1 The model: linear transport case

The basic setting is similar to that of Matsushima (2004). The only difference is that an upstream firm incurs a transport cost τs to supply a downstream firm at distance s from the upstream firm. That is, the upstream firm's transport cost is linear in distance. In the model, we only consider endogenous locations of upstream firms.

There are two downstream firms, D_1 and D_2, which produce the same physical product. A linear city of length 1 lies on a line, and consumers are uniformly distributed with density 1 along this interval. Suppose that D_1 (*resp.* D_2) is located at point $l_1 \in [0,1]$ (*resp.* $1 - l_2 \in [0,1]$). Without loss of generality, it is sufficient to consider only the case in which $l_1 \leq 1 - l_2$. A consumer living at $y \in [0,1]$ incurs a transport cost of $t(l_1 - y)^2$ (*resp.* $t(1 - l_2 - y)^2$) when purchasing a product from D_1 (*resp.* D_2). Consumers have unit demands, i.e., each consumes one unit of the product.

Two upstream firms, U_A and U_B, supply inputs to two downstream firms. Suppose that U_A (*resp.* U_B) is located at $h_A \in [0,1]$ (*resp.* $1 - h_B \in [0,1]$). Upstream firms produce a homogeneous output, which is the input to downstream firms. Upstream firms engage in price competition for the business of downstream firms. To supply an s distant downstream firm, an upstream firm incurs a transport cost τs. We assume that $\tau \leq t$. τ can be interpreted as the degree of difficulty, which is a reflection of the transport of downstream firms' inputs to their locations.

We analyze a four-stage game. In the first stage, downstream firms and upstream firms simultaneously choose their locations. In the second stage, each upstream firm, U_i $(i = A, B)$, simultaneously chooses its wholesale prices, $w_{ij} \in [0, \infty)$ $(j = 1, 2)$, where j is the index of the downstream firm, D_j $(j = 1, 2)$. For instance, w_{A2} is U_A's wholesale price for D_2. Each upstream firm engages in price competition for the business of downstream firms. In the third stage, observing the wholesale prices, each downstream firm chooses its supplier between U_A and U_B and then sets its retail price $p_i \in [0, \infty)$ $(i = 1, 2)$ simultaneously. In the fourth stage, observing the retail prices, consumers select between sellers D_1 and D_2.

2 Basic analysis: linear transport case

2.1 The third and fourth stages

In this paper, we only consider the case in which $l_1 \leq 1 - l_2$. For a consumer living at

$$x = \frac{1 + l_1 - l_2}{2} + \frac{p_2 - p_1}{2t(1 - l_1 - l_2)}, \tag{1}$$

the full price (transport cost plus price) is the same at either of the two firms. The profit of each downstream firm is given by

$$\pi_{d1} \equiv (p_1 - w_1)\left(\frac{1 + l_1 - l_2}{2} + \frac{p_2 - p_1}{2t(1 - l_1 - l_2)}\right), \tag{2}$$

$$\pi_{d2} \equiv (p_2 - w_2)\left(\frac{1 - l_1 + l_2}{2} + \frac{p_1 - p_2}{2t(1 - l_1 - l_2)}\right). \tag{3}$$

In (2) and (3), $w_1 = \min\{w_{A1}, w_{B1}\}$ and $w_2 = \min\{w_{A2}, w_{B2}\}$. These mean that D_1 (D_2) procures his or her input from the upstream firm (U_A or U_B) that offers the lower wholesale price. Inserting the first-order conditions of downstream firms, we have

$$\pi_{d1} = \frac{((1 - l_1 - l_2)(3 + l_1 - l_2)t - w_1 + w_2)^2}{18(1 - l_1 - l_2)t}, \tag{4}$$

$$\pi_{d2} = \frac{((1 - l_1 - l_2)(3 - l_1 + l_2)t + w_1 - w_2)^2}{18(1 - l_1 - l_2)t}. \tag{5}$$

2.2 The second stage

For each downstream firm, each upstream firm sets its price at the rival firm's transport cost if its cost is lower than the rival's (Bertrand competition). The prices set by U_A and U_B are as follows:

$$U_A: \; w_{A1} = \max\{\tau|l_1 - h_A|, \tau|1 - h_B - l_1|\}, \; w_{A2} = \max\{\tau|1 - l_2 - h_A|, \tau|l_2 - h_B|\}, \tag{6}$$

$$U_B: \; w_{B1} = \max\{\tau|l_1 - h_A|, \tau|1 - h_B - l_1|\}, \; w_{B2} = \max\{\tau|1 - l_2 - h_A|, \tau|l_2 - h_B|\}. \tag{7}$$

2.3 The first stage

In Figure 1-(1) and 1-(4), if U_B moves rightward, the following two changes occur: (a) U_B efficiently supplies D_2 without change of price w_{B2} (for $\tau|1 - l_2 - h_A| > \tau|l_2 - h_B|$); (b) w_{A1} increases (for $\tau|1 - h_B - l_1| > \tau|l_1 - h_A|$). Then, D_1 becomes less efficient and the quantity supplied by D_2 is higher. Both changes enlarge U_B's profit. Therefore, U_B has an incentive to move rightward. In Figure 1-(2) and 1-(3), U_A earns zero profit and has an incentive to relocate.

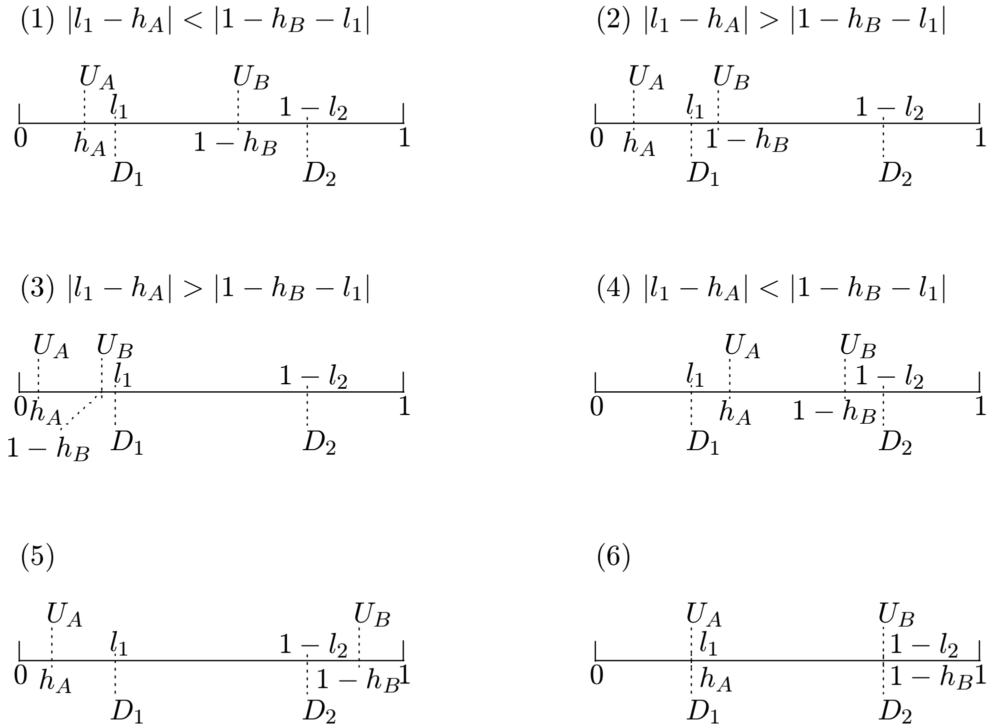


Figure 1: location patterns

Thus, we have only to consider cases (5) and (6). From the discussion above, we have the following lemma:

Lemma 1 *If a location pattern is an equilibrium outcome, the location pattern has to be that in which* $(0 \leq)h_A \leq l_1$ *and* $(0 \leq)h_B \leq l_2$.

We now consider the location pattern in which $(0 \leq)h_A \leq l_1$ and $(0 \leq)h_B \leq l_2$. From (1), (4), (5), (6), and (7), we derive the four firms' profits:

$$\pi_{d1} = \frac{[(3 + l_1 - l_2)(1 - l_1 - l_2)t + (l_1 - l_2 - h_A + h_B)\tau]^2}{18t(1 - l_1 - l_2)}, \tag{8}$$

$$\pi_{d2} = \frac{[(3 - l_1 + l_2)(1 - l_1 - l_2)t - (l_1 - l_2 - h_A + h_B)\tau]^2}{18t(1 - l_1 - l_2)}, \tag{9}$$

$$\pi_{uA} \equiv (w_{A1} - \tau|l_1 - h_A|)x$$

$$= \frac{\tau[1-2l_1+h_A-h_B][(3+l_1-l_2)(1-l_1-l_2)t+(l_1-l_2-h_A+h_B)\tau]}{6t(1-l_1-l_2)}, \quad (10)$$

$$\pi_{uB} \equiv (w_{B2}-\tau|l_2-h_B|)(1-x)$$

$$= \frac{\tau[1-2l_2-h_A+h_B][(3-l_1+l_2)(1-l_1-l_2)t-(l_1-l_2-h_A+h_B)\tau]}{6t(1-l_1-l_2)}. \quad (11)$$

The first-order conditions are as follows:

$$\frac{\partial \pi_{d1}}{\partial l_1} = [(3+l_1-l_2)(1-l_1-l_2)t+(l_1-l_2-h_A+h_B)\tau] \times \frac{(2-l_1-3l_2-h_A+h_B)\tau-(1-l_1-l_2)(1+3l_1+l_2)t}{18t(1-l_1-l_2)^2}, \quad (12)$$

$$\frac{\partial \pi_{d2}}{\partial l_2} = [(3-l_1+l_2)(1-l_1-l_2)t-(l_1-l_2-h_A+h_B)\tau] \times \frac{(2-3l_1-l_2+h_A-h_B)\tau-(1-l_1-l_2)(1+l_1+3l_2)t}{18t(1-l_1-l_2)^2}, \quad (13)$$

$$\frac{\partial \pi_{uA}}{\partial h_A} = \frac{\tau((3+l_1-l_2)(1-l_1-l_2)t-(1-3l_1+l_2+2h_A-2h_B)\tau)}{6t(1-l_1-l_2)}, \quad (14)$$

$$\frac{\partial \pi_{uB}}{\partial h_B} = \frac{\tau((3-l_1+l_2)(1-l_1-l_2)t-(1+l_1-3l_2-2h_A+2h_B)\tau)}{6t(1-l_1-l_2)}. \quad (15)$$

From (14) and (15), we have the following lemma:

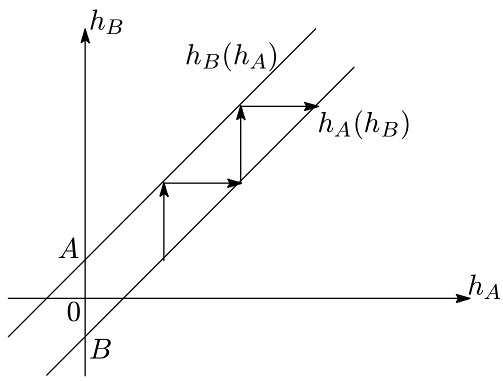
Lemma 2 *In an equilibrium outcome, the following location pattern must hold:* $h_A = l_1$ *and* $h_B = l_2$.

Proof: From (14) and (15), we have the reaction functions of the upstream firms:

$$h_A = h_B + \frac{(3+l_1-l_2)(1-l_1-l_2)t+(1-3l_1+l_2)\tau}{\tau}, \quad (14')$$

$$h_B = h_A + \frac{(3-l_1+l_2)(1-l_1-l_2)t+(1+l_1-3l_2)\tau}{\tau}. \quad (15')$$

¿From (14') and (15'), we find that the reaction functions are parallel. We depict them:



where, $A \equiv \frac{(3-l_1+l_2)(1-l_1-l_2)t+(1+l_1-3l_2)\tau}{\tau}$ and $B \equiv -\frac{(3+l_1-l_2)(1-l_1-l_2)t+(1-3l_1+l_2)\tau}{\tau}$. If $A > B$, the Lemma holds.

$$\begin{aligned} A - B &= \frac{(3-l_1+l_2)(1-l_1-l_2)t+(1+l_1-3l_2)\tau}{\tau} \\ &\quad - \left(-\frac{(3+l_1-l_2)(1-l_1-l_2)t+(1-3l_1+l_2)\tau}{\tau}\right) \\ &= \frac{6(1-l_1-l_2)t-2(1-l_1-l_2)\tau}{\tau} > 0. \end{aligned}$$

Q.E.D.

One of the upstream firms locates at the same point as one of the downstream firms and supplies its input to that downstream one, even though the upstream firm is independent of the downstream firms. Each upstream firm produces a *basic* input for each downstream firm, as if each of the upstream firms belonged to each adjacent downstream firm.

We now show the intuition behind Lemma 2. In Figure 1-(5), if U_A moves rightward, the following two changes occur: (a) U_A efficiently supplies D_1 without change of w_{A1} (for $\tau|1-h_B-l_1| > \tau|l_1-h_A|$). (b) w_{B2} decreases.[1] That is, it enhances the quantity supplied by D_2. The former (*resp.* latter) effect is positive (*resp.* negative) to U_A. In (10), h_A in the first (*resp.* second) pair of brackets represents the former (*resp.* latter) effect. The former (*resp.* latter) effect is the first (*resp.* second) order effect on U_A's profit. Therefore, the former effect dominates the latter one, and Lemma 2 holds.[2]

From Lemma 2 and the first-order conditions, we solve the following simultaneous equations:

$$\frac{\partial \pi_{d1}}{\partial l_1} = 0, \ \frac{\partial \pi_{d2}}{\partial l_2} = 0, \ h_A = l_1, \ h_B = l_2. \tag{18}$$

We derive the following proposition:

Proposition 1 *The following location pattern is an equilibrium outcome:*

$$l_1 = l_2 = h_A = h_B = 0, \quad \textit{if} \ \ \tau \leq \frac{t}{2}, \ \textit{and} \tag{19}$$

$$l_1 = l_2 = h_A = h_B = \frac{2\tau - t}{4t}, \quad \textit{if} \ \ \frac{t}{2} < \tau \leq \frac{(24\sqrt{2}-30)t}{4}. \tag{20}$$

[1] We implicitly assume that $|l_2 - h_B| < |1 - l_2 - h_A|$. If the assumption does not hold, U_B earns *zero* profits and has an incentive to move leftward.

[2] As shown by Matsushima (2004), the latter effect is not dominated by the former, if the transport costs of upstream firms are quadratic in distance.

Proof Substituting $h_A = l_1$ and $h_B = l_2$ into (12) and (13), we have

$$\frac{\partial \pi_{d1}}{\partial l_1} = -\frac{(3 + l_1 - l_2)((1 + 3l_1 + l_2)t - 2\tau)}{18}, \tag{21}$$

$$\frac{\partial \pi_{d2}}{\partial l_2} = -\frac{(3 - l_1 + l_2)((1 + l_1 + 3l_2)t - 2\tau)}{18}. \tag{22}$$

Solving the following simultaneous equations: $\partial \pi_{d1}/\partial l_1 = 0$ and $\partial \pi_{d2}/\partial l_2 = 0$, we have:

$$(l_1, l_2) = \left(-\frac{t - 2\tau}{4t}, -\frac{t - 2\tau}{4t}\right), \left(\frac{t + \tau}{2t}, -\frac{5t - \tau}{2t}\right), \left(-\frac{5t - \tau}{2t}, \frac{t + \tau}{2t}\right). \tag{23}$$

For any $\tau \leq t$, the second and the third pairs violate the boundary condition, $l_1 \geq 0$ and $l_2 \geq 0$.

If $\tau \leq t/2$, the first pair also violates the boundary condition, and the optimal location may be $l_1 = h_A = 0$ and $l_2 = h_B = 0$. We now check whether the location pattern is an equilibrium outcome when $\tau \leq t/2$. From Lemma 2, we find that neither upstream firm has an incentive to change its location. We now show that neither downstream firm has an incentive to change its location. By symmetry, it is sufficient to consider D_1's incentive. Given the locations of the other three firms, there exist two patterns of its location.

1. $0 \leq l_1 \leq 1/2$: U_A supplies D_1 and U_B supplies D_2.

   In this case, π_{d1} is (8) and the first order condition is (12). Substituting $l_2 = h_A = h_B = 0$ into (12), we have

   $$\frac{\partial \pi_{d1}}{\partial l_1} = -\frac{[(3 + l_1)(1 - l_1)t + l_1\tau][(7 + 5l_1)(1 - l_1)t - (2 - 3l_1)\tau]}{18t(1 - l_1)^2} < 0. \tag{24}$$

   For any $0 \leq l_1 \leq 1/2$, $l_1 = 0$ is optimal.

2. $1/2 \leq l_1 \leq 1$: U_A supplies D_1 and U_B supplies D_2.

   In this case, U_B supplies D_1 and D_2. The wholesale price for D_1 is not w_{A1} but w_{B1}. The wholesale price for D_2 is w_{B2}. From (4), (6), and (7), π_{d1} is:

   $$\pi_{d1} = \frac{((3 + l_1 - l_2)(1 - l_1 - l_2)t + (1 - l_1 - l_2)\tau)^2}{18(1 - l_1 - l_2)t}. \tag{25}$$

   The first order condition is

   $$\frac{\partial \pi_{d1}}{\partial l_1} = -\frac{(3t + l_1 t - l_2 t + \tau)(t + 3l_1 t + l_2 t + \tau)}{18t} < 0.$$

   We find that $l_1 = 1/2$ is optimal for any $1/2 \leq l_1 \leq 1$.

From the result of the two patterns, we find that $l_1 = 0$ is optimal.

If $\tau > t/2$, the first pair is an interior solution. We now check whether the first pair in (23) is an equilibrium outcome, that is, we check whether (26) is an equilibrium outcome:

$$(l_1, l_2, h_A, h_B) = \left(-\frac{t-2\tau}{4t}, -\frac{t-2\tau}{4t}, -\frac{t-2\tau}{4t}, -\frac{t-2\tau}{4t}\right). \tag{26}$$

From Lemma 1, we find that neither upstream firm has an incentive to change its location. We now investigate the condition that neither downstream firm has an incentive to change its location. By symmetry, it is sufficient for us to consider D_1's incentive. Given the locations of the other three firms, there exist three patterns of location:

1. $0 \le l_1 \le 1/2$: U_A supplies D_1 and U_B supplies D_2.

   In this case, π_{d1} is (8) and the first order condition is (12). Substituting $l_2 = h_A = h_B = -\frac{t-2\tau}{4t}$ into (12), we have

$$\frac{\partial \pi_{d1}}{\partial l_1} = \frac{(2\tau - t - 4tl_1)(15t - 10\tau - 12tl_1)(65t^2 - 32t\tau - 4\tau^2 - 16t(2t-\tau)l_1 + 16t^2 l_1^2)}{288(5t - 2\tau - 4tl_1)^2}. \tag{27}$$

   We now consider the following three cases. If $0 \le l_1 \le (2\tau - t)/4t$, the value is positive. If $(2\tau - t)/4t \le l_1 \le 5(3t - 2\tau)/12t$ and $5(3t - 2\tau)/12t < 1/2$, or $(2\tau - t)/4t \le l_1 \le 1/2$ and $5(3t - 2\tau)/12t > 1/2$, the value is negative. If $5(3t - 2\tau)/12t \le l_1 \le 1/2$ and $5(3t - 2\tau)/12t < 1/2$, the value is positive. Therefore, if π_{d1} in which $l_1 = (2\tau - t)/4t$ is larger than that in which $l_1 = 1/2$, the optimal location is $l_1 = (2\tau - t)/4t$ for any $0 \le l_1 \le 1/2$.

$$\pi_{d1}\left(\frac{2\tau - t}{4t}\right) = \frac{3t - 2\tau}{4}, \quad \pi_{d1}\left(\frac{1}{2}\right) = \frac{(3t - 2\tau)(15t + 2\tau)^2}{1152t^2}. \tag{28}$$

   We now compare $\pi_{d1}(\frac{2\tau - t}{4t})$ with $\pi_{d1}(\frac{1}{2})$:

$$\begin{aligned} \pi_{d1}\left(\frac{2\tau - t}{4t}\right) - \pi_{d1}\left(\frac{1}{2}\right) &= \frac{3t - 2\tau}{4} - \frac{(3t - 2\tau)(15t + 2\tau)^2}{1152t^2} \\ &= \frac{(3t - 2\tau)(63t^2 - 60t\tau - 4\tau^2)}{1152t^2}. \end{aligned} \tag{29}$$

   If $63t^2 - 60t\tau - 4\tau^2 > 0$, that is, if $\tau < (24\sqrt{2} - 30)t/4 (\sim 0.984t)$, the optimal location is $l_1 = (2\tau - t)/4t$ for any $0 \le l_1 \le 1/2$.

2. $1/2 \leq l_1 \leq 1 - (2\tau - t)/(4t)$: U_A does not supply but U_B supplies D_1 and D_2.

In this case, U_B supplies D_1 and D_2. The wholesale price for D_1 is not w_{A1} but w_{B1}. The wholesale price for D_2 is w_{B2}. From (4), (6), and (7), π_{d1} is:

$$\pi_{d1} = \frac{((3 + l_1 - l_2)(1 - l_1 - l_2)t + (1 - l_1 - l_2)\tau)^2}{18(1 - l_1 - l_2)t}. \tag{30}$$

The first order condition is

$$\frac{\partial \pi_{d1}}{\partial l_1} = -\frac{(3t + l_1 t - l_2 t + \tau)(t + 3l_1 t + l_2 t + \tau)}{18t} < 0.$$

We find that $l_1 = 1/2$ is optimal for any $1/2 \leq l_1 \leq 1 - (2\tau - t)/(4t)$.

3. $1 - (2\tau - t)/(4t) \leq l_1 \leq 1$: U_A does not supply but U_B supplies D_1 and D_2.

In this case, π_{d1} and π_{d2} are

$$\pi_{d1} = (p_1 - w_{B1})\left(1 - \left(\frac{1 + l_1 - l_2}{2} + \frac{p_2 - p_1}{2t(1 - l_1 - l_2)}\right)\right), \tag{31}$$

$$\pi_{d2} = (p_2 - w_{B2})\left(\frac{1 + l_1 - l_2}{2} + \frac{p_2 - p_1}{2t(1 - l_1 - l_2)}\right). \tag{32}$$

Each firm's pricing in the third stage is

$$p_1 = \frac{(l_1 + l_2 - 1)(3 - l_1 + l_2)t + 2w_{B1} + w_{B2}}{3}, \tag{33}$$

$$p_2 = \frac{(l_1 + l_2 - 1)(3 + l_1 - l_2)t + w_{B1} + 2w_{B2}}{3}. \tag{34}$$

π_{d1} and π_{d2} are

$$\pi_{d1} = \frac{((l_1 + l_2 - 1)(3 - l_1 + l_2)t - w_{B1} + w_{B2})^2}{18(l_1 + l_2 - 1)t} = \frac{((l_1 + l_2 - 1)(3 - l_1 + l_2)t - (l_1 + l_2 - 1)\tau)^2}{18(l_1 + l_2 - 1)t}, \tag{35}$$

$$\pi_{d2} = \frac{((l_1 + l_2 - 1)(3 + l_1 - l_2)t + w_{B1} - w_{B2})^2}{18(l_1 + l_2 - 1)t} = \frac{((l_1 + l_2 - 1)(3 + l_1 - l_2)t + (l_1 + l_2 - 1)\tau)^2}{18(l_1 + l_2 - 1)t}. \tag{36}$$

The first order condition is

$$\frac{\partial \pi_{d1}}{\partial l_1} = \frac{((5 - 3l_1 - l_2)t - \tau)((3 - l_1 + l_2)t - \tau)}{18} > 0. \tag{37}$$

For any $1-(2\tau-t)/(4t) \leq l_1 \leq 1$, $l_1 = 1$ is optimal. The profit is

$$\pi_{d1} = \frac{(2\tau-t)(7t-2\tau)}{1152t^2}. \tag{38}$$

This is smaller than $\pi_{d1}(\frac{2\tau-t}{4t})$ in (28).

From the results of the three patterns, we find that $l_1 = (2\tau-t)/4t$ is optimal if $\tau < (24\sqrt{2}-30)t/4 (\sim 0.984t)$. Q.E.D.