Appendix E: The Minimum Distance Estimator

In order to test if there is a common structure in the parameter estimates for the choice of the alternative vertical information sources, a Minimum Distance Estimator (MDE) is used. A thorough discussion of the MDE and applications are presented in Kodde et al. (1990). Minimum Distance Estimation involves the estimation of the R reduced form parameter vectors in a first stage. In the present case, these reduced form parameters are the parameter estimates obtained from running two separate OLS regressions for the innovation intensity of cooperating and non–cooperating firms. In the second stage, the Minimum Distance Estimator is derived by minimizing the weighted difference between the auxiliary parameter vectors obtained in the first stage.

Besides the practical advantage that the MDE can be easily implemented empirically, it has the further benefit that it provides the researcher with a formal test of common structures among the auxiliary parameter vectors. The MDE is derived by minimizing the distance between the auxiliary parameter vectors under the following set of restrictions:

$$f(\boldsymbol{\beta}, \hat{\boldsymbol{\theta}}) \;=\; \boldsymbol{H}\, \boldsymbol{\beta} \;-\; \hat{\boldsymbol{\theta}} \;=\; \mathbf{0}, \tag{1}$$

where the $R \cdot K \;\times\; K$ matrix $\boldsymbol{H}$ imposes $(R-1)\cdot K$ restrictions on $\boldsymbol{\theta}$. The $R \cdot K \times 1$ vector $\hat{\boldsymbol{\theta}}$ contains the R stacked auxiliary parameter vectors. In the present case, $\boldsymbol{H}$ is defined by a $R \cdot K \times K$–dimensional stacked identity matrix. The MDE is given by the minimization of:

$$D(\boldsymbol{\beta}) \;=\; f(\boldsymbol{\beta}, \hat{\boldsymbol{\theta}})'\, \hat{V}[\hat{\boldsymbol{\theta}}]^{-1}\, f(\boldsymbol{\beta}, \hat{\boldsymbol{\theta}}), \tag{2}$$

where $\hat{V}[\hat{\boldsymbol{\theta}}]$ denotes the common estimated variance–covariance matrix of the auxiliary parameter vectors. Minimization of D leads to

$$\hat{\boldsymbol{\beta}} \;=\; (H'\, \hat{V}[\hat{\boldsymbol{\theta}}]^{-1}\, H)^{-1} H'\, \hat{V}[\hat{\boldsymbol{\theta}}]^{-1}\, \hat{\boldsymbol{\theta}} \tag{3}$$

with variance–covariance matrix

$$\hat{V}[\hat{\boldsymbol{\beta}}] \;=\; \left(H'\, \hat{V}[\hat{\boldsymbol{\theta}}]^{-1}\, H\right)^{-1}. \tag{4}$$

In the present case, where the two equations were estimated using different samples, $V[\hat{\boldsymbol{\theta}}]$ is a matrix carrying the estimated variance–covariance matrices of the first stage parameter vectors on its diagonal blocks. The off–diagonal blocks consist of zero–matrices.

To test the null hypothesis that the R auxiliary parameter vectors coincide with one another, the following Wald–type test statistic can be applied:

$$W \;=\; f(\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\theta}})' \hat{V}[\hat{\boldsymbol{\theta}}]^{-1} f(\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\theta}}) \sim\; \chi^2_{(R-1)\cdot K}. \tag{5}$$