

TWO-PART TARIFF COMPETITION IN DUOPOLY

Xiangkang Yin*

La Trobe University

Abstract – This paper develops two models of two-part tariff competition. When consumers are differentiated *à la* Hotelling, equilibrium prices equal marginal cost if and only if the demand of the marginal consumer equals the average demand. Entry fees are socially optimal in a symmetric equilibrium if all consumers participate in the market. Two-part tariffs tend to result in lower prices, higher profits and social welfare relative to uniform pricing. In the logit model, marginal cost pricing holds but entry fees are higher than socially optimal, and two-part tariffs lead to lower aggregate net consumer surplus but higher profits than uniform pricing.

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* Department of Economics and Finance, La Trobe University, Victoria 3086, AUSTRALIA, Tel: 61-3-9479

2312, Fax: 61-3-9479 1654, Email: x.yin@latrobe.edu.au

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1. INTRODUCTION

Recent worldwide deregulation in telecommunication and other utilities has converted many traditional monopolies into oligopolies. Consequently, two-part tariffs, which were widely practiced in these industries, have extended to an imperfectly competitive environment. Elsewhere, competition with two-part tariffs also prevails. For instance, stockbrokers charge an annual maintenance fee on an account and a per-trade fee for each stock exchange; various clubs (health, golf, book and wine), and many websites levy a membership fee plus a per-use or per-unit charge. In all these circumstances, competitors in the same business provide close substitutes of products or services to well informed customers. What is the Nash equilibrium of two-part tariff competition in these markets? Can such an equilibrium be first-best optimal? Are two-part tariffs better than uniform pricing for consumers surplus, profits and social welfare? The purpose of this paper is to address these issues.

Two types of consumer preferences are under consideration. The first model is built on the location model of Hotelling (1929),¹ where a consumer purchases only one type of product produced by either firm in a duopolistic industry. For a product to be bought, it has to provide positive net consumer surplus (net of the purchase cost and lump-sum fee) and more surplus than the rival product as well. Although there is competition in duopoly, the necessary and sufficient condition for marginal cost pricing in equilibrium is the same as in monopoly; that is, the demand of the marginal consumer has to be equal to the average demand. In an equilibrium with some consumers not served by either firm, the lump-sum

¹ In contrast to conventional location models, we allow for elastic demand and focus on price and lump-sum fee competition.

entry fees are too high relative to the welfare maximum. But if all consumers participate in the market, the symmetric equilibrium entry fee maximizes social welfare. In comparison with uniform price competition, the two examples of the model show that two-part tariffs result in a lower marginal price, higher profits and social welfare. But the aggregate net consumer surplus is ambiguous.

A limitation of Hotelling preferences is that firms either compete against each other or compete against the outside option. To study the situation where a firm competes simultaneously with the other firm and the outside choice, this paper develops another model with logit demand. In such a setting, when a firm marginally raises its marginal price and/or entry fee, there is a marginal decline in the demand for its product and the profit function changes smoothly without kinks. It is found that two-part tariff competition leads to marginal cost pricing which is socially optimal, but yields entry fees which are too high. In comparison with uniform pricing, it yields more profits but less aggregate net consumer surplus in a symmetric equilibrium.

The effects of two-part tariffs on pricing strategy and social welfare have been of interest to economists since the seminal contribution of Oi (1971). However, the majority of the literature focuses on monopolistic two-part tariffs. Although two-part tariff competition has attracted more attention recently, formal analyses focusing on this issue are few. Armstrong and Vickers (2001) propose a framework of competition in utility space to investigate competitive price discrimination. Applying their analysis to Hotelling preferences, they find that two-part tariff competition is an equilibrium outcome of general non-linear price competition and they then specify the conditions for marginal cost pricing. While their model is more general in terms of considering multiproduct firms and allowing for both horizontal and vertical preference heterogeneity, they also restrict the transportation

cost to a form of shopping cost.² We consider more general horizontal preferences but exclude differences in vertical preferences and implicitly assume full non-linear pricing to be infeasible. In particular, our second example of the Hotelling model is a shipping model, and firms set price above marginal cost in equilibrium.

Another related work is Rochet and Stole (2002), which is very similar to Armstrong and Vickers (2001) if the quality in the Rochet-Stole model is interpreted as quantity. By considering both horizontal differentiation and vertical differentiation, the Rochet-Stole model predicts that there are no distortions in firms' quality choices, and tariffs in the equilibrium of general non-linear price competition are cost-based two-part tariffs. Again in a Hotelling setting the main difference between our analysis and theirs is that we focus on two-part tariff competition with a general form of horizontal product differentiation, while they pay more attention to two-dimensional preference heterogeneity and the optimal quality choice. Neither Armstrong-Vickers and Rochet-Stole formally analyze imperfect competition with logit demand.

The rest of the paper is organized as follows. Section 2 presents a model of two-part tariff competition with Hotelling preferences and characterizes the properties of equilibrium and social welfare. Then it compares two-part tariff competition with uniform price competition. Two examples in Section 3 illustrate the closed forms of the equilibrium discussed in Section 2 and allow more detailed analysis. Section 4 turns to the logit model and compares equilibrium marginal prices and entry fees and their welfare consequences resulting from two-part tariff competition and uniform price competition. The final section concludes the paper. All proofs of propositions, corollaries and lemmas are given in the Appendix.

² Anderson and Engers (1994) characterize two types of transportation costs. A shopping cost is independent of the amount purchased. A shipping cost is proportional to the quantity bought.

2. TWO-PART TARIFF COMPETITION WITH HOTELLING PREFERENCES

2.1) *The Model and Equilibrium*

Consider an industry with two substitutes, x and y , supplied by two firms at marginal prices, p_x and p_y , with lump-sum entry fees, e_x and e_y , respectively. Casual observation indicates that most customers purchase only one firm's product or service to save entry fees. For instance, most families have only one wired telephone number but can make as many phone calls as they want. We assume that each individual consumer pays only one lump-sum fee to purchase either type of good supplied by the firms.³ Consumers have diverse tastes over the goods and are indexed by a taste parameter τ , distributed on $[0, 1]$ with density and cumulative distribution functions $f(\tau)$ and $F(\tau)$. Firm x locates at $\tau = 0$ while firm y locates at $\tau = 1$.

Assume consumers have quasilinear utility and let $v_x(p_x, \tau)$ denote consumer τ 's consumer surplus function associated with demand function $x(p_x, \tau)$.^{4, 5} It is also assumed that a consumer with a larger taste parameter τ has a lower level of utility from the consumption of good x but a higher level of utility from the consumption of good y ; that is,

$$\partial v_x(p_x, \tau) / \partial \tau < 0 \quad \text{and} \quad \partial v_y(p_y, \tau) / \partial \tau > 0.$$

³ Such a simplification of "one-stop-shopping" is also adopted by other authors, for example, Armstrong and Vickers (2001), Anderson et al. (1995), and Anderson and de Palma (2000).

⁴ Because of quasilinear utility, to assume a consumer surplus function is equivalent to assuming a utility function. Let the utility of consuming good x be $u_x(x, \tau)$, then $v_x(p_x, \tau) = \max_x \{u_x(x, \tau) - xp_x\}$, where $u_x(0, \tau) = 0$.

⁵ Similar notations are applicable to good y . To avoid duplication, the analysis below will only give formulas for the x -good when their counterparts for the y -good can be obtained by exchanging subscripts.

A consumer buys good x if and only if the net consumer surplus obtained by such a purchase is non-negative and is not less than that of purchasing the y -good; i.e., the utilities and entry fees must satisfy the following two individual rationality constraints:

$$v_x(p_x, \tau) \geq e_x, \quad (\text{Individual Rationality constraint 1, IR1})$$

$$v_x(p_x, \tau) - e_x \geq v_y(p_y, \tau) - e_y \quad . \quad (\text{Individual Rationality constraint 2, IR2})$$

Let T_x be the marginal consumer of good x , who is indifferent between buying the x -good or not. Then T_x is equal to the smaller solution to the binding constraints IR1 and IR2. Among all potential marginal consumers, there is a special one, \bar{T}_x , who is indifferent between buying from firm x or buying from firm y or not buying from either of them; i.e., both IR1 and IR2 are binding. In other words, \bar{T}_x is the turning point where the binding constraint shifts from IR1 to IR2. Given the rival firm's tariffs, \bar{T}_x satisfies $v_y(p_y, \tau) = e_y$. If firm x has chosen its market size T_x , the maximum entry fee it can charge is⁶

$$e_x = \begin{cases} v_x(p_x, T_x) & \text{if } T_x \leq \bar{T}_x \\ v_x(p_x, T_x) - v_y(p_y, T_x) + e_y & \text{if } T_x \geq \bar{T}_x \end{cases} \quad (1)$$

Note that given p_x and competitor's tariffs, e_x is uniquely determined by T_x , and vice versa. In other words, the choice of entry fee is equivalent to choosing the marginal consumer. In this Hotelling model a firm competes in two regimes. Firm x is a local monopoly and competes with the outside choice when $T_x < \bar{T}_x$ (i.e., IR1 is binding) but it directly competes with firm y when $T_x > \bar{T}_x$ (i.e., IR2 is binding). At the turning point, $T_x = \bar{T}_x$, and the profit function is kinked.

Normalizing the total population to unity, the aggregate demands for goods x and y are, respectively,

⁶ Market size is defined as the number of clients a firm has. It is possible that $\bar{T}_x \leq 0$ or $\bar{T}_x \geq 1$ for some tariff pairs (p_y, e_y) , which means that firm x sets entry fee $v_x(p_x, T_x) - v_y(p_y, T_x) + e_y$ or $v_x(p_x, T_x)$ for all $T_x \in [0, 1]$.

$$X(p_x, T_x) = \int_0^{T_x} x(p_x, \tau) f(\tau) d\tau \quad \text{and} \quad Y(p_y, T_y) = \int_{T_y}^1 y(p_y, \tau) f(\tau) d\tau, \quad (2)$$

where each individual's demands $x(p_x, \tau)$ and $y(p_y, \tau)$ are obtained from Roy's identity.

Each firm incurs a constant marginal cost c_x or c_y but incurs no fixed costs.

Therefore, the profit functions of the two firms can be written as

$$\pi_x = e_x F(T_x) + (p_x - c_x) X(p_x, T_x) \quad \text{and} \quad \pi_y = e_y [1 - F(T_y)] + (p_y - c_y) Y(p_y, T_y). \quad (3)$$

The goal of a firm in two-part tariff competition is to choose tariffs (p_x, e_x) , or equivalently (p_x, T_x) , to maximize its profits. Substituting (1) into (3), we can see from Figure 1 (the solid lines) that given firm y 's tariffs, the profit function of firm x is continuous but kinked at $T_x = \bar{T}_x$, so that⁷

$$\pi_x = \begin{cases} \pi_{x1} \equiv v_x(p_x, T_x) F(T_x) + (p_x - c_x) X(p_x, T_x) & \text{if } T_x \leq \bar{T}_x \\ \pi_{x2} \equiv [v_x(p_x, T_x) - v_y(p_y, T_x) + e_y] F(T_x) + (p_x - c_x) X(p_x, T_x) & \text{if } T_x \geq \bar{T}_x \end{cases} \quad (4)$$

On the segment $\pi_x = \pi_{x1}$ (excluding the point $T_x = \bar{T}_x$), IR1 is binding so the marginal consumers of firm x and firm y are different (i.e., $T_x < T_y$). But on the segment $\pi_x = \pi_{x2}$, IR2 is binding so the two marginal consumers are the same person (i.e., $T_x = T_y$). To simplify analysis, it is assumed that π_{x1} and π_{x2} are concave functions of $T_x \in [0, 1]$.

INSERT FIGURE 1 (a) – (d) AROUND HERE

Taking the partial derivative of profit function (4) with respect to p_x yields the first-order condition (FOC) for profit maximization,

$$-x(p_x, T_x) F(T_x) + X(p_x, T_x) + (p_x - c_x) \partial X(p_x, T_x) / \partial p_x = 0. \quad (5)$$

On the other hand, the partial derivative with respect to T_x leads to

⁷ If $\bar{T}_x \geq 1$ or $\bar{T}_x \leq 0$, $\pi_x = \pi_{x1}$ or $\pi_x = \pi_{x2}$ on the whole interval $T_x \in [0, 1]$ and the profit function has no kinks.

$$\partial \pi_x / \partial T_x = F(T_x) \partial v_x(p_x, T_x) / \partial T_x + [v_x(p_x, T_x) + (p_x - c_x)x(p_x, T_x)]f(T_x) \quad \text{if } T_x \leq \bar{T}_x, \quad (6)$$

$$\begin{aligned} \partial \pi_x / \partial T_x = [\partial v_x(p_x, T_x) / \partial T_x - \partial v_y(p_y, T_x) / \partial T_x] F(T_x) + [v_x(p_x, T_x) - v_y(p_y, T_x) + e_y] f(T_x) \\ + (p_x - c_x)x(p_x, T_x) f(T_x) \quad \text{if } T_x \geq \bar{T}_x. \end{aligned} \quad (7)$$

Since π_{x1} and π_{x2} are assumed to be concave, (6) and (7) have a zero point, T_{x1} and T_{x2} , respectively (see Figure 1). However, it is quite likely that T_{x1} does not fall within the interval $(0, \bar{T}_x)$ (see Figure 1 (b) and (c)) and T_{x2} not within $(\bar{T}_x, 1)$ (see Figure 1 (b) and (d)). Thus, given firm y 's decision, there are four possible configurations of profit functions,⁸ and the possible candidates for firm x 's optimal marginal consumer are those who are located at $0, 1, T_{x1}, T_{x2}$ or \bar{T}_x . A further comparison between profit values at these points can determine the best response in the choice of marginal consumer. Since $T_x = 1$ or $T_x = 0$ implies that either firm x or firm y exits from the market so that the industry becomes monopolistic, these two trivial cases will be excluded from the analysis below.

To facilitate discussion, several definitions are introduced. A full-cover market means that each consumer is served by some firm while in a partial-cover market some consumers are not served by either firm. The equilibria of a full-cover market can be further divided into two types: a full competition equilibrium where the IR1 constraints of both goods are slack and a kinked equilibrium where the marginal consumer is at the kink of profit function. From (5), the following proposition is immediate.

Proposition 1. *When consumer preferences are differentiated à la Hotelling, equilibrium marginal prices are equal to marginal cost if and only if the demand of the marginal consumer is equal to the average demand, i.e., $x(p_x, T_x) = X(p_x, T_x)/F(T_x)$.*

⁸ But we can rule out the case of Figure 1 (a) for equilibrium because $\partial \pi_x / \partial T_x$ on the left of \bar{T}_x is smaller than its counterpart on the right of \bar{T}_x .

Proposition 1 shows that the condition for marginal cost pricing under two-part tariff competition and under two-part tariff monopoly is the same (for the case of monopoly, see Varian (1989)). This similarity between monopoly and duopoly is due to the lack of strategic effects of price decisions in both market structures, given each firm's market territory. The condition that the demand of the marginal consumer equals the average demand is not a knife-edge case. For instance, as long as the effect of horizontal preference parameter on utility is additively separate from price, then $x(p_x, T_x) = x(p_x)$ and $x(p_x, T_x) = X(p_x, T_x)/F(T_x)$. More generally, we have the following corollary.⁹

Corollary 1. *When consumers' marginal utilities are monotonic on the Hotelling line (i.e., $\partial^2 u_x(x, \tau)/\partial\tau\partial x \leq 0$ and $\partial^2 u_y(y, \tau)/\partial\tau\partial y \geq 0$) or conditional demands are monotonic (i.e., $\partial^2 v_x(p_x, \tau)/\partial\tau\partial p_x \geq 0$ and $\partial^2 v_y(p_y, \tau)/\partial\tau\partial p_y \leq 0$), firms set marginal prices equal to marginal cost if and only if the taste parameter does not interact with quantity (i.e., the transportation cost is a shopping cost).¹⁰*

Armstrong and Vickers (2001), Laffont et al. (1998), and Rochet and Stole (2002) show that the marginal price is (unconditionally) equal to marginal cost in full competition equilibrium.¹¹ In their models (like our example 1 below), there is no interaction between the location parameter and the quantity (or quality in the Rochet-Stole model), i.e., the transportation cost is a shopping cost. Consequently, all consumers of the same vertical type

⁹ I would like to thank Simon Anderson (the editor) for proposing this corollary.

¹⁰ It should be noticed that the monotonicity of marginal utility or conditional demand imposes a restriction on the model. In our setting, utility is monotonic but we do not impose any assumption on marginal utility elsewhere. Therefore, for any given price a consumer buying less (i.e., with lower marginal utility) is likely to have greater utility relative to other consumers. The condition that the marginal demand equals the average demand in Proposition 1 allows such non-monotonicity of demand.

¹¹ The cost structure is more complicated in Laffont et al. (1998) because of charges between competing networks. The marginal cost pricing rule holds if inter-firm charges are marginal cost based.

purchase the same amount of a good, if they decide to buy it, despite location differences. In other words, the particular utility functions they adopt ensure the demand of the marginal consumer to be equal to the average demand. However, divergence in the form of per-trip transportation costs is a special case of horizontal preference differentiation. In Example 2 below, where the location parameter interacts with quantity, the demand of the marginal consumer is smaller than the average demand and marginal cost pricing does not hold.

The welfare function is defined as the sum of consumers' surplus and firm profits:

$$W(p_x, p_y, T_x, T_y) = \int_0^{T_x} v_x(p_x, \tau) f(\tau) d\tau + \int_{T_y}^1 v_y(p_y, \tau) f(\tau) d\tau + (p_x - c_x)X(p_x, T_x) + (p_y - c_y)Y(p_y, T_y) \quad (8)$$

Since Proposition 1 gives the condition for marginal cost pricing, we focus on the welfare effects of entry fees. The partial derivative of (8) with respect T_x is

$$\partial W(p_x, p_y, T_x, T_y) / \partial T_x = [v_x(p_x, T_x) + (p_x - c_x)x(p_x, T_x)]f(T_x), \quad \text{if } T_x < T_y \quad (9)$$

$$\partial W(p_x, p_y, T, T) / \partial T = [v_x(p_x, T) - v_y(p_y, T) + (p_x - c_x)x(p_x, T) - (p_y - c_y)y(p_y, T)]f(T). \quad \text{if } T_x = T_y = T \quad (10)$$

Recalling (6), the value of (9) at the partial-cover equilibrium $(p_x, p_y, T_{x1}, T_{y1})$ is

$$\partial W(p_x, p_y, T_{x1}, T_{y1}) / \partial T_x = -F(T_{x1}) \partial v_x(p_x, T_{x1}) / \partial T_x > 0,$$

which implies that social welfare can be improved if each firm lowers the entry fee and extends their market coverage. Moreover, evaluating (10) at the full-cover equilibrium we find that it is not equal to zero in general, which implies that full-cover equilibrium entry fees are generally not socially optimal. It should be noticed that the absolute values of the entry fees are not critical to the social optimum when the market is fully covered because entry fees only transfer wealth from consumers to producers as long as the marginal prices and the marginal consumer are given. Rather the relative magnitude of the two entry fees plays a central role since it determines the marginal consumer. Market equilibrium does not maximize social welfare when the marginal consumer differs from the one determined by the

social optimum. However, in a symmetric setting, we have $c_x = c_y$ and $v_x(p, T) = v_y(p, 1 - T)$. A symmetric equilibrium implies both firms set the same marginal price and entry fee and have the same market size; i.e., $p_x = p_y$, $e_x = e_y$ and $T_x = 1 - T_y$. Furthermore, we have $T_x = T_y = \frac{1}{2}$ in a symmetric full-cover equilibrium. Thus, from (10) we have $\partial W/T = 0$ at a symmetric full-cover equilibrium, which implies the location of the marginal consumer maximizes social welfare and in turn the entry fee is socially optimal. The welfare analysis results are summarized in the following proposition.

Proposition 2. *(i) The equilibrium marginal price of each good under two-part tariff competition maximizes social welfare if and only if the demand of the marginal consumer is equal to the average demand.*

(ii) If two-part tariff competition leads to a partial-cover equilibrium, the market size is too small and entry fees are too high relative to the welfare optimum.

(iii) A full-cover equilibrium generally does not result in optimal entry fees. But in a symmetric full-cover equilibrium, the location of the marginal consumer and entry fees are socially optimal.

Part (ii) of the proposition resembles the conclusion obtained in monopoly (see Varian, 1989). This is not surprising because duopolistic firms in a partial-cover market are local monopolists in their market ranges. Recalling that entry fees are only a wealth transfer from consumers to producers and do not affect social welfare given the marginal prices and marginal consumer, the symmetric full-cover equilibrium in (iii) plus marginal cost pricing is only one of (infinitely) many social optima. The condition of symmetric equilibrium is important to the conclusion in (iii). As Example 1 below shows, two firms in a kinked equilibrium are likely to charge different entry fees within a symmetric setting. In that case reducing the higher entry fee to allow consumers to purchase from the closer firm can obviously save transportation costs and improve social welfare.

Another welfare implication of this model is that the partial-cover equilibrium and symmetric kinked equilibrium maximize joint profits; that is the duopolistic competition yields the same outcome as two-part tariff cartelization (for the proof, see the working paper Yin, 2000). The intuition behind the similarity is straightforward. In the partial-cover equilibrium both firms operate in their monopoly ranges and one firm's tariffs do not affect the demand for the other firm's product. Thus, the outcome of two local monopolists is equivalent to that of a single cartel. In the kinked equilibrium, the symmetry induces the duopoly to select the same marginal consumer regardless of being imperfectly competitive or collusive.¹² When profit functions reach a maximum at the kinked point, competition actually makes firms stick to the strategy of setting the entry fee equal to the marginal consumer's surplus in response to a range of the rival firm's entry fee. So, imperfectly competitive firms in equilibrium charge the same entry fee as the cartel does.

2.2) *Comparison with Uniform Pricing*

Linear prices can be considered as a special case of two-part tariffs where the entry fees are restricted to zero. The marginal consumer under this pricing regime is characterized by

$$v_x(p_x, T_x) = \max\{v_y(p_y, T_x), 0\}.$$

This condition determines T_x as a function of p_x , given p_y . The derivative of T_x with respect to p_x is

$$\frac{dT_x}{dp_x} = \begin{cases} x(p_x, T_x)/[\partial v_x(p_x, T_x)/\partial T_x] & \text{if } v_y(p_y, T_x) \leq 0 \\ x(p_x, T_x)/[\partial v_x(p_x, T_x)/\partial T_x - \partial v_y(p_y, T_x)/\partial T_x] & \text{if } v_y(p_y, T_x) \geq 0 \end{cases} \quad (11)$$

Given p_y , let $\bar{T}_x = \arg_t\{v_y(p_y, t) = 0\}$ be the marginal consumer who is indifferent between buying x and buying y and buying neither x nor y (i.e., both IR1 and IR2 are binding)

¹² The FOCs for equilibrium marginal prices of competition and cartelization are identical in all circumstances.

and let $\bar{p}_x = \arg_p v_x(p, \bar{T}_x) = 0$. Then, profit $\pi_x = (p_x - c_x)X(p_x, T_x)$ is kinked at $p_x = \bar{p}_x$ and \bar{T}_x

is the marginal consumer at the kink. The first-order derivative of π_x is equal to

$$\partial\pi_x/\partial p_x = X(p_x, T_x) + (p_x - c_x)\partial X(p_x, T_x)/\partial p_x + (p_x - c_x)x(p_x, T_x)f(T_x)dT_x/dp_x. \quad (12)$$

Evaluating dT_x/dp_x in (12) by the first or second equation in (11) and setting (12) and its counterpart for firm y equal to zero, the solution generates the partial-cover equilibrium or full competition equilibrium. Moreover, there is a kink equilibrium if (12) is positive on the left of \bar{p}_x and negative on the right of \bar{p}_x . Comparing (12) with (5), it is clear that the equilibrium price of uniform price competition can be either higher or lower than that of two-part tariff competition. Without more detailed specifications of the model, it is impossible to draw unambiguous conclusions on prices and welfare. However, the lemma below shows that price is a good indicator of welfare performance, which is helpful in our welfare analysis of two examples in the next section.

Lemma 1. *In a symmetric full-cover equilibrium, two-part tariff competition yields greater social welfare than uniform pricing if and only if it results in a lower marginal price, provided the equilibrium marginal price of two-part tariff competition is not smaller than marginal cost.*

The intuition of the lemma is obvious. If the market is covered, the aggregate welfare increases as social welfare (consumer surplus plus profit) obtained from the sales to each consumer is improved, which can be achieved with a lower marginal prices provided it is not below marginal cost.

3. TWO EXAMPLES OF HOTELLING PREFERENCES

This section presents two examples of Hotelling preferences. In both examples, consumers are uniformly distributed on $[0, 1]$ and two firms have the same marginal cost; i.e.,

$c_x = c_y = c$. The main difference between the two examples is their transportation costs, which induces divergence in demand structure and equilibrium characteristics.

3.1) *Example 1: Shopping Model*

This example adopts a simple shopping model so that utility is of the form

$$u_x(x, \tau) = u(x) - t\tau \quad \text{and} \quad u_y(y, \tau) = u(y) - t(1 - \tau), \quad a \geq 0 \quad (13)$$

where $u(x)$ and $u(y)$ are utilities of consuming goods x and y , respectively. The transportation costs for a consumer located at τ to firm x and firm y are $t\tau$ and $t(1 - \tau)$, respectively, which are independent of the amount shipped. This utility specification is similar to Armstrong and Vickers (2001) and Rochet and Stole (2002) if vertical preferences in their models are abstracted. It can also be considered as a simplified version of the model studied by Laffont et al. (1998) when one network has free access to the other network. With specification (13) the demand function is independent of a consumer's location as long as consumers decide to purchase, i.e., $x(p_x, \tau) = x(p_x)$. Consequently, consumer surplus functions are

$$v_x(p_x, t) = v(p_x) - t\tau \quad \text{and} \quad v_y(p_y, t) = v(p_y) - t(1 - \tau).$$

Since the demand of the marginal consumer is equal to the average demand, the firms price their products at their marginal costs in equilibrium. To simplify notation, write $v(c)$ as v in the analysis of entry fees below. Thus, t/v can be considered as a measure of product differentiation and/or consumer diversity. Given marginal cost pricing, we can interpret the model as if consumers have unit demand with reservation price v and the entry fee is the price for one unit.¹³ With standard calculation (which is available upon request) it can be shown that there are three types of equilibria depending on the parameters of the model:

- (a) A unique full competition equilibrium $e_x = e_y = t$ with marginal consumer $T_{x2} = T_{y2} = 1/2$ when $t/v \in [0, 2/3)$.

¹³ I would like to thank two anonymous referees for pointing out this analogue.

- (b) A continuum of kinked equilibria with $e_x + e_y = 2v - t$ for all $(e_x, e_y) \in [4/3v - t, 3/2v - t] \times [4/3v - t, 3/2v - t]$ when $t/v \in [2/3, 1]$. Any $\tau \in [v/3t, 1 - v/3t]$ can be the equilibrium marginal consumer if $t/v \in [2/3, 5/6]$, and any $\tau \in [1 - v/2t, v/2t]$ is the equilibrium marginal consumer if $t/v \in [5/6, 1]$.

The symmetric kinked equilibrium is $e_x = e_y = v - t/2$ where the marginal consumer is located at $\bar{T} = 1/2$.

- (c) A unique partial-cover equilibrium $e_x = e_y = v/2$ with marginal consumers $T_{x1} = v/2t$ and $T_{y1} = 1 - v/2t$ when $t/v \in (1, +\infty)$.

These results show that as t/v increases, the equilibrium moves from the full competition equilibrium to one of kinked equilibria and then to the partial-cover equilibrium. There are three interesting observations about entry fees. The first is that equilibrium entry fees tend to zero as t tends to zero. This demonstrates that if consumers are homogenous firms cannot charge two-part tariffs. Second, the entry fees increase as v is larger (for the partial and kinked equilibria) or t is larger (for the full competition equilibrium). This is obvious because as consumers value the commodity higher, they are willing to pay more. On the other hand, more diversified consumers implies firms have more market power and consequently they can set a higher entry fee. Finally, the entry fees are strategic complements in the full competition equilibrium range but they are strategic substitutes in the kinked equilibrium range.

From now on, we focus on the symmetric equilibrium. In light of Proposition 2, we have the corollary below.

Corollary 2. *With the utility specifications in (13), two-part tariff competition results in a socially optimal outcome if $t/v \in (0, 1]$ but its entry fee is higher than the social optimal level if $t/v \in (1, +\infty)$.*

Turning to comparison with uniform pricing, it is possible that a symmetric equilibrium under one tariff regime is fully covered but it turns out to be a partial-cover equilibrium under the other tariff regime. Since the full-cover equilibrium is more interesting, we consider such a demand and cost structure that results in full-cover equilibrium in both tariff regimes. Recalling $X(p_x, T_x) = x(p_x)T_x$, (12) shows that the symmetric full-cover equilibrium price under uniform pricing, p^u , is characterized by

$$x(p^u) + (p^u - c)dx(p^u)/dp - (p^u - c)x^2(p^u)/t = 0 \quad (14)$$

or

$$\begin{aligned} x(p^u) + (p^u - c)dx(p^u)/dp - (p^u - c)x^2(p^u)/t &> 0, \\ x(p^u) + (p^u - c)dx(p^u)/dp - 2(p^u - c)x^2(p^u)/t &< 0. \end{aligned} \quad (15)$$

Proposition 3. *Assume consumers have the utility functions in (13). In comparison with uniform price competition, two-part tariff competition results in a lower marginal price but higher profits and social welfare in symmetric full-cover equilibrium. The aggregate net consumer surplus is smaller if two-part tariff competition leads to a kinked equilibrium or if the demand function $x(p)$ is concave or if $x(p)$ is convex but $dx(c)/dp \geq 2dx(p^u)/dp$.*

The results regarding profits and social welfare in Proposition 3 are similar to those of Armstrong and Vickers (2001) except for our inclusion of the kinked equilibrium. The result on consumer surplus is slightly different. Although they do not explicitly spell out the condition for their conclusion (Corollary 1, Armstrong and Vickers, 2001), an implicit presumption in their analysis is that the profit function is concave in utility. While the conditions specified in Proposition 3 and assumed by Armstrong and Vickers are moderate, it seems unlikely to obtain an unambiguous conclusion on the relative magnitude of consumer surplus without these conditions. From the proof of Proposition 3 it is not difficult to see that two-part tariff competition (in full competition equilibrium) leads to greater aggregate net consumer surplus than uniform pricing if $v(p^u) < v/3$.

Monopolistic two-part tariffs definitely make the marginal consumer worse off in contrast to uniform pricing since the monopolistic entry fee drives his surplus to zero (see Philips, 1982, Wilson, 1993 and Yin, 2001). However, the marginal consumer is likely to be better off in two-part tariff duopoly than in uniform price duopoly. In this example, the marginal consumer's net surplus under the two-part tariff regime is $v - 3t/2$ if $t/v \leq 2/3$ while the surplus under the uniform pricing regime is $v(p^u) - t/2$. Thus, if $v(p^u) < v/3$, the two-part tariff makes the marginal consumer better off.

3.2) *Example 2: Shipping Model with Linear Demand*

Suppose now that the transportation cost is proportionate to the distance and amount shipped. More specifically, the utility function is assumed to be

$$u_x(x, \tau) = (b - x/2)x - \delta x \tau, \quad u_y(y, \tau) = (b - y/2)y - \delta y(1 - \tau), \quad (16)$$

where $(b - x/2)x$ and $(b - y/2)y$ are, respectively, the utilities from the consumption of the x -good and the y -good by the consumer located at τ , and $\delta x \tau$ and $\delta y(1 - \tau)$ are his shipping costs. Thus, the demands for goods x and y by the consumer located at τ , respectively, are

$$x(p_x, \tau) = b - p_x - \delta \tau \quad \text{and} \quad y(p_y, \tau) = b - p_y - \delta(1 - \tau). \quad (17)$$

To ensure the example is meaningful, it is assumed that $b > c$. For consumers who buy the x -good (y -good), the larger is τ , the smaller (larger) is demand; that is, the quantity demanded is determined not only by the price but also the location. Aggregate demand is

$$X(p_x, T_x) = \int_0^{T_x} (b - p_x - \delta \tau) d\tau = (b - p_x - \delta T_x/2)T_x. \quad (18)$$

The consumer surplus functions are

$$v_x(p_x, \tau) = (b - p_x - \delta \tau)^2/2 \quad \text{and} \quad v_y(p_y, \tau) = [b - p_y - \delta(1 - \tau)]^2/2. \quad (19)$$

Proposition 4. *For the specification of Example 2, there is a unique symmetric partial-cover equilibrium $p_x = p_y = (b + 4c)/5 > c$, $T_x = 2(b - c)/5\delta$, $T_y = 1 - 2(b - c)/5\delta$ and $e_x = e_y =$*

$2(b-c)^2/25$ when $(b-c) < 5\delta/4$. If $(b-c) \geq 5\delta/4$, the market is fully covered and symmetric equilibrium marginal prices are $p_x = p_y = c + \delta/4$. For $5\delta/4 \leq (b-c) < 9\delta/4$, the symmetric equilibrium rests at the kink with entry fees $e_x = e_y = (b-c-3\delta/4)^2/2$. For $(b-c) \geq 9\delta/4$, a symmetric full competition equilibrium obtains with entry fees $e_x = e_y = 3\delta(b-c-3\delta/4)/4$.

For uniform pricing, recalling (11)-(12) and (17)-(19), the symmetric full competition equilibrium price p^u is determined by

$$(b-p^u-\delta T_x/2)T_x-(p^u-c)\{T_x+(b-p^u-\delta T_x)^2/[b-p^u-\delta T_x+b-p^u-\delta(1-T_x)]\delta\}=0.$$

Since $T_x = 1/2$, it leads to the solution

$$p^u = \frac{1}{2} \left[b+c+3\delta/2 - \sqrt{(b+c+3\delta/2)^2 - 4(b\delta+bc+c\delta/2-\delta^2/4)} \right].$$

On the other hand, consider the FOC for partial-cover equilibrium,

$$(b-p-\delta T_x/2)T_x-(p-c)\{T_x+(b-p-\delta T_x)^2/(b-p-\delta T_x)\}=0.$$

Setting $T_x = 1/2$, we obtain

$$p^\# = \frac{1}{4} \left[2b+2c+\delta - \sqrt{(2b+2c+\delta)^2 - 8(b\delta+2bc-\delta^2/4)} \right].$$

So, if the kinked equilibrium exists, the symmetric equilibrium price is greater than $p^\#$ but smaller than p^u . The results of a comparison between two-part tariffs and linear pricing in symmetric full-cover equilibrium are given below.

Proposition 5. *With demand functions (17), two-part tariff competition leads to a lower marginal price and aggregate net consumer surplus, but higher industry profits and social welfare than uniform price competition.*

4. Two-Part Tariff Competition with Logit Demand

4.1) The Model and Equilibrium

A feature of Hotelling preferences is that each firm either competes with the outside choice (in a partial-cover equilibrium) or competes with the other firm's product (in a full-

cover equilibrium). It excludes the possibility that a firm competes with the other firm at the same time as it competes with the outside choice. To account for this simultaneous competition on two fronts, an alternative is to assume that the taste parameter τ is randomly drawn from a population. When a consumer faces prices p_x and p_y and entry fees e_x and e_y , he derives the following consumer surplus, respectively, through the consumption of the x -, y - or z - (outside) good:

$$v_x(p_x, \tau) = v_x(p_x) + \tau_x, \quad v_y(p_y, \tau) = v_y(p_y) + \tau_y, \quad v_z(\tau) = v_z + \tau_z.$$

In these equations, $v_x(p_x)$, $v_y(p_y)$ and v_z are nonstochastic and reflect the population's tastes¹⁴ and τ_x , τ_y and τ_z are stochastic and reflect the idiosyncrasies of this individual's tastes for goods x , y and z . Similar to the Hotelling model, individual rationality conditions, $v_x(p_x, \tau) - e_x \geq v_z(\tau)$ and $v_x(p_x, \tau) - e_x \geq v_y(p_y, \tau) - e_y$, must hold for the x -good to be attractive to consumer τ . But consumers steadily leave the x -good market as e_x rises so that there are no kinks in the demand function. To obtain logit demand functions, it is assumed that τ_i ($i = x, y, z$) are i.i.d and follow the double exponential distribution with mean zero and variance $\sigma^2 \pi^2/6$ so that the cumulative distribution function is

$$F(\omega) = \exp\{-[\exp(-\gamma + \omega/\sigma)]\},$$

where γ is Euler's constant and σ is a positive constant. It can be shown (see, for example, Anderson et al. (1992) Chapter 2) that the *fractions* of consumers, who choose the x -, y - or z -good, respectively, are

$$S_i = \frac{\exp[(v_i(p_i) - e_i)/\sigma]}{\exp(v_z/\sigma) + \sum_{j=x,y} \exp[(v_j(p_j) - e_j)/\sigma]}, \quad i = x, y,$$

$$S_z = \frac{\exp(v_z/\sigma)}{\exp(v_z/\sigma) + \sum_{j=x,y} \exp[(v_j(p_j) - e_j)/\sigma]}. \quad (20)$$

¹⁴ If the utility of consuming good x is $u_x(x)$, then $v_x(p_x) = \max_x\{u_x(x) - p_x x\}$.

Thus, the demand functions for the x -good and y -good are xS_x and yS_y , respectively, where each consumer's demand, x or y , is constant across consumers who decide to buy, and can be obtained by Roy's identity. As (20) illustrates there are always some consumers who purchase neither good x nor good y when v_z is a finite constant. But no consumers purchase z and the market is fully covered if $v_z \rightarrow -\infty$.

Firm x 's profit function can be written as¹⁵

$$\pi_x = e_x S_x + (p_x - c_x)x(p_x)S_x.$$

Its partial derivatives with respect to p_x and e_x yield the following FOCs:

$$x(p_x) + (p_x - c_x)dx(p_x)/dp_x - [e_x + (p_x - c_x)x(p_x)](1 - S_x)x(p_x)\sigma^{-1} = 0, \quad (21)$$

$$1 - [e_x + (p_x - c_x)x(p_x)](1 - S_x)\sigma^{-1} = 0. \quad (22)$$

Define the symmetric case as in Sections 2 and 3 so that $v_x(\cdot) = v_y(\cdot) = v(\cdot)$ and $c_x = c_y = c$ and let $v(c) \equiv v$. From (21) and (22), we can immediately obtain the proposition below.

Proposition 6. *In the logit model, two-part tariff competition leads to marginal cost pricing in equilibrium and in the symmetric case the equilibrium entry fee, e , can be characterized by*

$$e = \sigma(1 - S_x) = \sigma \left\{ 1 + \frac{\exp[(v - e)/\sigma]}{\exp(v_z/\sigma) + \exp[(v - e)/\sigma]} \right\}. \quad (23)$$

Moreover $de/dv_z < 0$ and $e = 2\sigma$ when $v_z \rightarrow -\infty$.

The result $de/dv_z < 0$ is intuitive: when the outside option becomes more attractive (i.e., v_z is larger) the firms have less market power so that they have to lower entry fees to attract consumers. At the extreme (i.e., $v_z \rightarrow -\infty$), consumers do not purchase the outside option and the firms can set the highest entry fee $e = 2\sigma$.

¹⁵ Note that S_x is a function of p_x, p_y, e_x, e_y . To simplify notation, these variables are dropped in the expressions.

A consumer purchasing good x , y or z has net consumer surplus $v_x(p_x) + \tau_x - e_x$, $v_y(p_y) + \tau_y - e_y$ or $v_z + \tau_z$, respectively. Following the same proof in Anderson et al. (1992, Chapter 2), it can be shown that the aggregate net consumer surplus under the two-part tariff regime is

$$CS = \sigma \ln \left\{ \exp(v_z / \sigma) + \sum_{j=x,y} \exp[(v_j(p_j) - e_j) / \sigma] \right\}.$$

By definition the welfare function is $W = CS + (\pi_x + \pi_y)$. Since two-part tariff competition leads to marginal cost pricing, we need only investigate the welfare effects of the entry fee:

$$\partial W / \partial e_x = -S_x + S_x \{ 1 - [e_x + (p_x - c_x)x](1 - S_x)\sigma^{-1} + [e_y + (p_y - c_y)y]S_y\sigma^{-1} \}.$$

Substituting the marginal price and entry fee into the symmetric equilibrium yields

$$\partial W / \partial e_x = S_x(e_y S_y \sigma^{-1} - 1) < 0.$$

However, if there is no outside option (i.e., $v_z \rightarrow -\infty$), $e_y S_y = \sigma$ by (23), so $\partial W / \partial e_x = 0$.

Proposition 7. *The symmetric equilibrium of two-part tariff competition with logit demand leads to a socially optimal marginal price but the entry fee is too high and the market coverage too small.¹⁶ If there is no outside option, the equilibrium is socially optimal.*

4.2) Comparison with Uniform Pricing

In uniform price competition, the equilibrium price, p_x^u , is characterized by

$$x(p_x^u) + (p_x^u - c_x)dx(p_x^u)/dp_x - (p_x^u - c_x)x(p_x^u)(1 - S_x)x(p_x^u)\sigma^{-1} = 0, \quad (24)$$

which implies above-marginal-cost pricing. There is an unambiguous conclusion for the comparison between the welfare outcomes of two-part tariffs and uniform pricing if there is no outside option. However, with an outside option, only aggregate net consumer surplus and profits can be unambiguously compared.

Proposition 8. *If $x(p)$ and $y(p)$ are log-concave, two-part tariff competition results in a smaller market coverage, less aggregate net consumer surplus and more profits than uniform*

¹⁶ When v_z is finite the welfare maximizing entry fee is $e_x = 0$ since $\partial W / \partial e_x < 0$ for all positive e_x .

price competition in symmetric equilibrium. If there is no outside option, it also yields greater social welfare.

5. Closing Discussion

The utility functions in the logit model are similar to those in Example 1 in the sense that preference divergence is an additive term in the utility functions and there is no interaction between the preference parameter and the quantity demanded. This similarity implies that consumers in both settings have identical demand functions if they buy a product, which in turn results in marginal cost pricing. Moreover, both settings show that the symmetric full-cover equilibrium is socially optimal but the partial-cover equilibrium is not. On the other hand, the random distribution of consumer preferences of the logit model makes it differ substantially from Example 1. There is no systematic relationship between the tastes of any two consumers in the logit model. Consequently, firms do not have exclusive market territories and cannot behave as local monopolists.

For applications, it is hard to say whether the assumption of Hotelling preferences is better than the assumption of logit demand or not. Hotelling preferences may be appealing for industries such as wired telephone services where the market is completely covered and variations in tariffs only make consumers switch between firms. On the other hand, logit demand may be more plausible for other situations such as luxury club memberships where a reduction in price and/or entry fee attracts consumers from other suppliers as well as new customers. The most appropriate assumption for a particular industry or market is an empirical question.

Appendix

Proof of Corollary 1

Since $p_x = \partial u_x(x, \tau)/\partial x$, the demand function is independent of τ when τ does not interact with x in the utility function. So, $x(p_x, T_x) = X(p_x, T_x)/F(T_x)$.

On the other hand, $\partial^2 u_x(x, \tau)/\partial \tau \partial x \leq 0$ or $\partial^2 v_x(p_x, \tau)/\partial \tau \partial p_x \geq 0$ implies $\partial x(p_x, \tau)/\partial \tau \leq 0$. It is obvious that the demand function is independent of τ when $\partial x(p_x, \tau)/\partial \tau \leq 0$ and $x(p_x, T_x) = X(p_x, T_x)/F(T_x)$. Hence, there is no interaction between τ and x in the utility function.

Proof of Lemma 1

Let c be the marginal cost and p the equilibrium marginal price in the symmetric case. Superscripts t and u indicate two-part tariffs and uniform pricing, respectively. Then

$$\begin{aligned} (W^t - W^u)/2 &= \int_0^{0.5} [v_x(p^t, \tau) - v_x(p^u, \tau)] d\tau + (p^t - c)X(p^t, 0.5) - (p^u - c)X(p^u, 0.5) \\ &= \int_{p^t}^{p^u} X(p, 0.5) dp - (p^u - p^t)X(p^u, 0.5) + (p^t - c)[X(p^t, 0.5) - X(p^u, 0.5)]. \end{aligned}$$

Because $X(\cdot, 0.5)$ is downward sloping and $p^t > c$, $W^t > W^u$ if and only if $p^t < p^u$.

Proof of Proposition 3

From (14)-(15), $p^u > c$ is obvious. By Lemma 1 the conclusion of total welfare is immediate.

Let CS and Π be aggregate net consumer surplus and industry profits, respectively. Then,

$$\Pi^t = \begin{cases} t & \text{if } 0 < t/v < 2/3 \\ v - t/2 & \text{if } 2/3 \leq t/v \leq 1 \end{cases},$$

$$\Pi^u = (p^u - c)x(p^u).$$

Recalling (14)-(15) and $dx(p^u)/dp < 0$, we have $\Pi^u = (p^u - c)x(p^u) < a$. Moreover, $v(p^u) \geq t/2$

since all consumers participate in the market. Thus, $v - t/2 = v(p^u) + \int_c^{p^u} x(p) dp - t/2 > (p^u -$

$c)x(p^u)$. So, $\Pi^u < \Pi^l$. On the other hand, $CS^u - CS^l = t - \int_c^{p^u} x(p)dp$. Equations (14) and

(15) imply that

$$t \geq (p^u - c)x^2(p^u)/[x(p^u) + (p^u - c)dx(p^u)/dp].$$

When $x(p)$ is concave, we have

$$\int_c^{p^u} x(p)dp < (p^u - c)x(c) \leq (p^u - c)[x(p^u) - (p^u - c)dx(p^u)/dp],$$

A direct comparison shows $t \geq \int_c^{p^u} x(p)dp$. When $x(p)$ is convex, we obtain

$$\begin{aligned} \int_c^{p^u} x(p)dp &\leq (p^u - c)[x(p^u) + x(c)]/2 = (p^u - c)[2x(p^u) - (p^u - c)dx(p^u)/dp]/2 \\ &\leq (p^u - c)[x(p^u) - (p^u - c)^2dx(p^u)/dp]. \end{aligned}$$

where $p' \in (c, p^u)$. In the last inequality, $dx(p')/dp \geq dx(c)/dp \geq 2dx(p^u)/dp$ is applied.

Proof of Proposition 4

Substituting (17) and (18) into (5) yields $p_x = c + \delta T_x/2$. Similarly, setting (6) equal to zero yields $T_{x1} = (b + p_x - 2c)/3\delta$. Solving these two equations and their counterparts for firm y yields a unique symmetric partial-cover equilibrium $p_x = p_y = (b + 4c)/5 > c$, $T_x = 2(b - c)/5\delta$, $T_y = 1 - 2(b - c)/5\delta$ and $e_x = e_y = 2(b - c)^2/25$ when $(b - c) < 5\delta/4$.

The above proof also shows that there is no partial-cover symmetric equilibrium when $(b - c) \geq 5\delta/4$. Since $T_x = T_y = 1/2$ in a full-cover symmetric equilibrium, the equilibrium marginal prices are $p_x = p_y = c + \delta/4$. Substituting these values into (7) and setting it equal to zero yield symmetric-full-competition-equilibrium entry fees $e_x = e_y = 3\delta(b - c - 3\delta/4)/4$. Recalling $e_y \leq v(c + \delta/4, 1/2) = (b - c - 3\delta/4)^2/2$, the condition for the existence of the symmetric full-competition equilibrium is $(b - c) \geq 9\delta/4$. For $5\delta/4 \leq (b - c) < 9\delta/4$, substituting $p_x = p_y = c + \delta/4$ and $T_y = 1/2$ into (6) and (7), we obtain

$$\frac{\partial \pi_x}{\partial T_x} \Big|_{T_x=\frac{1}{2}-0} = (b-c-3\delta/4)(b-c-5\delta/4)/2 \geq 0, \quad \frac{\partial \pi_x}{\partial T_x} \Big|_{T_x=\frac{1}{2}+0} = e_y - 3\delta(b-c-3\delta/4)/4 < 0.$$

So, the symmetric full-cover equilibrium is at the kink and equilibrium entry fees are $e_x = e_y = (b-c-3\delta/4)^2/2$.

Proof of Proposition 5

First, consider the case where uniform pricing results in a symmetric full competition equilibrium. Define $\Delta \equiv \sqrt{(b+c+3\delta/2)^2 - 4(b\delta+bc+c\delta/2-\delta^2/4)}$, then $p^u > p^l \Leftrightarrow (b-c+\delta) > \Delta \Leftrightarrow (b-c) > 3\delta/4$. Since a full-cover equilibrium of two-part tariff completion implies $(b-c) \geq 5\delta/4$, $p^u > p^l$ is true. Recalling Lemma 1 we have $W^l > W^u$. Note, the full competition equilibrium results in higher entry fees than the kinked equilibrium under two-part tariffs and $2 \int_0^{0.5} v(p, \tau) d\tau = (b-p)(b-p-\delta/2)/2 + \delta^2/24$. Routine calculation shows,

$$CS^l \leq 2 \int_0^{0.5} v(c+\delta/4, \tau) d\tau - 3\delta(b-c-3\delta/4)/4 = (b-c-7\delta/4)(b-c-3\delta/4)/2 + \delta^2/24,$$

$$CS^u = 2 \int_0^{0.5} v(p^u, \tau) d\tau = (b-c-3\delta/2+\Delta)(b-c-5\delta/2+\Delta)/8 + \delta^2/24.$$

Because $\Delta > b-c-\delta/2$, it is immediately apparent that $CS^l < CS^u$.

$$\Pi^l = e + 2(p^l - c)X(p^l, 1/2) \geq \delta(b-c-11\delta/16),$$

$$\Pi^u = 2(p^u - c)X(p^u, 1/2)/2 = \delta(b-c-\delta/4) - 7\delta(b-c+3\delta/2-\Delta)/8.$$

$\Pi^l - \Pi^u > 0 \Leftrightarrow 7\delta(b-c+2\delta-\Delta)/8 > 0 \Leftrightarrow (b-c+2\delta) > \Delta$. Since we have shown $(b-c+\delta) > \Delta$, it is true that $\Pi^l > \Pi^u$.

If uniform pricing leads to a symmetric kinked equilibrium, then the equilibrium price is between $p^\#$ and p^u . With the similar calculation, we can prove $p^\# \geq p^l$ and the proposition also holds when the equilibrium of uniform price competition rests on the kink.

Proof of Proposition 6

Marginal cost pricing is clear by substituting (22) into (21). The first equality in (23) is obtained from (22) by setting $p_x = c_x$ and the second by substituting symmetric (20) into $\sigma/(1 - S_x)$. It is obvious that

$$e - \sigma \left\{ 1 + \frac{\exp[(v - e)/\sigma]}{\exp(v_z/\sigma) + \exp[(v - e)/\sigma]} \right\}$$

is continuous and increasing in e . Moreover, it is positive when $e = 0$ and tends to $-\sigma$ when e tends to positive infinity, so (23) has a unique positive solution.

Routine calculation shows that $de/dv_z < 0$ and $e = 2\sigma$ when $v_z \rightarrow -\infty$.

Proof of Proposition 8

Let S be a firm's market share in a symmetric equilibrium. Then,

$$S'' = \exp[v(p'')/\sigma] / \{ \exp(v_z/\sigma) + 2\exp[v(p'')/\sigma] \},$$

$$S' = \exp[(v - e)/\sigma] / \{ \exp(v_z/\sigma) + 2\exp[(v - e)/\sigma] \},$$

$$CS'' = \sigma \ln \{ \exp(v_z/\sigma) + 2\exp[v(p'')/\sigma] \}, \quad CS' = \sigma \ln \{ \exp(v_z/\sigma) + 2\exp[(v - e)/\sigma] \},$$

$$\Pi'' = 2(p'' - c)x(p'')S'', \quad \Pi' = 2eS'.$$

First, let us suppose no outside option ($v_z \rightarrow -\infty$). Then, $S'' = S' = 1/2$ and $CS'' - CS' > e - (p'' - c)x(c)$. Recalling $e = 2\sigma$ and applying (24), we have $CS'' - CS' > 2\sigma - 2\sigma[x(p'') + (p'' - c)x'(p'')]x(c)/x^2(p'')$. By the log-concavity of the demand function,

$$x(p'') - x(c) = (p'' - c)x'(\hat{p}) \geq (p'' - c)x'(p'')x(\hat{p})/x(p'') > (p'' - c)x'(p'')x(c)/x(p''),$$

where $\hat{p} \in (c, p'')$. Therefore, $CS'' - CS' > 2\sigma - 2\sigma\{x(c)/x(p'') + [x(p'') - x(c)]/x(p'')\} = 0$.

Recalling (21), $\Pi'' = (p'' - c)x(p'') = 2\sigma[x(p'') + (p'' - c)x'(p'')]x(p'') < 2\sigma = e = \Pi'$.

With fully covered markets, $W'' - W' = v(p'') + (p'' - c)x(p'') - v < 0$.

Now consider the case where the outside option exists. Note $CS''(v_z)$ and $CS'(v_z)$ are continuous functions of v_z and $CS''(-\infty) > CS'(-\infty)$. Suppose that there is a finite \tilde{v}_z such that $CS''(\tilde{v}_z) < CS'(\tilde{v}_z)$. Then, $CS''(v_z) - CS'(v_z)$ has at least one zero point. Let \hat{v}_z be the smallest zero point. We obtain $CS''(v_z) < CS'(v_z)$ for $v_z \in (\hat{v}_z, \hat{v}_z + \varepsilon)$, where ε is a sufficiently small positive constant. In turn, $dCS''(v_z)/dv_z = \exp(v_z/\sigma)/\{\exp(v_z/\sigma) + 2\exp[v(p'')/\sigma]\} > \exp(v_z/\sigma)/\{\exp(v_z/\sigma) + 2\exp[(v - e)/\sigma]\} = dCS'(v_z)/dv_z$ for $v_z \in (\hat{v}_z, \hat{v}_z + \varepsilon)$. This implies $CS''(v_z) > CS'(v_z)$ on $\in (\hat{v}_z, \hat{v}_z + \varepsilon)$ and leads to a contradiction. Therefore, it is impossible to have $CS''(v_z) < CS'(v_z)$ for all $v_z \in (-\infty, \infty)$.

Turning to the profit comparison, (21)-(22) show that in response to firm y's tariffs (p_y, e_y) , firm x chooses

$$e_x = \sigma(1 - S_x), \quad (\text{A1})$$

where

$$S_x = \exp[(v - e_x)/\sigma]/\{\exp(v_z/\sigma) + \exp[(v - e_x)/\sigma] + \exp(\bar{v}/\sigma)\} \quad (\text{A2})$$

and $\bar{v} \equiv v(p_y) - e_y$. In particular, let \tilde{e}_x be the solution to (A1) when firm y charges tariffs $(p'', 0)$ and $\bar{v} = v(p'')$. Since firm x prefers to charge tariffs (c, \tilde{e}_x) resulting in profit $\tilde{\pi}_x$ rather than to set $(p'', 0)$ to yield profit $\Pi''/2$, it implies that $\tilde{\pi}_x > \Pi''/2$. When firm y sets tariffs (c, e) and $\bar{v} = v - e$, (21)-(22) show that firm x sets (c, e) to earn $\Pi'/2$. From (A1) and (A2),

$$\frac{de_x}{d\bar{v}} = \frac{\sigma}{(1 - S_x)^2} \frac{dS_x}{d\bar{v}}, \quad \frac{dS_x}{d\bar{v}} = \frac{-\exp[(v - e_x)/\sigma] \left\{ [\exp(v_z/\sigma) + \exp(\bar{v}/\sigma)] \frac{de_x}{d\bar{v}} + \exp(\bar{v}/\sigma) \right\}}{\sigma \{ \exp(v_z/\sigma) + \exp[(v - e_x)/\sigma] + \exp(\bar{v}/\sigma) \}^2}.$$

Solving these two equations, we obtain $de_x/d\bar{v} < 0$ and $dS_x/d\bar{v} < 0$. Thus, as \bar{v} falls from $v(p'')$ to $v - e$, profit increases from $\tilde{\pi}_x$ to $\Pi'/2$, which proves that $\Pi'/2 > \tilde{\pi}_x > \Pi''/2$.

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REFERENCES

- Anderson, S. and A. de Palma, 2000, From local to global competition, *European Economic Review* 44, 423-448.
- Anderson, S., A. de Palma and Y. Nesterov, 1995, Oligopolistic competition and the optimal provision of products, *Econometrica* 63, 1281-1301.
- Anderson, S., A. de Palma and J. Thisse, 1992, *Discrete choice theory of product differentiation* (MIT Press, Cambridge).
- Anderson, S. and M. Engers, 1994, Spatial competition with price-taking firms, *Economica* 61, 125-136.
- Armstrong, M. and J. Vickers, 2001, Competitive price discrimination, *Rand Journal of Economics* 32, 579-605.
- Hotelling, H., 1929, Stability in competition, *Economic Journal* 39, 41-37.
- Laffont, J., P. Rey and J. Tirole, 1998, Network computation: Overview and nondiscriminatory pricing, *Rand Journal of Economics* 29, 1-37.
- Oi, W., 1971, A Disneyland dilemma: Two-part tariffs for a Mickey Mouse monopoly, *Quarterly Journal of Economics* 85, 77-90.
- Phlips, L., 1983, *The economics of price discrimination* (Cambridge University Press, Cambridge).
- Rochet, J. and L. Stole, 2002, Nonlinear pricing with random participation, *Review of Economic Studies* 69, 277-311.
- Varian, H., 1989, Price discrimination, in R. Schmalensee and R. Willig, eds., *Handbook of Industrial Organization*, Vol. 1 (North-Holland, Amsterdam) 598-654.
- Wilson, R., 1993, *Nonlinear pricing* (Oxford University Press, New York).
- Yin, X., 2001, A model of shareholder discounts, *Economic Record* 77, 89-102.

Yin, X., 2000, Two-part tariff competition in a duopoly,
http://papers.ssrn.com/sol3/papers.cfm?abstract_id=249075

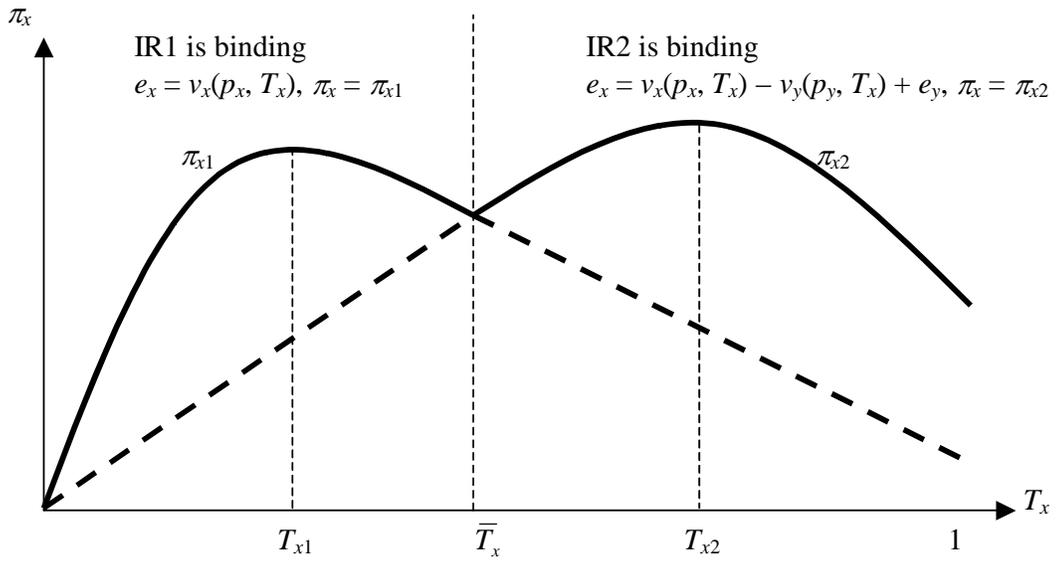


Figure 1 (a)

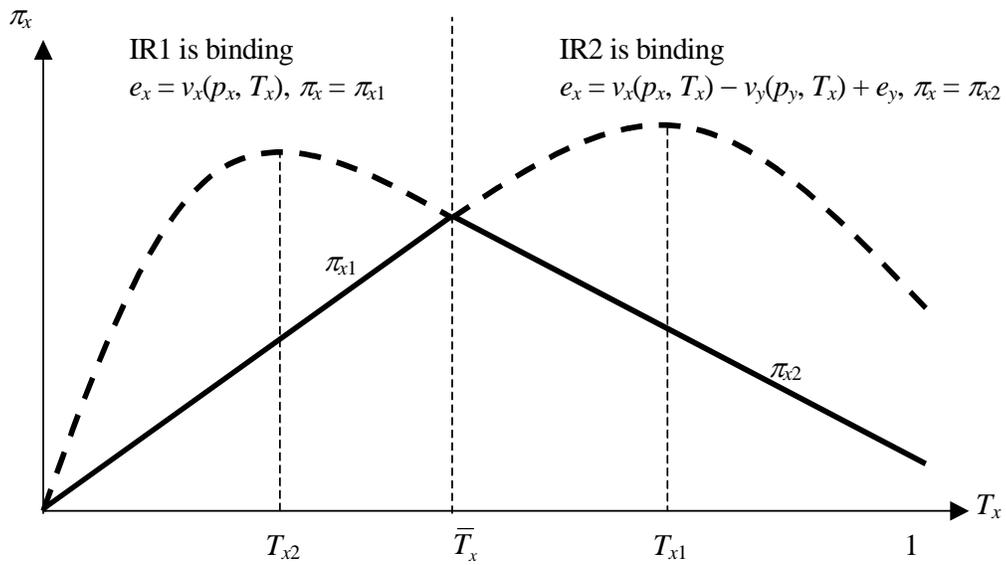


Figure 1 (b)

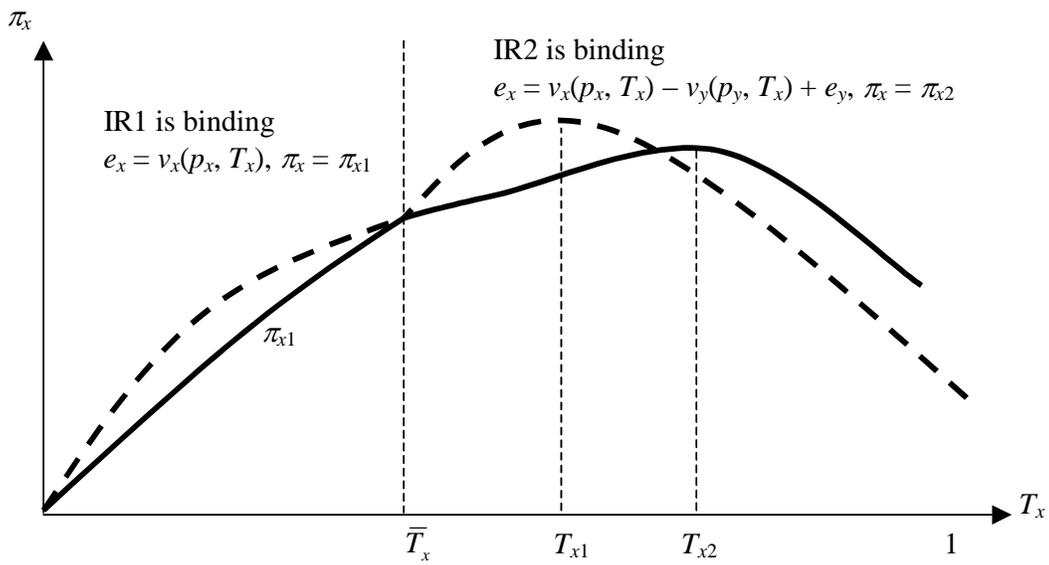


Figure 1 (c)

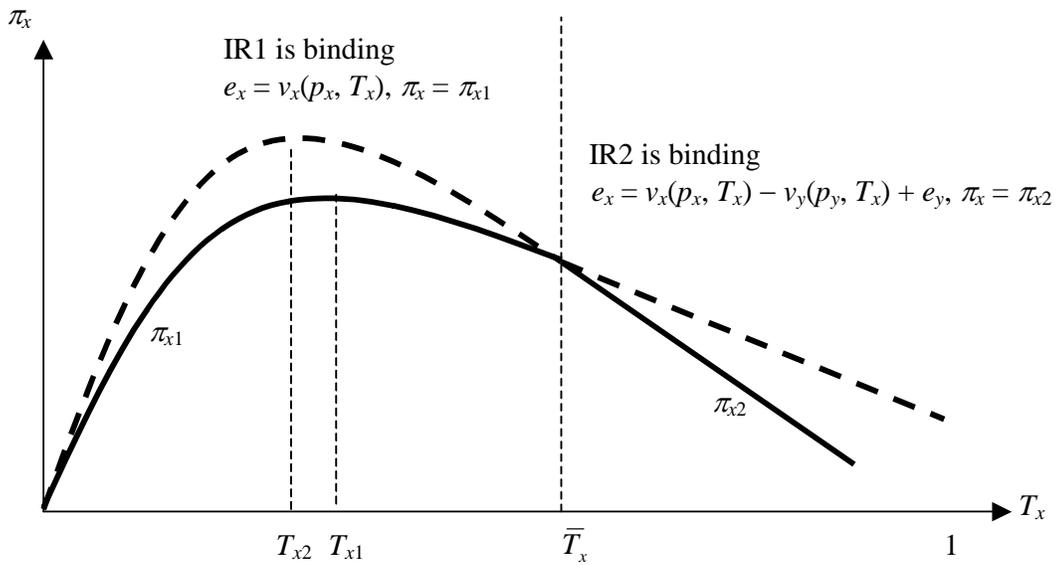


Figure 1 (d)