms respond to p_2 when costs are low with mixed strategies, restoring the relevant focal price only with some probability β_L less than one.[11] Once one firm sets p_{10} the low-cost focal price would then continue to be played. Similar behavior enforces the high cost monopoly price p_{11} as a focal price when costs are high and $\alpha = 1$; a deviation from the focal price is responded to with a price sufficiently low to force a restoration of the high-cost monopoly price with some probability.

These strategies, with adjustments to the mixing probabilities, also support each focal price when the probability that marginal costs will switch states is low (that is, for α sufficiently near 1). Intuitively, if the likelihood of a cost change is sufficiently small, then the present value payoffs that can be earned from each possible response will not change substantially, and the specified behavior will remain preferred to any deviations. In our example, $\alpha > 0.84$ is sufficient for the specified strategies to be an equilibrium.

Similar equilibrium strategies, presented in Table 2, support a single focal price of p_{10} for $\alpha > 0.92$. The response functions are similar those discussed above. If a firm's rival sets

[11] The use of randomization is a result of the large grid size - as the grid size decreases, these responses become replaced with pure strategies, restoring the focal price with probability one.

p_{10}, the firm will match. If it observes a price slightly below p_{10}, the optimal response is to set a price low enough to force the rival to restore the focal price p_{10} with some probability if costs remain low.

While α determines the payoffs obtained from several different deviations, a key role of α is in the payoffs earned by a firm facing low costs responding to a deviation from the current focal price. Suppose, in either equilibrium, that marginal costs are currently low and that a firm's rival has deviated from the focal price by setting a price of p_9. To force a rival facing low costs to restore the focal price with some probability, the firm must set the lowest price that earns a greater stream of profits than restoring the focal price; in the examples this price was p_2. For k near zero, this earns the firm approximately what it would earn from restoring the low-cost focal price.

Alternatively, suppose the firm undercuts to p_8, and restores the relevant focal price at the next opportunity. The firm would serve the entire market for one period at low costs, followed by the payoffs earned two periods later from restoring the focal price. When α is near 1, the payoff from forcing a restoration of the focal price exceeds the payoffs from serving the entire market at a high price but delaying the price restoration. However, as α falls, the relative payoff from serving the entire market at low costs at the price p_8 increases. Intuitively, as α decreases, the temptation to take advantage of current low costs by undercutting a deviation by a grid size increases. By doing so, a firm can enjoy the entire market at a near-monopoly price and low costs, which is more attractive than setting a very low price in order to restore the long run equilibrium price.[12] This problem, that a low cost firm would rather serve the

[12] In the example, this deviation determines the lower bound of α in the alternating focal price equilibrium. In the single focal price equilibrium, however, the coarse grid means that the lower bound of α is determined

entire market at a high price than respond aggressively to deviations from the focal price, motivates a consideration of focal price equilibria when marginal cost persistence is low.

4 The Case of $\alpha = 1/2$

In this section I consider the opposite extreme, in which costs are high or low with equal probability in any given period ($\alpha = 0.5$), so that current costs are not informative about future costs. I demonstrate two key results. First, I show that, for c_H and c_L not too far apart, alternating focal price equilibria supporting the monopoly prices cannot exist in this setting. Second, I show that single focal price equilibria can still be constructed, using responses off of the equilibrium path that are more complicated than in the previous section. When costs are low, a firm responds to certain prices below the focal price by undercutting by a grid size to take advantage of temporary low costs, rather than inducing a restoration of the focal price with certainty. Therefore, a deviation from the focal price causes a lengthy "price war" period, in which firms cycle through a large number of prices.

First, I demonstrate that there do not exist alternating focal price equilibria supporting p_H^M and p_L^M under high and low costs, provided $c_H - c_L$ is not too large.[13]

Proposition 1: Suppose $\alpha = 1/2$. For $c_H - c_L$ sufficiently small, there are no alternating focal price equilibria supporting p_H^M and p_L^M under high and low costs.

The formal proof of Proposition 1 is given on this journal's webpage. The intuition behind

in order for the mixing probability to lie between zero and one.

[13] If the firms are restricted to using only pure strategies, one can show that for δ near 1 and k near zero, there can be no alternating focal price equilibrium supporting $p_H^f > p_L^f$ (provided that p_H^f and p_L^f are not too far apart). A proof of this result can be obtained from the author on request.

the proof as is follows. In an alternating focal price equilibrium supporting the monopoly prices, the best response to the high-cost monopoly price when costs are low must be the low-cost monopoly price. That is, if costs switch from high to low, the firm immediately sets the new focal price. This implies that if a firm faces high costs and the high-cost monopoly price, matching yields positive profits next period only in the event that costs remain high. Alternatively, if the firm chooses to drop down immediately to the low-cost focal price, it serves the entire market in the current period, and is matched by its rival next period in the event costs are low. This deviation earns higher profits than matching the high-cost monopoly price, provided the distance between the two marginal costs is not too large. For example, suppose $D(p) = 1 - p$ and $c_L = 0$. As δ approaches 1, an approximate sufficient condition for there not to exist an alternating focal price equilibrium is that $c_H < 0.25$.

Finally, a similar result can be shown for a model in which marginal cost is constant over time, but demand is either high or low each period with probability 1/2.[14] These results suggest that when the monopoly price can be high or low each period with probability 1/2, and the difference between the monopoly prices is not too large, Markov Perfect Equilibria cannot be constructed in which the firms track the monopoly prices.

I now consider the existence of single focal price equilibria. Recall that in the previous section a single focal price equilibrium was supported through strategies in which a firm responds to deviations from the focal price by setting the lowest price that earns it a higher payoff in present value than immediately restoring the focal price. This response induces the firm's rival to raise its price, and the firms return to the equilibrium path.

It was argued earlier that as the probability that costs change from low to high increases,

[14] A proof of this result is available from the author upon request.

the temptation to respond to a deviation by undercutting by a grid size increases. When marginal cost is low a firm would rather serve the entire market at a high price, than squander temporary low costs through a price so low that it induces a restoration of the focal price.[15] This means that a single focal price equilibrium requires responses to $p^f - k$ that yield a payoff sufficiently larger than that earned by restoring the focal price.

Such strategies involve extended periods of undercutting. Suppose again that $D(p) = 1 - p$, $c_L = 0$ and $c_H = 0.1$. As well, fix δ at 0.99, and set $k = 1/20$ initially. I index the prices 0, 1/10, 2/10,...1 by $p_0, p_1, p_2, ...p_{20}$. Table 3 presents equilibrium dynamic reactions supporting the low-cost monopoly price as a single focal price. The first column indicates the price, from p_0 to p_{10}. The second and third columns contain the one-period total operating profits (multiplied by 400).

The fourth and fifth columns contain the dynamic reaction functions, under low and high marginal costs. The responses to prices greater than p_{10} have been omitted. If the current price is greater than p_{10} the best response is to set a price of p_{10}, regardless of current costs. First, responding to a price above p_{10} with a price above p_{10}, thus serving the entire market one period and no customers the next, is clearly less profitable than setting the focal price, and serving customers in both periods. Second, setting a price less than p_{10} is not preferred because p_{10} is the best response to p_{10}.

Suppose next that $p \leq p_2$. Here, a firm's best response is to restore the focal price. Suppose cost is currently low and $p = p_2$. A firm's most attractive deviation would be to set

[15] This reasoning is similar to that of Rotemberg and Saloner (1986). In their model, demand each period is either high or low with equal probability. In periods in which demand is high, the temptation to undercut the monopoly price is greater than when demand is low.

a price of p_1, which yields 19 immediately, 0 next period in expectation, and the expected value of $V(p_{10}, c)$ two periods hence. This is less than $45\delta + \delta^2 EV(p_{10}, c)$, the expected payoffs from restoring the focal price. Similar arguments hold when $c = c_H$.

The lowest price that a firm facing low costs would be willing to set before restoring the focal price is p_2. Setting a price of p_2 and serving the entire market for two periods, given that the rival will respond by restoring the focal price, earns higher continuation profits than restoring the focal price. When costs are high, however, it is still optimal to restore the focal price, since the profits from setting p_2 are lower. For these reasons, a low-cost firm responds to p_4 and p_3 with p_2, whereas a firm facing high costs restores the focal price.

At a price of p_5, for a low cost firm the payoff from setting p_4 is greater than the payoff from setting p_2. If the firm sets a price of p_4, it serves the entire market in the current period, and also in the next period if costs increase (because the rival will respond by raising price). Setting a price of p_4 forces its rival to raise price only if costs are high next period, whereas a price of p_2 causes the rival to restore the focal price with certainty. Because the gains to serving the entire market at a price of p_4 are less for a firm facing high costs, such a firm would still choose to restore the focal price.

Finally, in response to any price between p_6 and p_9, a firm responds by undercutting to p_5. For a high cost firm, p_5 is the lowest price it is willing to set before restoring the focal price. When costs are low, setting p_5 is preferred to p_4, since the immediate profits are higher and the firm would rather face p_4 than p_2 in two periods.

It follows that if a firm undercuts its rival to p_9 it starts a price war of stochastic length. If costs remain high for two periods, the rival will reduce price to p_5 and prompt a restoration of the focal price. If costs fall and remain low a period of undercutting begins, with each firm wanting to serve the entire market at low cost, rather than restore higher prices. Therefore, the length of the "price war" after a deviation from the focal price depends upon marginal costs, with low costs leading to longer wars.

It turns out, however, that the conclusion that lengthy price wars occur only if costs are low is a product of the large grid size. Suppose that the grid size is decreased to $k = 1/100$. Because tables are now inconvenient, Figures 1 and 2 plot equilibrium dynamic reaction functions $R(p, c_L)$ and $R(p, c_H)$. The horizontal axis plots the price p set by the rival in the previous period, with p_{50} at the point furthest *left* and p_0 furthest *right*. One can think of time proceeding from left to right in a price war triggered by a deviation. The downward sloping straight line on each graph has a slope of -1, and represents price matching. Where the reaction function is below this line corresponds to responses below the rival's price, and where the reaction function is above this line corresponds to price restorations.

As illustrated in Figure 1, $R(p, c_L)$ resembles the reaction function described in Table 3. In response to an undercut below the focal price, a firm facing low costs drops its price to that which induces the rival to restore the focal price only when costs are high. If costs remain low, firms wish to take advantage of the temporarily low costs by serving the entire market. As a result, a period of undercutting begins, ending when p_9 is reached. At p_9, a low cost firm strictly prefers to restore the focal price than to continue undercutting its rival.

For prices less than or equal to p_{46}, $R(p, c_H)$ describes the same sort of behavior for a firm facing high costs as was observed in the case where $k = 1/20$. The firm will drop its price to p_{24}, but would rather restore the focal price than reduce price any further. However, the best response when costs are high to prices slightly below the focal price is not to reduce price to p_{24}. For example, suppose that $p = p_{47}$. In this case, setting p_{24} would earn

$1064/2 + \delta 1064/2 + \delta^2(V(p_{50}, c_H) + V(p_{50}, c_L) + V(p_{23}, c_L) + V(p_{23}, c_H))/4$. Alternatively, by undercutting to $p = p_{46}$, the firm earns continuation payoffs of $1944 + \delta^2(V(p_{24}, c_L)/2 + V(p_{24}, c_H)/2$, which can be shown to be larger. Intuitively, setting p_{24} yields approximately the same profit for a firm facing high costs as restoring the focal price. However, a low cost firm facing p_{24} earns strictly greater profits than what it would from restoring the focal price. Therefore, a firm facing high costs and p_{47} has an incentive to set a price of p_{46}, in the hopes that at its next turn it will face low costs, and earn greater than the payoffs from restoring the focal price. Similar arguments apply to prices $\in [p_{47}, p_{49}]$.

Such undercutting behavior is not optimal for a firm facing low costs. Suppose that $p = p_{47}$ and costs are low. Setting p_{46} earns approximately the same as restoring the focal price, plus $\Pi(p_{46}, c_L) - \Pi(p_{46}, c_H)$. Alternatively, setting p_{24} earns approximately the same as restoring the focal price plus $\Pi(p_{24}, c_L) - \Pi(p_{24}, c_H)$. Since demand is downward sloping, the former is less than the latter. Similar arguments apply to p_{48} and p_{49}.

Because of the undercutting triggered by a deviation if costs are high, the price war that results from a deviation can be lengthy even if marginal costs are high. Suppose that in response to p_{50}, a firm sets a price of p_{49}. A rival facing low costs will drop the price immediately to p_{24}, whereas if costs are high, it will initiate a period of undercutting, until eventually p_{24} was set. At this point, a high cost firm will restore the focal price, while if costs are low, firms will start another period of undercutting.

Equilibrium strategies of the form illustrated in Figures 1 and 2 can be shown to be equilibria for the general model, for $c_H - c_L$ sufficiently small:

Proposition 2: Consider a price p^f, and suppose that c_H and c_L satisfy

(i) $\Pi(c_H, c_L) < \frac{\Pi(p^f, c_L)}{2}$

(ii) $\Pi(p^f, c_H) + \frac{1}{2}E\Pi(p^f, c) > \Pi(p_H^M, c_H)$

and

(iii) $\Pi(p^f, c_H) > E\frac{\Pi(p^f,c)}{2}$.

Then for δ *sufficiently near* 1 *and* k *sufficiently near* 0*, the following strategies constitute an equilibrium:*

$$R(p, c_L) = \begin{cases} p^f & \text{for } p > p^f, \\ p^f & \text{for } p = p^f, \\ \underline{p}_H & \text{for } p \in (\underline{p}_H, p^f), \\ p - k & \text{for } p \in (\underline{\underline{p}}_L, \underline{p}_H), \\ \underline{p}_L & \text{for } \underline{\underline{p}}_L > p > \underline{p}_L, \\ p^f & \text{for } \underline{p}_L \geq p. \end{cases}$$

$$R(p, c_H) = \begin{cases} p^f & \text{for } p > p^f, \\ p^f & \text{for } p = p^f, \\ p - k & \text{for } p \in (\underline{\underline{p}}_H, p^f), \\ \underline{p}_H & \text{for } p \in (\underline{p}_H, \underline{\underline{p}}_H], \\ p^f & \text{for } \underline{p}_H \geq p. \end{cases}$$

for p^f *in an interval about* p_L^M*, and where* $\underline{\underline{p}}_H, \underline{p}_H, \underline{\underline{p}}_L$*, and* $\underline{p}_L$ *are defined such that* $\underline{p}_L < \underline{\underline{p}}_L < \underline{p}_H < \underline{\underline{p}}_H < p^f$.

These strategies dictate the same behavior as Figures 1 and 2. In response to a deviation from the focal price, a complex phase of undercutting is initiated, with firms undercutting each other at high prices if costs are high, and at lower prices if costs are low. The range of $|c_H - c_L|$ for which these strategies are an equilibrium will depend upon the demand curve and the price being supported. As an example suppose $D(p) = 1 - p$, $c_L = 0$ and $p^f = 0.5$. Then for conditions (i) to (iii) to be satisfied we require c_H less than approximately 0.145.

The proof of Proposition 2 is given on this journal's website. First, a simplified proposition is proven, in which the focal price is the monopoly price under low marginal costs. The proof of this result has three stages. In the first stage, the parameters $\underline{p}_L, \underline{p}_H, \underline{\underline{p}}_L$ and $\underline{\underline{p}}_H$ are defined. These represent, respectively, the lowest prices that a firm would be willing to set under low and high costs, the price at which a low cost firm ceases undercutting by a grid size in order to induce its rival to restore the focal price, and the price at which a high cost firm ceases undercutting by a grid size. In the second stage, we consider each possible price set in the previous period and show that a firm has no incentive to deviate regardless of marginal costs. Finally, it is shown that for a grid size sufficiently small, the parameters $\underline{p}_L, \underline{p}_H, \underline{\underline{p}}_L$ and $\underline{\underline{p}}_H$ are well defined. Once this result is proven we demonstrate that similar arguments apply to other focal prices near the low-cost monopoly price.

As a final comment, note that the model likely has other equilibria, supporting constant prices outside of the range defined in the proof of Proposition 2. As in Maskin and Tirole (1988), equilibria supporting prices far from the low-cost monopoly price will likely require mixed strategies, significantly complicating the analysis. While equilibria supporting other focal prices may exist, the above discussion demonstrates that constant price equilibria can exist, and may be supported using the undercutting price wars described above.

5 Price Wars in Equilibrium

In the previous section I generated equilibria in which the firms never deviate from the focal price because deviation results in an extended period of undercutting. However, because deviations are never optimal, such periods of undercutting are not observed. The purpose of this section is to discuss examples in which such price wars can be observed in equilibrium.

As Maskin and Tirole(1988) and the first example considered in section 3 suggest, if firms are alternating between a high and low focal price, infrequent and near-permanent shocks to demand or cost that trigger a change from the low focal price to the high focal price would result in a firm immediately *lowering* price, to induce its rival to establish the new higher focal price. If marginal costs are also subject to shocks with low permanence, the shock increasing the focal price could trigger a prolonged period of undercutting before the new focal price is established.

As an illustration, suppose again that $k = 1/20$, δ is near 1, $c_L = 0$, $c_H = 0.1$, and $\alpha = 0.5$. As well, suppose that there are two possible demand functions, $D_L(p) = 1 - p$, and $D_H(p) = 1.6 - p$. The probability of demand remaining in its current state is γ and the probability of demand switching to the other state is $1 - \gamma$. Throughout this section, I will consider only γ sufficiently near 1. That is, I introduce infrequent demand shocks, which shift the monopoly price under low costs from 0.5 to 0.8, and under high costs from 0.55 to 0.85. There are now *three* payoff relevant variables: the rival's price, current marginal cost, and current demand. The strategies played by the firms are therefore allowed to depend on all of these variables.

Suppose first that γ equals one, so that demand never leaves its current state. In this circumstance, in the low demand state, the dynamic reaction functions given in Table 3 constitute an equilibrium, as indicated in the previous section. Similar reaction functions are given in Table 4 supporting the low-cost monopoly price as a focal price under high demand.[16] In this equilibrium, an undercut below the focal price results in a period of undercutting, with prices falling potentially as far as p_3 before the focal price is restored.

[16] Following previous examples, the equilibrium response to all prices above p_{16} is p_{16}.

For γ less than but sufficiently near one, one equilibrium is for the firms to play according to Table 3 when demand is low and according to Table 4 when demand is high. For example, such behavior is an equilibrium for $\gamma = 0.99$. As a result, one would occasionally observe extended price wars, in which firms undercut each other through a range of prices. Suppose that the firms are in the low-demand regime and setting the focal price of p_{10}, when demand shifts to the high-demand regime. Suppose it is firm i's turn to set price. Because it observes high demand and a rival price of p_{10}, firm i prefers to set p_7 in the hope that demand will stay high, costs will be high and the rival will initiate the new focal price of p_{16}, rather than restoring the new focal price itself. If costs shift to low, however, the rival will undercut to p_6. Supposing demand remains high, firm i will then either set p_3 or the focal price, depending upon marginal costs. Once p_3 has been set, the new focal price is restored with certainty.

When demand shifts from high to low, no price war is observed, since if a firm observes low demand and a price of p_{16}, the best response is to drop immediately to the new focal price. This suggestion that price wars would result from almost unanticipated *positive* demand shocks but not from negative shocks is similar to that of Maskin and Tirole (1988), in which a firm responds to a demand decrease by reducing price so far that its rival restores the new focal price with certainty in the following period. This prediction differs from Green and Porter (1984), in which price wars result from negative demand shocks. Slade (1989) predicts that periods of price volatility can result from either positive or negative demand shocks, although the severe price wars are anticipated when the shock is negative.[17]

Equilibrium price wars could also result from additional complexity in the marginal cost

[17] Rotemberg and Saloner (1986) present a model in which the collusive markup is greatest when demand is lowest; however, in this model, there are no price wars.

process. Suppose that demand is given every period by $D(p) = 1 - p$, but that marginal costs are subject to both low-persistence shocks and high-persistence shocks. In particular, marginal costs each period can be in either state A or state B. In state A, marginal costs are 0 (denoted c_L^A) with probability 0.5, and equal to 0.1 (c_H^A) with probability 0.5. In state B, marginal costs are higher, equaling 0.2 (c_L^B) with probability 0.5 and 0.3 (c_H^B) with probability 0.5. Costs remain in their previous state with probability γ, and switch states with probability $1 - \gamma$. In this example, γ will take on values near one.

I construct focal price equilibria supporting a focal price of 0.52 in state A and a focal price of 0.62 in state B (these prices were chosen as being between the high and low monopoly prices in each state; strategies could also be constructed to support other focal prices.). In this example, I suppose that $k = 1/100$. Equilibria are characterized in a fashion similar to that used in the previous section. In state A, a firm's strategy is described by four parameters with the same interpretation as in the previous section. The parameter $\underline{p}_L^A$, is the lowest price a firm in state A facing marginal costs of 0 would be willing to set before restoring the focal price of 0.52. $\underline{\underline{p}}_L^A$ is the lowest price this firm would set before dropping its price all the way to price $\underline{p}_L^A$. The lowest price a firm in state A with costs of 0.1 would be willing to set before restoring the focal price is $\underline{p}_H^A$. Finally, $\underline{\underline{p}}_H^A$ represents the lowest price the firm with costs of 0.1 in state A will set before dropping its price to $\underline{p}_H^A$. The parameters $\underline{p}_L^B, \underline{\underline{p}}_L^B, \underline{p}_H^B$ and $\underline{\underline{p}}_H^B$ are defined similarly for state B. Formally, the reaction function for a firm in state s is given by:

$$R(p, c_L^s, s) = \begin{cases} p^{fs} & \text{for } p > p^{fs}, \\ p^{fs} & \text{for } p = p^{fs}, \\ \underline{p}_H^s & \text{for } p \in (\underline{p}_H^s, p^{fs}), \\ p - k & \text{for } p \in (\underline{\underline{p}}_L^s, \underline{p}_H^s), \\ \underline{p}_L^s & \text{for } \underline{\underline{p}}_L^s > p > \underline{p}_L^s, \\ p^{fs} & \text{for } \underline{p}_L^s \geq p. \end{cases}$$

$$R(p, c_H^s, s) = \begin{cases} p^{fs} & \text{for } p > p^{fs}, \\ p^{fs} & \text{for } p = p^{fs}, \\ p - k & \text{for } p \in (\underline{\underline{p}}_H^s, p^{fs}), \\ \underline{p}_H^s & \text{for } p \in (\underline{p}_H^s, \underline{\underline{p}}_H^s], \\ p^{fs} & \text{for } \underline{p}_H^s \geq p. \end{cases}$$

Here, the focal price in state s is represented by p^{fs}. Equilibrium parameters for three different values of γ are given in table 5. For $\gamma = 0.995$, the equilibrium strategies are the same that would be used if $\gamma = 1$; that is, if switches between states A and B were completely unexpected. As γ is reduced, new equilibria can be constructed by adjusting the parameters of the reaction functions. This suggests that equilibrium strategies can be constructed when the probability of switching states is not so low that it does not affect the reaction functions.[18] As in the previous example, if costs switch from state B to state A, thus lowering the focal price, firms would immediately drop to the new focal price - the best response to a price above the current focal price is to set the current focal price. However, as costs switch from B to A, firms would enter into an extensive price war before setting the

[18]In this paper, I have not addressed whether price wars would be observed for intermediate values of persistence, but have been content with the intuition provided by considering shocks with a very small probability. Analysis of the effects of more likely cost or demand shocks would be complex and is a subject of future research.

higher focal price. This price war would be characterized by repeated undercutting, as in the equilibria in Section 4.

To date, the causes of periodic price wars that interrupt collusive prices[19] appear to be largely undetermined empirically (see Slade (1990) for a review of the small number of empirical studies of collusion in particular industries). The theoretical literature offers models in which price wars follow only positive demand shocks, only negative demand shocks, or both. Therefore, the question of the link between demand shocks and price wars is now an empirical one. Such a study would likely require a panel data set of price levels in local markets for similar products, and is a possible direction of future work.

6 Conclusions

This paper examines an alternating move price-setting duopoly model, in which stochastic marginal costs are modeled in a stylized way. The following intuition emerges. As the probability that costs will remain in their current state decreases, the incentive of a firm facing low costs to serve the entire market at the highest price possible increases. While this incentive is not sufficient to induce a firm facing low costs to deviate from the focal price, it can influence the off equilibrium path behavior, so that deviations from the focal price are met with lengthy bouts of undercutting. It is also shown that equilibria in which firms always match the current monopoly price cannot be constructed when the probability that marginal costs remain in their current state is low, provided the difference between high

[19] As opposed to price wars that ultimately lead to the end of a cartel; see for example Suslow(1988) and Dick(1996).

and low marginal costs is not excessive. Finally, this paper discusses how extended price wars may be observed in equilibria in models in which demand or cost are also subject to infrequent shocks.

The model presented in this paper is stylized, and has been studied to obtain intuition about the role of cost changes in pricing behavior. In terms of understanding the relationship between retail prices and marginal costs, this paper represents a first step. By introducing persistence into the marginal cost process, one could determine the degree of persistence required to support price equilibria in which the focal price depends on marginal cost. Such analysis could potentially give predictions regarding when price adjustment or price rigidity would be observed.

The theoretical model in this paper also suggests testable hypotheses concerning the nature of price wars, and the conditions that lead to them. Specifically, it suggests that one would observe wars when low probability demand or marginal cost shocks increase the monopoly prices, but not when they decrease the monopoly prices. Such a hypothesis is differs from those suggested by other models, and therefore allows for empirical work to determine which models seem most appropriate in different circumstances.

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Table 1: Alternating Focal Price

p	$\Pi(p, c_L)$	$\Pi(p, c_H)$	$R(p, c_L)$	$R(p, c_H)$
p_{20}	0	0	p_{10}	p_{11}
p_{19}	19	17	p_{10}	p_{11}
p_{18}	36	32	p_{10}	p_{11}
p_{17}	51	45	p_{10}	p_{11}
p_{16}	64	56	p_{10}	p_{11}
p_{15}	75	65	p_{10}	p_{11}
p_{14}	84	72	p_{10}	p_{11}
p_{13}	91	77	p_{10}	p_{11}
p_{12}	96	80	p_{10}	p_{11}
p_{11}	99	81	p_{10}	p_{11}
p_{10}	100	80	p_{10}	p_4
p_9	99	77	p_2	p_4
p_8	96	72	p_2	p_4
p_7	91	65	p_2	p_4
p_6	84	56	p_2	p_4
p_5	75	45	p_2	p_4
p_4	64	32	p_2	p_{11} w.p. β_H p_4 w.p.$1-\beta_H$
p_3	51	17	p_2	p_{11}
p_2	36	0	p_{10} w.p. β_L p_2 w.p. $1-\beta_L$	p_{11}
p_1	19	-19	p_{10}	p_{11}
p_0	0	-40	p_{10}	p_{11}

Table 2: Single Focal Price

p	$\Pi(p, c_L)$	$\Pi(p, c_H)$	$R(p, c_L)$	$R(p, c_H)$
p_{20}	0	0	p_{10}	p_{10}
p_{19}	19	17	p_{10}	p_{10}
p_{18}	36	32	p_{10}	p_{10}
p_{17}	51	45	p_{10}	p_{10}
p_{16}	64	56	p_{10}	p_{10}
p_{15}	75	65	p_{10}	p_{10}
p_{14}	84	72	p_{10}	p_{10}
p_{13}	91	77	p_{10}	p_{10}
p_{12}	96	80	p_{10}	p_{10}
p_{11}	99	81	p_{10}	p_{10}
p_{10}	100	80	p_{10}	p_{10}
p_9	99	77	p_2	p_4
p_8	96	72	p_2	p_4
p_7	91	65	p_2	p_4
p_6	84	56	p_2	p_4
p_5	75	45	p_2	p_4
p_4	64	32	p_2	p_{10} w.p. β_H p_4 w.p.$1-\beta_H$
p_3	51	17	p_2	p_{10}
p_2	36	0	p_{10} w.p. β_L p_2 w.p. $1-\beta_L$	p_{10}
p_1	19	-19	p_{10}	p_{10}
p_0	0	-40	p_{10}	p_{10}

Table 3: Single Focal Price

p	$\Pi(p, c_L)$	$\Pi(p, c_H)$	$R(p, c_L)$	$R(p, c_H)$
p_{10}	100	80	p_{10}	p_{10}
p_9	99	77	p_5	p_5
p_8	96	72	p_5	p_5
p_7	91	65	p_5	p_5
p_6	84	56	p_5	p_5
p_5	75	45	p_4	p_{10}
p_4	64	32	p_2	p_{10}
p_3	51	17	p_2	p_{10}
p_2	36	0	p_{10}	p_{10}
p_1	19	-19	p_{10}	p_{10}
p_0	0	-40	p_{10}	p_{10}

Table 4: Focal Price Equilibrium Strategies: High Demand

p	$R(p, c_L)$	$R(p, c_H)$
p_{16}	p_{16}	p_{16}
p_{15}	p_7	p_7
p_{14}	p_7	p_7
p_{13}	p_7	p_7
p_{12}	p_7	p_7
p_{11}	p_7	p_7
p_{10}	p_7	p_7
p_9	p_7	p_7
p_8	p_7	p_7
p_7	p_6	p_{16}
p_6	p_3	p_{16}
p_5	p_3	p_{16}
p_4	p_3	p_{16}
p_3	p_{16}	p_{16}
p_2	p_{16}	p_{16}
p_1	p_{16}	p_{16}
p_0	p_{16}	p_{16}

Table 5: Equilibrium Parameters for the Model With Persistent Cost Shocks

	$\gamma = 0.995$	$\gamma = 0.99$	$\gamma = 0.98$
$\underline{p}_L^A$	p_9	p_9	p_9
$\underline{\underline{p}}_L^A$	p_{18}	p_{18}	p_{17}
$\underline{p}_H^A$	p_{24}	p_{24}	p_{24}
$\underline{\underline{p}}_H^A$	p_{46}	p_{46}	p_{45}
$\underline{p}_L^B$	p_{28}	p_{28}	p_{28}
$\underline{\underline{p}}_L^B$	p_{35}	p_{36}	p_{36}
$\underline{p}_H^B$	p_{41}	p_{41}	p_{41}
$\underline{\underline{p}}_H^B$	p_{57}	p_{57}	p_{57}