Technology of upstream firms and equilibrium product differentiation*

Noriaki Matsushima†

Faculty of Economics, Shinshu University

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Abstract

Using a Hotelling-type product differentiation model (linear city model), we investigate the location strategies of upstream and downstream firms. We show that when transport costs of upstream firms are large, higher transport costs decrease the level of product differentiation of downstream firms. We also show that more inefficient transport technologies of upstream firms may enhance welfare. We briefly discuss vertical mergers and show that vertical mergers occur if the transport costs of upstream firms are large enough.

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†Corresponding author: Noriaki Matsushima, Faculty of Economics, Shinshu University, 3-1-1 Asahi, Matsumoto-City, Nagano 390-8621, Japan. Phone: +81-263-37-2951, Fax: +81-263-37-2344, E-mail: nmatsush@econ.shinshu-u.ac.jp

1 Introduction

Since the seminal work of Hotelling (1929), the literature on spatial competition has been rich and diverse.[1] Most studies assume that the marginal costs of the (downstream) firms are exogenous. Furthermore, most economists do not consider the vertical relationship between upstream and downstream firms. That is, the literature does not take into account the possibility that transactions with upstream firms might affect the location strategy of downstream firms.

Gupta *et al.* (1994) consider the situation in which an upstream monopoly sets the price of its output based on the locations chosen by downstream firms. The downstream firms compete on price for each point on a linear city, that is, each downstream firm price discriminates. On the contrary, we consider an upstream duopoly and mill pricing by downstream firms.

Belleflamme and Toulemonde (2003) consider how the endogenous choice of input specificity in the upstream industry affects the choice of product differentiation in the downstream industry. Using a linear demand system, they allow for two possible differentiated brands. Each upstream and downstream firm chooses one brand simultaneously. They show that three possible location patterns can arise: two patterns in which only one brand is chosen by all firms, and one pattern in which some firms choose one brand and some choose the other. In their model, the degree of product differentiation of the downstream firms is exogenous.

Pepall and Norman (2001) also consider vertical relationships and product differentiation. They consider vertical mergers and vertical foreclosure, but not endogenous product differentiation. In their model, the degree of product differentiation of the downstream firms is exogenous.

Other literature considers buyer-seller networks. In networks, buyers are downstream firms and sellers are upstream ones. Kranton and Minehart (2000) state that "Case studies show an abundance of industries organized as networks: manufactures procure specialized inputs from suppliers that, in turn, sell to several other manufacturers." Kranton and Mine-

[1] See, for example, d'Aspremont *et al.* (1979) and Anderson *et al.* (1992), among others.

hart (2000) also point out that "a link between a buyer and seller allows the seller to make specialized inputs to the buyer's specifications." In manufacturing, customized equipment or any specific asset is a link between two firms. Such links are similar to the network in a vertical exchange that would connect the firms.

For instance, in the aircraft industry, the jet (turboprop) engine and the jet (turboprop) aircraft industries are vertically related. To produce a differentiated product, aircraft firms have to procure suitable equipment. Bonaccorsi and Giuri (2001) point out that "In the presence of economies of scope, engine programs are potentially applicable to different aircraft programs of different manufactures. This allows engine companies to relate to many buyers, and potentially to all of them."

In the automobile industry, Toyota procures electric parts from Denso, which is one of the largest auto-part manufactures. Denso sometimes supplies to other automobile manufacturers, for instance, Daimler Chrysler.[2] Thus, electric parts may be used in automobiles of different manufactures.

To explain these tasks of upstream firms, some economists suppose that they are able to use a flexible manufacturing system (FMS).[3] Eaton and Schmitt's (1994) framework of FMS is as follows: "By incurring a sunk cost of product development, upstream firms develop the ability to produce a *basic product*, described by a point in Hotelling's attribute space, at a constant marginal cost. A basic product can be modified to produce any other variant in the attribute space, but such modification involves additional costs; the costs of switching the production process from one variant to another, and a per-unit cost of modification that is proportional to the difference between the basic product and the variant (1994, p. 875)."[4]

Using the linear city model, we consider the relationship between differentiation strategies and vertical relations. Our setting is as follows. There are two upstream firms and two

[2] Ahmadjian and Lincoln (2001) research the vertical relationship between Toyota and Denso.

[3] Norman and Thisse (1999) say: "The essence of FMS is that it allows firms to customize their products to the requirements of heterogeneous consumers at little or no cost penalty."

[4] Eaton and Schmitt (1994) use a spatial price discrimination model based on Lederer and Hurter (1986), MacLeod *et al.* (1988), and Hamilton *et al.* (1989). For the spatial quantity discrimination model, see Anderson and Neven (1991) for instance.

downstream firms. Each upstream firm symmetrically locates in a linear city. The location of the upstream firms is exogenously given.[5] Each downstream firm chooses a location in the city, and has to buy input from upstream firms. Each upstream firm engages in price competition for the business of downstream firms. The inputs produced by the upstream firms are perfect substitutes. To supply one unit of input for a downstream firm, each upstream firm incurs a transport cost quadratic in distance.[6] We can interpret the transport costs of upstream firms as the conversion cost from the product of an upstream firm into an input suitable for a downstream firm. After purchasing its input from an upstream firm, each downstream firm sets its retail price. Observing these retail prices, consumers select a downstream firm. As in d'Aspremont *et al.* (1979), each consumer pays a travelling cost that is quadratic in distance between the downstream firm and the consumer's location.

In this paper, it is shown that, when the transport costs of upstream firms are large, the degree of product differentiation of downstream firms decreases as transport costs increase. That is, the maximum differentiation result of d'Aspremont *et al.* (1979) need not arise in equilibrium. When each upstream firm is located close to the mid-point, the downstream firms maximally differentiate. On the other hand, when the upstream firms are located far apart, the downstream firms locate near the center.

We also discuss vertical mergers between downstream and upstream firms. To develop an equilibrium theory of the vertical merger, we follow the method of Ordover *et al.* (1990). We consider a special case in which upstream firms are located at ends of the linear city. We show that if the per unit transport costs of upstream firms are large, a vertical merger occurs, and the merged downstream firm locates at the end that is nearer to the merged upstream firm. We also show that if the per unit transport costs of the upstream firms are large enough, both downstream firms merge vertically, and each merged downstream firm locates at an end.

[5] We consider endogenous location choice of the upstream firms in Section 4.

[6] It is assumed here that both firms' and consumers' transport costs are quadratic. In Eaton and Schmitt (1994), Norman and Thisse (1999), and Belleflamme and Toulemonde (2003), the upstream firm's transport cost is linear in distance. Even if we employ the same transport technology as in the related papers, several results of our paper still hold.

In Section 4, we extend the basic model to consider endogenous location choice by upstream firms. As the transport technologies of the upstream firms become worse, the social surplus rises as long as the technologies are not too inefficient. That is, we show that the flexibility of production of upstream firms (efficient transport technology) does not always enhance social welfare. Inefficient transport technology induces moderate product differentiation in downstream firms because the per unit production cost is higher when the downstream firm differentiates its product more.

The remainder of this paper is organized as follows. Section 2 presents the basic model. Section 3 shows the results. Section 4 extends the basic model. Section 5 concludes. All proofs of propositions are presented in the Appendix.

2 The model

There are two downstream firms, D_1 and D_2, which produce the same physical product. A linear city of length 1 lies on a line, and consumers are uniformly distributed with density 1 along this interval.[7] Suppose that D_1 (*resp.* D_2) is located at point $l_1 \in [0,1]$ (*resp.* $1 - l_2 \in [0,1]$). Without loss of generality, it is sufficient to consider only the case in which $l_1 \leq 1 - l_2$. A consumer living at $y \in [0,1]$ incurs a transport cost of $t(l_1 - y)^2$ (*resp.* $t(1 - l_2 - y)^2$) when purchasing a product from D_1 (*resp.* D_2). Consumers have unit demands, i.e., each consumes one unit of the product.

Two upstream firms, U_A and U_B, supply inputs to the two downstream firms. Suppose that U_A (*resp.* U_B) is located at $h_A \in [0,1/2]$ (*resp.* $1-h_B \in [1/2,1]$) and $h_A = h_B = h$ ($h \leq 1/2$).[8] Upstream firms produce a homogenous output, which is the input to downstream firms. Upstream firms engage in price competition for the business of downstream firms. To supply an s distant downstream firm, an upstream firm incurs a transport cost τs^2. We assume that $\tau \leq t$. τ can be interpreted as the degree of difficulty, which is a reflection of the transport of downstream firms' inputs to their locations.

[7] This formulation is due to Hotelling (1929) and d'Aspremont *et al.* (1979).

[8] In Section 3.4, we discuss a case in which the firms locate asymmetrically. In Section 4, we discuss endogenous location choice of upstream firms.

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Figure 1

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We analyze a four-stage game. In the first stage, downstream firms simultaneously choose their locations. In the second stage, each upstream firm, U_i $(i = A, B)$, simultaneously chooses its wholesale prices, $w_{ij} \in [0, \infty)$ $(j = 1, 2)$, where j is the index of the downstream firm, D_j $(j = 1, 2)$. For instance, w_{A2} is U_A's wholesale price for D_2. Each upstream firm engages in price competition for the business of downstream firms. In the third stage, observing the wholesale prices, each downstream firm chooses its supplier between U_A and U_B and then sets its retail price $p_i \in [0, \infty)$ $(i = 1, 2)$ simultaneously. In the fourth stage, observing the retail prices, consumers select between the sellers D_1 and D_2.

3 Basic analysis

3.1 The third and fourth stages

As assumed in Section 2, we only consider the case in which $l_1 \leq 1 - l_2$. For a consumer living at

$$x = \frac{1 + l_1 - l_2}{2} + \frac{p_2 - p_1}{2t(1 - l_1 - l_2)}, \tag{1}$$

the full price (transport cost plus price) is the same at either of the two firms.[9] The profit of each downstream firm is given by

$$\pi_{d1} \equiv (p_1 - w_1)\left(\frac{1 + l_1 - l_2}{2} + \frac{p_2 - p_1}{2t(1 - l_1 - l_2)}\right), \tag{2}$$

$$\pi_{d2} \equiv (p_2 - w_2)\left(\frac{1 - l_1 + l_2}{2} + \frac{p_1 - p_2}{2t(1 - l_1 - l_2)}\right). \tag{3}$$

In (2) and (3), $w_1 = \min\{w_{A1}, w_{B1}\}$ and $w_2 = \min\{w_{A2}, w_{B2}\}$. These mean that D_1 (D_2) procures its input from the upstream firm that bids the lower wholesale price. Inserting the first-order conditions of downstream firms, we have

$$\pi_{d1} = \frac{((1 - l_1 - l_2)(3 + l_1 - l_2)t - w_1 + w_2)^2}{18(1 - l_1 - l_2)t}, \tag{4}$$

[9] Equation (1) is derived from the following equation: $t(x - l_1)^2 + p_1 = t(1 - l_2 - x)^2 + p_2$.

$$\pi_{d2} = \frac{((1-l_1-l_2)(3-l_1+l_2)t+w_1-w_2)^2}{18(1-l_1-l_2)t}. \quad (5)$$

3.2 The second stage

For each downstream firm, each upstream firm sets its price at the rival firm's transport cost if its cost is lower than the rival's (Bertrand competition).[10] The prices set by U_A and U_B are as follows:

$$U_A: \ w_{A1} = \max\{\tau(l_1-h)^2, \tau(1-h-l_1)^2\}, \ w_{A2} = \max\{\tau(1-l_2-h)^2, \tau(l_2-h)^2\}, \quad (6)$$

$$U_B: \ w_{B1} = \max\{\tau(l_1-h)^2, \tau(1-h-l_1)^2\}, \ w_{B2} = \max\{\tau(1-l_2-h)^2, \tau(l_2-h)^2\}. \quad (7)$$

3.3 The first stage

It is sufficient to consider only the location pattern in which $l_1 \leq 1/2$ and $l_2 \leq 1/2$.[11] From (1), (4), (5), (6), and (7), the profits of the downstream firms are:

$$\pi_{d1} = \frac{[(3+l_1-l_2)(1-l_1-l_2)t+(l_1-l_2)(2-2h-l_1-l_2)\tau]^2}{18t(1-l_1-l_2)}, \quad (8)$$

$$\pi_{d2} = \frac{[(3-l_1+l_2)(1-l_1-l_2)t+(l_2-l_1)(2-2h-l_1-l_2)\tau]^2}{18t(1-l_1-l_2)}. \quad (9)$$

The first-order conditions lead to the following proposition:

Proposition 1 *The following location pattern is an equilibrium outcome:*[12]

$$l_1 = l_2 = 0, \quad \text{if } \tau \leq \frac{t}{4(1-h)}, \text{ and} \quad (10)$$

$$l_1 = l_2 = \frac{4(1-h)\tau - t}{4(t+\tau)}, \quad \text{if } \frac{t}{4(1-h)} < \tau < \frac{3(4\sqrt{2}-5)t}{2(1-2h)}. \quad (11)$$

[10] In this paper, we only focus on equilibrium outcomes in which each upstream firm sets its price at the rival firm's transport cost if its cost is lower than the rival's. However, when τ is large enough, an upstream firm's optimal price may be lower than the rival firm's transport costs. Our propositions do not change, even though we take into account this pricing strategy of an upstream firm. This is discussed in the Appendix (proof of Proposition 1).

[11] We can show that the following location pattern does not occur as an equilibrium outcome: $l_1 < 1/2$ and $l_2 > 1/2$ (or $l_1 > l/2$ and $l_2 < 1/2$). The proof is presented on the Web.

[12] $\frac{3(4\sqrt{2}-5)t}{2(1-2h)}$ in (11) is smaller than t if and only if $h < (17-12\sqrt{2})/4 (\sim 0.008)$.

At the equilibrium outcome, each firm's profit is:

$$\pi_{d1} = \pi_{d2} = \frac{t}{2}, \quad \textit{if} \;\; \tau \leq \frac{t}{4(1-h)}, \;\; \textit{and} \tag{12}$$

$$\pi_{d1} = \pi_{d2} = \frac{t(3t - 2(1-2h)\tau)}{4(t+\tau)}, \quad \textit{if} \;\; \frac{t}{4(1-h)} < \tau < \frac{3(4\sqrt{2}-5)t}{2(1-2h)}. \tag{13}$$

We now show the intuition behind Proposition 1. In this model, given the locations of the other three firms, when a downstream firm moves farther away from its rival, three effects occur. First, price competition between downstream firms is softened ("price effect"). The price effect, which is similar to that in d'Aspremont *et al.* (1979), enhances the profit of the downstream firm (D_1). Second, the demand for the downstream firm falls ("demand effect"). The demand effect, which is similar to that in d'Aspremont *et al.* (1979), diminishes the profit of the downstream firm. Third, the wholesale price for the downstream firm rises ("input price effect") because the distance increases between the downstream firm and other supplier (eg. D_1 and U_B).

The input price effect is affected by the value of τ (see (6) and (7)). The larger is τ, the larger are wholesale prices. On the other hand, the price and demand effects are unaffected by the value of τ because the cost of each downstream firm is equal to that of its rival in equilibrium. From (6) and (7), wholesale prices are $\tau(1-h-l_1)^2 (= \tau(1-h-l_2)^2)$ in a symmetric equilibrium. Therefore, when τ is large, the third effect is important for downstream firms, and the maximal differentiation result does not hold. From (11), the larger the value of τ, the more efficient the location pattern. If $\frac{t}{4(1-h)} < \tau$, the larger the value of τ, the smaller the profit of each downstream firm.

From Proposition 1 and (11), we have:

Corollary 1 *If* $\tau < \frac{3(4\sqrt{2}-5)t}{2(1-2h)}$, *the larger* h, *the smaller* l_1 *and* l_2.

When upstream firms locate near the center, downstream firms locate far apart. On the other hand, when upstream firms locate far apart, downstream firms locate near the center. When upstream firms locate close together, mitigating price competition is more important than reducing procurement costs because an input cost reduction is negligible. As a result, each downstream firm is far from the central point.[13]

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Figure 2

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Social welfare is discussed briefly here. When $\tau \leq \frac{t}{4(1-h)}$, social welfare is:[14]

$$\begin{aligned} SW &= -\int_0^{\frac{1}{2}} t(x-l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1-l_2-x)^2 dx - \frac{1}{2}\tau(l_1-h)^2 - \frac{1}{2}\tau(l_2-h)^2 \\ &= -\frac{t+12h^2\tau}{12}. \end{aligned}$$

This is decreasing with respect to τ. When $\frac{t}{4(1-h)} < \tau$, social welfare is:

$$\begin{aligned} SW &= -\int_0^{\frac{1}{2}} t(x-l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1-l_2-x)^2 dx - \frac{1}{2}\tau(l_1-h)^2 - \frac{1}{2}\tau(l_2-h)^2 \\ &= -\frac{13t^2 - 4(11-18h-12h^2)t\tau + 48(1-2h)^2\tau^2}{48(t+\tau)}. \end{aligned}$$

When h is small and τ is large, social welfare is larger than when $\tau = 0$. This location pattern is efficient from the viewpoint of social welfare, because consumers' aggregate transport cost is lower than when $\tau = 0$ (which leads to the maximum differentiation).

3.4 Asymmetric location

Assume that $h_A < h_B < 1/2$, so U_B is closer to the central point. Suppose that the downstream firms locate symmetrically. Then, D_2 (the right-hand-side downstream firm) faces higher procurement cost than D_1.

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Figure 3

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[13] Corollary 1 is somewhat restrictive. If we assume that upstream firms have *linear* transport technology, the location of the upstream firms does not affect the difference in procurement costs. Therefore, l_1 and l_2 are independent of h if upstream firms have linear transport costs. Convexity of the transport function may give the results we see in Corollary 1.

[14] It is sufficient to consider only the social costs of serving the market.

Consider the location choice of the downstream firms. First, the price effect is more severe for the inefficient downstream firm because it has a cost disadvantage. The price effect depends on τ. Second, τ directly affects the efficiency of the downstream firms (the input price effect) because it affects their procurement costs.

On the one hand, when τ is small, the price effect is almost the same for both downstream firms, and the input price effect is more important to the less efficient downstream firm. Therefore, D_2 is closer to the central point. On the other hand, when τ is large, the price effect does not effect the efficient downstream firm, D_1; however, the price effect strongly effects D_2 because the difference between the procurement costs increases. Therefore, D_2 tends to be far from the center, but D_1 tends to be close to the center.

3.5 Integrated firms

Suppose that each pair of downstream and upstream firms integrate. Suppose that firm 1's (*resp.* firm 2's) input source, U_A (*resp.* U_B), is located at 1 (*resp.* 0), then

Proposition 2 *Suppose that U_A and U_B locate at $h_A = 1$ and $1 - h_B = 0$ ($h_B = 1$), respectively. The following location pattern is an equilibrium outcome:*

$$l_1 = l_2 = 0, \quad \textit{if} \;\; \tau \leq \frac{t}{4}, \;\; \textit{and} \tag{14}$$

$$l_1 = l_2 = \frac{4\tau - t}{4(t+\tau)}, \quad \textit{if} \;\; \frac{t}{4} < \tau < \frac{t}{2}. \tag{15}$$

The following location pattern is also an equilibrium outcome:

$$l_1 = l_2 = 1. \tag{16}$$

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Figure 4

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The location pattern in (14) and (15) is somewhat curious and inefficient. The pattern can appear as a coordination failure of the downstream firms. We now show the intuition behind the proposition.

For a downstream firm, moving from its location to the other side of the market has two effects. On the one hand, the movement makes the downstream firm more efficient because the new location is closer to its own input source. The effect is positive. On the other hand, the movement induces fierce price competition because the new location is closer to the rival firm. The effect is negative. However, as mentioned earlier, the price effect on the efficient downstream is slight.

If τ rises, the first (positive) effect is larger because savings in transport cost increase. The second effect is less important because the difference between input costs increases; that is, the (Bertrand) price competition is not so fierce for the efficient downstream firm. Therefore, for any $\tau < t/2$, the location pattern (which is similar to that in proposition 1) holds.

3.6 Vertical merger

In this subsection, we briefly discuss vertical mergers between downstream and upstream firms. To develop an equilibrium theory of vertical merger, we follow Ordover *et al.* (1990) and consider a game with the following stages. In stage 1, downstream firms can bid to acquire U_B. When a vertical merger occurs, we assume, without loss of generality, that it involves D_2, and the integrated firm is called I. In stage 2, D_1 can counter the merger of D_2 and U_B, if there is one, by a merger with the un-integrated upstream firm, U_A. In stage 3, the firms play the four-stage game as in Section 2.

For simplicity, we suppose that $h = 0$, that is, each upstream firm locates at each end of the linear city. After tedious calculations, we have the following result:

Proposition 3 *Suppose that* U_A *and* U_B *locate at* $h_A = 0$ *and* $1 - h_B = 1$ $(h_B = 0)$*, respectively. No vertical merger occurs, if and only if* $\tau < 3(3\sqrt{3} - 5)t/2(\sim 0.294t)$*. One vertical merger occurs, if and only if* $3(3\sqrt{3} - 5)t/2 \leq \tau < (39 + \sqrt{2769})t/144(\sim 0.636t)$*. Two vertical mergers occur, if and only if* $(39 + \sqrt{2769})t/144 < \tau$*. The location patterns of* *the firms are as follows:*[15]

$$\begin{aligned}
&l_1 = l_2 = 0 && \textit{(no merger)}, && \textit{if } \tau \le \frac{t}{4}, \\
&l_1 = l_2 = \frac{4\tau - t}{4(t+\tau)} && \textit{(no merger)}, && \textit{if } \frac{t}{4} < \tau < \frac{3(3\sqrt{3}-5)t}{2}, \\
&l_1 = l_2 = 0 && \textit{(one merger)}, && \textit{if } \frac{3(3\sqrt{3}-5)t}{2} \le \tau \le \frac{t}{3}, \\
&l_1 = \frac{3\tau - t}{3(t+\tau)},\ l_2 = 0 && \textit{(one merger)}, && \textit{if } \frac{t}{3} \le \tau < \frac{(39+\sqrt{2769})t}{144}, \\
&l_1 = l_2 = 0 && \textit{(two mergers)}, && \textit{if } \frac{(39+\sqrt{2769})t}{144} \le \tau.
\end{aligned} \tag{17}$$

If τ is large, a downstream firm buys an upstream firm, and then the integrated downstream firm tends to be far away from its rival (note that $l_2 = 0$ for any $\tau \ge 3(3\sqrt{3}-5)t/2$). As mentioned earlier, as τ increases, the competition between the (non-integrated) downstream firms is tougher. To avoid this competition, the downstream firm merges.

From (17), we find that a vertical merger induces the integrated downstream firm to locate at the end. This is because the integrated downstream firm procures its input from its own upstream firm at its transport cost. Access to the other supplier has no strategic effect for the integrated downstream firm, but the price effect is important for it. Therefore, the integrated downstream firm locates at the end.

4 Endogenous location choice

We now analyze a four-stage game. In the first stage, downstream firms and upstream firms simultaneously choose their locations. The second, third, and fourth stages are similar to Section 3.

[15] We exclude the inefficient location patterns that are discussed in Section 3.5.

Proposition 4 *The following location pattern is an equilibrium outcome:*

$$\begin{aligned}
&l_1 = l_2 = 0,\; h_A = h_B = 0, && \textit{if } \tau \le \frac{t}{4}, \\
&l_1 = l_2 = \frac{4\tau - t}{4(t+\tau)},\; h_A = h_B = 0, && \textit{if } \frac{t}{4} < \tau \le \frac{3t}{7}, \textit{ and} \\
&l_1 = l_2 = \frac{4t + 13\tau - 3\sqrt{4t^2 + 4t\tau + 9\tau^2}}{8(t+\tau)}, && \textit{if } \frac{3t}{7} < \tau. \\
&h_A = h_B = \frac{3\sqrt{4t^2 + 4t\tau + 9\tau^2} - (6t + 5\tau)}{8\tau}, &&
\end{aligned} \tag{18}$$

In the equilibrium outcome, the profit of each firm is:

$$\begin{aligned}
&\pi_{d1} = \pi_{d2} = \frac{t}{2}, && \textit{if } \tau \le \frac{t}{4}, \\
&\pi_{d1} = \pi_{d2} = \frac{t(3t - 2\tau)}{4(t+\tau)}, && \textit{if } \frac{t}{4} < \tau \le \frac{3t}{7}, \textit{ and} \\
&\pi_{d1} = \pi_{d2} = \frac{3t(\sqrt{4t^2 + 4t\tau + 9\tau^2} - 3\tau)}{8(t+\tau)}, && \textit{if } \frac{3t}{7} < \tau.
\end{aligned} \tag{19}$$

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Figure 5

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As τ increases, the downstream firms tend to move toward the center. Now consider the locations of upstream firms. By (18), when $\tau > 3t/7$, an upstream firm tends to move towards its downstream client. It then more efficiently supplies the downstream client ("cost reducing effect") because the distance between them decreases. Moreover, the price paid by its downstream client does not change.

However, there is another effect. The rival's input price falls because the distance between the upstream firm and the other downstream firm decreases. The decrease causes a cost disadvantage for the downstream client. The quantity demanded by its downstream client decreases ("supply effect"). The cost-reducing effect and the supply effect are traded off.[16]

We now briefly discuss social welfare. When $\tau \le t/4$, social welfare is:

$$SW = -\int_0^{\frac{1}{2}} t(x - l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1 - l_2 - x)^2 dx - \frac{1}{2}\tau(l_1 - h_A)^2 - \frac{1}{2}\tau(l_2 - h_B)^2$$

[16] If we assume that upstream firms have linear transport technology ($\tau(l_i - h_i)$), the cost-reducing effect dominates the supply effect. The upstream firms locate at the same place as the downstream ones. The proof of the linear city case is presented on the Web.

$$= -\frac{t}{12}.$$

When $t/4 < \tau \leq 3t/7$, social welfare is:

$$\begin{aligned} SW &= -\int_0^{\frac{1}{2}} t(x-l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1-l_2-x)^2 dx - \frac{1}{2}\tau(l_1-h_A)^2 - \frac{1}{2}\tau(l_2-h_B)^2 \\ &= -\frac{13t^2 - 44t\tau + 48\tau^2}{48(t+\tau)}. \end{aligned}$$

This is increasing with respect to τ. When $3t/7 < \tau$, social welfare is:

$$\begin{aligned} SW &= -\int_0^{\frac{1}{2}} t(x-l_1)^2 dx - \int_{\frac{1}{2}}^1 t(1-l_2-x)^2 dx - \frac{1}{2}\tau(l_1-h_A)^2 - \frac{1}{2}\tau(l_2-h_B)^2 \\ &= -\frac{(108t^3 + 494t^2\tau + 845t\tau^2 + 972\tau^3) - 9(6t^2 + 23t\tau + 36\tau^2)\sqrt{4t^2 + 4t\tau + 9\tau^2}}{96\tau(t+\tau)}. \end{aligned}$$

After tedious calculations, we find that for $\tau > t/4$ social welfare is larger than when $\tau = 0$. As the transport technologies of the upstream firms become worse, the social surplus becomes larger, when the technologies are not too inefficient. Inefficient transport technologies induce moderate product differentiation between the downstream firms because production costs of the downstream firms increase when the downstream firms differentiate more.

We now briefly mention some welfare implication of the result. Upstream firms have two ways to improve their efficiency: improving transport technologies and reducing (marginal) production costs. When they improve transport technologies, they are able to meet more demands from downstream firms. These enable the downstream firms to make their product more differentiated. In this setting, however, the strategies of product differentiation are not as beneficial for social welfare. For the social surplus, it is not as important to improve the transport technologies of upstream firms. Therefore, it might be more important to reduce production costs than to improve transport technologies.

5 Concluding remarks

Using a Hotelling-type product differentiation model, we have investigated the relationship between product differentiation strategies of downstream firms and their vertical relationship. Given relatively large transport costs born by upstream firms, the larger they are, the smaller the level of product differentiation between downstream firms. When each upstream firm locates near the center, the downstream firms locate at opposite ends. On the other hand, when the upstream firms locate at opposite ends, downstream firms locate near the center.

The result presented here may not be the unique equilibrium outcome. First- and the second-order conditions are somewhat complex, and it is difficult to establish uniqueness. This will be a significant undertaking for future research.

The effect of a vertical merger between an upstream firm and a downstream firm could be treated more elaborately. The wider problem includes the possibility of foreclosure: see Ordover *et al.* (1990), Hart and Tirole (1990), and Choi and Yi (2000), among others. These authors assume that the substitutability of the downstream firms' products is exogenous. When the problem of foreclosure is considered within our framework, the situation in which a vertical merger effects the strategies of a firm's product differentiation should be taken into account. These themes would be interesting subjects for future research.

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Appendix

The Appendices are on the web:

http://www.mgmt.purdue.edu/centers/ijio/sup/sup.htm

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