Appendix C: The van Ophem and Schram estimator

The indirect utilities $y^*_{d,i}$ of the choices 'cooperation' (*coop*), 'no cooperation' (*no coop*), 'vertical cooperation' (*vert*), and 'mixed cooperation' (*mix*) for firm i ($d = coop, no\ coop, vert, mix$) are assumed to be linearly dependent on a set of explanatory variables summarized in row vector $\boldsymbol{x_t}$:

$$\begin{array}{lcllll} y^*_{coop,i} & = & \boldsymbol{x_t \vartheta} & + & \kappa\ I_i & + & \omega_{no\ coop,i}, \\ y^*_{no\ coop,i} & = & \boldsymbol{x_t \tau} & & & + & \omega_{coop,i}, \\ y^*_{vert(coop),i} & = & \boldsymbol{x_t \alpha} & & & + & \omega_{vert(coop),i}, \\ y^*_{mix(coop),i} & = & \boldsymbol{x_t \gamma} & & & + & \omega_{mix(coop),i}, \end{array} \tag{1}$$

where the inclusive value I_i is given by $I_i = log[exp(\boldsymbol{x_t\alpha})+exp(\boldsymbol{x_t\gamma})]$. The error terms are type I extreme value distributed. Error term $\omega_{no\ coop,i}$ is independent of $\omega_{coop,i}$. Further, $\omega_{no\ coop,i}$, $\omega_{vert(coop),i}$ and $\omega_{mix(coop),i}$ are independent. Unless $\kappa = 0$, $\omega_{coop,i}$ is correlated with $\omega_{vert(coop),i}$ and $\omega_{mix(coop),i}$. The indicator variables $y_{d,i}$ take on the value 1 if the dth option is chosen, and 0 otherwise. It follows that

$$\begin{array}{lclcl} P_{coop,i} & = & P[y_{coop,i} = 1] & = & \frac{exp(\boldsymbol{x_t\vartheta}+\kappa I_i)}{exp(\boldsymbol{x_t\tau})+exp(\boldsymbol{x_t\vartheta}+\kappa I_i)} \\ P_{no\ coop,i} & = & P[y_{no\ coop,i} = 1] & = & \frac{exp(\boldsymbol{x_t\tau})}{exp(\boldsymbol{x_t\tau})+exp(\boldsymbol{x_t\vartheta}+\kappa I_i)} \\ P_{vert(coop),i} & = & P[y_{vert} = 1 | y_{coop} = 1] & = & \frac{exp(\boldsymbol{\alpha x_t})}{exp(\boldsymbol{x_t\alpha})+exp(\boldsymbol{x_t\gamma})} \\ P_{mix(coop),i} & = & P[y_{mix} = 1 | y_{coop} = 1] & = & \frac{exp(\boldsymbol{\gamma x_t})}{exp(\boldsymbol{x_t\alpha})+exp(\boldsymbol{x_t\gamma})}. \end{array} \tag{2}$$

In order to achieve identification, the following restrictions are imposed: $\boldsymbol{\alpha}$=0 and $\boldsymbol{\vartheta}$=0. The loglikelihoodfunction corresponding to firm i is:

$$log\ L_i = \sum_{d=coop,no\ coop} y_{d,i}\ P(d)_i + \sum_{d=vert,mix} y_{d,i}\ P(d)_i, \tag{3}$$

where the first part of equation (3) corresponds to the choice between cooperation and no cooperation and the second part corresponds to the choice between vertical, horizontal and mixed cooperation, given the firm decided to cooperate at all in the first stage. Equation (3) could be estimated by a two–step procedure which yielded consistent estimates for the coefficients but not for the variance–covariance matrix since the information matrix related to (3) is not block–diagonal. Thus, I estimated the model using a full information maximum likelihood procedure.[1]

[1]The estimation of the van Ophem and Schram (1997) procedure and the simultaneous equation model as well as the Minimum Distance Estimation were performed using my own GAUSS program. A copy of the programs can be obtained from the author upon request.

The gradients corresponding to equation (3) are given by:

$$\begin{aligned} \frac{\partial log\ L_i}{\partial \boldsymbol{\tau}} &= \boldsymbol{x_t} \odot (y_{no\ coop,i} - P_{no\ coop,i}) \\ \frac{\partial log\ L_i}{\partial \kappa} &= I_i(y_{coop,i} - P_{coop,i}) \\ \frac{\partial log\ L_i}{\partial \boldsymbol{\gamma}} &= \boldsymbol{x_t} \odot \Big(y_{mix,i}P_{vert,i} + P_{mix,i}\big(\kappa(y_{coop,i} - P_{coop,i}) - y_{vert,i}\big)\Big). \end{aligned} \tag{4}$$

The marginal effects corresponding to the probabilities shown in equations (2) are:

$$\begin{aligned} \frac{\partial P_{coop,i}}{\partial \boldsymbol{x_t}} &= -\Big(P_{coop,i}\ P_{no\ coop,i}\ P_{vert,i}\Big) \odot \Big(\boldsymbol{\tau} + exp(\boldsymbol{x_t}\boldsymbol{\gamma}) \odot (\boldsymbol{\tau} - \boldsymbol{\gamma}\kappa)\Big) \\ \frac{\partial P_{no\ coop,i}}{\partial \boldsymbol{x_t}} &= -\frac{\partial P_{coop,i}}{\partial \boldsymbol{x_t}}, \\ \frac{\partial P_{vert,i}}{\partial \boldsymbol{x_t}} &= -(P_{vert,i}\ P_{mix,i}) \odot \boldsymbol{\gamma} \\ \frac{\partial P_{mix,i}}{\partial \boldsymbol{x_t}} &= -\frac{\partial P_{vert,i}}{\partial \boldsymbol{x_t}}. \end{aligned} \tag{5}$$