Innovation, Market Structure and the Holdup Problem: Investment Incentives and Coordination

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Abstract: I analyze the innovation incentives under monopoly and duopoly provision of horizontally differentiated products purchased via bilateral negotiations, integrating the market structure and innovation literature with the holdup literature. I show that competition can improve local incentives for non-contractible investment. Because innovation levels are generally strategic substitutes, however, there can be multiple duopoly equilibria. In some circumstances, monopoly can provide a coordination device that can lead to greater expected welfare despite inferior local innovation incentives. The conditions for this to be the case, however, are quite restrictive.

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I. Introduction

While many papers have analyzed the effect of market structure on innovation incentives (Arrow, 1962; Dasgupta and Stiglitz, 1980; Bulow, 1982; and many others), there has been very little analysis of this issue in markets where trade occurs in bilateral contracts rather than in a spot market. Most prior papers dealing with bilateral contracts and non-contractible investments have assumed the existence of bilateral monopoly (Williamson, 1985; Tirole, 1986; Grossman and Hart, 1986; Hart and Moore, 1990 among others), bypassing the question of how market structure affects investment incentives.

When customers are not final consumers, but rather firms purchasing inputs, these firms will often negotiate with multiple suppliers. In fact, this paper grows out of analysis of a proposed merger between the two dominant suppliers of accounting software for large law firms. In that market, law firms would negotiate with two different software firms over prices for a customized version of that firm’s software. The welfare consequences of this merger hinged to a large extent on how the merger would affect the incentives of the merged firm to improve each product. Similar negotiations occur in all types of business planning software. Companies such as Oracle or PeopleSoft offer large, customizable software packages that perform a myriad of essential tasks such as billing and accounting, human resources management, supply chain management and many others. Large companies do not buy these products "off the shelf." Rather, they send out a request for proposal to several firms and negotiate with each one. Health maintenance organizations negotiate with multiple hospitals in an area to choose which one will be their exclusive provider. Corporations often bargain with many different law firms before choosing which one will represent them in a certain class of cases. Indeed, any time a business buys a service or a customized good that cannot be easily resold, there can be individualized pricing resulting from direct negotiations with the supplier.
In markets where trade is governed by bilateral contracts, the issue of output distortion does not arise because trade is negotiated individually.\(^1\) The effect of market structure on welfare is only through its effect on non-contractible investments. Moreover, the effect of market structure on product innovation incentives is substantially different when there are no set prices. For example, the “replacement effect” and the “product inertia effect” that greatly influence innovation incentives in standard models (Greenstein and Ramey, 1998 is one such example) are not present in this paper.

I use a model with a continuum of consumers uniformly distributed along the unit interval and two products, A and B, located at the endpoints. Consumers demand at most one unit of one product. Each product has a common value to all customers, which a firm can increase through quality-improving investment. Each product also has an idiosyncratic value to each consumer based on the consumer’s location. When different firms own A and B, each firm bargains with each customer (under complete information), who chooses the product that provides her with the greatest net surplus. If one firm owns both products, the firm offers each consumer only the product that a given consumer values more.

Consumers always have the option of buying a third product (which could be in-house production), but this product provides less gross value (i.e., not net of price) to all consumers than does either product A and B. So, while this third option is every consumer’s outside option under monopoly ownership, it has no effect on the duopoly bargaining.

In the context of this model, I show that competition alleviates the holdup problem by providing superior incentives for non-contractible investment. Since only the sellers are making non-contractible investments, if they had all the bargaining power, investment incentives would be optimal. Because the split of the surplus is determined by an outside option bargaining game, however, a seller only gets the entire marginal surplus from product improvements (though, not the entire total surplus) if the buyer has a

\(^1\) Of course, there can be bargaining failures. In the model below, I assume that bargaining always results in the efficient transaction taking place. Even when bargaining failures can occur, however, market structure still won’t affect the degree of output distortion unless the probability of a bargaining failure is correlated with market structure.
binding outside option (an option that gives the buyer at least half the total surplus available from trade with its preferred seller). Otherwise, the seller only receives half the marginal surplus from product improvements. Competition alleviates the holdup problem because competition increases the options available to buyers, making it more likely that a buyer will have a binding outside option when negotiating with its preferred supplier. When more buyers have binding outside options, the marginal return (to the seller) from product improvement is closer to the social optimum.

Competition may not always increase total welfare, however, because it can create a coordination problem. There can be multiple equilibria because the firms’ innovation levels are (usually) strategic substitutes. This is due to a market share effect; the more my rival innovates the larger is her market share and the smaller is mine, reducing my incentive to innovate. If the degree of product differentiation and innovation costs are small, then the incentive to concentrate all innovation on one product is strong enough that in monopoly and duopoly, only one product is developed. Similarly, when product differentiation and innovation costs are large, the benefits to innovating both products are large, both in terms of meeting the needs of all customers and for keeping innovation costs down (since these costs are convex). So, in duopoly and monopoly both products are developed. For intermediate levels of product differentiation and innovation costs, however, there can be multiple duopoly equilibria. In this case, the equilibrium with the most asymmetric product development generates the most social welfare because it has the least duplicative investment. If the value of the third option is large enough (the monopolist gets a large enough share of the marginal benefits from innovation), then the monopolist may choose asymmetric development when a social planner would. Because monopoly innovation incentives are worse (locally) than in duopoly, this outcome is still worse than the asymmetric duopoly equilibrium, even if it is better than the symmetric duopoly equilibrium.

This is not the first paper to study the relationship between competition and non-contractible investment incentives. Felli and Roberts (2000) also show that competition can alleviate the holdup problem when only sellers invest. Their model assumes that sellers can make take it or leave it offers to buyers, guaranteeing that sellers get the entire marginal surplus from the transaction (whether there is
competition or not). Like Felli and Roberts (2000), Cole et. al. (2001) model the effect of competition in
a model with match specific investments (not product innovation investments as in this paper). An
additional difference between their paper and this one is that their analysis of competition considers the
effect of adding closer substitutes, whereas in this paper, the number of available products is fixed, but
there is more competition when the products are under separate ownership. Neither of the above papers
considers the effect of ownership on the holdup problem. Che and Gale (2000) show that competition in
the form of contests can help improve sellers’ incentives to make non-contractible cooperative
investments.² Inderst and Wey (2003) analyze the effect of market structure on production technology
choice with negotiated prices. Their model uses a different bargaining solution (Shapley value) and
focuses on how different upstream and downstream market structures affects the seller’s choice between a
high fixed and low marginal cost technology versus low fixed cost and high marginal cost technology
rather than on how market structure affects a seller’s incentive to invest in product quality.

The plan of the rest of the paper is as follows. Section II develops the duopoly model, while
Section III develops the monopoly model. Section IV discusses the welfare comparisons between the
two. Section V concludes. Appendix A describes the monopoly outcome for all possible parameter
constellations. The proof of the second proposition is in Appendix B.

II. The Duopoly Model

Consider a linear city model where product A is located at point zero and B is at point one.³⁴
There are two periods in the model. In period 0, products A and B start with an equal general value

² The second sourcing literature (Demski et al 1987; Riordon and Sappington 1989; and Anton and Yao 1987, 1989,
and 1992) is also related. That literature, however, focuses on when it is optimal for a single buyer to use more than
one supplier. Thus, the questions it addresses and its conclusions are quite different from those of this paper.
³ Allowing the firms to choose their locations would not qualitatively change the greater incentive to innovate with
competition due to the greater value of consumers’ outside option. If location remained fixed after a change in
market structure, it would not affect the nature of the welfare comparisons either. If a monopolists could easily
change the product locations after buying the other product, then the welfare effects of monopoly versus duopoly
(gross of price paid) to customers, $V$. During period 0, the suppliers of product A and B choose an amount of, non-stochastic, product development innovation. If supplier $i$ develops its product by an amount $d_i$, then it increases the general value of its product by that amount.\(^5\) This costs the supplier $C(d_i) = c d_i^2 / 2$.\(^6\) The products are differentiated by their location in product space. In period 1, customers make purchase decisions. A customer at point $\epsilon$ between 0 and 1 that purchases product A receives a value of $V + d_A - k \epsilon$, whereas, if it purchases product B it receives a value of $V + d_B - k (1- \epsilon)$. $k$ measures the cost of purchasing a product whose specifications are one unit away from one’s optimal specifications.\(^7\)

\[\begin{array}{ll}
\text{Period 0} & \text{Period 1} \\
\text{Firms choose $d_A$ and $d_B$} & \text{Consumer at $\epsilon$ values $A$ at $V + d_A - k \epsilon$ and $B$ at $V + d_B - k (1- \epsilon)$} & \text{Firms bargain with consumers over prices. Consumers make purchase decisions}
\end{array}\]

I make the following additional assumptions:

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4 Analyzing differentiated products is natural in this setting since negotiated trade is far less likely in a market for homogenous products.

5 Notice, I am following Che and Hausch (1999) in considering the case of cooperative investments rather than selfish investments, i.e., investments that benefit one’s trading partner rather than oneself. In the markets I have in mind (such as software markets), the dominant form of innovation is product improvement innovation, which is fully cooperative.

6 I assume quadratic costs for tractability. The key assumption is that innovation costs are convex and that there are no innovation synergies between the products, even if they are under common ownership.

7 Since price is determined individually for each consumer, the particular form of transportation costs does not qualitatively affect the results. The key effect of transport costs is that consumers closer to a given product have an outside option that is less desirable relative to the product they purchase.
• There is a unit mass of customers uniformly distributed between zero to one (I discuss the effect of relaxing this assumption below).

• Customers only need one unit of one of the two products.

• Each product is produced with zero marginal cost.

• There is a third product that provides a fixed value $v$ to all customers regardless of location. One can think of $v$ as being the value consumers get by making this product themselves.

• The third product provides less gross value than either A and B to all customers, $v \leq V - k$.  

• Consumers can only buy the products from suppliers; resale is infeasible or prohibited.

• Sellers negotiate with each buyer individually in Period 1 and can decide what product(s) to offer.

• Each customer’s value of $\epsilon$ is common knowledge to the customer and both suppliers. Notice that I do not allow contracts in period 0 between customers and firms. There are several reasons for this. First, firms may not know the identity of all their potential customers in period 0, making ex ante contracts infeasible. Second, the type of contract suggested by MacLeod and Malcomson (1993) for inducing efficient investments in a similar situation requires (in addition to the seller knowing the identity of its customers) ensuring that a customer’s outside option always binds. This necessitates payments from seller to buyer if the buyer does not trade with the seller. Unless the seller gets the entire surplus from their interaction, up front payments from the buyer to the seller will be strictly less than this

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8 Law firms could build their own accounting and billing software, but this is typically much less efficient than buying this software from an outside producer.

9 This assumption allows me to use a bargaining solution without uncertainty to determine the price that each customer pays for the product and eliminates the possibility of bargaining failures. While this will not exactly reflect reality, for many markets it is not that far off. In the business planning software market, for example, the customers send out detailed requests for proposals, host demonstrations where they inquire about the capabilities that they are most interested in, and ask for specific customization of the software to meet their needs. All of this provides the supplier with detailed information about how important different capabilities are to that customer, giving it a very good idea of how the customer will value its product relative to its competitors.
payment. If potential buyers can misrepresent (in period 0) their desire to purchase this product, then buyers with no intention to trade could induce the seller to give them this contract that gives them a positive surplus.\textsuperscript{10} These losses could easily exceed the seller’s share of the added surplus from more efficient development incentives. Finally, in many cases, such as the market for accounting software for law firms, we do not observe such contracts.

Since $\epsilon$ is common knowledge in period 1, and purchase decisions are made by bargaining without transactions costs, each customer chooses the product that maximizes the surplus from the transaction. That is, it will purchase the product that gives it the greatest gross value: a customer chooses product $A$ if and only if its $\epsilon < (d_A - d_B + k)/2k \equiv \epsilon^*$.\textsuperscript{11}

A bargaining game between the consumer and the two potential suppliers determines the price each consumer pays.\textsuperscript{12} I posit an alternating offer bargaining game of the type used in Bolton and Whinston (1993). The only difference between that bargaining game and this one is that in their game there are two buyers and one seller rather than two sellers and one buyer. Thus, the equilibrium of this bargaining game is a straightforward application of their equilibrium. They show that the equilibrium of the three player bargaining game is identical to the equilibrium of an outside option bargaining game between the two parties with the largest joint surplus where the party with the alternative trading partner has an outside option of trading with its less preferred partner and obtaining the entire surplus from that trade. This structure is appropriate since buyers demand at most one unit of one product, so accepting one

\textsuperscript{10} This is not inconsistent with the assumption that consumer valuations are common knowledge in period 1. In fact, one would expect much of the seller’s knowledge of a buyer’s valuation to come via the negotiation process.

\textsuperscript{11} Of course, a customer purchases the product that provides it with the greatest net value. The outcome of the bargaining game guarantees that the product with the greatest gross value will always have weakly the greatest net value.

\textsuperscript{12} Bargaining over price is common for intermediate goods. It is certainly the dominant method of trade when large firms purchase major software systems.
offer necessarily means terminating bargaining with the other seller. This means that buying from the alternate seller is very much like exercising an outside option.

It is well known that the solution to this outside option bargaining game (in the current application) gives the buyer the larger of half the surplus from the transaction and the surplus it could get from its outside option (getting the other product for free) (Rubenstein 1982; Shaked and Sutton, 1984). This result is due to the fact that exercising this option (buying from the other seller) requires terminating the bargaining with the other seller. If the bargaining equilibrium gives the buyer more surplus than she can get from exercising her outside option, then the outside option is no longer relevant.

The fact that outside options create such a constraint is what drives the difference in monopoly and duopoly investment incentives. If one used the Nash bargaining solution, where outside options are incorporated into the disagreement payoffs, then local innovations incentives would be independent of market structure. The reason for this is that while monopoly reduces the buyer’s disagreement payoff, it does not change the buyer’s share of the surplus above the disagreement payoffs in the Nash bargaining solution (where each side always receives half of this surplus). Thus, the seller’s incentive to invest to increase this surplus is not affected by market structure. In the outside option bargaining game, by contrast, if changing from duopoly to monopoly makes the buyer’s outside option no longer binding, this increases the buyer’s share of the marginal surplus from zero to one-half, thereby reducing the seller’s incentive to invest to increase this surplus.

In this outside option bargaining game, a buyer located at $\varepsilon \leq \varepsilon^*$ will pay the following price for product A:

$$p_A(\varepsilon) = \text{Min} \left\{ \frac{1}{2} (V + d_A - k\varepsilon), (V + d_A - k\varepsilon) - (V + d_B - (1 - \varepsilon)k) \right\}.$$  

(1a)

Similarly, a buyer located at $\varepsilon > \varepsilon^*$ will pay the following price for product B:

$$p_B(\varepsilon) = \text{Min} \left\{ \frac{1}{2} (V + d_B - (1 - \varepsilon)k), (V + d_B - (1 - \varepsilon)k) - (V + d_A - k\varepsilon) \right\}.$$  

(1b)
Given these prices, A and B in period 0 choose $d_A$ and $d_B$ respectively to maximize their profits.

The profit functions are given below, where $f$ is the probability density function (and $F$ the cumulative distribution function) for the uniform distribution ($f(\epsilon) = 1$ if $\epsilon \in [0,1]$, otherwise 0).

\[
\pi_A = \int_0^{\epsilon^*_A} p_A(\epsilon)f(\epsilon)d\epsilon - C(d_A) \quad (2a)
\]

\[
\pi_B = \int_{\epsilon^*_B}^{1} p_B(\epsilon)f(\epsilon)d\epsilon - C(d_B). \quad (2b)
\]

Differentiating (2), with these pricing functions, gives the following first order conditions:

\[
cd_A = F(\frac{d_A - d_B + k}{2k}) - \frac{1}{2} F(\frac{d_A - 2d_B - V + 2k}{3k}) \quad \text{and} \quad (3a)
\]

\[
cd_B = 1 - F(\frac{d_A - d_B + k}{2k}) - \frac{1}{2} (1 - F(\frac{2d_A - d_B + V + k}{3k})). \quad (3b)
\]

Any Nash equilibrium product development levels must satisfy these two equations, which simply equate the marginal cost and marginal benefit of product development for each firm. Firm A’s benefit from developing its product comes solely from the higher price its customers (those located to the left of $(d_A - d_B + k)/2k$) will pay when its product is more attractive. For those A customers whose preference for A over B is not too great, i.e., those located between $(d_A - 2d_B - V + 2k)/3k$ and $(d_A - d_B + k)/2k$, product B provides a binding outside option. While for those customers whose preference for product A is strong, i.e., those located to the left of $(d_A - 2d_B - V + 2k)/3k$, product B is not a binding outside option. Equations (3a) and (3b) indicate that a producer obtains the full marginal increase in value from product development from its customers with binding outside options but only half the increase in value from its customers whose outside options are not binding.

While in models of innovation with non-negotiated prices, innovative investments are strategic substitutes, this is not always the case here. Typically, innovative investments are strategic substitutes because increased innovation by the rival decreases one’s market share, reducing one’s incentive to invest. With negotiated prices, however, there is an additional effect. When one’s rival innovates, this improves the outside option for one’s customers. This increases the fraction of one’s customers whose
outside option is binding, which (because of the effect of outside options on the bargaining equilibrium) increases the marginal return to innovation. Of course, if product development enhanced the value of a product much more for customers nearby (in product space) than for more distant customers, then this effect would be diminished. Innovation targeting more distant customers would enhance this effect.

Because customers are uniformly distributed between zero and one, the market share effect always dominates (innovative investments are strategic substitutes) whenever some consumers buy each product. If all consumers buy the same product, then increased investment in that product only affects the outside option for the other firm’s customers, but does not reduce its market share. This makes investment by the non-selling firm a strategic complement for investment by the selling firm. If the density of consumers were greater near the middle of the unit interval than at the extremes, then investment by the firm selling to a positive (but small) fraction of consumers could be a strategic complement for investment by the firm selling to the bulk of the customers.

Differentiating the marginal benefit functions (the right hand sides of (3a) and (3b)) with respect to rival’s development gives the precise conditions for when innovation is a strategic substitute or complement. For \( d_A \), \( d_B \) will be a strategic substitute for \( d_A \) if and only if:

\[
\frac{f(d_A - d_B + k)}{2k} > \frac{2}{3} \cdot \frac{f(d_A - 2d_B - V + 2k)}{3k}
\]

Similarly, \( d_A \) will be a strategic substitute for \( d_B \) if and only if:

\[
\frac{f(d_A - d_B + k)}{2k} > \frac{2}{3} \cdot \frac{f(2d_A - d_B + V + k)}{3k}
\]

Figure 1 depicts the reaction functions for \( d_A \) and \( d_B \) for one particular set of parameter values.

**Figure 1 Here**

Notice that there are three possible equilibria in the figure (the reaction functions intersect three times). There is a symmetric equilibrium where both products are developed equally and two equilibria where only one product is developed and sold. In the software example, in the symmetric equilibrium both firms would continue to make enhancement to their programs and thus both firms would continue to make new sales. In the asymmetric equilibria, only one firm would invest in improving its program and,
as a result, only this firm would make new sales; the other firm would simply survive on supporting its installed base of customers. Of course, the number of equilibria is dependent on the values of the parameters. If product development is very costly \((c)\) is large), then there will only be a symmetric equilibrium. In Figure 1, however, product development is cheap enough that if one firm (say A) is not going to develop its product, then B’s level of development will be large enough that no customers will buy product A, giving A no incentive to develop its product. One can also see in Figure 1 that when one firm’s investment is very small, it becomes a strategic complement for the other firm’s investment (the reaction functions have a positive slope in these regions).

Figure 2 depicts the reaction functions when product development is relatively more important (it is cheaper and the initial value of the product is smaller). This creates five, rather than three, equilibria. In addition to the symmetric and corner equilibria, there are two interior asymmetric equilibria.

**Figure 2 Here**

The interior asymmetric equilibria occur when one product (say B) is developed enough more than A that the outside option of purchasing product B is binding for all customers that purchase product A. The reverse, however, is not true. Thus, even though A has a smaller market share, its development incentive is only slightly smaller than B’s since A receives the entire surplus from added development from all its customers. This ensures that its market share is large enough to justify its development level.

Solving for the Nash equilibrium development levels requires explicitly solving the two duopoly first order conditions, equations (3a) and (3b). Notice that there are three important expressions in these equations: \((d_A - d_B + k)/2k, (d_A - 2d_B - V + 2k)/3k, \) and \((2d_A - d_B + V + k)/3k\). The first gives the location of customer who is indifferent between the two products. The second and third are the locations of the customer whose outside option is just binding (purchasing her less preferred product gives her exactly half the gross utility she gets from purchasing her preferred product). I call these the outside option cutoffs. Solving equations (3a) and (3b) requires separately considering whether each of these locations are inside or outside the unit interval. To simplify the exposition, from here on out I will make the following assumption:
Assumption (*): \[ V \geq (2ck + 1)/c \]

Because this assumption guarantees that every consumer’s outside option is always binding (the outside option cutoffs are always outside the unit interval in equilibrium), it collapses the six possible cases into two.\(^{13}\) Either both firms have sales and all their customers have a binding outside option, or only one firm has sales but all its customers have a binding outside option. To see that this is the case, I now solve equations (3a) and (3b) assuming that the outside option cutoffs are outside the unit interval and then verify that this is the case whenever \( V \geq (2ck + 1)/c \). The first order conditions are now:

\[
 cd_A = F\left(\frac{d_A - d_B + k}{2k}\right) \quad \text{and} \quad cd_B = 1 - F\left(\frac{d_A - d_B + k}{2k}\right) \quad (5) 
\]

If both products are sold, the solution is:

\[
 d_A = d_B = 1/2c \quad (6)
\]

Each firm’s second order condition is \( 2ck - 1 > 0 \), so this equilibrium only exists if \( ck > 1/2 \).

If only B is sold, the solution to the first order conditions is:

\[
 d_A = 0; d_B = 1/c \quad (7)
\]

For only B to have sales in equilibrium, the indifferent consumer must be located to the left of zero. That is, \( (d_A - d_B + k)/2k \leq 0 \), which means that this equilibrium only exists if \( ck \leq 1 \).

It is easy to verify that in both equilibria \( (2d_A - d_B + V + k)/3k \geq 1 \) and \( (d_A - 2d_B - V + 2k)/3k \leq 0 \) provided \( V \geq (2ck + 1)/c \). There can be no other equilibrium with interior outside option cutoffs since these reduce development incentives, which increases the value of \( (2d_A - d_B + V + k)/3k \) and decreases the value of \( (d_A - 2d_B - V + 2k)/3k \).

\(^{13}\) There would be three cases where the indifferent customer is inside the unit interval, so that both products have strictly positive sales in equilibrium, and three cases where it is outside the unit interval, so that only one product has strictly positive sales in equilibrium. I consider cases that are identical except that the names of the firms are switched to be the same case.
While Assumption (*) greatly simplifies the analysis, there are still multiple equilibria whenever \( \frac{1}{2} < ck \leq 1 \). Section IV discusses how the equilibria compare in terms of social welfare.

**III. The Monopoly Model**

Because trade is negotiated individually under perfect information, market structure does not affect the purchase decisions of any customers (though, it does affect the price). The key difference between monopoly and duopoly is that when one firm owns both A and B, it offers each customer only that customer’s most preferred product. If that is product B, then the monopolist does not offer that customer product A. Of course, the customer knows that the monopolist sells product A to other customers, but she cannot threaten to buy product A if the monopolist does not give her a good deal on product B since there is not a separate firm that sells A. In negotiated sales markets, the products are not available “off the shelf” at fixed prices. So, the customer has to negotiate the purchase of either product with the monopolist. Since everyone knows that this customer prefers product B, the monopolist has no reason to ever negotiate over the sale of product A.\(^{14}\) This customer will always pay more for B. Also, note that a customer cannot buy either product from another consumer. If that were possible, then negotiated sales (where each customer is charged a different price) would be infeasible under any market structure. That is why one sees negotiated sales mostly in markets such as complex software systems, where consumers require that the product be customized for their individual needs and that the seller provide extensive installation assistance. Furthermore, copyright restrictions often make resale of such products illegal, even if it were feasible. The fact that a multi-product monopolist can eliminate competition between substitute products is one of the key distinguishing features of these markets.

Now that customers are only offered A or B, but not both, the third product (or in-house production) that gives all customers a value of \( v \) serves as every customer’s outside option. As a result, a customer at \( \epsilon \leq \epsilon^* \) now pays the following price for product A:

\(^{14}\) Even if the monopolist was uncertain as to which product the customer preferred, it would not let the products compete with each other. It would offer both at high prices since the consumer’s only option to dealing with the monopolist is now the third, inferior, product.
\[ p_A(\varepsilon) = \min\left\{ \frac{1}{2}(V + d_A - k\varepsilon), (V + d_A - k\varepsilon) - v \right\} . \]  

(8a)

Similarly, a buyer located at \( \varepsilon > \varepsilon^* \) will pay the following price for product B:

\[ p_B(\varepsilon) = \min\left\{ \frac{1}{2}(V + d_B - (1 - \varepsilon)k), (V + d_B - (1 - \varepsilon)k) - v \right\} . \]  

(8b)

With these pricing functions, the monopolist’s profit function is the following:

\[ \pi_M = \int_{0}^{\varepsilon^*} p_A(\varepsilon)f(\varepsilon)d\varepsilon - C(d_A) + \int_{\varepsilon^*}^{1} p_B(\varepsilon)f(\varepsilon)d\varepsilon - C(d_B) \]  

(9)

Differentiating (9) gives the following first order conditions in the monopoly case:

\[ \begin{align*}
    cd_A &= F\left( \frac{d_A - d_B + k}{2k} \right) - \frac{1}{2} F\left( \min\left( \frac{d_A - 2V + 2V}{k}, \frac{d_A - d_B + k}{2k} \right) \right) \quad \text{and} \\
    cd_B &= 1 - F\left( \frac{d_A - d_B + k}{2k} \right) - \frac{1}{2} \left( 1 - F\left( \max\left( \frac{-d_B + k + 2V}{k}, \frac{d_A - d_B + k}{2k} \right) \right) \right) .
\end{align*} \]  

(10a)

Because \( v \leq V - k \), the right hand sides of (10) are (at least weakly) smaller than the right hand sides of (3), the first order conditions in the duopoly case. The reason the marginal benefit to innovating is smaller with monopoly than duopoly is that, since the value of every consumer’s outside option is smaller, a smaller fraction of consumers will have a binding outside option. Thus, the seller will more often split the gross social surplus from the transaction rather than get the residual surplus over the consumer’s outside option. So, even though the total price the producer receives for the product is (weakly) greater for every consumer, the marginal effect of innovation on price is (weakly) smaller with monopoly than with duopoly. This is due to the fact that outside options operate as constraints on the equilibrium outcome in the bargaining game rather than as threat points.

Investment incentives are distorted the least the larger is the value of the third product. Monopoly generates the most social welfare when \( v = V - k \). In this case, the first order conditions are:

\[ \begin{align*}
    cd_A &= F\left( \frac{d_A - d_B + k}{2k} \right) - \frac{1}{2} F\left( \frac{d_A + 2k - V}{k} \right) \quad \text{and} \\
    cd_B &= 1 - F\left( \frac{d_A - d_B + k}{2k} \right) - \frac{1}{2} \left( 1 - F\left( \frac{-d_B - k + V}{k} \right) \right) .
\end{align*} \]  

(11a)
Any solution to these equations clearly requires that \( d_A, d_B \leq 1/c \). This guarantees that 
\[(d_A + 2k - V)/k < 0 \text{ and } (-d_B - k + V)/k \geq 1\] whenever Assumption (*) holds. That is, if \( v = V - k \) then every consumer’s outside option is binding. Thus, the first order conditions are identical to those under duopoly, leading to the same solutions. Recall, however, that when \( 1/2 < ck \leq 1 \), there were two possible solutions to these first order conditions. To determine which one the monopolist chooses, one must compare its profits in each case. Evaluating (9) when \( d_A = d_B = 1/2c \) reveals that the monopolist’s profit from developing both products is \( V - v + (1 - ck)/4c \). Alternatively, evaluating (9) when \( d_A = 0, d_B = 1/c \) reveals that the monopolist’s profit from developing only products is \( V - v + (1 - ck)/2c \). If \( 1/2 < ck \leq 1 \), then the monopolist will develop only one product whereas with duopoly either one or both products may be developed.

If the third product is less valuable than \( V - k \), then it is possible that not all consumers will have a binding outside option. This occurs for product \( i \) if \( v < (d_i + V)/2 \). This leads to a greater distortion (downward) of development incentives since the right hand side of the first order conditions are strictly smaller if some consumers do not have a binding outside option. As a result, the monopoly outcome is not identical to one of the duopoly equilibria. A complete description of the monopolist’s optimal level of product development for all parameter values is in Appendix A.

**IV. Welfare Comparisons**

Social welfare is the total value consumers receive from their purchases less innovation costs.

\[
SW = \int_0^{\epsilon*} (V + d_A - ke) f(\varepsilon) d\varepsilon + \int_{\epsilon*}^1 (V + d_B - (1 - k)e) f(\varepsilon) d\varepsilon - C(d_A) - C(d_B).
\] (12)

The first order conditions for social welfare maximization are the following:

\[
cc d_A = F\left(\frac{d_A - d_B + k}{2k}\right) \quad \text{and} \quad (13a)
\]

\[
cc d_B = (1 - F\left(\frac{d_A - d_B + k}{2k}\right)) \quad \text{.} \quad (13b)
\]
By comparing these first order conditions with those of the duopoly and monopoly models ((3) and (10) respectively) one can see that the social planner wants (weakly) larger local innovation incentives than exist in either duopoly or monopoly, but that local innovation incentives are closer to the social optimum under duopoly than monopoly. In fact, under Assumption (*), the duopoly first order conditions are identical to those of the social planner. This occurs because when the outside options are always binding, the firms get all the increased surplus from additional product development.

Of course, socially inferior local innovation incentives do not necessarily imply socially inferior innovation levels. If there are multiple duopoly equilibria, welfare depends on which duopoly equilibria obtains. The first proposition demonstrates that welfare is greater in the more asymmetric equilibrium.

Proposition 1. Say Assumption (*) holds. Whenever there are multiple duopoly equilibria, social welfare is always largest when only one product is developed

Proof. Evaluate the social welfare function, (12), for each equilibria. The social welfare when both products are developed is \( V + (1 - ck)/4c \). The social welfare when one product is developed is \( V + (1 - ck)/2c \). This is greater than the social welfare when only one product is developed if and only if \( ck < 1 \), a necessary condition for developing only one product to be a duopoly equilibrium. Q.E.D.

Proposition 1 holds even if Assumption (*) does not, but the proof is much more tedious and requires examination of the many other possible duopoly equilibria that can exist when this assumption does not hold. The advantage of asymmetric equilibria is that they have less duplicative investment and more customers are taking advantage of the product that has had more quality-enhancing investment. The disadvantage is that customers are, on the average, purchasing products further away from their ideal location in product space. If an asymmetric equilibrium exists, however, this effect cannot be that strong or the firm that does not sell its product would be able to do so. Thus, the first effect dominates whenever there are multiple duopoly equilibria.
Because, if multiple equilibria exist, the asymmetric equilibrium generates more social welfare than the symmetric one, the level of social welfare in a duopoly is uncertain. When one firm owns both products, however, this uncertainty is absent. If asymmetric development levels maximize monopoly profits, then monopoly could have the advantage of ensuring that this more efficient distribution of development occurs. Of course, even if monopoly guarantees asymmetric development, the asymmetric monopoly outcome will often be inferior to the asymmetric duopoly outcome because of the inferior local development incentives (fewer customers have binding outside options). Just how inferior these incentives are depends on \( v \), the value of the third product that provides the outside option for buyers once A and B are under common ownership.

As we saw in the last section, if \( v = V - k \), indeed any time it exceeds \((1/c + V)/2\), then the local innovation incentives are identical under monopoly and duopoly. If \( ck < 1/2 \) or \( ck > 1 \), then, since there is an unique duopoly equilibrium which corresponds exactly to the monopoly outcome, social welfare is identical in each market structure. For intermediate values of \( ck \), \( 1/2 \leq ck \leq 1 \), monopoly can generate more social welfare than duopoly, but can never generate less. In this range, the monopoly outcome is the asymmetric duopoly equilibrium, which generates more social welfare than the symmetric duopoly equilibrium.\(^{15} \) For smaller values of \( v \), however, since the monopoly product development is not necessarily identical to one duopoly equilibrium, monopoly can generate less social welfare than one or all possible duopoly equilibria. The following proposition precisely describes the social welfare rankings of the two possible market structures for different parameter values.

\(^{15} \) This is always the case when \( v = V - k \) only because of Assumption (*). For smaller \( V \), monopoly can generate less welfare than the asymmetric duopoly equilibrium even if \( v = V - k \).
Proposition 2. Say Assumption (*) holds and say $X \succ (\sim) Y$ if and only if welfare in outcome $X$ is strictly greater than (equal to) welfare in outcome $Y$. The following table compares the welfare in duopoly and monopoly for different parameter values.

<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>Duopoly Eq.</th>
<th>Welfare Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V - 2v &lt; -1/c &amp; , ck \leq 1/2$</td>
<td>Asymmetric Eq.</td>
<td>Monopoly $\sim$ Duopoly</td>
</tr>
<tr>
<td>$V - 2v &lt; -1/c &amp; , ck \in (1/2,1]$</td>
<td>Asymmetric &amp; Symmetric Eq</td>
<td>Monopoly $\sim$ Asymmetric Eq $\succ$ Symmetric Eq</td>
</tr>
<tr>
<td>$V - 2v &lt; -1/2c &amp; , ck &gt;1$</td>
<td>Symmetric Eq.</td>
<td>Monopoly $\sim$ Duopoly</td>
</tr>
<tr>
<td>$-1/c \leq V - 2v &amp; , ck \leq 1/2$</td>
<td>Asymmetric Eq.</td>
<td>Duopoly $\succ$ Monopoly</td>
</tr>
<tr>
<td>$-\frac{1}{c} \leq V - 2v &lt; \text{Max}[\frac{-1}{2c}, -\sqrt{\frac{k(2ck - 1)}{2c}}] &amp; , ck \in (\frac{1}{2}, 1]$</td>
<td>Asymmetric &amp; Symmetric Eq</td>
<td>Asymmetric Eq $\succ$ Monopoly $\succ$ Symmetric Eq</td>
</tr>
<tr>
<td>$\text{Max}[\frac{-1}{2c}, -\sqrt{\frac{k(2ck - 1)}{2c}}] \leq V - 2v &amp; , ck \in (1/2,1]$</td>
<td>Asymmetric &amp; Symmetric Eq</td>
<td>Asymmetric Eq $\succ$ Symmetric Eq $\succ$ Monopoly</td>
</tr>
<tr>
<td>$-\frac{1}{2c} \leq V - 2v &amp; , ck &gt;1$</td>
<td>Symmetric Eq</td>
<td>Duopoly $\succ$ Monopoly</td>
</tr>
</tbody>
</table>

What Proposition 2 says is that, given the assumptions of the model, the value of the consumers’ alternative ($v$) to the monopolist’s products, is the key to comparing the welfare of duopoly versus monopoly. It may seem obvious that monopoly is relatively better when the competition from outside the market is greater. Notice, however, that in this model the third product has no sales in this market and is not necessary to reduce output distortion. Moreover, unlike markets with fixed prices, while monopoly can never produce more social welfare than the best (or only) duopoly equilibrium, if the value of $v$ is large enough, monopoly can guarantee the same social welfare as under the best duopoly equilibrium (even without any production or innovation synergies). Since, for some parameter values, there can be multiple duopoly equilibria, this represents an expected improvement in social welfare. As the extent of outside competition declines ($v$ decreases), so does the social welfare produced by monopoly because of
insufficient monopoly innovation. This ensures that the best duopoly equilibrium generates more welfare than monopoly. With multiple duopoly equilibria, however, welfare under monopoly can exceed the welfare in the bad duopoly equilibrium. For very low values of values of \( v \), the monopoly outcome is worse than even the bad duopoly equilibrium. This relationship is depicted in the following graphs.

**Figure 3 Here**

These graphs depict the relationship between social welfare and \( v \), the value of the third product. Duopoly welfare does not depend on \( v \) because no consumers buy this product and because this product is not any consumer’s second choice. Under monopoly, the third product is every consumer’s outside option. So, the larger is \( v \), the larger the fraction of consumers with a binding outside option, increasing monopoly innovation incentives and aligning profit incentives more closely with social welfare.

The graphs also depict how the relationship between market structure and welfare depends on \( ck \), the product of the innovation cost parameter and the product differentiation parameter. When this is less than \( \frac{1}{2} \), then the only duopoly equilibrium is where just one product is developed. This is also the monopoly outcome. Similarly, when \( ck \) is greater than 1, the only duopoly equilibrium involves equal development of both products. When innovation costs and product differentiation are this substantial, a monopolist also develops both products equally. So, in these instances, monopoly never provides a coordination advantage. For intermediate values of \( ck \), however, monopoly can provide a coordination benefit provided \( v \) is large enough. In this region, there are multiple duopoly equilibrium and, if \( v \) is large, the monopolist will choose to develop the two products asymmetrically. Even if \( v \) is not so large that this asymmetric monopoly outcome is as good as the asymmetric duopoly equilibrium (because not all customers have a binding outside option under monopoly), the asymmetric development still improves welfare over the symmetric duopoly outcome. That is, if the probability of the bad duopoly equilibrium occurring is large enough, monopoly can increase expected welfare for large enough \( v \).\(^{16}\)

\(^{16}\) It is important to note that the existence of cases where monopoly can never be worse than duopoly depends on the assumption of a finite distribution of consumer types. If the support of consumer types extended across the real line, then there might not be any regions where local development incentives were identical in monopoly and
V. Conclusion

Analyzing the welfare effects of market structure in markets where bilateral contracting is the norm is very different from analyzing these effects in markets where spot market transactions predominate. With efficient bilateral contracting, there is no ex post allocation inefficiency; market structure only affects welfare when it affects the hold up problem. Because of the holdup problem, firms will have insufficient incentives to improve the quality of their products. In this paper’s model, competition among suppliers reduces this problem.

Competition, however, brings with it the problem of multiple equilibria. When that is the case, the more asymmetric is the development of the two products the greater is welfare. If the value of consumers’ third option is large enough, then monopoly can lead to an asymmetric development outcome that is superior to the symmetric duopoly equilibrium. If the symmetric duopoly equilibrium occurs with large enough probability, then the coordination benefits of monopoly might make it preferable to duopoly.

Of course, the paper assumes that a firm’s ability to develop its product(s) (the development cost function) is independent of market structure. If innovation synergies occur when one firm owns both products, then this makes monopoly more advantageous than the model in this paper predicts. In fact, because (efficiently) negotiated trade eliminates the effect of market structure on allocation efficiency, it will be much easier for synergies to result in greater welfare with less competition than in markets governed by fixed price transactions.

The paper also assumes that $\varepsilon$, the relative preference parameter, is common knowledge. While the suppliers often know much more about their products than customers, and often learn a great deal about how the customer will use the product in the sales process, the customer will always know more about its valuation for each product than the suppliers. Because I assumed away this type of information asymmetry, there were no bargaining failures in the model. Since bargaining failures reduce welfare, the duopoly. If this distribution had very small density in the tails, however, there would be cases where the local innovation incentives were almost identical. Thus, the probability that duopoly would result in both products being developed would not have been much greater than zero for monopoly to generate more expected welfare than duopoly.
effect of market structure on the incidence of bargaining failures is important. The direction of this effect is unclear. On the one hand, under monopoly the disagreement option is worse for the buyer, so it has a greater incentive to avoid a bargaining failure. On the other hand, the consequences of bargaining failure in this situation are more severe.

Bargaining failures could also impact the results to the extent they affect innovation incentives. The social planner will always weigh bargaining failures more severely than the monopolist since the monopolist only bears part of the cost of a failure. Asymmetric information could also affect innovation incentives if it creates a business stealing incentive in duopoly. This is unlikely to occur, however, since it can only happen if an indifferent customer must pay a positive price for either product. Since the most likely form of bargaining failure from asymmetric information is delay, there should be no business stealing incentive even with asymmetric information. That said, analyzing more precisely the effect of asymmetric information would be quite interesting.
### APPENDIX A—MONOPOLY OUTCOME

<table>
<thead>
<tr>
<th>Case</th>
<th>Develop. Levels</th>
<th>Conditions on (ck)</th>
<th>Conditions on (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both Developed</td>
<td>(d_A = d_B = \frac{1}{2c})</td>
<td>(ck \geq 1)</td>
<td>(V - 2v \leq -\frac{1}{2c})</td>
</tr>
<tr>
<td>Both Developed</td>
<td>(d_A = d_B = \frac{1}{2c})</td>
<td>(ck \geq 1)</td>
<td>(V - 2v \leq -\frac{1}{2c})</td>
</tr>
<tr>
<td>Only One</td>
<td>(d_A = \frac{k(1 - 2ck) - V + 2v}{1 + 2ck - 4(ck)^2}; d_B = \frac{2k(1 - ck) - (V - 2v)(1 - 2ck)}{1 + 2ck - 4(ck)^2})</td>
<td>(1 + \sqrt{\frac{5}{4}} \leq ck &lt; 1)</td>
<td>(k(1 - 2ck) \leq V - 2v &lt; -1/2c)</td>
</tr>
<tr>
<td>No Outside Options</td>
<td>(d_A = d_B = \frac{1}{4c})</td>
<td>(ck &gt; 1/2)</td>
<td>(\frac{2ck - 1}{4c} \leq V - 2v)</td>
</tr>
<tr>
<td>Outside Option Cutoff Interior</td>
<td>(d_A = 0)</td>
<td>(0 \leq ck \leq \frac{1}{2}) or (1 + \frac{\sqrt{5}}{4} \leq ck &lt; 1)</td>
<td>(-\frac{1}{c} \leq V - 2v \leq \frac{2ck - 1}{2c}) or (-\frac{1}{c} \leq V - 2v \leq \frac{k(2ck - 1)}{2c}) or (-\frac{1}{c} \leq V - 2v \leq k(1 - 2ck))</td>
</tr>
<tr>
<td>All Outside Options</td>
<td>(d_A = 0) ; (d_B = \frac{1}{c})</td>
<td>(ck \leq 1)</td>
<td>(V - 2v &lt; -\frac{1}{c})</td>
</tr>
<tr>
<td>Binding</td>
<td>(d_A = 0) ; (d_B = \frac{1}{2c})</td>
<td>(ck \leq 1/2)</td>
<td>(\frac{2ck - 1}{2c} &lt; V - 2v)</td>
</tr>
</tbody>
</table>

\(^{17}\) I do not need to worry about second order conditions in Table 1 since the development levels have been compared to all other possible local maxima.
APPENDIX B—PROOF OF PROPOSITION 2

(a) \( V - 2v < -\frac{1}{c} \). If \( ck \notin [1/2,1] \), then the duopoly equilibrium is unique. Table 1 shows that the monopoly outcome is identical to the duopoly equilibrium in these cases. If \( ck \in [1/2,1] \), then Table 1 shows that the monopoly outcome is identical to the asymmetric duopoly equilibrium. By Proposition 1, this generates more welfare than the symmetric duopoly equilibrium.

(b) \( -\frac{1}{c} \leq V - 2v < -\frac{1}{2c} \). Table 1 shows that the monopoly outcome is the unique duopoly equilibrium if \( ck > 1 \). If \( ck < 1/2 \), then Table 1 shows that the monopoly outcome has only one product developed, just as in the unique duopoly equilibrium, but the level of development is strictly smaller with monopoly. Since the duopoly development outcome is identical to the social planner’s optimum when only one product is developed, this duopoly equilibrium must generate more social welfare. If \( ck \in [1/2,1] \), then Table 1 shows that the monopoly outcome either has only one product developed or has two products developed but one much more than the other (if \((1 + \sqrt{5})/4 \leq ck < 1\) and \( k(1 - 2ck) \leq V - 2v < -1/2c \)). If only one product is developed, this clearly generates less welfare than the asymmetric duopoly equilibrium by the same argument as above.

If the monopolist develops two products, welfare is still less than in the asymmetric duopoly equilibrium. To see this, evaluate the social welfare, using (12), in the monopoly outcome. This is:

\[
V + \frac{(1 - ck)[k(3 - 2ck + 8ck^2 - 8ck^3) + 2(1 - 2ck)(V - 2v)(1 + c(V - 2v))]}{2(1 + 2ck - 4ck^2)^2} \tag{B1}
\]

Social welfare in the asymmetric duopoly equilibrium (also calculated using (12)) is:

\[
V + \frac{1 - ck}{2c} \tag{B2}
\]

Subtracting (B1) from (B2) gives:

\[
\frac{(1 - ck)[(1 - ck)(1 + 2ck - 8ck^3) + 2c(1 - 2ck)(V - 2v)(1 + c(V - 2v))]}{2c(1 + 2ck - 4ck^2)^2} \tag{B3}
\]
(B3) is positive if and only if the term in square brackets is positive. Differentiating this term with respect to \( V-2v \) gives \( 2c(1-2ck)(1+2c(V-2v)) \). This is positive for \( (1+\sqrt{5})/4 \leq ck < 1 \) and \( k(1-2ck) \leq V-2v < -1/2c \), so the square bracket term is smallest when \( V-2v = k(1-2ck) \). At this point, it is:

\[
(1 - ck)(1 + 2ck - 4(ck)^2)^2 > 0
\]  

(B4)

This proves that welfare is greater in the asymmetric duopoly equilibrium.

It only remains to show that welfare from the monopoly outcome exceeds welfare in the symmetric duopoly equilibrium. Welfare in the symmetric duopoly equilibrium is:

\[
V + \frac{1 - ck}{4c}
\]

(B5)

Welfare when the monopolist develops only one product in this parameter range is

\[
V - \frac{k}{2} + \frac{2k - (V-2v)}{1+2ck} - \frac{c(V-2v-2k)^2}{2(1+2ck)^2}
\]

(B6)

Subtracting (B6) from (B5) gives:

\[
\frac{(1-2ck)^2(1+ck) + 2c(V-2v)(2 + c(V-2v))}{4c(1+2ck)^2}
\]

(B7)

(B7) is increasing in \( V-2v \) so long as \( V-2v > -1/c \). So, if the symmetric duopoly will ever give more welfare, it must do so when \( V-2v \) is at its maximum. If \( 1/2 \leq ck < \frac{1+\sqrt{5}}{4} \), this maximum is \( -1/2c \). At this value of \( V-2v \), (B7) is \( -1 - 6ck + 8(ck)^3 < 0 \) for \( 1/2 \leq ck < \frac{1+\sqrt{5}}{4} \). If \( \frac{1+\sqrt{5}}{4} \leq ck < 1 \), then this maximum is \( k(l-2ck) \) (because for larger \( V-2v \) the monopolist does develop the other product somewhat). At this value, (B7) is \( \frac{(1-ck)(1-2ck)}{4c} < 0 \). So the monopoly outcome with one product developed always gives more welfare than the symmetric duopoly equilibrium.
If the parameters are such that the monopolists develops both products (albeit, asymmetrically), then the welfare difference between the symmetric duopoly and monopoly is obtained by subtracting \((B1)\) from \((B5)\). This gives:

\[
\frac{(1 - ck)(1 - 2ck)(1 + 2c(V - 2v))^2}{4c(1 + 2ck - 4(ck)^2)^2} < 0 \tag{B8}
\]

Thus, symmetric duopoly is always worse than the monopoly outcome in this parameter region.

(c) \(-\frac{1}{2c} \leq V - 2v\). If \(ck < 1/2\) then Table 1 shows that the monopolist will develop only one product, but will develop that product less than in the unique duopoly equilibrium. Since the unique duopoly equilibrium is also the socially optimum, the monopoly outcome must generate less welfare. If

\[
ck > \frac{1 + \sqrt{5}}{4} \quad \text{or if} \quad ck \in \left[1/2, \frac{1 + \sqrt{5}}{4}\right] \quad \text{and} \quad V - 2v > -\frac{k(2ck - 1)}{2c},
\]

then Table 1 shows that the monopolist will develop both products equally, and strictly less than in the symmetric duopoly equilibrium. Since development in the symmetric duopoly equilibrium satisfies the social planner’s first order conditions, it generates more welfare than any other symmetric development outcome. Thus, the monopoly outcome is strictly worse than the symmetric duopoly equilibrium, which is strictly worse than the asymmetric duopoly equilibrium. If \(ck \in \left[1/2, \frac{1 + \sqrt{5}}{4}\right] \) and \(-\frac{1}{2c} \leq V - 2v \leq -\frac{k(2ck - 1)}{2c}\), Table 1 shows that monopoly outcome is identical to what it was in case (b) (only one product is developed, development is less than in the asymmetric duopoly equilibrium). Thus, the results from the proof of (b) show that the monopoly outcome generates more welfare than the symmetric duopoly equilibrium but less welfare than the asymmetric duopoly equilibrium. This completes the proof. Q.E.D.

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REFERENCES


Figure 1

The dashed line is B’s reaction function. The solid line is A’s reaction function.

$V = 2.5, c = 0.75, k = 1$
Figure 2

$V = 1, c = .6, k = 1$
Figure 3: Welfare by Market Structure and $v^{18}$

18 Welfare levels across the four graphs should not be compared since the x-axis is not in the same place in each graph.