Endogenous Sequential Entry in a Spatial Model Revisited: *Mathematica* Simulation

Code © Georg Götz
Caveat: While I tried to document the steps of the simulation, I am not sure that everything comes out clearly. For details on *Mathematica*, see: Stephen Wolfram, The *Mathematica* Book, 3rd edition, Cambridge University Press, Cambridge 1996. The results have been derived with *Mathematica* version 4.1.

Starting from Neven 1987, p. 422: The consumer who is indifferent. The notation is like in the paper except that I always use small letters. Instead of subscripts I put the firm indices in brackets .

```
a[i_,j_]:=(p[j]-p[i])/(2(x[j]-x[i]))+(x[i]+x[j])/2
```

```
a[1,2]
```

$$\frac{-p[1]+p[2]}{2\,(-x[1]+x[2])}+\frac{1}{2}\,(x[1]+x[2])$$

```
demand[i_]:=(a[i,i+1]-a[i,i-1])n
```

```
profit[i_]:=(p[i]-c[i])demand[i]-f[i]
```

```
profit[5]
```

$$-f[5]+ n\,(-c[5]+p[5])\left(\frac{1}{2}\,(-x[4]-x[5])-\frac{p[4]-p[5]}{2\,(x[4]-x[5])}+\frac{-p[5]+p[6]}{2\,(-x[5]+x[6])}+\frac{1}{2}\,(x[5]+x[6])\right)$$

```
foc[i_]:=D[profit[i],p[i]]
```

■ The game with 2 firms

■ Equilibrium of the price-setting game with 2 firms 1 and 2, locations given as well as technology

The definition of the boundaries of the market.

```
a[1,0]=0;a[2,3]=1;
```

```
demand[1]+demand[2]//Simplify
```

n

```
foc[1]
```

$$-\frac{n\,(-c[1]+p[1])}{2\,(-x[1]+x[2])}+n\left(\frac{-p[1]+p[2]}{2\,(-x[1]+x[2])}+\frac{1}{2}\,(x[1]+x[2])\right)$$

```
prices2Firms=Solve[{foc[1]==0,foc[2]==0},{p[1],p[2]}]//Simplify
```

$$\left\{\left\{p[1] \to \frac{1}{3}\,(2\,c[1] + c[2] - 2\,x[1] - x[1]^2 + 2\,x[2] + x[2]^2),\ p[2] \to \frac{1}{3}\,(c[1] + 2\,c[2] - 4\,x[1] + x[1]^2 + 4\,x[2] - x[2]^2)\right\}\right\}$$

The reduced profit function taking into account the pricing rules.

```
redProfit[1]=(profit[1]/.prices2Firms//Simplify)[[1]]
```

$$-\left(18\,f[1]\,(x[1] - x[2]) + n\,(c[1] - c[2] + 2\,x[1] + x[1]^2 - 2\,x[2] - x[2]^2)^2\right) / (18\,(x[1] - x[2]))$$

Operating profits of firm 1 with locations x[1] and x[2]. Note: % gives the last output generated. /. applies a rule. -> represents a rule which transforms lhs (here, for instance, f[1]) to rhs (here 0).

```
%/.{f[1]->0,c[1]->c[2]}
```

$$-\frac{n\,(2\,x[1] + x[1]^2 - 2\,x[2] - x[2]^2)^2}{18\,(x[1] - x[2])}$$

```
Simplify[%]
```

$$-\frac{1}{18}\,n\,(x[1] - x[2])\,(2 + x[1] + x[2])^2$$

```
redProfit[2]=(profit[2]/.prices2Firms//Simplify)[[1]]
```

$$-\left(18\,f[2]\,(x[1] - x[2]) + n\,(c[1] - c[2] - 4\,x[1] + x[1]^2 + 4\,x[2] - x[2]^2)^2\right) / (18\,(x[1] - x[2]))$$

```
%/.{f[2]->0,c[1]->c[2]}//Simplify
```

$$-\frac{1}{18}\,n\,(x[1] - x[2])\,(-4 + x[1] + x[2])^2$$

■ Different profit configurations

The following calculations present the profits for various constellations when to firms are active. The first element gives the profit of firm 1, the second that of firm 2. Locations are 0 and 1, respectively. Both firms marginal costs are the same.

```
{redProfit[1], redProfit[2]} /. {x[1] -> 0, c[1] -> c[2], x[2] -> 1} // Expand
```

$$\left\{\frac{n}{2} - f[1],\ \frac{n}{2} - f[2]\right\}$$

Locations are 1/2 and 1

```
{redProfit[1], redProfit[2]} /. {x[1] -> 1 / 2, c[1] -> c[2], x[2] -> 1} // Expand
```

$$\left\{\frac{49\,n}{144} - f[1],\ \frac{25\,n}{144} - f[2]\right\}$$

A useful definition, covers also the case of different costs.

```
profits2Firms[x1_, x2_, costdif_] :=
 {redProfit[1], redProfit[2]} /. {x[1] -> x1, c[1] -> c[2] - costdif, x[2] -> x2}
```

■ The game with 3 firms

■ Equilibrium of the price-setting game with 3 firms 1, 2 and 3, locations given. Note firm 2 is the center firm, firm 1 is located to its left, firm 3 to the right.

```
a[1,0]=0;a[3,4]=1;a[2,3]=.
```

```
demand[1]+demand[2]+demand[3]//Simplify
```

```
n
```

```
foc[2]
```

$$n\,(-c[2]+p[2])\left(\frac{1}{2\,(x[1]-x[2])}-\frac{1}{2\,(-x[2]+x[3])}\right)+ n\left(\frac{1}{2}\,(-x[1]-x[2])-\frac{p[1]-p[2]}{2\,(x[1]-x[2])}+\frac{-p[2]+p[3]}{2\,(-x[2]+x[3])}+\frac{1}{2}\,(x[2]+x[3])\right)$$

The equilibrium prices depending on locations and technologies.

```
prices3Firms=Solve[{foc[1]==0,foc[2]==0,foc[3]==0},
    {p[1],p[2],p[3]}]//Simplify
```

$$\{\{p[1]\to(c[1]\,(3\,x[1]+x[2]-4\,x[3])+2\,c[2]\,(x[1]-x[3])+(x[1]-x[2])\,(c[3]-3\,x[1]^2-2\,x[2]-2\,x[1]\,x[2]+2\,x[3]+2\,x[1]\,x[3]+2\,x[2]\,x[3]+x[3]^2))\,/\,(6\,(x[1]-x[3])),\ p[2]\to(c[3]\,(x[1]-x[2])+2\,c[2]\,(x[1]-x[3])+(c[1]+(x[1]-x[2])\,(-2+x[1]-x[3]))\,(x[2]-x[3]))\,/\,(3\,(x[1]-x[3])),\ p[3]\to(c[3]\,(4\,x[1]-x[2]-3\,x[3])+2\,c[2]\,(x[1]-x[3])+(x[2]-x[3])\,(c[1]+x[1]^2+2\,x[2]+6\,x[3]-2\,x[2]\,x[3]-3\,x[3]^2+2\,x[1]\,(-4+x[2]+x[3])))\,/\,(6\,(x[1]-x[3]))\}\}$$

The equilibrium prices when the marginal costs are 0 for all firms.

```
priceC0=prices3Firms/.{c[1]->0,c[2]->0,c[3]->0}//Simplify
```

$$\Big\{\Big\{p[1]\to -((x[1]-x[2])\,(3\,x[1]^2+2\,x[1]\,(x[2]-x[3])-2\,x[2]\,(-1+x[3])-x[3]\,(2+x[3])))\,/\,(6\,(x[1]-x[3])),\ p[2]\to\frac{(x[1]-x[2])\,(-2+x[1]-x[3])\,(x[2]-x[3])}{3\,(x[1]-x[3])},\ p[3]\to -((-x[2]+x[3])\,(x[1]^2-2\,x[2]\,(-1+x[3])-3\,(-2+x[3])\,x[3]+2\,x[1]\,(-4+x[2]+x[3])))\,/\,(6\,(x[1]-x[3]))\Big\}\Big\}$$

```
priceC0/.{x[1]->0,x[2]->1/2,x[3]->1}
```

$$\Big\{\Big\{p[1]\to\frac{1}{4},\ p[2]\to\frac{1}{4},\ p[3]\to\frac{1}{4}\Big\}\Big\}$$

Neven's result in his Section 3

```
priceC0/.{x[1]->.068,x[2]->.42,x[3]->.891}
```

```
{{p[1] → 0.180669, p[2] → 0.189563, p[3] → 0.257041}}
```

This result is different from that reported by Neven, p. 425! The order and the magnitudes fit, however.

The reduced profit functions in the case of three firms: Profits as a function of locations (and costs) only.

```
redProfit3[i_]:=redProfit3[i]=(profit[i]/.prices3Firms//Simplify)[[1]]

redProfit3[1]
redProfit3[2]
redProfit3[3]

-f[1] -
 (n (2 c[2] (x[1] - x[3]) + c[1] (-3 x[1] + x[2] + 2 x[3]) + (x[1] - x[2]) (c[3] - 3 x[1]^2 -
          2 x[2] - 2 x[1] x[2] + 2 x[3] + 2 x[1] x[3] + 2 x[2] x[3] +
          x[3]^2))^2) / (72 (x[1] - x[2]) (x[1] - x[3])^2)

-f[2] - (n (c[3] (-x[1] + x[2]) + c[2] (x[1] - x[3]) -
        (c[1] + (x[1] - x[2]) (-2 + x[1] - x[3])) (x[2] - x[3]))^2) /
  (18 (x[1] - x[2]) (x[1] - x[3]) (x[2] - x[3]))

-f[3] + (n (-c[3] (2 x[1] + x[2] - 3 x[3]) + 2 c[2] (x[1] - x[3]) +
        (x[2] - x[3]) (c[1] + x[1]^2 + 2 x[2] + 6 x[3] - 2 x[2] x[3] - 3 x[3]^2 +
          2 x[1] (-4 + x[2] + x[3])))^2) / (72 (x[1] - x[3])^2 (-x[2] + x[3]))

redProfitSym[i_]:=redProfitSym[i]=redProfit3[i]/.\
    {c[1]->0,c[2]->0,c[3]->0}//Simplify

redProfitSym[1]
redProfitSym[2]
redProfitSym[3]

-f[1] -
 (n (x[1] - x[2]) (-3 x[1]^2 - 2 x[1] (x[2] - x[3]) + 2 x[2] (-1 + x[3]) + x[3] (2 + x[3]))^2) /
  (72 (x[1] - x[3])^2)

-f[2] - n (x[1] - x[2]) (x[2] - x[3]) (2 - x[1] + x[3])^2
        ------------------------------------------------
                     18 (x[1] - x[3])

-f[3] + (n (-x[2] + x[3])
     (x[1]^2 - 2 x[2] (-1 + x[3]) - 3 (-2 + x[3]) x[3] + 2 x[1] (-4 + x[2] + x[3]))^2) / (72
     (x[1] - x[3])^2)
```

Evaluation of the first order condition of the firm located at the left edge of the market. This firm is the last entrant in Neven's paper. The positive value in the next expression shows that the locations cannot be equilibrium locations.

```
D[redProfitSym[1],x[1]]/.{x[1]->.068,x[2]->.42,x[3]->.891}//Simplify

0.000491876 n
```

The equilibrium with simultaneous locational choice.

```
symEqu=Solve[{D[redProfitSym[1],x[1]]==0,D[redProfitSym[2],x[2]]==0,
    D[redProfitSym[3],x[3]]==0},{x[1],x[2],x[3]}]
```

```
N[%]
```

```
{{x[2.] → 0.0666667, x[1.] → -0.633333, x[3.] → 0.766667},
 {x[2.] → 0.5, x[1.] → -1.5, x[3.] → 2.5}, {x[2.] → 0.5, x[1.] → 0.125, x[3.] → 0.875},
 {x[2.] → 0.933333, x[1.] → 0.233333, x[3.] → 1.63333},
 {x[2.] → -2. - 1.41421 i, x[1.] → 2. + 1.41421 i, x[3.] → 0. + 1.41421 i},
 {x[2.] → -2. - 1.41421 i, x[1.] → 2. + 1.41421 i, x[3.] → 0. + 1.41421 i},
 {x[2.] → 3. - 1.41421 i, x[1.] → 1. + 1.41421 i, x[3.] → -1. + 1.41421 i},
 {x[2.] → 3. - 1.41421 i, x[1.] → 1. + 1.41421 i, x[3.] → -1. + 1.41421 i},
 {x[2.] → -2. + 1.41421 i, x[1.] → 2. - 1.41421 i, x[3.] → 0. - 1.41421 i},
 {x[2.] → -2. + 1.41421 i, x[1.] → 2. - 1.41421 i, x[3.] → 0. - 1.41421 i},
 {x[2.] → 3. + 1.41421 i, x[1.] → 1. - 1.41421 i, x[3.] → -1. - 1.41421 i},
 {x[2.] → 3. + 1.41421 i, x[1.] → 1. - 1.41421 i, x[3.] → -1. - 1.41421 i},
 {x[2.] → 0.5 - 4.97494 i, x[1.] → 1.5 - 1.65831 i, x[3.] → -0.5 - 1.65831 i},
 {x[2.] → 0.5 - 4.97494 i, x[1.] → 1.5 - 1.65831 i, x[3.] → -0.5 - 1.65831 i},
 {x[2.] → 0.5 + 4.97494 i, x[1.] → 1.5 + 1.65831 i, x[3.] → -0.5 + 1.65831 i},
 {x[2.] → 0.5 + 4.97494 i, x[1.] → 1.5 + 1.65831 i, x[3.] → -0.5 + 1.65831 i}}
```

The third element of the list gives the result for the simultaneous choice of locations! Mentioned by Neven!

```
symEqu[[3]]
```

$$\left\{x[2] \to \frac{1}{2},\ x[1] \to \frac{1}{8},\ x[3] \to \frac{7}{8}\right\}$$

Profits in the case of the simultaneous choice:

```
{redProfitSym[1], redProfitSym[2], redProfitSym[3]} /. symEqu[[3]]
```

$$\left\{\frac{169\,n}{3072} - f[1],\ \frac{121\,n}{1536} - f[2],\ \frac{169\,n}{3072} - f[3]\right\}$$

```
priceC0 /. symEqu[[3]]
```

$$\left\{\left\{p[1] \to \frac{13}{64},\ p[2] \to \frac{11}{64},\ p[3] \to \frac{13}{64}\right\}\right\}$$

The profit is highest at the centre location, the price of the respective firm is lower than that of its rivals.

■ Sequential entry

For the sequential entry scenario the reaction functions must be determined. The sequence of entry is as follows: 2,3,1 as used by Neven.

Derivation of the reaction functions. The first order conditon with respect to locational choice. At the moment I use the condition for firm 1 only. Later on, I need the other conditions when evaluating entry deterrence in the case of two active firms.

```
focLoc[1] = D[redProfit3[1], x[1]] // Simplify
focLoc[2] = D[redProfit3[2], x[2]] // Simplify
focLoc[3] = D[redProfit3[3], x[3]] // Simplify
```

```
-(n (c[1] (3 x[1] - x[2] - 2 x[3]) - 2 c[2] (x[1] - x[3]) - (x[1] - x[2])
        (c[3] - 3 x[1]^2 - 2 x[2] - 2 x[1] x[2] + 2 x[3] + 2 x[1] x[3] + 2 x[2] x[3] + x[3]^2))
     (2 c[2] (x[1] - x[3])^2 + c[1] (-3 x[1]^2 - 2 x[2]^2 + x[2] x[3] - 2 x[3]^2 +
          3 x[1] (x[2] + x[3])) + (x[1] - x[2]) (9 x[1]^3 + 4 x[2]^2 - 6 x[2] x[3] + 2 x[3]^2 -
          4 x[2] x[3]^2 + x[3]^3 + c[3] (x[1] - 2 x[2] + x[3]) - x[1]^2 (4 x[2] + 17 x[3]) +
          x[1] (x[3] (2 + 7 x[3]) + x[2] (-2 + 8 x[3]))))) / (72 (x[1] - x[2])^2 (x[1] - x[3])^3)
```

```
(n (c[3] (-x[1] + x[2]) + c[2] (x[1] - x[3]) -
      (c[1] + (x[1] - x[2]) (-2 + x[1] - x[3])) (x[2] - x[3]))
    (-c[3] (x[1] - x[2]) (x[1] - x[3]) + c[2] (x[1] - x[3]) (x[1] - 2 x[2] + x[3]) +
      (x[2] - x[3])
       (c[1] (x[1] - x[3]) + (x[1] - x[2]) (-2 + x[1] - x[3]) (x[1] - 2 x[2] + x[3])))) /
 (18 (x[1] - x[2])^2 (x[1] - x[3]) (x[2] - x[3])^2)
```

```
(n (c[3] (2 x[1] + x[2] - 3 x[3]) - 2 c[2] (x[1] - x[3]) - (x[2] - x[3])
       (c[1] + x[1]^2 + 2 x[2] + 6 x[3] - 2 x[2] x[3] - 3 x[3]^2 + 2 x[1] (-4 + x[2] + x[3])))
    (2 c[2] (x[1] - x[3])^2 + c[3] (-2 x[1]^2 - 2 x[2]^2 + 3 x[2] x[3] - 3 x[3]^2 +
         x[1] (x[2] + 3 x[3])) - (x[2] - x[3]) (x[1]^3 - 4 x[2]^2 + 2 x[2] x[3] - 6 x[3]^2 -
         4 x[2] x[3]^2 + 9 x[3]^3 + c[1] (x[1] - 2 x[2] + x[3]) + x[1]^2 (-8 - 4 x[2] + 7 x[3]) +
         x[1] ((10 - 17 x[3]) x[3] + x[2] (6 + 8 x[3]))))) / (72 (x[1] - x[3])^3 (x[2] - x[3])^2)
```

The reaction function of firm 1 (i.e. of the third entrant!) determines the location of this firm as a function of the other firms' locations and of the technologies.

```
react1 = Solve[focLoc[1] == 0, x[1]];
```

Mathematica finds not only the (real valued) maximum, but a large number of solutions. The respective expression is very long. Rather than printing it out, I insert locations for the two remaining firms in order to check which solution gives the maximum profit, given the constraints on the locations.

```
react1 /. {x[2] -> 1 / 2, x[3] -> 1, c[1] → 0, c[2] → 0, c[3] → 0} // N
```

```
{{x[1.] → 1.18046 - 2.22045×10^-16 i}, {x[1.] → -0.847127 - 1.66533×10^-16 i},
 {x[1.] → 0.5 + 3.33067×10^-16 i}, {x[1.] → 0.0977664}, {x[1.] → 0.5},
 {x[1.] → 1.00667 - 0.350866 i}, {x[1.] → 1.00667 + 0.350866 i}}
```

I pick out the solution in the next step (element 4).

```
react1 = react1[[4]];
```

```
react1 /. {x[2] -> 1 / 2, x[3] -> 1, c[1] → 0, c[2] → 0, c[3] → 0} // N
```

```
{x[1.] → 0.0977664}
```

Consistency check: The values from the symmetric equilibrium.

```
react1 /. {x[2] -> 1 / 2, x[3] -> 7 / 8, c[1] → 0, c[2] → 0, c[3] → 0}
```

$\{x[1] \to \frac{1}{8}\}$

Further analytical results like the reaction function of firm 3 (i.e, the second entrant) taking into account the reaction function of firm 1 (the third entrant) are not available. Therefore I start with numerical solutions.

■ Numerical solutions.

All firms use the same technology. Fixed costs are 25, marginal costs are 0.

```
f[_] = 25; c[_] = 0;
```

■ The equilibrium locations when three firms are active and there is no further entry

First I derive the equilibrium locations for the case with 3 active firms. To do this I must specify the market size as well. The market size has only an effect on profits, not on the equilibrium locations.

```
n = 1000;
```

The following function gives the first order constraint for the locational choice of firm 3 (the second entrant), taking into account the reaction function of firm 1 (the third entrant). It is a function of i, the locational choice of firm 2 (Firm 2 is the first entrant!).

```
function[i_] := function[i] =
    ((D[redProfit3[3] /. react1, x[3]]) /. {x[3] -> x3, x[2] -> i});
```

Given the location of firm 2, the function can be solved for the location of firm 3 (here x3), and also for x[1].

```
FindRoot[function[.5] == 0, {x3, .9}]
```

```
{x3 → 0.918677}
```

```
react1 /. {x[2] -> .5, x[3] -> .918677}
```

```
{x[1] → 0.115256}
```

The solution is defined in the next routine.

```
reactNum3[i_] := reactNum3[i] = FindRoot[function[i] == 0, {x3, .9}]
```

```
reactNum3[.4]
```

```
{x3 → 0.878158}
```

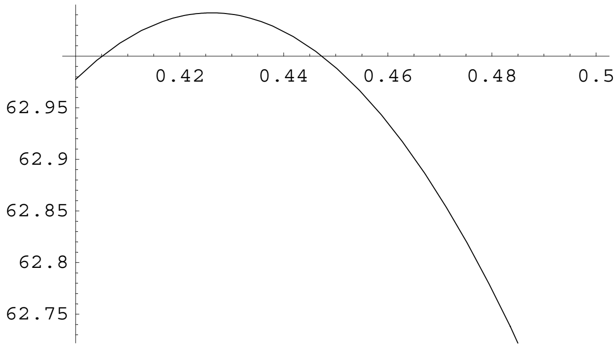
The next line (and the plot following) gives the profit of the first entrant as a function of her location (here .5) taking into account the reaction functions of firms 1 and 3 (that is entrants 2 and 3).

```
((redProfit3[2] /. react1) /. {x[3] -> x3}) /. Join[reactNum3[.5], {x[2] -> .5}]
```

```
62.5412
```

It takes quite long to calculate the next plot. In the following I derive the first entrant's optimum choice by calculating the profits for different locations. General routines do not work, therefore I approximate "by hand".

```
Plot[((redProfit3[2] /. react1) /. {x[3] -> x3}) /.
  Join[reactNum3[j], {x[2] -> j}], {j, .4, .5}]
```




```
- Graphics -

InputForm[%]

valuesSym = %[[1, 1, 1, 1]]

{{0.4, 62.9774}, {0.404057, 62.9959}, {0.408481, 63.0125}, {0.412636, 63.0248},
 {0.416632, 63.0334}, {0.418687, 63.0367}, {0.420885, 63.0394},
 {0.421958, 63.0403}, {0.422523, 63.0407}, {0.423122, 63.0411},
 {0.423646, 63.0414}, {0.424222, 63.0416}, {0.424474, 63.0417}, {0.424746, 63.0418},
 {0.425004, 63.0418}, {0.425237, 63.0419}, {0.425503, 63.0419}, {0.42565, 63.0419},
 {0.425787, 63.0419}, {0.425915, 63.042}, {0.426033, 63.042}, {0.426162, 63.042},
 {0.426298, 63.042}, {0.426436, 63.042}, {0.426513, 63.0419}, {0.426584, 63.0419},
 {0.42685, 63.0419}, {0.426999, 63.0419}, {0.427161, 63.0419}, {0.427456, 63.0418},
 {0.427987, 63.0416}, {0.428552, 63.0414}, {0.429566, 63.0409},
 {0.430549, 63.0401}, {0.43161, 63.0392}, {0.433529, 63.0369}, {0.435634, 63.0335},
 {0.437849, 63.0291}, {0.44185, 63.0189}, {0.446109, 63.0048}, {0.450209, 62.988},
 {0.454566, 62.9668}, {0.458765, 62.943}, {0.462803, 62.9172}, {0.4671, 62.8864},
 {0.471237, 62.8536}, {0.475215, 62.8192}, {0.479451, 62.7794}, {0.483528, 62.7381},
 {0.487862, 62.6909}, {0.492037, 62.6423}, {0.496052, 62.5927}, {0.5, 62.5412}}

Max[valuesSym]

63.042

FullForm[%]

63.04195934723515`

Position[valuesSym, Max[valuesSym]]

{{22, 2}}

FullForm[valuesSym[[22]]]

List[0.4261616680713123`, 63.04195934723515`]

Precision[%]

16
```

In the next step I calculate the profits of the first entrant (firm 2) for different locations arond the maximum value derived in the the above diagram. The increment is 0.00001.

```
N[Table[
   {j, ((redProfit3[2] /. react1) /. {x[3] -> x3}) /. Join[reactNum3[j], {x[2] -> j}]},
   {j, 0.42610, 0.42620, 0.00001}], 16] // FullForm
```

```
List[List[0.4261`, 63.04195917592847`], List[0.42611`, 63.04195925219177`],
 List[0.42612`, 63.0419593096869`], List[0.42612999999999995`, 63.041959348414395`],
 List[0.42613999999999996`, 63.04195936837462`],
 List[0.42615`, 63.04195936956738`], List[0.42616`, 63.04195935199331`],
 List[0.42617`, 63.04195931565175`], List[0.42618`, 63.04195926054378`],
 List[0.42618999999999996`, 63.041959186668365`],
 List[0.42619999999999997`, 63.04195909402685`]]
```

```
Max[%] // FullForm
```

```
63.04195936956738`
```

```
locationsSym = Flatten[{react1 /. {x[2] -> 0.42615, x[3] -> x3 /. reactNum3[0.42615]},
   x[2] -> 0.42615, x[3] -> x3 /. reactNum3[0.42615]}]
```

```
{x[1] → 0.0736691, x[2] → 0.42615, x[3] → 0.888526}
```

```
locationsSym = {x[1] → 0.07367, x[2] → 0.42615, x[3] → 0.88853}
```

```
{x[1] → 0.07367, x[2] → 0.42615, x[3] → 0.88853}
```

locationsSym

These are the equilibrium locations in the benchmark case when three firms are active and further entry is blockaded.

```
{redProfit3[1], redProfit3[2], redProfit3[3]} /. locationsSym
```

```
{21.9465, 63.0423, 43.8192}
```

```
locationsNeven = { x[1] -> 0.068, x[2] ->  0.420, x[3] ->  0.891}
```

```
{x[1] → 0.068, x[2] → 0.42, x[3] → 0.891}
```

```
{redProfit3[1], redProfit3[2], redProfit3[3]} /. locationsNeven
```

```
{21.3657, 64.1893, 45.1381}
```

```
prices3Firms /. locationsSym
prices3Firms /. locationsNeven
```

```
{{p[1] → 0.181921, p[2] → 0.187666, p[3] → 0.252272}}
```

```
{{p[1] → 0.180669, p[2] → 0.189563, p[3] → 0.257041}}
```

Compared to Neven's equilibrium the last entrant has a higher profit in the equilibrium I derive, the two earlier entrants have lower profits. The same holds for the prices.
Curiously, the prices I get for the Neven locations are greater by an order of at least 0.005 than the prices Neven reports (p.429, Table 2). In general, however, the magnitudes are not too different.

The results really differ when it comes to entry deterrence in the 2 firms case.

■ Entry deterrence in the case of two active firms

At first I undo the definition of the parameter n.

```
n =.
```

Here I show first that the profit a potential entrant at the center is not changed with a change in the locations of the two incumbents as long as the distance between the two incumbents stays the same.

```
Solve[(focLoc[2] /. {x[1] -> i, x[3] -> k}) == 0, x[2]] // Simplify
```

$$\left\{\left\{x[2] \to \frac{i+k}{2}\right\}\right\}$$

```
(redProfit3[2] /. {x[1] -> i, x[2] -> (1 + i) / 2, x[3] -> 1}) -
  (redProfit3[2] /. {x[1] -> i / 2, x[2] -> 1 / 2, x[3] -> 1 - i / 2}) // Simplify
```

0

```
profits2Firms[i, 1, 0] - profits2Firms[i / 2, 1 - i / 2, 0] // Simplify
```

$$\left\{-\frac{1}{18}(-1+i)\,i\,(6+i)\,n,\ -\frac{1}{18}(-6+i)(-1+i)\,i\,n\right\}$$

The profit difference is positive for the first entrant (the first element) and negative for the second entrant. This result proofs that the first entrant will deter further entry alone as long as it is possible.

Next I determine the optimum location of a potential entrant in the case of two incumbents. She locates halfway between the incumbents.

```
x2opt[i_] =
 x[2] /. Solve[(focLoc[2] /. {x[1] -> i, x[3] -> 1}) == 0, x[2]][[-1]] // Simplify
```

$$\frac{1+i}{2}$$

The profit of the entrant as a function of the incumbents' locations and of market size n.

```
function1[i_, j_] := redProfit3[2] /. {x[1] -> i, x[2] -> x2opt[i], x[3] -> 1, n -> j}
```

```
Solve[function1[i, n] == 0, i]
```

```
N[% /. n -> 300]
```

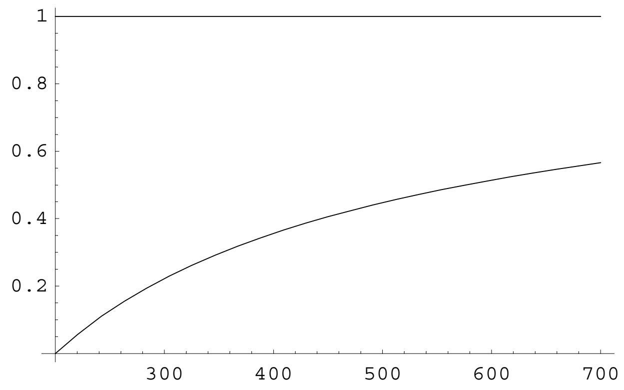
{{i → 0.222336}, {i → 3.38883 − 1.41736 ⅈ}, {i → 3.38883 + 1.41736 ⅈ}}

This solution gives the entry deterring location of the first entrant, given market size n and a location at the edge of the second entrant. The first solution is the right one, it yields real numbers. (Attention: Different version of *Mathematica* order the solutions differently!)

```
deter[j_] = i /. Solve[function1[i, j] == 0, i][[1]]
```

$$\frac{7}{3} - \frac{2\,2^{1/3}\,j^{1/3}}{3\left(6075 + 2\,j + 45\sqrt{3}\,\sqrt{6075 + 4\,j}\right)^{1/3}} - \frac{2^{2/3}\left(6075 + 2\,j + 45\sqrt{3}\,\sqrt{6075 + 4\,j}\right)^{1/3}}{3\,j^{1/3}}$$

```
fig1 = Plot[{deter[n], 1}, {n, 200, 700}]
```




```
- Graphics -
```

This the pattern of entry deterrence for n>200.

Now I calculate the value of n for which the entry deterrence constraint for an entrant at the left edge becomes binding. As the following calculation shows the potential entrant will not always choose maximum product differentiation, that is the location 0.

```
Solve[D[(redProfit3[1] /. {x[1] -> i, x[2] -> j, x[3] -> k, n -> 1}), i] == 0, i]

% /. {j -> .4, k -> 1}
```

$\{\{i \to -0.819804\}, \{i \to 1.2198\}, \{i \to 1.01665 - 0.406504\, i\},$
$\{i \to 0.033366 - 2.08167 \times 10^{-17}\, i\}, \{i \to 1.01665 + 0.406504\, i\}\}$

```
FindRoot[(i /. (%% /. {k -> 1}))[[4]]) == 0, {j, .33}]
```

$\{j \to 0.348612 + 2.18894 \times 10^{-16}\, i\}$

The third entrant locates inside the market area for locations of the first entrant to the right of .348612.

```
xOptAt0[j_, k_] =
 i /. Solve[D[(redProfit3[1] /. {x[1] -> i, x[2] -> j, x[3] -> k}), i] == 0, i][[4]]
```

$$\frac{1}{27}(4j+17k) + \left(\left(1+i\sqrt{3}\right)\left(-54j-16j^2+54k+80jk-100k^2\right)\right) \Big/ \left(54\, 2^{1/3}\left(-2025j^2+32j^3+3807jk-240j^2k-1782k^2+600jk^2-500k^3+27\sqrt{3}\sqrt{(-18j^3+1859j^4-64j^5+54j^2k-6938j^3k+608j^4k-54jk^2+9651j^2k^2-2224j^3k^2+18k^3-5924jk^3+3880j^2k^3+1352k^4-3200jk^4+1000k^5)}\right)^{1/3}\right) - \frac{1}{27\, 2^{2/3}}\left(\left(1-i\sqrt{3}\right)\left(-2025j^2+32j^3+3807jk-240j^2k-1782k^2+600jk^2-500k^3+27\sqrt{3}\sqrt{(-18j^3+1859j^4-64j^5+54j^2k-6938j^3k+608j^4k-54jk^2+9651j^2k^2-2224j^3k^2+18k^3-5924jk^3+3880j^2k^3+1352k^4-3200jk^4+1000k^5)}\right)^{1/3}\right)$$

```
xOptAt0[.4, 1]
```

$0.033366 - 2.08167 \times 10^{-17}\, i$

```
deter[350.]
```

0.296407

```
xOptAt0[deter[440.], 1]
```

$0.0310802 - 2.08167\times 10^{-17}\,i$

Next I determine the market size for which the first entrant can no longer deter entry alone.

```
FindRoot[
 (redProfit3[1] /. {x[1] -> xOptAt0[deter[n], 1], x[2] -> deter[n], x[3] -> 1}) == 0,
 {n, 450., 451.}, MaxIterations -> 50, AccuracyGoal -> 12]
```

$\{n \to 468.946 - 2.95345\times 10^{-29}\,i\}$

```
nDeter0 = Chop[n /. %]
```

468.946

```
(redProfit3[1] /. {x[1] -> xOptAt0[deter[n], 1], x[2] -> deter[n], x[3] -> 1}) /.
 n -> nDeter0
```

$5.32907\times 10^{-14} - 9.0719\times 10^{-30}\,i$

```
deter[nDeter0]
```

0.422316

```
xOptAt0[deter[nDeter0], 1]
```

$0.0478158 - 2.77556\times 10^{-17}\,i$

Deterring an entrant at the center and at 0.

```
function2[i_, j_, k] := redProfit3[2] /. {x[1] -> i, x[2] -> k / 2 + i / 2, x[3] -> k, n -> j}

Solve[function2[i, n, k] == 0, k]
```

$$\Big\{\Big\{k \to -\frac{4n-3in}{3n} + \frac{2\,2^{1/3} n^{1/3}}{3\left(6075+2n+45\sqrt{3}\sqrt{6075+4n}\right)^{1/3}} + \frac{2^{2/3}\left(6075+2n+45\sqrt{3}\sqrt{6075+4n}\right)^{1/3}}{3n^{1/3}}\Big\},$$

$$\Big\{k \to -\frac{4n-3in}{3n} - \frac{2^{1/3}\left(1+i\sqrt{3}\right)n^{1/3}}{3\left(6075+2n+45\sqrt{3}\sqrt{6075+4n}\right)^{1/3}} - \frac{\left(1-i\sqrt{3}\right)\left(6075+2n+45\sqrt{3}\sqrt{6075+4n}\right)^{1/3}}{3\,2^{1/3}n^{1/3}}\Big\},$$

$$\Big\{k \to -\frac{4n-3in}{3n} - \frac{2^{1/3}\left(1-i\sqrt{3}\right)n^{1/3}}{3\left(6075+2n+45\sqrt{3}\sqrt{6075+4n}\right)^{1/3}} - \frac{\left(1+i\sqrt{3}\right)\left(6075+2n+45\sqrt{3}\sqrt{6075+4n}\right)^{1/3}}{3\,2^{1/3}n^{1/3}}\Big\}\Big\}$$

The first element gives the real valued solution. In the next step I define this as the location of the second entrant (i.e.k), which deters entry at the center, given the first entrant locates at i.

```
deterCenter[i_, n_] = k /. Solve[function2[i, n, k] == 0, k][[1]] // FullSimplify
```

$$\frac{1}{3}\left(-4+3i+\frac{2\,2^{1/3}n^{2/3}+\left(12150+4n+90\sqrt{3}\sqrt{6075+4n}\right)^{2/3}}{n^{1/3}\left(6075+2n+45\sqrt{3}\sqrt{6075+4n}\right)^{1/3}}\right)$$

The next two steps calculate the profit of an entrant at the left edge (i.e close to 0), given that the entrant locates optimally at xOptAt0 and that the first and second entrant locate at i and at deterCenter, respectively. Note that this function is also a function of market size n. For technical reasons I replace market size n by the variable n1.

```
function2a[n1_] = (redProfit3[1] /. {x[1] -> xOptAt0[i, deterCenter[i, n1]],
     x[2] -> i, x[3] -> deterCenter[i, n1], n -> n1});
```

```
General::spell1 : Possible spelling error: new
   symbol name "function2a" is similar to existing symbol "function2".
```

deterLeft gives the location for the first entrant which prevents entry at the left edge, taking into account that the second entrant locates at deterCenter.

```
deterLeft[n1_] := i /. Chop[FindRoot[function2a[n1] == 0, {i, .35, .4}][[1]]]
```

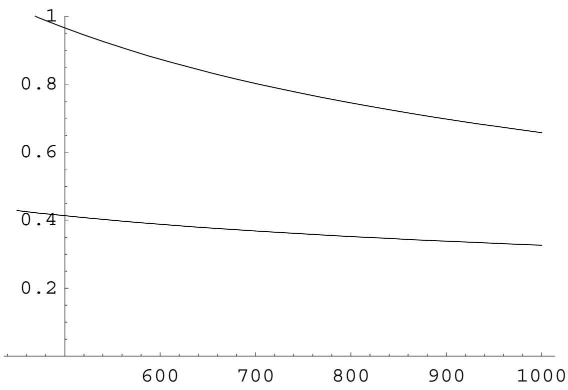
```
deterLeft[450]
```

```
0.428357
```

```
deterLeft[1000.]
```

```
0.326482
```

```
fig2 = Plot[{deterCenter[deterLeft[n], n], deterLeft[n]},
  {n, 450, 1000}, PlotRange -> {0, 1}]
```




```
- Graphics -
```

Next, I calculate the market size, from which on deterrence of a third entrant (locating at 1) is no longer possible.

```
function3[j_] :=
 (redProfit3[3] /. {x[1] -> deterLeft[j], x[2] -> deterCenter[deterLeft[j], j],
   x[3] -> 1 - xOptAt0[1 - deterCenter[deterLeft[j], j], 1 - deterLeft[j]], n -> j})
```

```
1 - xOptAt0[1 - deterCenter[deterLeft[900], 900], 1 - deterLeft[900]]
```

$0.959283 + 2.60209 \times 10^{-17}\, i$

```
function3[1000]
```

$1.87828 + 6.67376 \times 10^{-28}\, i$

```
FindRoot[function3[n] == 0, {n, 900., 910.}]
```

$\{n \to 967.58\}$

```
nNoDeter = n /. %
```

```
967.58
```

```
f[1] / nNoDeter
```

```
0.0258377
```

Neven's value here is 0.0255

```
deterLeft[nNoDeter]
deterCenter[deterLeft[nNoDeter], nNoDeter]
```

```
0.330099
```

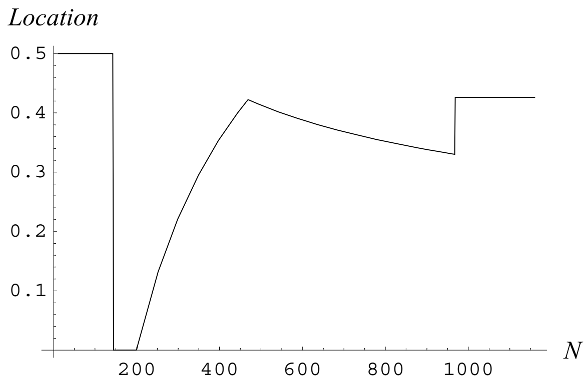
```
0.669901
```

As I show below in the section with four firms, entry of a fourth entrant is blockaded up to n=1136.86. I do not consider the industry structure for greater values of n.

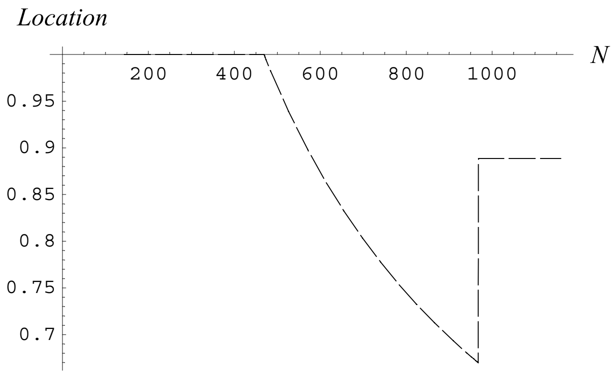
```
deter1stEntrant[n_] := Which[n < 144, 1 / 2, n < 200, 0, n < nDeter0,
  deter[n], n < nNoDeter, deterLeft[n], True, x[2] /. locationsSym]
```

```
deter2ndEntrant[n_] := Which[n < nDeter0, 1, n < nNoDeter,
  deterCenter[deterLeft[n], n], True, x[3] /. locationsSym]
```

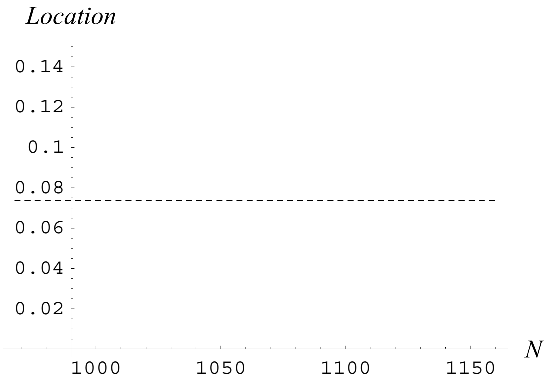
```
Plot[deter1stEntrant[n], {n, 10, 1160}, PlotStyle -> GrayLevel[0], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Location",
{"Times-Italic", 12}]}]
Plot[deter2ndEntrant[n], {n, 144, 1160},
 PlotStyle -> Dashing[{.05, .01}], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Location",
{"Times-Italic", 12}]}]
Plot[x[1] /. locationsSym, {n, nNoDeter, 1160},
 PlotStyle -> Dashing[{.01, .01}], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Location",
{"Times-Italic", 12}]}]
figurEntry = Show[%, %%, %%%]
```



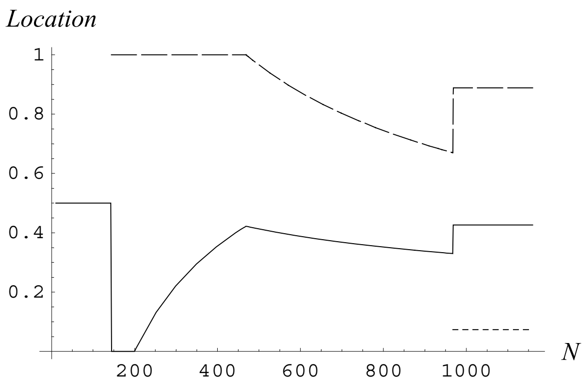

```
- Graphics -
```



- Graphics -



- Graphics -



- Graphics -

The profits:

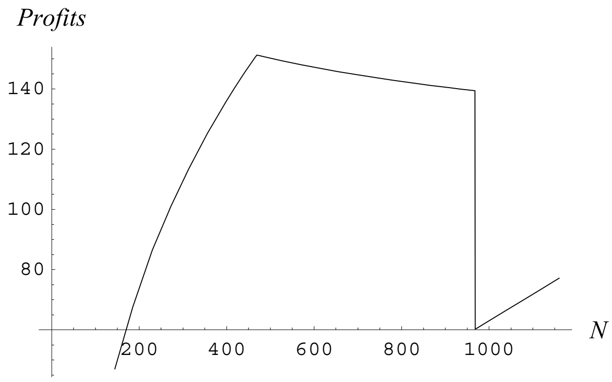
```
{redProfit3[1], redProfit3[2], redProfit3[3]} /. locationsSym
```

```
{-25 + 0.0469465 n, -25 + 0.0880423 n, -25 + 0.0688192 n}
```

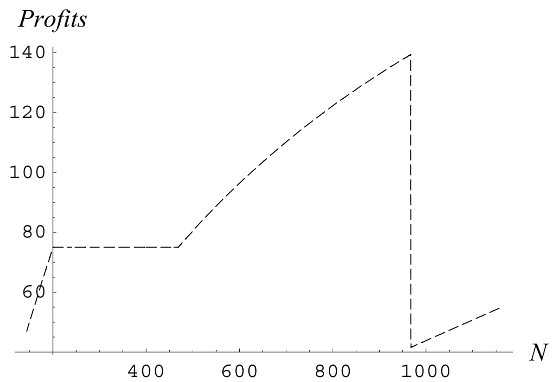
```
prof1[n_] := redProfit[1] /. {x[1] -> deter[n], x[2] -> 1}
prof2[n_] := redProfit[2] /. {x[1] -> deter[n], x[2] -> 1.}
```

```
prof3[n_] :=
 redProfit[1] /. {x[1] -> deterLeft[n], x[2] -> deterCenter[deterLeft[n], n]}
prof4[n_] := redProfit[2] /.
  {x[1] -> deterLeft[n], x[2] -> deterCenter[deterLeft[n], n]}
```

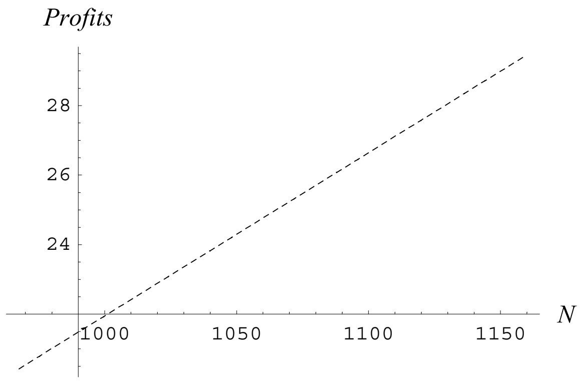
```
profit1stEntrant[n_] := Which[n < 200, (redProfit[1] /. {x[1] -> 0, x[2] -> 1}),
  n < nDeter0, prof1[n], n < nNoDeter, prof3[n],
        True, (redProfit3[2] /. locationsSym)]

profit2ndEntrant[n_] := Which[n < 200, (redProfit[2] /. {x[1] -> 0, x[2] -> 1}),
  n < nDeter0, prof2[n], n < nNoDeter, prof4[n], True, (redProfit3[3] /. locationsSym)]


Plot[profit1stEntrant[n], {n, 144, 1160}, PlotStyle -> GrayLevel[0], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
{"Times-Italic", 12}]}]
Plot[profit2ndEntrant[n], {n, 144, 1160},
 PlotRange -> All, PlotStyle -> Dashing[{.02, .01}], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
{"Times-Italic", 12}]}]
Plot[(redProfit3[1] /. locationsSym),
 {n, nNoDeter, 1160}, PlotStyle -> Dashing[{.01, .01}], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
{"Times-Italic", 12}]}]
figurEntryProfits =
 Show[%, %%, %%%, PlotRange -> {{100, 1160}, {0, 160}}, AxesOrigin -> {144, 0}]
```



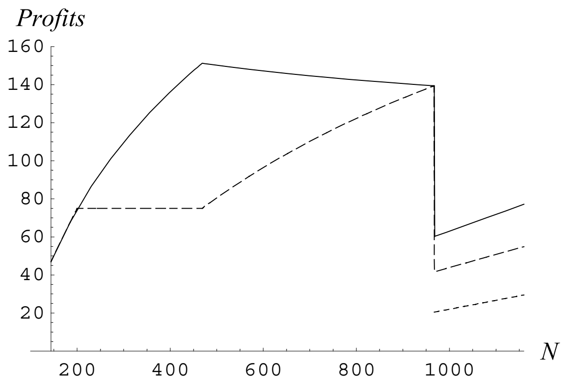
- Graphics -



- Graphics -



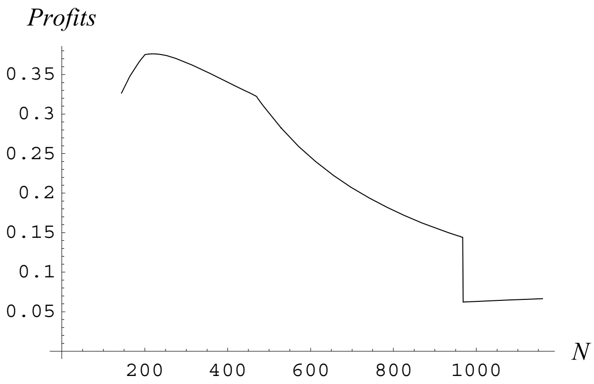
- Graphics -



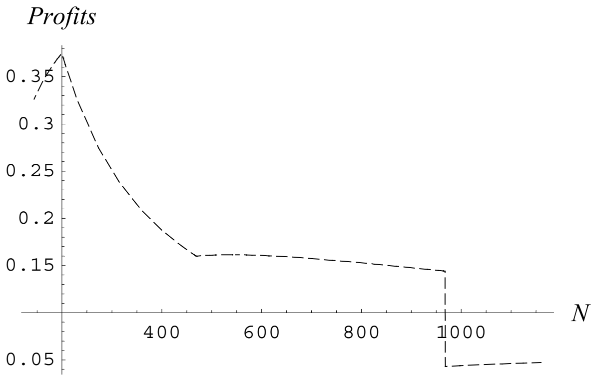
- Graphics -

The profits as a function of fixed costs

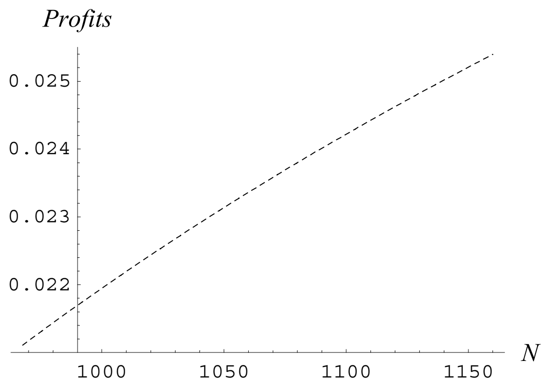
```
Plot[profit1stEntrant[n] / n, {n, 144, 1160}, PlotStyle -> GrayLevel[0], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
{"Times-Italic", 12}]}]
Plot[profit2ndEntrant[n] / n, {n, 144, 1160},
 PlotRange -> All, PlotStyle -> Dashing[{.02, .01}], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
{"Times-Italic", 12}]}]
Plot[((redProfit3[1] / n) /. locationsSym),
 {n, nNoDeter, 1160}, PlotStyle -> Dashing[{.01, .01}], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
{"Times-Italic", 12}]}]
figurEntryProfitsOfFixedCosts =
 Show[%, %%, %%%, PlotRange -> {{100, 1160}, Automatic}, AxesOrigin -> {144, 0}]
```



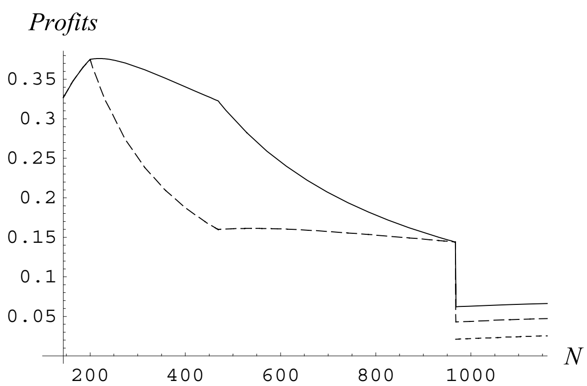
- Graphics -



- Graphics -



- Graphics -



- Graphics -

The prices

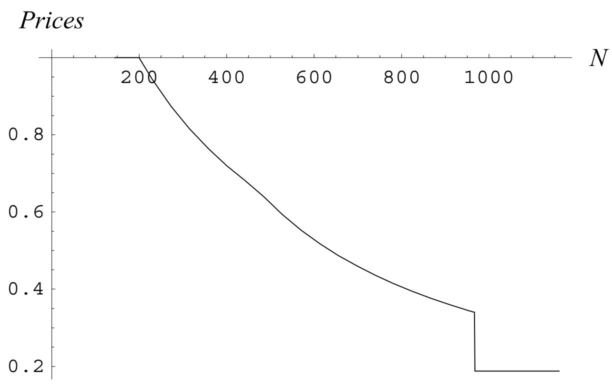
Attention! The price of firm 3 is the price of the 2nd entrant!! Therefore I redefine manually the prices! That is, in the case of 3 firms in equPrices I p[1](2,3) denotes the price the 1st(2nd, 3rd) entrant charges.

```
prices3Firms /. locationsSym
```

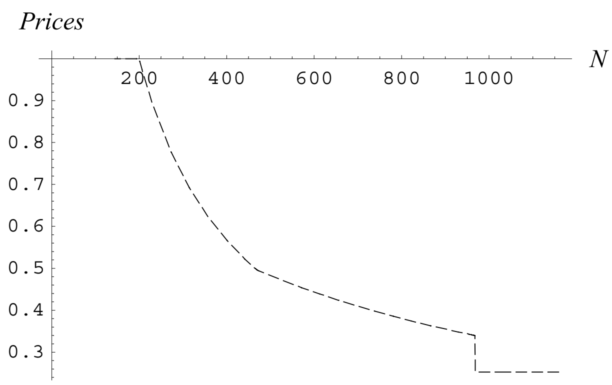
```
{{p[1] → 0.181921, p[2] → 0.187666, p[3] → 0.252272}}
```

```
equPrices[n_] := Which[n < 200, (prices2Firms /. {x[1] -> 0, x[2] -> 1}),
  n < nDeter0, prices2Firms /. {x[1] -> deter[n], x[2] -> 1}, n < nNoDeter,
  prices2Firms /. {x[1] -> deterLeft[n], x[2] -> deterCenter[deterLeft[n], n]},
  True, {{p[1] → 0.187666, p[2] → 0.252272, p[3] → 0.181921}}]
```

```
Plot[{p[1] /. equPrices[n][[1]]},
 {n, 144, 1160}, PlotStyle -> GrayLevel[0], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Prices",
{"Times-Italic", 12}]}]
Plot[{p[2] /. equPrices[n][[1]]}, {n, 144, 1160},
 PlotStyle -> Dashing[{.02, .01}], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Prices",
{"Times-Italic", 12}]}]
Plot[{p[3] /. equPrices[n][[1]]}, {n, 144, 1160},
 PlotStyle -> Dashing[{.005, .01}], AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Prices",
{"Times-Italic", 12}]}]
figurEntryPrices =
 Show[%, %%, %%%, PlotRange -> {{144, 1160}, {0, 1.1}}, AxesOrigin -> {144, 0}]
```

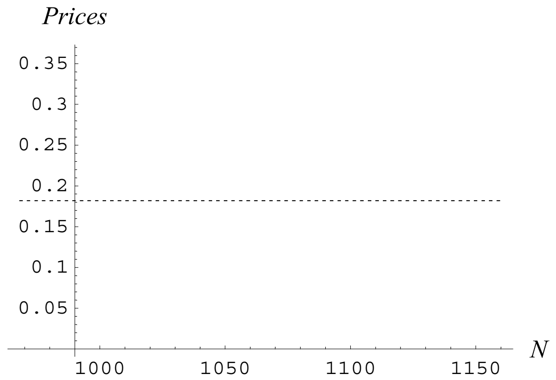


- Graphics -

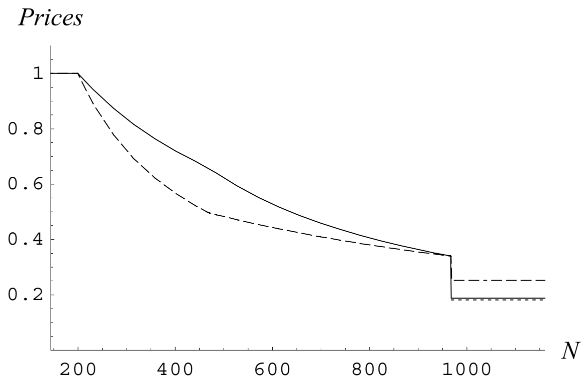


- Graphics -

```
Plot::plnr :
 p[3] /. equPrices[n]⟦1⟧ is not a machine-size real number at n = 144.00004233333334`.

Plot::plnr :
 p[3] /. equPrices[n]⟦1⟧ is not a machine-size real number at n = 185.21606343808247`.

Plot::plnr :
 p[3] /. equPrices[n]⟦1⟧ is not a machine-size real number at n = 230.16574065712368`.

General::stop : Further output of Plot::plnr will be suppressed during this calculation.
```



- Graphics -



- Graphics -

■ Existence of equilibrium and derivation of third mover advantage

```
Clear[f]
Clear[n]
```

```
focLoc[1] = D[redProfitSym[1], x[1]] // Simplify
focLoc[2] = D[redProfitSym[2], x[2]] // Simplify
focLoc[3] = D[redProfitSym[3], x[3]] // Simplify
```

```
-(n (3 x[1]^2 + 2 x[1] (x[2] - x[3]) - 2 x[2] (-1 + x[3]) - x[3] (2 + x[3]))
    (9 x[1]^3 + 4 x[2]^2 + x[3]^2 (2 + x[3]) - 2 x[2] x[3] (3 + 2 x[3]) - x[1]^2 (4 x[2] + 17 x[3]) +
      x[1] (x[3] (2 + 7 x[3]) + x[2] (-2 + 8 x[3])))) / (72 (x[1] - x[3])^3)
```

$$-\frac{n\,(2 - x[1] + x[3])^2\,(x[1] - 2\,x[2] + x[3])}{18\,(x[1] - x[3])}$$

```
(n (x[1]^2 - 2 x[2] (-1 + x[3]) - 3 (-2 + x[3]) x[3] + 2 x[1] (-4 + x[2] + x[3])) (x[1]^3 - 4 x[2]^2 +
     2 x[2] (1 - 2 x[3]) x[3] + 3 x[3]^2 (-2 + 3 x[3]) + x[1]^2 (-8 - 4 x[2] + 7 x[3]) +
     x[1] ((10 - 17 x[3]) x[3] + x[2] (6 + 8 x[3])))) / (72 (x[1] - x[3])^3)
```

```
react1 = Solve[focLoc[1] == 0, x[1]];

react1 = react1[[4]];

react1 /. {x[2] -> 1 / 2, x[3] -> 1} // N
```

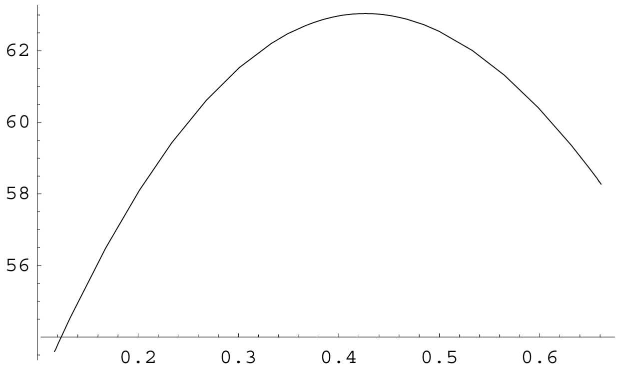
{x[1.] → 0.0977664 + 1.47255 × 10^{-16} ⅈ}

```
Clear[function]

function[i_] := function[i] =
    ((D[redProfitSym[3] /. react1, x[3]]) /. {x[3] -> x3, x[2] -> i});

f[_] = 25; c[_] = 0;

n = 1000;
```

The diagrams below depict the profit functions. They are quasi-concave implying that the calculated equilibrium values are indeed globally optimal. Existence is therefore not a problem.

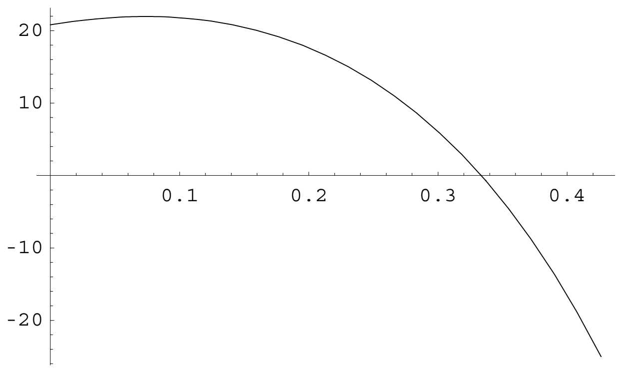
```
Plot[((redProfitSym[2] /. react1) /. {x[3] -> x3}) /.
  Join[reactNum3[j], {x[2] -> j}], {j, .1, .9}]
```

```
Plot::plnr : redProfitSym[2] /. react1 /. {x[3] → x3} /. Join[≪1≫]
   is not a machine-size real number at j = 0.10000003333333334`.

Plot::plnr : redProfitSym[2] /. react1 /. {x[3] → x3} /. Join[≪1≫]
   is not a machine-size real number at j = 0.11579853661041614`.

Plot::plnr : redProfitSym[2] /. react1 /. {x[3] → x3} /. Join[≪1≫]
   is not a machine-size real number at j = 0.665848750541516`.

General::stop : Further output of Plot::plnr will be suppressed during this calculation.
```



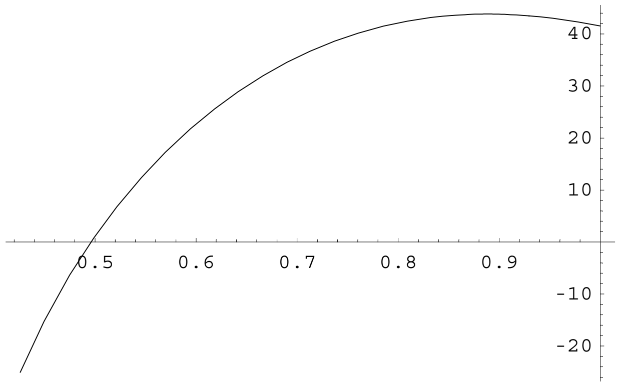
- Graphics -

```
Plot[(redProfitSym[1] /. {x[1] -> j}) /. locationsSym, {j, 0, .42615}, PlotRange -> All]
```



- Graphics -

```
Plot[(redProfitSym[3] /. react1) /. {x[2] -> 0.42615, x[3] -> j},
 {j, .42615, 1}, PlotRange -> All]
```

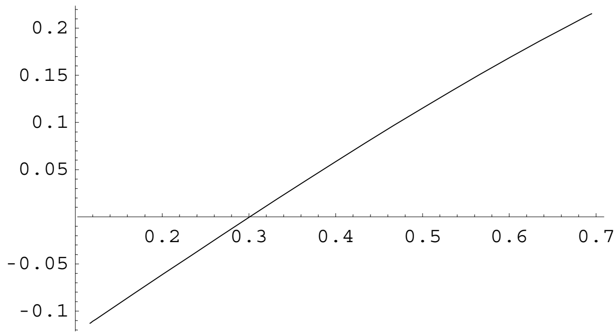


- Graphics -

Below I plot the reaction functions of firms 1 and 3, respectively. They enter in the derivation of the equilibrium. They are well-shaped and show that locations sym is indeed an equilibrium.

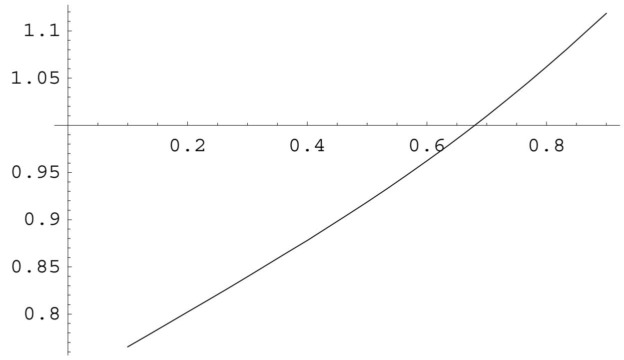
```
Plot[x[1] /. (react1 /. {x[3] -> x3}) /. Join[reactNum3[j], {x[2] -> j}], {j, .1, .9}]
```

```
Plot::plnr : «1» is not a machine-size real number at j = 0.10000003333333334`.

Plot::plnr : «1» is not a machine-size real number at j = 0.11579853661041614`.

Plot::plnr : «1» is not a machine-size real number at j = 0.7017169499874066`.

General::stop : Further output of Plot::plnr will be suppressed during this calculation.
```



- Graphics -

```
Plot[x3 /. (reactNum3[j] /. {x[2] -> j}), {j, .1, .9}]
```

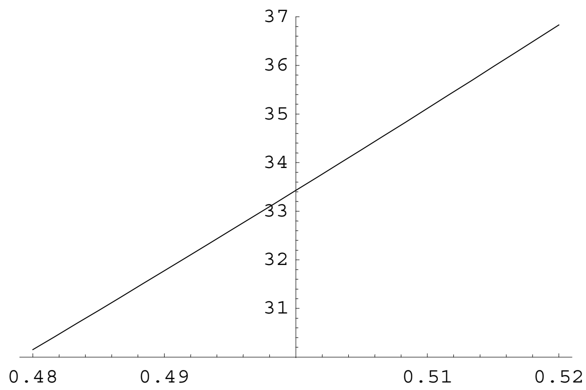


- Graphics -

Now I check whether the second entrant would earn lower profits than the third if the first locates around .5. This is important in the derivation of the entry deterrence equilibrium with three firms. The diagrams show profits of the second and third entrant as a function of the first entrants location.

Profit of the third entrant.

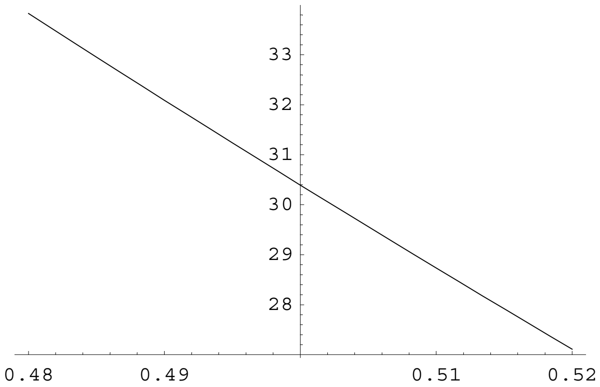
```
Plot[((redProfitSym[1] /. react1) /. {x[3] -> x3}) /. Join[reactNum3[j], {x[2] -> j}],
 {j, .48, .52}]
```



- Graphics -

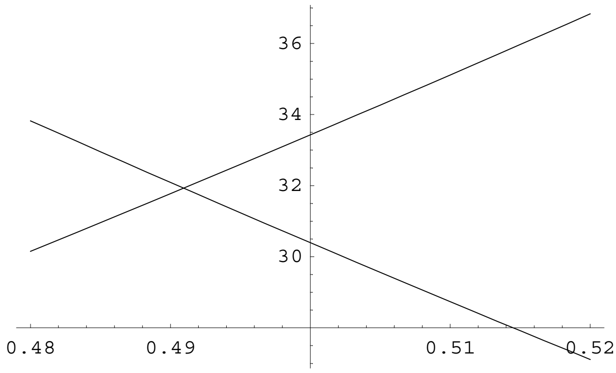
Profit of the second entrant.

```
Plot[((redProfitSym[3] /. react1) /. {x[3] -> x3}) /. Join[reactNum3[j], {x[2] -> j}],
 {j, .48, .52}]
```



- Graphics -

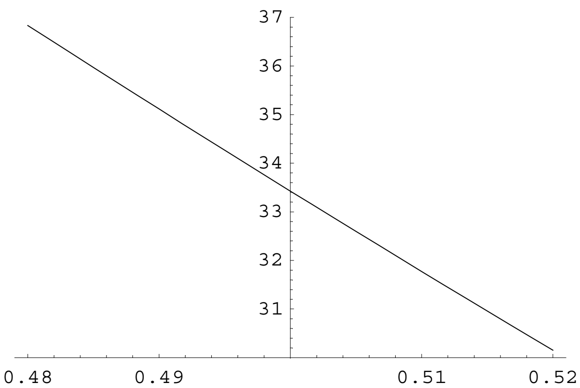
```
Show[%, %%]
```



- Graphics -

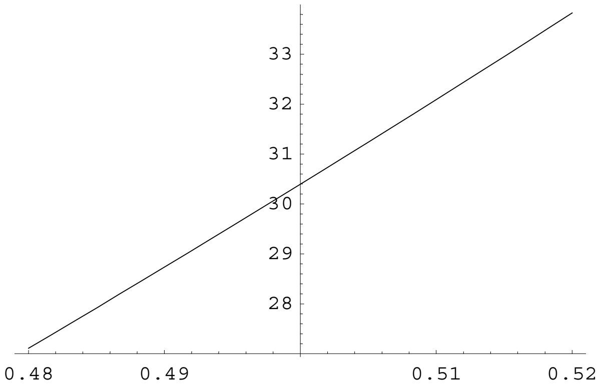
Third entrant's profit greater if the location of the first entrant is to the right of .49. The next diagrams switch positions between the second and the third entrant.

```
Plot[((redProfitSym[1] /. react1) /. {x[3] -> x3}) /.
  Join[reactNum3[1 - j], {x[2] -> 1 - j}], {j, .48, .52}]
```



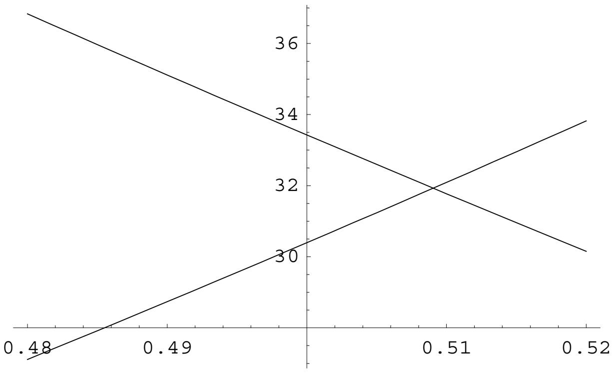
- Graphics -

```
Plot[((redProfitSym[3] /. react1) /. {x[3] -> x3}) /.
  Join[reactNum3[1 - j], {x[2] -> 1 - j}], {j, .48, .52}]
```



- Graphics -

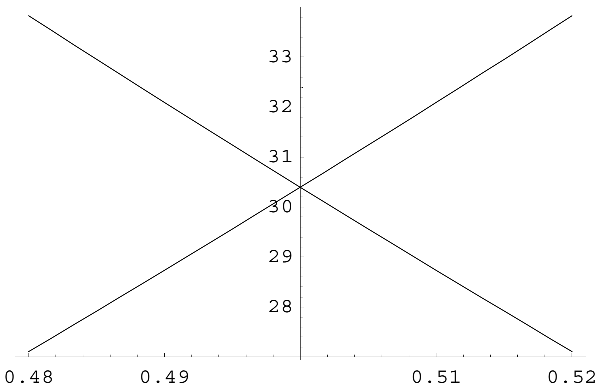
```
Show[%, %%]
```



- Graphics -

It wouldn't help to switch to the other side!

```
Show[%%, %%%%%]
```

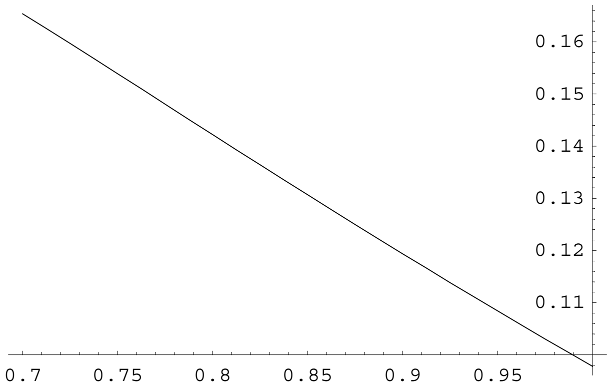


- Graphics -

Of course, the second entrant would switch sides at .5. However, this wouldn't prevent that she earns less than the third entrant in the respective region.

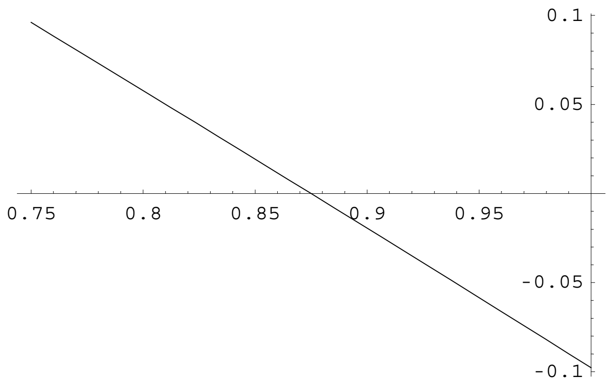
An explanation of the result can be found by looking at the reaction function of the third entrant. For locations of the second entrant greater 7/8, the location from the equilibrium of the simultaneous entry game, the third entrant locates further away from the edge of the market than the second.

```
Plot[x[1] /. react1 /. {x[2] -> 1 / 2, x[3] -> i, c[1] → 0, c[2] → 0, c[3] → 0}, {i, .7, 1}]
```



- Graphics -

```
Plot[1 - i - x[1] /. react1 /.
  {x[2] -> 1 / 2, x[3] -> i, c[1] → 0, c[2] → 0, c[3] → 0}, {i, .75, 1}]
```



- Graphics -

```
n =.
```

■ Case with 3 firms and deterred entry

Now I determine the market size at which entry of a fourth entrant is no longer blockaded. The entrant would locate between the first and the second entrant, i.e. between firms 2 and 3.

```
a[3, 4] =.
```

```
a[3, 4]
```

$$\frac{-p[3] + p[4]}{2\,(-x[3] + x[4])} + \frac{1}{2}\,(x[3] + x[4])$$

```
a[4, 5] = 1
```

1

```
foc[3]
```

$$n\,p[3]\left(\frac{1}{2\,(x[2]-x[3])}-\frac{1}{2\,(-x[3]+x[4])}\right)+ n\left(\frac{1}{2}\,(-x[2]-x[3])-\frac{p[2]-p[3]}{2\,(x[2]-x[3])}+\frac{-p[3]+p[4]}{2\,(-x[3]+x[4])}+\frac{1}{2}\,(x[3]+x[4])\right)$$

```
foc[4]
```

$$n\left(1+\frac{1}{2}\,(-x[3]-x[4])-\frac{p[3]-p[4]}{2\,(x[3]-x[4])}\right)+\frac{n\,p[4]}{2\,(x[3]-x[4])}$$

```
prices4Firms = Solve[{foc[1] == 0, foc[2] == 0, foc[3] == 0, foc[4] == 0},
    {p[1], p[2], p[3], p[4]}] // Simplify
```

```
{{p[1] → ((x[1] - x[2]) (-3 x[2]^2 x[3] + x[1] (x[2] - x[3]) (3 x[2] + x[3] - 4 x[4]) +
        6 x[1]^2 (x[2] - x[4]) + x[3] (2 x[3] (-1 + x[4]) + x[4] (2 + x[4])) -
        x[2] (3 x[3]^2 + x[4] (2 + x[4]) - 2 x[3] (1 + 2 x[4])))) /
    (3 (x[2]^2 + 2 x[2] x[3] + x[3] (x[3] - 4 x[4]) - 4 x[1] (x[2] - x[4]))),
  p[2] → -((x[1] - x[2]) (x[2] - x[3]) (-3 x[2]^2 - 4 x[3] - 3 x[2] x[3] +
        x[1] (3 x[2] + x[3] - 4 x[4]) + 4 x[4] + 4 x[3] x[4] + 2 x[4]^2)) /
     (3 (x[2]^2 + 2 x[2] x[3] + x[3] (x[3] - 4 x[4]) - 4 x[1] (x[2] - x[4]))),
  p[3] → ((x[2] - x[3]) (x[3] - x[4]) (-2 x[1]^2 + 3 x[3] (-2 + x[3] - x[4]) +
        x[2] (-2 + 3 x[3] - x[4]) + x[1] (8 - 4 x[2] + 4 x[4]))) /
    (3 (x[2]^2 + 2 x[2] x[3] + x[3] (x[3] - 4 x[4]) - 4 x[1] (x[2] - x[4]))),
  p[4] → ((x[3] - x[4]) (x[1]^2 (-x[2] + x[3]) + x[2]^2 (-4 + 3 x[3] + x[4]) -
        3 x[3] x[4] (-4 + x[3] + 2 x[4]) + x[2] x[3] (-8 + 3 x[3] + 2 x[4]) -
        2 x[1] (x[2]^2 - 2 x[3] (-1 + x[4]) - 3 (-2 + x[4]) x[4] + 2 x[2] (-4 + x[3] + x[4])))) /
    (3 (x[2]^2 + 2 x[2] x[3] + x[3] (x[3] - 4 x[4]) - 4 x[1] (x[2] - x[4])))}}
```

```
profit[i] /. prices4Firms
```

$$\left\{-25+n\,p[i]\left(\frac{1}{2}\,(-x[-1+i]-x[i])-\frac{p[-1+i]-p[i]}{2\,(x[-1+i]-x[i])}+\frac{-p[i]+p[1+i]}{2\,(-x[i]+x[1+i])}+\frac{1}{2}\,(x[i]+x[1+i])\right)\right\}$$

```
redProfit4[i_] := redProfit4[i] = Simplify[profit[i] /. prices4Firms]
```

The reduced profit function of the third firm (fourth entrant) in the 4 firms case:

```
redProfit4[3]
```

$$\left\{-25-\left(n\,(x[2]-x[3])\,(x[2]-x[4])\,(x[3]-x[4])\,(-2\,x[1]^2+3\,x[3]\,(-2+x[3]-x[4])+x[2]\,(-2+3\,x[3]-x[4])+x[1]\,(8-4\,x[2]+4\,x[4]))^2\right)\Big/\left(18\,(x[2]^2+2\,x[2]\,x[3]+x[3]\,(x[3]-4\,x[4])-4\,x[1]\,(x[2]-x[4]))^2\right)\right\}$$

```
redProfit4[3] /. {x[1] -> 0, x[2] -> 1/3, x[3] -> 2/3, x[4] -> 1} // Expand
```

$$\left\{-25+\frac{n}{27}\right\}$$

```
redProfit4[2]
```

```
{-25 - (n (x[1] - x[2]) (x[2] - x[3])
      (-3 x[2]^2 - 4 x[3] - 3 x[2] x[3] + x[1] (3 x[2] + x[3] - 4 x[4]) +
        4 x[4] + 4 x[3] x[4] + 2 x[4]^2) (x[1]^2 (3 x[2] + x[3] - 4 x[4]) -
        x[1] (3 x[2]^2 + 6 x[2] x[3] + x[3]^2 + x[3] (4 - 8 x[4]) - 2 x[4] (2 + x[4])) +
        x[3] (3 x[2]^2 + 3 x[2] x[3] - 2 (2 x[3] (-1 + x[4]) + x[4] (2 + x[4]))))) /
    (18 (x[2]^2 + 2 x[2] x[3] + x[3] (x[3] - 4 x[4]) - 4 x[1] (x[2] - x[4]))^2)}
```

The optimal location given the equilibrium locations in the three firms case. As the forth entrant would locate between firms 2 and 3, I redefine firm 3 as firm 4.

```
Solve[D[redProfit4[3], x[3]] == 0, x[3]];
```

```
% /. (locationsSym /. x[3] -> x[4])
```

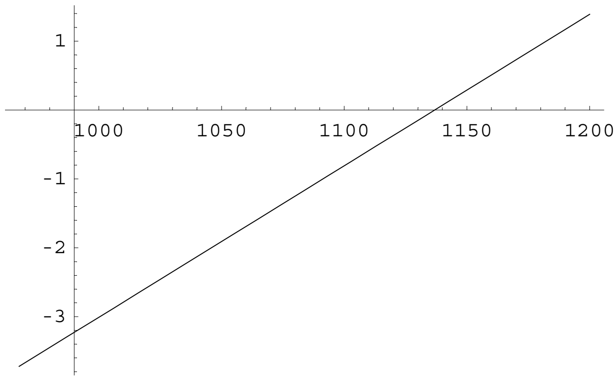
```
{{x[3] → 0.628026}, {x[3] → 2.269}, {x[3] → 2.8903}, {x[3] → 0.136829 - 0.295371 i},
 {x[3] → 0.136829 + 0.295371 i}, {x[3] → -0.0679987}, {x[3] → 2.53038}}
```

```
x3opt = x[3] /. %[[1]]
```

```
0.628026
```

The 4th entrant would locate quite close to the center firm, not halfway between the two adjacent firms.

```
Plot[(redProfit4[3] /. (locationsSym /. x[3] -> x[4])) /. x[3] → x3opt,
 {n, nNoDeter, 1200}]
```




```
- Graphics -
```

```
FindRoot[
 ((redProfit4[3] /. (locationsSym /. x[3] -> x[4])) /. x[3] → x3opt)[[1]] == 0, {n, 1000}]
```

```
{n → 1136.86}
```

```
nOfEntryBlockade3Firms = n /. %
```

```
1136.86
```

From this value onwards entry of a fourth firm is no longer blockaded.

```
f[2] / nOfEntryBlockade3Firms
```

```
0.0219904
```

Result differs from Neven! He gets 0.0245.

■ Case with three firms and deterred entry: Entry between first and second entrant must be deterred.

```
Solve[D[redProfit4[3], x[3]] == 0, x[3]];

% /. {x[1] -> 0.1, x[2] -> .5, x[4] -> .88}

{{x[3] → 0.667817}, {x[3] → 0.185084 - 0.33734 i},
 {x[3] → 0.185084 + 0.33734 i}, {x[3] → 2.34601 - 0.421761 i},
 {x[3] → 2.34601 + 0.421761 i}, {x[3] → -0.0691399}, {x[3] → 2.44914}}

x3opt4firms[i_, j_, k_] = x[3] /.
  (Solve[D[redProfit4[3], x[3]] == 0, x[3]][[1]] /. {x[1] -> i, x[2] -> j, x[4] -> k})

Root[32 i^2 j^2 - 8 i^3 j^2 - 16 i j^3 - 14 i^2 j^3 + 2 j^4 + 4 i j^4 + 8 i j^2 k + 26 i^2 j^2 k - 2 j^3 k - 12 i j^3 k +
   7 j^4 k - 32 i^2 k^2 + 8 i^3 k^2 + 24 i j k^2 - 16 j^2 k^2 - 4 i j^2 k^2 - j^3 k^2 - 16 i^2 k^3 + 12 i j k^3 - 8 j^2 k^3 -
   64 i^2 j #1 + 16 i^3 j #1 - 24 i j^2 #1 + 24 i^2 j^2 #1 + 10 j^3 #1 + 20 i j^3 #1 - 9 j^4 #1 + 64 i^2 k #1 -
   16 i^3 k #1 - 64 i j k #1 - 52 i^2 j k #1 + 30 j^2 k #1 - 36 i j^2 k #1 + 8 j^3 k #1 + 40 i k^2 #1 +
   40 i^2 k^2 #1 + 8 j k^2 #1 - 4 i j k^2 #1 + 15 j^2 k^2 #1 + 20 i k^3 #1 + 4 j k^3 #1 + 120 i j #1^2 -
   6 i^2 j #1^2 - 18 j^2 #1^2 - 9 j^3 #1^2 - 72 i k #1^2 - 6 i^2 k #1^2 - 6 j k #1^2 + 96 i j k #1^2 - 12 j^2 k #1^2 -
   24 k^2 #1^2 - 96 i k^2 #1^2 - 15 j k^2 #1^2 - 12 k^3 #1^2 - 16 i #1^3 + 4 i^2 #1^3 - 26 j #1^3 - 64 i j #1^3 +
   15 j^2 #1^3 + 42 k #1^3 + 64 i k #1^3 - 16 j k #1^3 + 57 k^2 #1^3 + 21 j #1^4 - 51 k #1^4 + 6 #1^5 &, 1]

x3opt4firms[.07, .43, .88]

0.627103

n = 1130

1130

Flatten[{((react1 /. {x[3] -> x3}) /. Join[reactNum3[.42], {x[2] -> .42}]),
  x[2] -> .42, x[4] -> (x3 /. reactNum3[.42])}]

{x[1] → 0.0701351 - 6.07766×10^-17 i, x[2] → 0.42, x[4] → 0.886075 - 1.4061×10^-16 i}

(x[3] -> x3opt4firms[x[1], x[2], x[4]]) /.
 Chop[Flatten[{((react1 /. {x[3] -> x3}) /. Join[reactNum3[.42], {x[2] -> .42}]),
    x[2] -> .42, x[4] -> (x3 /. reactNum3[.42])}]]

x[3] → 0.623544

(redProfit4[3] /. x[3] -> x3opt4firms[x[1], x[2], x[4]])

((redProfit4[3] /. x[3] -> x3opt4firms[x[1], x[2], x[4]]) /.
   Chop[Flatten[{((react1 /. {x[3] -> x3}) /. Join[reactNum3[.42], {x[2] -> .42}]),
      x[2] -> .42, x[4] -> (x3 /. reactNum3[.42])}]])[[1]]

0.214756

redProfit4[3] /. {x[1] -> 0, x[2] -> .3, x[3] -> .6, x[4] -> 1}

{20.686}

reactNum3[.42]

{x3 → 0.886075 - 1.4061×10^-16 i}
```

```
n =.
```

The function below calculates the optimal location of the first entrant which deters entry of the fourth entrant.

```
x1stEntrantDeters4thEntrant[n1_] := x1stEntrantDeters4thEntrant[n1] = (n = n1; result =
    FindRoot[((redProfit4[3] /. x[3] -> x3opt4firms[x[1], x[2], x[4]]) /. Chop[Flatten[
          {((react1 /. {x[3] -> x3}) /. Join[reactNum3[i], {x[2] -> i}]), x[2] -> i,
           x[4] -> (x3 /. reactNum3[i])}]])[[1]] == 0, {i, .4, .48}]; n =.; i /. result)
```

```
x1stEntrantDeters4thEntrant[1136.86]
```

```
0.426147
```

```
n
```

```
n
```

```
FindRoot[x1stEntrantDeters4thEntrant[n] == .5, {n, 1250, 1400}]
```

```
{n → 1359.64}
```

```
nOffirstEntrantDeterAtonehalf = n /. %
```

```
1359.64
```

From the above value of market size onwards, the first entrant can no longer deter entry alone. the second entrant would switch to the other side.

Value at which Economides et al. find a discontinuity.

```
25 / .018
```

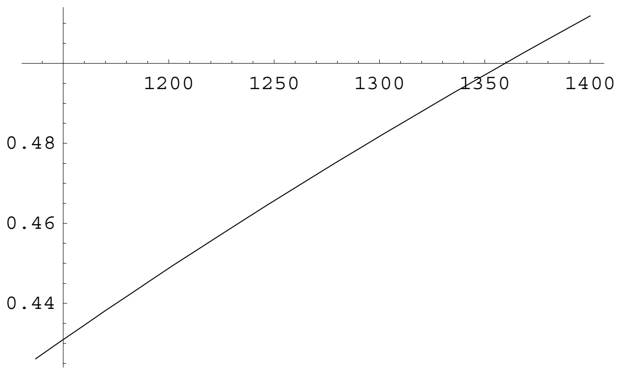
```
1388.89
```

```
25 / .022
```

```
1136.36
```

Below the equilibrium locations as a function of market size in the relevant range.

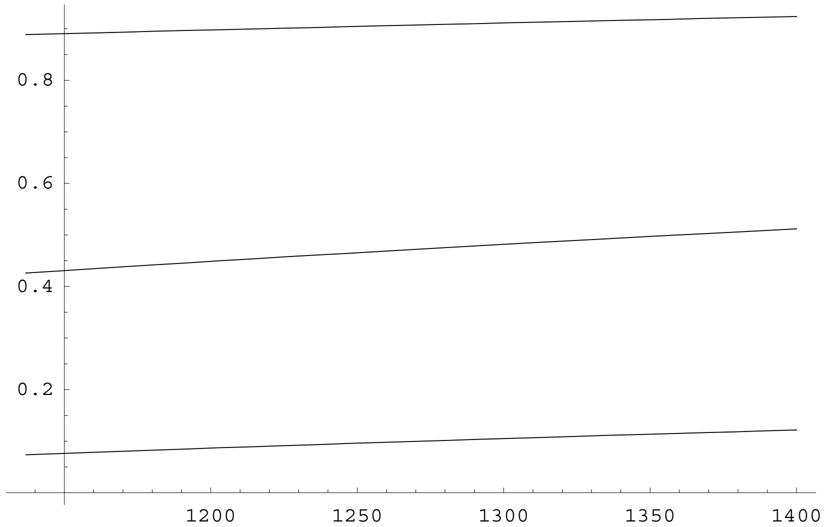
```
Plot[x1stEntrantDeters4thEntrant[n1], {n1, 1136.86, 1400}]
```




```
- Graphics -
```

```
n = 1000;
```

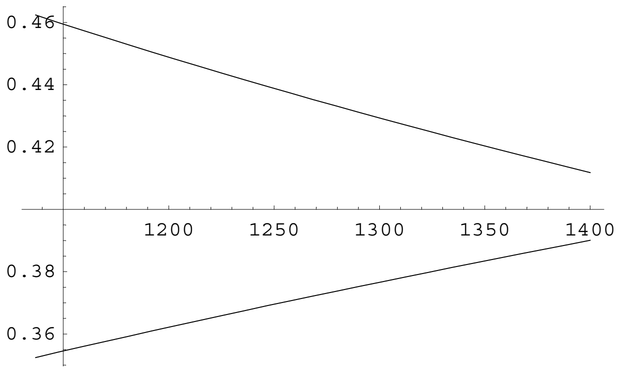
```
Plot[
 {x[1] /. ((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
      {x[2] -> x1stEntrantDeters4thEntrant[n1]}]), x1stEntrantDeters4thEntrant[n1],
  (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])}, {n1, 1136.86, 1400}]
```



- Graphics -

The distance between the the center and the right and the left firm, resp.

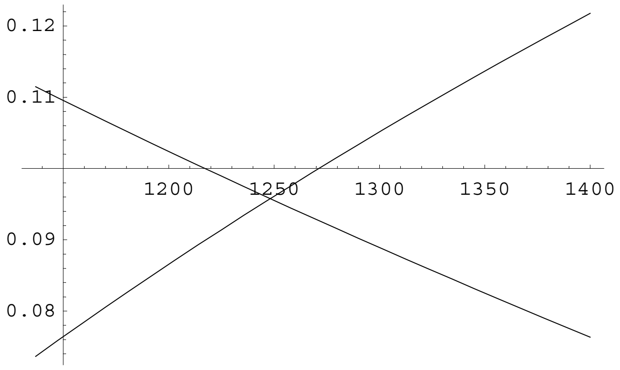
```
Plot[{x1stEntrantDeters4thEntrant[n1] - x[1] /.
   ((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
      {x[2] -> x1stEntrantDeters4thEntrant[n1]}]), -x1stEntrantDeters4thEntrant[n1] +
   (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])}, {n1, 1136.86, 1400}]
```



- Graphics -

The distance of the second and the third entrant from the edge of the market.

```
Plot[
 {x[1] /. ((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
      {x[2] -> x1stEntrantDeters4thEntrant[n1]}]),
  1 - (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])}, {n1, 1136.86, 1400}]
```




```
- Graphics -
```

```
x1stEntrantDeters4thEntrant[1350]
```

```
0.497112
```

```
n =.
```

```
n
```

```
n
```

```
redProfitSym[2]
```

$$-25 - \frac{n\,(x[1]-x[2])\,(x[2]-x[3])\,(2-x[1]+x[3])^2}{18\,(x[1]-x[3])}$$

```
redProfitSym[1]
redProfitSym[3]
```

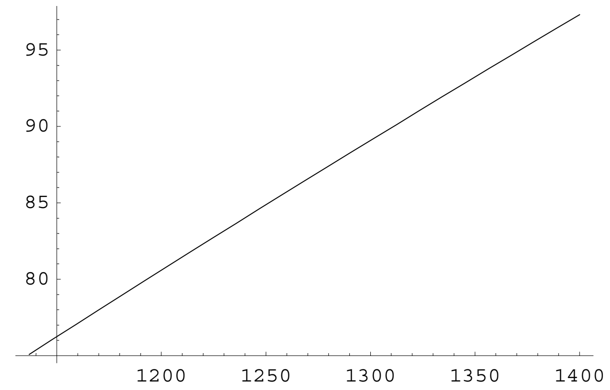
$$-25 - \left(n\,(x[1]-x[2])\,(-3\,x[1]^2-2\,x[1]\,(x[2]-x[3])+2\,x[2]\,(-1+x[3])+x[3]\,(2+x[3]))^2\right) \Big/ \left(72\,(x[1]-x[3])^2\right)$$

$$-25 + \left(n\,(-x[2]+x[3])\,(x[1]^2-2\,x[2]\,(-1+x[3])-3\,(-2+x[3])\,x[3]+2\,x[1]\,(-4+x[2]+x[3]))^2\right) \Big/ \left(72\,(x[1]-x[3])^2\right)$$

```
n = 1000;
```

Profit of the first entrant

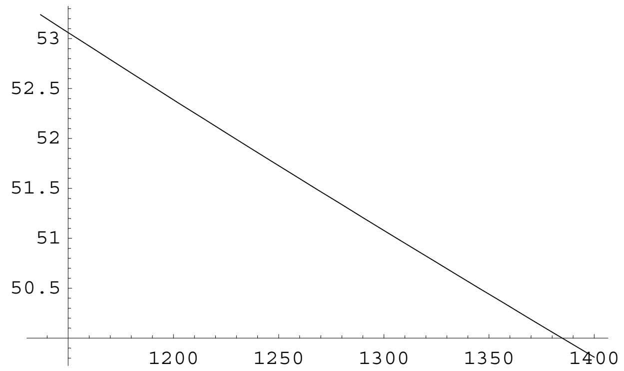
```
Plot[-25 - (n1 (x[1] - x[2]) (x[2] - x[3]) (2 - x[1] + x[3])^2)/(18 (x[1] - x[3])) /.
  {((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
       {x[2] -> x1stEntrantDeters4thEntrant[n1]}]) [[1]],
   x[2] -> x1stEntrantDeters4thEntrant[n1], x[3] ->
    (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])}, {n1, 1136.86, 1400}]
```



- Graphics -

Profit of the second entrant

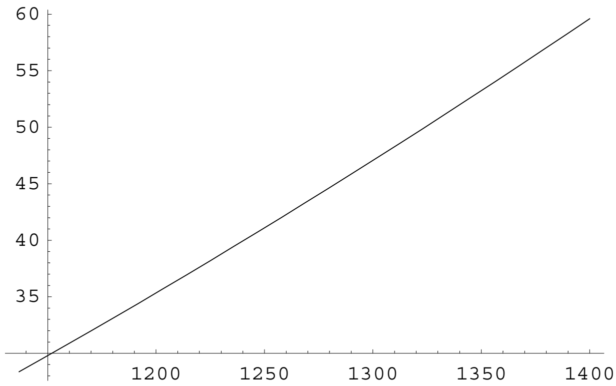
```
Plot[-25 + 1/(72 (x[1] - x[3])^2) (n1 (-x[2] + x[3])
      (x[1]^2 - 2 x[2] (-1 + x[3]) - 3 (-2 + x[3]) x[3] + 2 x[1] (-4 + x[2] + x[3]))^2) /.
  {((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
       {x[2] -> x1stEntrantDeters4thEntrant[n1]}]) [[1]],
   x[2] -> x1stEntrantDeters4thEntrant[n1], x[3] ->
    (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])}, {n1, 1136.86, 1400}]
```



- Graphics -

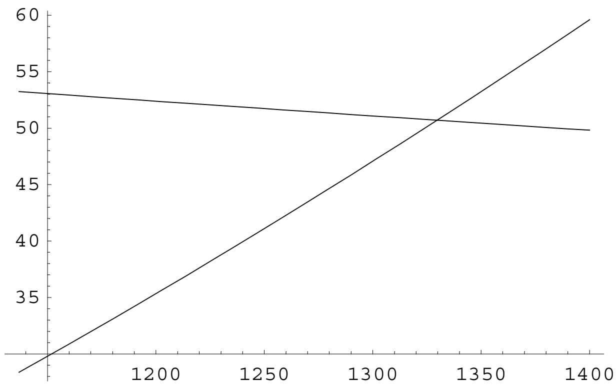
Profit of the third entrant

```
Plot[-25 - 1/(72 (x[1] - x[3])^2) (n1 (x[1] - x[2])
      (-3 x[1]^2 - 2 x[1] (x[2] - x[3]) + 2 x[2] (-1 + x[3]) + x[3] (2 + x[3]))^2) /.
  {((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
       {x[2] -> x1stEntrantDeters4thEntrant[n1]}])[[1]],
   x[2] -> x1stEntrantDeters4thEntrant[n1], x[3] ->
    (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])}, {n1, 1136.86, 1400}]
```



- Graphics -

```
Show[%, %%]
```



- Graphics -

```
n =.
```

```
redProfitSym[1] -
 redProfitSym[3]
```

```
-(n (x[1] - x[2]) (-3 x[1]^2 - 2 x[1] (x[2] - x[3]) + 2 x[2] (-1 + x[3]) + x[3] (2 + x[3]))^2) /
   (72 (x[1] - x[3])^2) - (n (-x[2] + x[3])
    (x[1]^2 - 2 x[2] (-1 + x[3]) - 3 (-2 + x[3]) x[3] + 2 x[1] (-4 + x[2] + x[3]))^2) / (72
    (x[1] - x[3])^2)
```

```
      1
--------------- (n (- (-x[2] + x[3])
72 (x[1] - x[3])^2
    (x[1]^2 - 2 x[2] (-1 + x[3]) - 3 (-2 + x[3]) x[3] + 2 x[1] (-4 + x[2] + x[3]))^2 -
   (x[1] - x[2]) (2 x[2] - 2 x[3] + (x[1] - x[3]) (3 x[1] + 2 x[2] + x[3]))^2))

(n ((x[2] - x[3]) (x[1]^2 - 2 x[2] (-1 + x[3]) - 3 (-2 + x[3]) x[3] + 2 x[1] (-4 + x[2] + x[3]))^2 -
    (x[1] - x[2])
     (2 x[2] - 2 x[3] + (x[1] - x[3]) (3 x[1] + 2 x[2] + x[3]))^2)) / (72 (x[1] - x[3])^2)

FindRoot[
 ((        1
   --------------- (n (- (-x[2] + x[3]) (x[1]^2 - 2 x[2] (-1 + x[3]) - 3 (-2 + x[3]) x[3] + 2
   72 (x[1] - x[3])^2
              x[1] (-4 + x[2] + x[3]))^2 - (x[1] - x[2])
           (2 x[2] - 2 x[3] + (x[1] - x[3]) (3 x[1] + 2 x[2] + x[3]))^2))) /.
    Chop[{((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n]],
          {x[2] -> x1stEntrantDeters4thEntrant[n]}])[[1]],
      x[2] -> x1stEntrantDeters4thEntrant[n], x[3] ->
       (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n]])}]) == 0, {n, 1300., 1400}]

{n → 1329.69}
```

The market size at which the profit of the third entrant becomes greater than that of the second.

```
nOf2ndEntrantLowerProfitThan3rd = n /. %

1329.69

25 / 1329.69

0.0188014

n1 = 1329.6865876896738`

1329.69
```

Locations and profits, resp. at the resp. market size.

```
{x[1] /. ((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
      {x[2] -> x1stEntrantDeters4thEntrant[n1]}]),
 x1stEntrantDeters4thEntrant[n1], (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])}

{0.110248 - 6.81673×10^-18 i, 0.490946, 0.914902 - 1.43074×10^-16 i}

{redProfitSym[1], redProfitSym[3]} /.
 {((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
      {x[2] -> x1stEntrantDeters4thEntrant[n1]}])[[1]],
  x[2] -> x1stEntrantDeters4thEntrant[n1],
  x[3] -> (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])}

{-25 + (0.0569298 - 1.08028×10^-17 i) n, -25 + (0.0569298 + 3.24235×10^-18 i) n}
```

From the derivation in the existence section, we know that the second entrant would switch to the other side of the market if x[2]>.5. As long as x[2]<=.5, the second entrant will make lower profits than the third!

If x[2]=.5, the first entrant can no longer deter further entry along by moving towrds the second entrant, because the second entrant can and would switch to the other side. The question is whether the first entrant has an incentive to move closer to the third in order to make the third entrant to move closer to the edge.
The next input determines the location of the second entrant when it also deters entry.

```
xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n1_, i_] :=
 xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n1, i] =
  (n = n1; result = FindRoot[((redProfit4[3] /. x[3] -> x3opt4firms[x[1], x[2], x[4]]) /.
         Chop[Flatten[{((react1 /. {x[3] -> x3}) /. {x3 -> j, x[2] -> i}),
            x[2] -> i, x[4] -> j}]])[[1]] == 0, {j, .85, .9}]; n =.; j /. result)
```

```
xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1250, .43]
```

```
0.859731
```

```
xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1250, .46]
```

```
0.897389
```

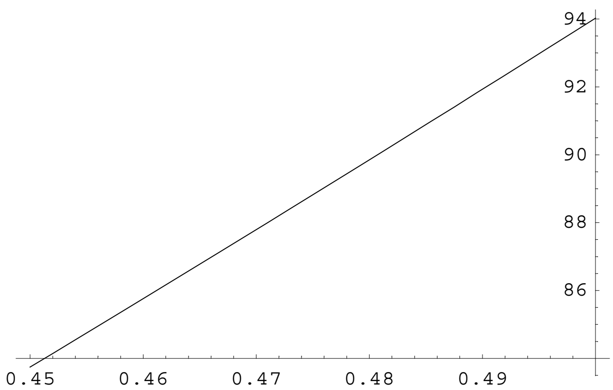
```
n1 =.
n =.
```

```
xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[nOffirstEntrantDeterAtonehalf, .5]
```

```
0.918677
```

Profit of first entrant if second moves as well to deter entry

```
Plot[-25 - (nOffirstEntrantDeterAtonehalf (x[1] - x[2]) (x[2] - x[3]) (2 - x[1] + x[3])^2) /
    (18 (x[1] - x[3])) /.
  {((react1 /. {x[3] -> x3}) /. {x3 -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
         nOffirstEntrantDeterAtonehalf, i], x[2] -> i})[[1]],
   x[2] -> i, x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
     nOffirstEntrantDeterAtonehalf, i]}, {i, .45, .5}]
```

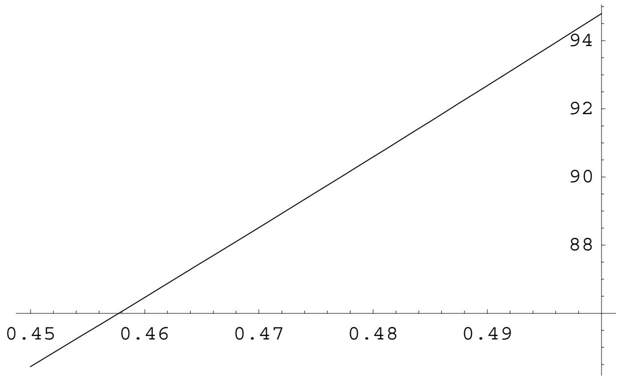



```
- Graphics -
```

```
nOffirstEntrantDeterAtonehalf
```

```
1359.64
```

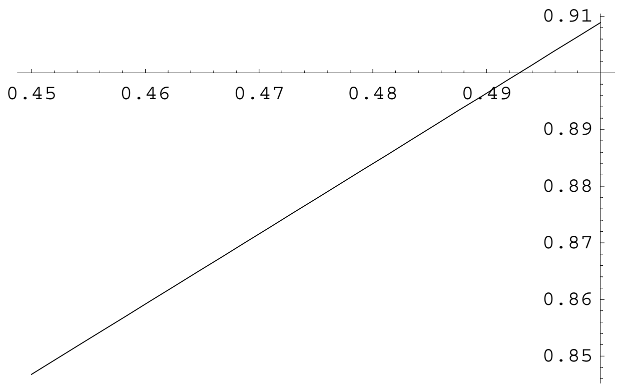
```
Plot[
 -25 - 1400 (x[1] - x[2]) (x[2] - x[3]) (2 - x[1] + x[3])^2 / (18 (x[1] - x[3])) /. {((react1 /. {x[3] -> x3}) /.
      {x3 -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1400, i], x[2] -> i})[[1]],
   x[2] -> i, x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1400, i]}, {i,
  .45, .5}]
```



- Graphics -

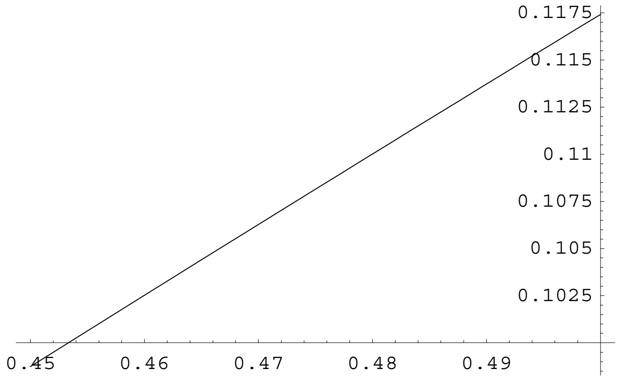
This diagram shows that the first entrant is best off by locating at .5, given that the second entrant also deters entry and the third entrant locates according to her reaction function. The first entrant cannot prevent the second from "sharing the burden of entry deterrence", since she can just switch to the other side. Moving away from .5 reduces profits. The first entrant gains less from the move of the third entrant towards the edge than she loses to the second from her move towards the center.

```
Plot[x[3] /. {((react1 /. {x[3] -> x3}) /.
      {x3 -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1400, i], x[2] -> i})[[1]],
   x[2] -> i, x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1400, i]}, {i,
  .45, .5}]
```



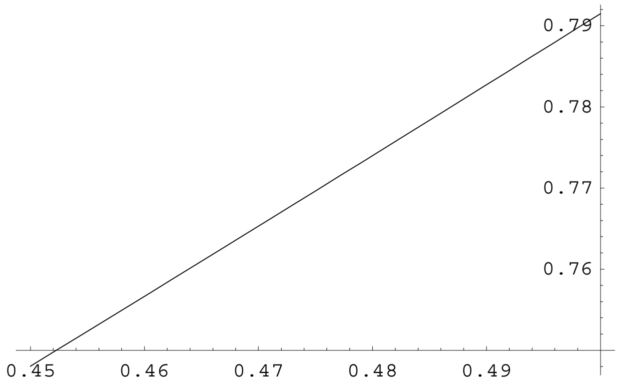
- Graphics -

```
Plot[x[1] /. {((react1 /. {x[3] -> x3}) /.
      {x3 -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1400, i], x[2] -> i})[[1]],
   x[2] -> i, x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1400, i]}, {i,
  .45, .5}]
```



- Graphics -

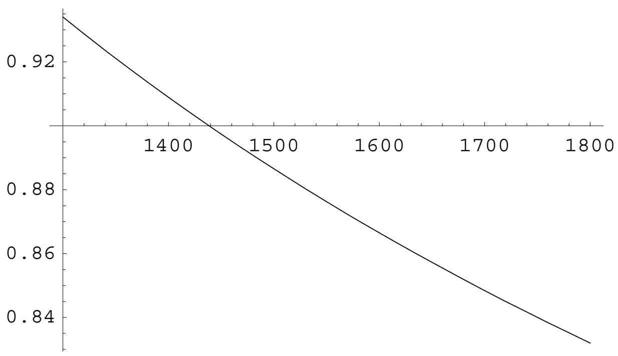
```
Plot[x[3] - x[1] /. {((react1 /. {x[3] -> x3}) /.
      {x3 -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1400, i], x[2] -> i})[[1]],
   x[2] -> i, x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1400, i]}, {i,
  .45, .5}]
```



- Graphics -

This diagram shows that the distance between the second and third entrant is at a maximum if the first entrant locates at .5.

```
Plot[xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5], {n, 1300, 1800}]
```



- Graphics -

Discontinuity of Economides et al. seems to yield lower profits for the first entrant.

```
{redProfitSym[1], redProfitSym[2], redProfitSym[3]} /.
 {x[1] -> .06, x[2] -> .35, x[3] -> .71}
```

```
{-25 + 0.0293123 n, -25 + 0.0626623 n, -25 + 0.0800923 n}
```

```
% /. n -> 1400
```

```
{16.0373, 62.7272, 87.1293}
```

```
25 + %% /. n -> 1
```

```
{0.0293123, 0.0626623, 0.0800923}
```

```
25 / .018
```

```
1388.89
```

The profit of the first entrant at .018

```
i = .5
```

```
0.5
```

```
-25 -
  (1388.888888888889` (x[1] - x[2]) (x[2] - x[3]) (2 - x[1] + x[3])^2) / (18 (x[1] - x[3])) /.
 {((react1 /. {x[3] -> x3}) /. {x3 -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
        1388.888888888889`, i], x[2] -> i})[[1]], x[2] -> i,
  x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1388.888888888889`, i]}
```

$94.582 + 3.43048 \times 10^{-15}\, i$

```
% / 1388.888888888889`
```

$0.068099 + 2.46994 \times 10^{-18}\, i$

Note that Economides et al and Neven calculate gross profits only

```
% + .018
```

$0.086099 + 2.46994 \times 10^{-18}\, i$

The above value is the profit of the first entrant in my case. The firm locates at the center. The profit is greater than in the case EHM consider. They claim that the first entrant locates close to the edge and makes profits of 0.0800923. Clearly smaller than what I obtain.

```
i =.
```

- **When will the entry deterring constraint for entry between the first and the third entrant become binding?**

```
n
```

```
n
```

```
x2opt4firms[i_, j_, k_] = x[2] /.
  (Solve[D[redProfit4[2], x[2]] == 0, x[2]][[1]] /. {x[1] -> i, x[3] -> j, x[4] -> k})
```

```
Root[32 i^2 j^2 - 8 i^3 j^2 - 20 i j^3 - i^2 j^3 - 4 j^4 + 7 i j^4 - 48 i^2 j k + 12 i^3 j k + 20 i j^2 k - 4 i^2 j^2 k +
   20 j^3 k - 12 i j^3 k + 4 j^4 k + 16 i^2 k^2 - 16 i^3 k^2 - 16 j^2 k^2 + 26 i j^2 k^2 - 14 j^3 k^2 + 8 i^2 k^3 -
   8 j^2 k^3 - 16 i^2 j #1 + 4 i^3 j #1 - 24 i j^2 #1 + 15 i^2 j^2 #1 + 16 j^3 #1 + 8 i j^3 #1 - 9 j^4 #1 +
   16 i^2 k #1 + 20 i^3 k #1 + 56 i j k #1 - 4 i^2 j k #1 - 48 j^2 k #1 - 36 i j^2 k #1 + 20 j^3 k #1 -
   32 i k^2 #1 + 40 i^2 k^2 #1 + 32 j k^2 #1 - 52 i j k^2 #1 + 24 j^2 k^2 #1 - 16 i k^3 #1 + 16 j k^3 #1 -
   12 i^3 #1^2 + 12 i j #1^2 - 15 i^2 j #1^2 + 12 j^2 #1^2 - 12 i j^2 #1^2 - 9 j^3 #1^2 - 12 i k #1^2 -
   96 i^2 k #1^2 - 12 j k #1^2 + 96 i j k #1^2 - 6 i k^2 #1^2 - 6 j k^2 #1^2 + 57 i^2 #1^3 - 8 j #1^3 - 16 i j #1^3 +
   15 j^2 #1^3 + 8 k #1^3 + 64 i k #1^3 - 64 j k #1^3 + 4 k^2 #1^3 - 51 i #1^4 + 21 j #1^4 + 6 #1^5 &, 1]
```

```
x2opt4firms[.07, .43, .88]
```

```
0.271187
```

```
x2opt4firms[.1, .5, .9]
```

```
0.324322
```

```
n
```

```
n
```

Nowthat the first entrant locates at .5 and that the second also deters.

```
n
```

```
n
```

```
n1 = 1400;
```

```
Chop[
 Flatten[{(react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n1, .5]
      , x[2] -> .5}), x[3] -> .5,
   x[4] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n1, .5]}]]
```

```
{x[1] → 0.117417, x[3] → 0.5, x[4] → 0.908897}
```

```
n =.
```

```
n1 =.
```

```
FindRoot[((redProfit4[2] /. x[2] -> x2opt4firms[x[1], x[3], x[4]]) /. Chop[Flatten[
       {(react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
           , x[2] -> .5}), x[3] -> .5, x[4] ->
         xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
          n, .5]}]])[[1]] == 0, {n, 1350, 1500}]
```

```
{n → 1556.19}
```

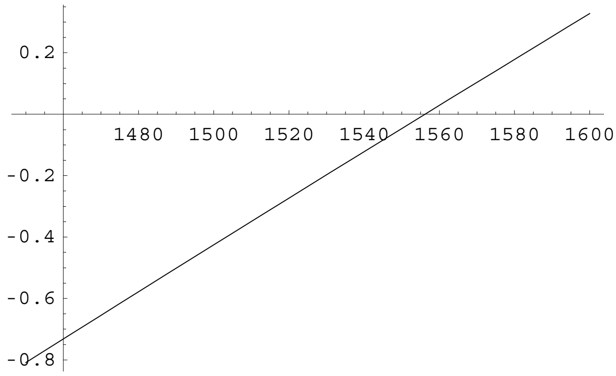
```
nOfEntryBetween1stAnd3rdEntrandMustBeDeterred = n /. %
```

```
1556.19
```

```
25 / 1556.1936410600297`
```

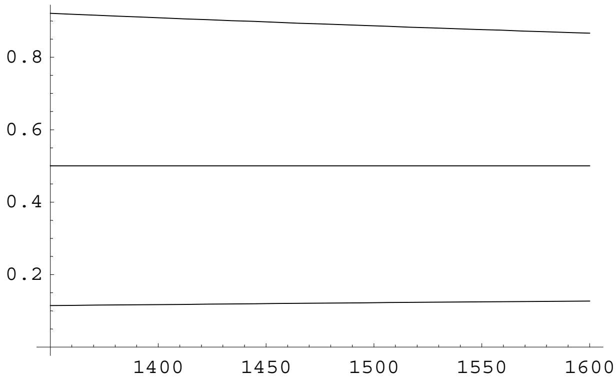
```
0.0160648
```

```
Plot[((redProfit4[2] /. x[2] -> x2opt4firms[x[1], x[3], x[4]]) /. Chop[
     Flatten[{(react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
          , x[2] -> .5}), x[3] -> .5,
        x[4] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
          n, .5]}]])[[1]], {n, 1450, 1600}]
```



- Graphics -

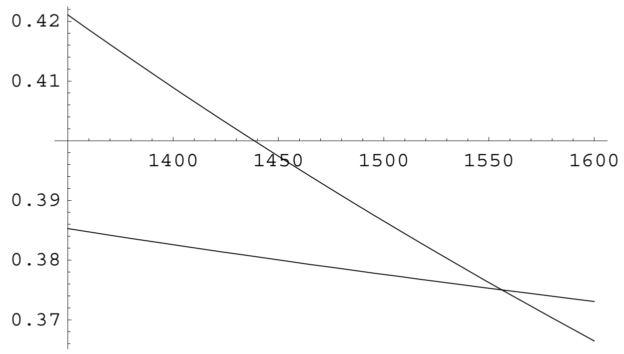
```
Plot[{x[1] /. (react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
      , x[2] -> .5}), .5, xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]},
  {n, 1350, 1600}]
```



- Graphics -

The below diagram shows the distance between the first and the second and third entrant, resp.

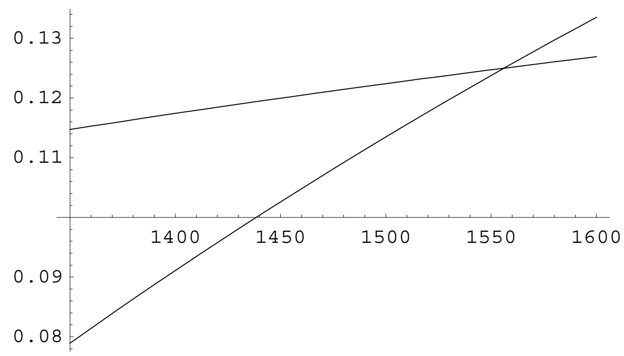
```
Plot[
 {-x[1] + .5 /. (react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
      , x[2] -> .5}), xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5] - .5},
 {n, 1350, 1600}]
```




```
- Graphics -
```

The distances of the second and third entrant from the edges.

```
Plot[{x[1] /. (react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
      , x[2] -> .5}), 1 - xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]},
 {n, 1350, 1600}]
```




```
- Graphics -
```

```
x[1] /. (react1 /.
   {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1556.1936410600308`, .5]
    , x[2] -> .5})
```

$0.125 - 1.38778 \times 10^{-17}\, i$

```
xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1556.1936410600308`, .5]
```

```
0.875
```

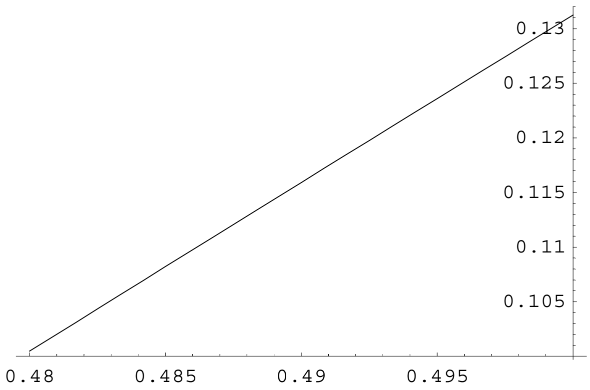
Next I show that the first entrant moves towards the third entrant in the scenario in which entry must be prevented at two locations.

```
Clear[xOf3rdEntrant1stAnd3rdEntrantDeter4thEntrant]

n1 =.
n =.
```

```
i =.

xOf3rdEntrant1stAnd3rdEntrantDeter4thEntrant[n1_, i_] :=
 xOf3rdEntrant1stAnd3rdEntrantDeter4thEntrant[n1, i] =
  j /. FindRoot[((Evaluate[redProfit4[2] /.
           {n -> n1, x[2] -> x2opt4firms[x[1], x[3], x[4]]}]) /. Chop[
         {x[1] -> j, x[3] -> i, x[4] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
            n1, i]}])[[1]] == 0, {j, .1, .25}]

xOf3rdEntrant1stAnd3rdEntrantDeter4thEntrant[1700, .5]

0.144625
```

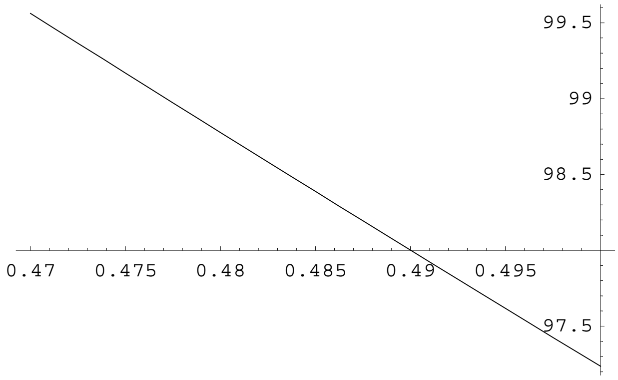
Profit of the first entrant

```
xOf3rdEntrant1stAnd3rdEntrantDeter4thEntrant[1600, .45]
xOf3rdEntrant1stAnd3rdEntrantDeter4thEntrant[1600, .48]

FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 15 iterations.

0.0733577

0.100474

Plot[xOf3rdEntrant1stAnd3rdEntrantDeter4thEntrant[1600, i], {i, 0.48, .5}]
```




```
- Graphics -
```

```
Plot[-25 - (1600 (x[1] - x[2]) (x[2] - x[3]) (2 - x[1] + x[3])^2)/(18 (x[1] - x[3])) /.
  {x[1] -> xOf3rdEntrant1stAnd3rdEntrantDeter4thEntrant[1600, i], x[2] -> i,
   x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[1600, i]}, {i, 0.47, .5}]
```

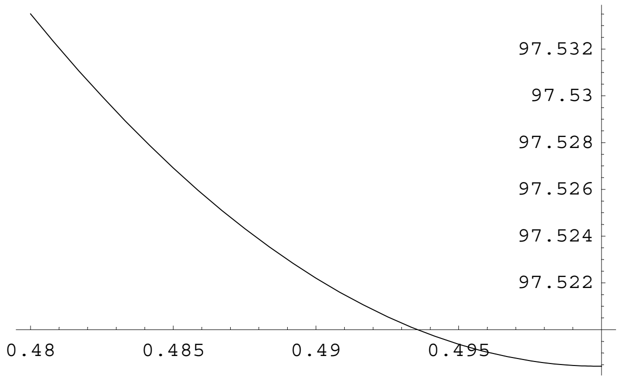



```
- Graphics -
```

```
Clear[xOf2ndAnd3rdEntrantBothDeter1stAroundPoint5]

xOf2ndAnd3rdEntrantBothDeter1stAroundPoint5[n1_, i_] :=
 xOf2ndAnd3rdEntrantBothDeter1stAroundPoint5[n1, i] = {j, k} /. (FindRoot[
     {(Evaluate[redProfit4[2] /. {n -> n1, x[2] -> x2opt4firms[x[1], x[3], x[4]]}] /.
          {x[1] -> j, x[3] -> i, x[4] -> k})[[1]] == 0,
      (Evaluate[redProfit4[3] /. {n -> n1, x[3] -> x3opt4firms[x[1], x[2], x[4]]}] /.
          {x[1] -> j, x[2] -> i, x[4] -> k})[[1]] == 0}, {j, .1, .2}, {k, .85, .9}])

xOf2ndAnd3rdEntrantBothDeter1stAroundPoint5[1600, .5]
xOf2ndAnd3rdEntrantBothDeter1stAroundPoint5[1600, .48]
xOf2ndAnd3rdEntrantBothDeter1stAroundPoint5[1600, .47]
```

```
{0.131833, 0.868167}

{0.0977841, 0.834839}

FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 15 iterations.

{0.0827245, 0.82102}
```

```
Plot[-25 - 1600 (x[1] - x[2]) (x[2] - x[3]) (2 - x[1] + x[3])^2 / (18 (x[1] - x[3])) /.
  {x[1] -> xOf2ndAnd3rdEntrantBothDeter1stAroundPoint5[1600, i][[1]], x[2] -> i,
   x[3] -> xOf2ndAnd3rdEntrantBothDeter1stAroundPoint5[1600, i][[2]]}, {i, 0.48, .5}]
```

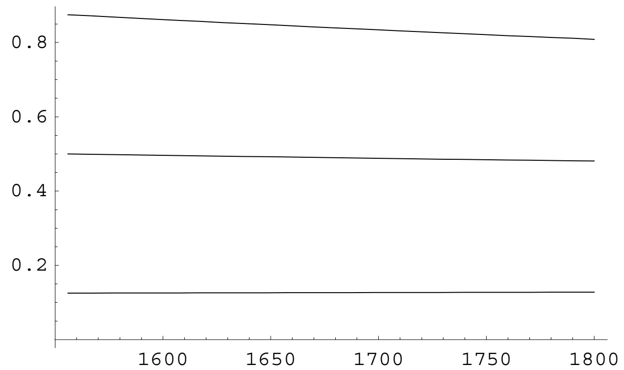



```
- Graphics -
```

The first entrant will deter entry as long as possible alone or together with the second entrant. As soon as the the entry deterrence constraint between the first and the third entrant becomes binding, the first entrant moves towards the third. Therefore, the third entrant locates according to her reaction function

```
xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1_] :=
 xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1] =
  i /. FindRoot[(Evaluate[redProfit4[2] /.
          {n -> n1, x[2] -> x2opt4firms[x[1], x[3], x[4]]}] /. Chop[Flatten[{react1 /.
            {x[2] -> i, x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n1, i]},
           x[3] -> i, x[4] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
             n1, i]}]]) [[1]] == 0, {i, .47, .5}]

xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[1600]

0.496285

xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[1556.1936410600308`]

0.5

n = 1600;

Chop[Flatten[{react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
      x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
        n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]},
    x[3] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
    x[4] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
      n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}]]

{x[1] → 0.125563, x[3] → 0.496285, x[4] → 0.861923}

n =.
```

```
Plot[
 {x[1] /. (react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n], x[3] ->
       xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
        xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}),
  xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
  xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
   xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}, {n, 1556, 1800}]
```




```
- Graphics -

25 / 1800.

0.0138889

i

i

n

n

xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[1800]
xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
 1800, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[1800]]

0.480826

0.808847

n = 1800;
Chop[Flatten[{react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
     x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
       n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]},
   x[3] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
   x[4] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
     n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}]]
n =.

{x[1] → 0.127763, x[3] → 0.480826, x[4] → 0.808847}

n

n
```

■ The first entrant has an incentive to switch to the edge (The entry deterring constraint for entry at 1 is not binding)

I first derive the market size for which entry at 1 must be deterred given that the first and the second entrant deter entry in the two ranges between the incumbents. Then I show that given that behavior the profit of the first and the second entrants are equalized for smaller values of market size. As a result, the first entrant will switch to the edge before the the entry deterring constraint at 1 becomes binding.

```
n
```

```
n
```

```
redProfit4[4]
```

```
{-25 - (n (x[3] - x[4]) (x[1]^2 (x[2] - x[3]) + x[2] x[3] (8 - 3 x[3] - 2 x[4]) -
        x[2]^2 (-4 + 3 x[3] + x[4]) + 3 x[3] x[4] (-4 + x[3] + 2 x[4]) + 2 x[1]
          (x[2]^2 - 2 x[3] (-1 + x[4]) - 3 (-2 + x[4]) x[4] + 2 x[2] (-4 + x[3] + x[4])))^2) /
   (18 (x[2]^2 + 2 x[2] x[3] + x[3] (x[3] - 4 x[4]) - 4 x[1] (x[2] - x[4]))^2)}
```

```
Solve[D[redProfit4[4], x[4]] == 0, x[4]];
```

```
% /. {x[1] -> 0.13, x[2] -> .48, x[3] -> .8}
```

```
{{x[4] → 0.380729}, {x[4] → 1.43141}, {x[4] → 0.983612},
 {x[4] → 0.508724 + 0.224916 i}, {x[4] → 0.508724 - 0.224916 i}}
```

```
x4opt4firms[i_, j_, k_] = x[4] /.
   (Solve[D[redProfit4[4], x[4]] == 0, x[4]][[3]] /. {x[1] -> i, x[2] -> j, x[3] -> k});
```

```
x4opt4firms[.13, .48, .80]
```

```
0.983612
```

```
x4opt4firms[.125, .5, .875]
```

```
1.02954
```

Profit of a potential entrant entering at the right edge, but in many instances not exactly at one as the above calculations show.

```
FindRoot[((redProfit4[4] /. x[4] -> x4opt4firms[x[1], x[2], x[3]]) /. Chop[
      Flatten[{react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
          x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
            n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]},
         x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
         x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
           n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[
            n]]}]])[[1]] == 0, {n, 1650, 1700}]
```

```
{n → 2185.04}
```

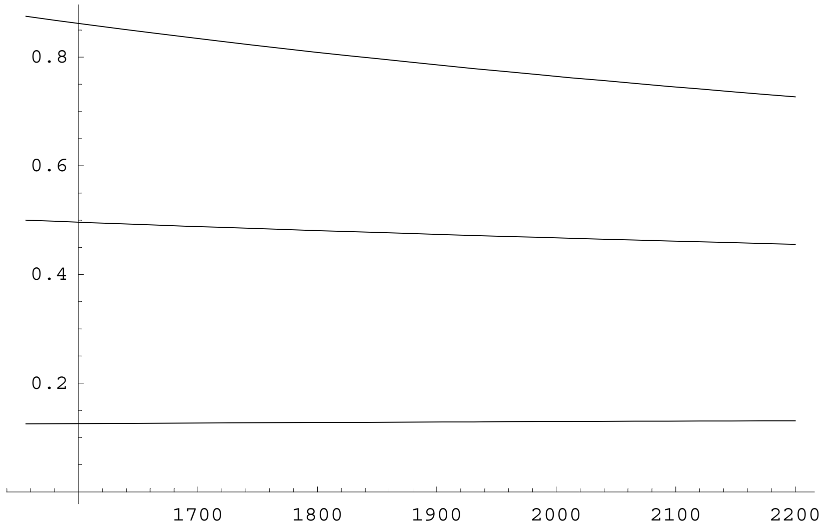
```
nOfEntryAt1MustBeDeterred = n /. %
```

```
2185.04
```

```
25 / nOfEntryAt1MustBeDeterred
```

```
0.0114415
```

```
Plot[
 {x[1] /. (react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n], x[3] ->
       xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
        xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}),
  xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
  xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
   xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}, {n, 1556, 2200}]
```




```
- Graphics -
```

```
n = 2185.0369332221053;
Chop[Flatten[{react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
     x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
       n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]},
   x[3] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
   x[4] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
     n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}]]
n =.
```

```
{x[1] → 0.13068, x[3] → 0.456401, x[4] → 0.729554}
```

```
x4opt4firms[0.13068027012917455`, 0.45640095555395416`, 0.7295540018075098`]
```

```
0.942085
```

Now I calculate the point where profits of first and second entrant are equal.

```
test2[n_] = redProfitSym[2]
test3[n_] = redProfitSym[3]
```

$$-25 - \frac{n\,(x[1]-x[2])\,(x[2]-x[3])\,(2-x[1]+x[3])^2}{18\,(x[1]-x[3])}$$

$$-25 + \left(n\,(-x[2]+x[3])\,(x[1]^2 - 2\,x[2]\,(-1+x[3]) - 3\,(-2+x[3])\,x[3] + 2\,x[1]\,(-4+x[2]+x[3]))^2\right) \Big/ \left(72\,(x[1]-x[3])^2\right)$$

```
FindRoot[
 (test2[n] /.
     {Chop[(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
           x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
             xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]})][[1]],
      x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
      x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
        xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}) - (test3[n] /.
     {Chop[(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
           x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
             xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]})][[1]],
      x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
      x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
        xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}) == 0, {n, 2100, 2200}]
```

```
{n → 2157.67}
```

```
nOfProfits1stAnd2ndEntrantEqual = n /. %
```

```
2157.67
```

Since nOfProfits1stAnd2ndEntrantEqual is smaller than the value at which the entry deterrence constraint (nOfEntryAt1MustBeDeterred) becomes binding, the first entry has an incentive to move to the edge. The switch will happen when the profit from the optimal locations at the edge are equal to the profit obtained using the locations of this section.

```
25 / 2200.
```

```
0.0113636
```

■ The equilibrium locations when three firms are active and the first entrant locates at the edge and the second in the center

■ Digression: The equilibrium locations in the case of three active firms and blockaded entry when the first entrant locates at the edge.

This section serves as a reference point. This section also provides calculations which are needed in the next section to show which constraints are binding.

I redefine the reaction functions and proceed as above in the cae where the first entrant locates in the center.

```
n = 1000;
```

The following function gives the first order constraint for the locational choice of firm 2 (the second entrant as the center firm), taking into account the reaction function of firm 1 (the third entrant). It is a function of i, the locational choice of firm 3 (Firm 3 is the first entrant locating close to 1!).

```
function1stEdge[i_] := function1stEdge[i] =
    ((D[redProfitSym[2] /. react1, x[2]]) /. {x[3] -> i, x[2] -> x2});
```

Given the location of firm 2, the function can be solved for the location of firm 3 (here x3), and also for x[1].

```
FindRoot[function1stEdge[.9] == 0, {x2, .5}]
```

$\{x2 \to 0.20043 - 2.84946 \times 10^{-16}\, i\}$

```
react1 /. {x[2] -> .2, x[3] -> .9}
```

$\{x[1] \to -0.0793378 - 5.20417 \times 10^{-17}\, i\}$

In principle, the location of the third entrant must always be calculated explicitly, because it will always locate strictly inside the market area. However, from the optimum location of firm 2, it is clear that this cannot be an equilibrium. The third entrant would rather enter between the first and the second entrant, occupying the center position. Therefore, the second entrant must locate in order to deter the third entrant from locating in the center. I define this next. The third entrant would locate according to the function xOptAt0 if it locates at the edge. The location is different from 0 for all 'sensible' locations of the first and the second entrant. This is shown by the function reactNum2a[k]. For a given location k of the first entrant (at the edge) it gives the location of the center firm such that the third entrant would locate exactly at 0 (if it were to choose the postion at the edge).

```
xOptAt0[.4, .9]
```

$0.0538928 - 7.63278 \times 10^{-17}\, i$

```
Clear[reactNum2a]
```

```
reactNum2a[i_] :=
 reactNum2a[i] = FindRoot[xOptAt0[k, i] == 0, {k, .25000, .3001}, MaxIterations -> 500]
```

```
reactNum2a[.8]
```

$\{k \to 0.288813 + 1.72877 \times 10^{-166}\, i\}$

```
xOptAt0[0.2888133041747487`, .8]
```

$1.82567 \times 10^{-6} - 5.20417 \times 10^{-17}\, i$

```
reactNum2a[.71]
```

$\{k \to 0.261109 - 4.89326 \times 10^{-18}\, i\}$

```
reactNum2a[.99]
```

```
FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 500 iterations.
```

$\{k \to 0.345668 + 1.82069 \times 10^{-166}\, i\}$

```
reactNum2a[.9]
```

```
FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 500 iterations.
```

$\{k \to 0.318978 + 1.9069 \times 10^{-166}\, i\}$

Profit of the third entrant if it locates in the center (function2 was defined above, it gives the profit of the center firm if the rivals locate at i and k, resp.)

```
function2[i, n, k]
```

$$-25 - \frac{500\,(-2 + i - k)^2\,\left(\frac{i}{2} - \frac{k}{2}\right)^2}{9\,(i - k)}$$

```
n =.
```

```
test =.
```

The function test[k] calculates the location i of the center firm (second entrant) such that the third entrant is indifferent between locating and the center and at (i.e. close to) 0, given the first entrant's location k.

```
test[k_] =
  (function2[i, n, k] - redProfitSym[1] /. {x[1] -> xOptAt0[i, k], x[2] -> i, x[3] -> k})
```

```
n = 1000;
```

```
Clear[reactNum2b]
```

```
General::spell1 : Possible spelling error: new
   symbol name "reactNum2b" is similar to existing symbol "reactNum2a".
```

```
reactNum2b[k_] := reactNum2b[k] = i /. Chop[FindRoot[test[k] == 0, {i, .3, .4}]]
```

```
reactNum2b[1.]
```

```
0.422316
```

```
reactNum2b[.7]
```

```
0.338935
```

The next calculation calculates the location i if one were to neglect that the third entrant does not locate at 0 but strictly inside the market. As can be seen there would hardly be a difference if one were to use EHM's approach.

```
Solve[(-4 i k^2 (1 + k) (2 + k) + k^3 (2 + k)^2 + i^3 (-4 + (8 - 5 k) k) - i^2 k (-8 + k^2)) == 0, i]
```

```
% /. k -> 1.
```

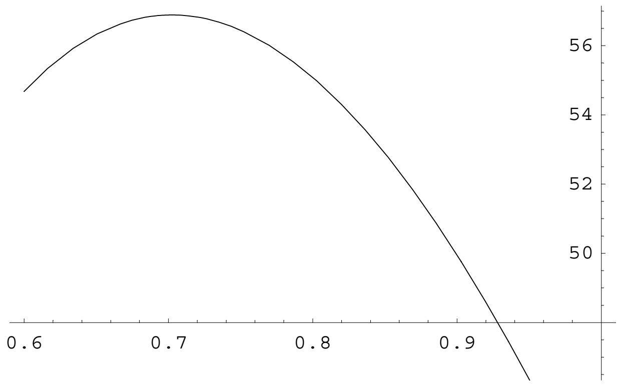
```
{{i → 0.424334}, {i → 3.28783 - 3.22488 ⅈ}, {i → 3.28783 + 3.22488 ⅈ}}
```

```
%% /. k -> .7
```

```
{{i → 0.342804}, {i → 2.92095 - 0.222302 ⅈ}, {i → 2.92095 + 0.222302 ⅈ}}
```

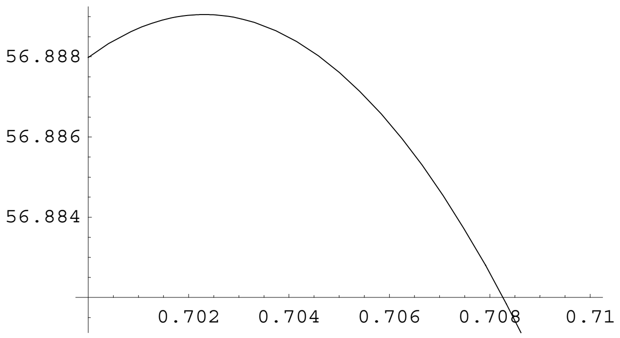
I omit the proof that the second entrant will not locate at the edge. It is straightforward, but seems obviuos when looking at the profits in Figure 5 of the main text.
Next I derive the equilibrium locations in the three firm case with blockaded entry when the first entrant locates close to one. For that purpose I plot the profit of the first entrant as a function of her location taking into account the choices of the third and the second entrant. The latter locates such that the third entrant is deterred from locating in the center.

```
Plot[(redProfitSym[3] /. x[1] -> xOptAt0[x[2], x[3]]) /.
  {x[3] -> i, x[2] -> reactNum2b[i]}, {i, .6, 1}]
```



- Graphics -

```
Plot[(redProfitSym[3] /. x[1] -> xOptAt0[x[2], x[3]]) /.
  {x[3] -> i, x[2] -> reactNum2b[i]}, {i, .7, .71}]
```



- Graphics -

```
(x[1] -> xOptAt0[x[2], x[3]]) /. {x[3] -> .702, x[2] -> reactNum2b[.702]}
```

$x[1] \to 0.0573337 - 4.85723 \times 10^{-17}\, i$

```
reactNum2b[.702]
```

0.339518

The (approximate) locations are .057, .340 and .702.

■ First entrant locates at the edge (first entrant becomes firm 3), the second entrant locates in the center (becomes firm 2).

From the unconstrained location above it is obvious that now entry deterrence between the second and the third firm as well as at 1 becomes binding. Since both firm 2 and firm 3 have an incentive to move towards 0 compared to the case in which the first entrant locates at the center, entry deterrence between firms 1 and 2 is no longer binding as will be shown below. There, I calculate the market size when this condition becomes binding explicitly.

```
n =.
```

```
n

n
```

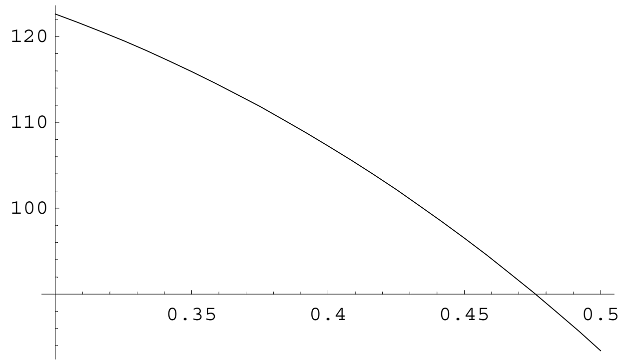
The function xOf1stAnd2ndEntrant1stAtEdge calculates the locations of firms 2 and 3 such that entry at the mentioned positions is deterred taking into account both optimal locations of the potential entrants as well as the reaction function of the third entrant. The former are strictly inside the market area as will be proofed below.

```
Clear[xOf1stAnd2ndEntrant1stAtEdge]

xOf1stAnd2ndEntrant1stAtEdge[n1_] :=
 xOf1stAnd2ndEntrant1stAtEdge[n1] = {i, k} /. (FindRoot[
     {((redProfit4[3] /. {n -> n1, x[3] -> x3opt4firms[x[1], x[2], x[4]]} /. (Chop[
              react1 /. {x[2] -> i, x[3] -> k}])) /. {x[2] -> i, x[4] -> k})[[1]],
       ((redProfit4[4] /. {n -> n1, x[4] -> x4opt4firms[x[1], x[2], x[3]]} /.
            (Chop[react1 /. {x[2] -> i, x[3] -> k}])) /.
          {x[2] -> i, x[3] -> k})[[1]]} == {0, 0}, {i, .48, .52}, {k, .85, .9}])

xOf1stAnd2ndEntrant1stAtEdge[2185]

{0.456396, 0.729552}

xOf1stAnd2ndEntrant1stAtEdge[2160]

{0.453117, 0.728464}

react1 /. {x[2] -> xOf1stAnd2ndEntrant1stAtEdge[2160][[1]],
  x[3] -> xOf1stAnd2ndEntrant1stAtEdge[2160][[2]]}

{x[1] → 0.12878 - 2.77556×10^-17 ⅈ}

n1 = 2160;
{(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1],
     x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n1,
       xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1]]})[[1]],
 x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1],
 x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
   n1, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1]]}
n1 =.

{x[1] → 0.135046 - 2.08167×10^-17 ⅈ, x[2] → 0.470001, x[3] → 0.748359}
```

The comparison of the above locations with the locations in the case with blockaded entry shows that the third entrant will not enter between the first and second entrant.

```
n1 = 2185;
{(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1],
     x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n1,
       xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1]]})[[1]],
 x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1],
 x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
   n1, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1]]}
n1 =.

{x[1] → 0.13068 - 2.08167×10^-17 ⅈ, x[2] → 0.456403, x[3] → 0.729561}
```

```
n = 2160;
Plot[(redProfitSym[2] /. x[1] -> xOptAt0[x[2], x[3]]) /.
  {x[3] -> 0.7284640930669652`, x[2] -> i}, {i, .3, .5}]
n =.
```

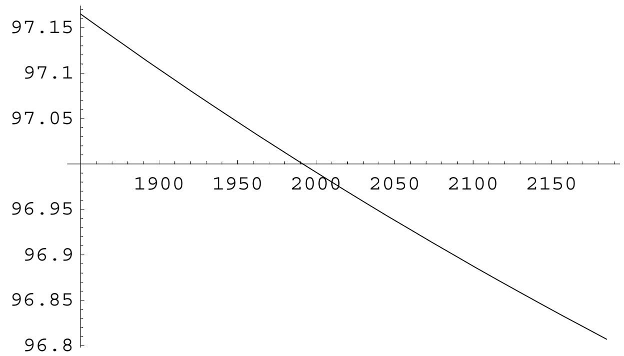


- Graphics -

The above figure shows the profit of the second entrant given that the first locates at .72846 and the third according to her reaction function. The second entrant would choose the maximum distance from the first. Therefore the entry deterrence condition is clearly binding.

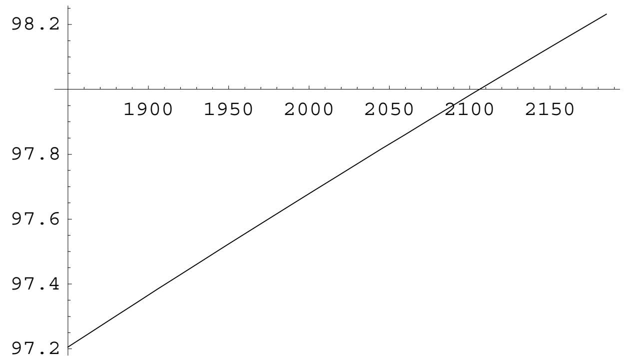
Next I calculate the market size at which the the first entrant switches to the edge. The next to figures show the profit of the first entrant if it locates at the center and at the edge, resp (taking into account the equilibrium locations).

```
Plot[
 redProfitSym[2] /.
  {(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
       x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
         xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]})[[1]],
   x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
   x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
     n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]},
 {n, 1850, nOfEntryAt1MustBeDeterred}]
```



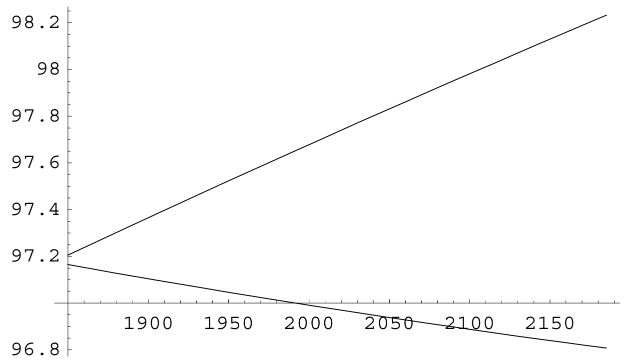
- Graphics -

```
Plot[
 redProfitSym[3] /. {(react1 /. {x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
      x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n][[2]]}) [[1]],
   x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
   x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n][[2]]},
 {n, 1850, nOfEntryAt1MustBeDeterred}]
```



- Graphics -

```
Show[%, %%]
```



- Graphics -

Now Icalculate the market size with the switch, nOf1stEntrantSwitchesToEdge is the result.

```
test2[n_] = redProfitSym[2]
test3[n_] = redProfitSym[3]
```

$$-25 - \frac{n\,(x[1]-x[2])\,(x[2]-x[3])\,(2-x[1]+x[3])^2}{18\,(x[1]-x[3])}$$

$$-25 + \left(n\,(-x[2]+x[3])\,(x[1]^2 - 2\,x[2]\,(-1+x[3]) - 3\,(-2+x[3])\,x[3] + 2\,x[1]\,(-4+x[2]+x[3]))^2\right) \Big/ (72\,(x[1]-x[3])^2)$$

```
FindRoot[
 (test2[n] /.
    {Chop[(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
          x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
            xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]})][[1]],
     x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
     x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
       xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}) - (
   test3[n] /. {Chop[(react1 /. {x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
          x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n][[2]]})][[1]],
     x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]], x[3] ->
      xOf1stAnd2ndEntrant1stAtEdge[n][[2]]}) == 0, {n, 1700, 1850}]
```

```
{n → 1840.94}
```

```
nOf1stEntrantSwitchesToEdge = n /. %
```

```
1840.94
```

```
25 / %
```

```
0.01358
```

```
xOf1stAnd2ndEntrant1stAtEdge[nOf1stEntrantSwitchesToEdge]
```

```
{0.405957, 0.712948}
```

```
react1 /. {x[2] -> xOf1stAnd2ndEntrant1stAtEdge[nOf1stEntrantSwitchesToEdge][[1]],
  x[3] -> xOf1stAnd2ndEntrant1stAtEdge[nOf1stEntrantSwitchesToEdge][[2]]}
```

```
{x[1] → 0.100863 - 2.08167×10^-17 i}
```

```
prices3Firms /.
 {x[1] → 0.100863, x[2] -> 0.40595689871291885`, x[3] -> 0.7129476716869781`}
```

```
{{p[1] → 0.143931, p[2] → 0.133233, p[3] → 0.201861}}
```

Price of the first entrant is more than 50% higher than that of the second entrant.

```
0.2018608182309887` / 0.1332334694651309`
```

```
1.51509
```

The consumer indifferent between buying at the first and the second entrant.

```
a[2, 3] /. {x[1] → 0.100863, x[2] -> 0.40595689871291885`, x[3] -> 0.7129476716869781,
  p[1] → 0.143930564154371`, p[2] → 0.1332334694651309`, p[3] → 0.2018608182309887`}
```

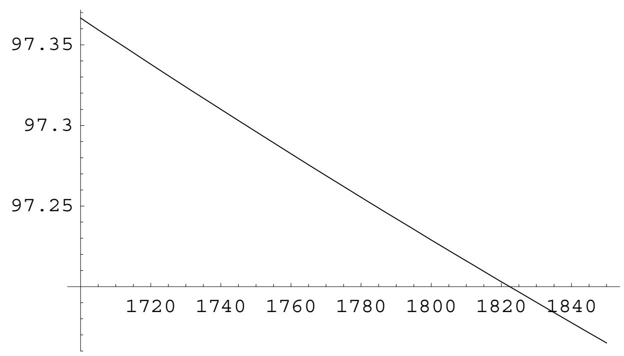
```
0.671227
```

```
n1 = nOf1stEntrantSwitchesToEdge;
{(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1],
      x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n1,
        xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1]]}) [[1]],
 x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1],
 x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
   n1, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n1]]}
n1 =.
```

{x[1] → 0.128148 − 2.08167×10^{-17} i, x[2] → 0.477929, x[3] → 0.799146}

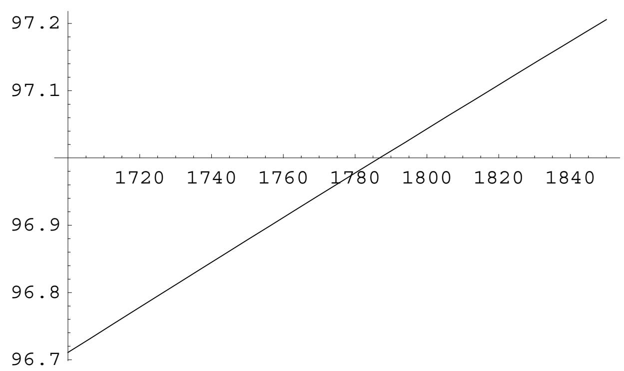
The above locations are those just before the switch takes place.

```
Plot[
 redProfitSym[2] /.
  {(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
        x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
          xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}) [[1]],
   x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
   x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
     xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}, {n, 1700, 1850}]
```



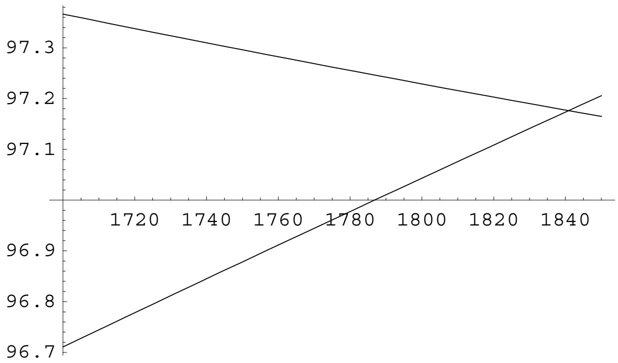
- Graphics -

```
Plot[
 redProfitSym[3] /. {(react1 /. {x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n] [[1]],
        x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n] [[2]]}) [[1]],
   x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n] [[1]],
   x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n] [[2]]}, {n, 1700, 1850}]
```



- Graphics -

```
Show[%, %%]
```




```
- Graphics -
```

■ All incumbents must deter

Next I calculate when the entry deterring constraint between the second and third entrant becomes binding. The entrant would be firm 2.

```
FindRoot[((redProfit4[2] /. x[2] -> x2opt4firms[x[1], x[3], x[4]]) /.
     Chop[Flatten[{(react1 /. {x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
            x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n][[2]]})[[1]],
        x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
        x[4] -> xOf1stAnd2ndEntrant1stAtEdge[n][[2]]}]])[[1]] == 0, {n, 2100, 2300.}]

{n → 2185.04}
```

Identical to the case in which the first entrant locates in the center (the resp. value was nOfEntryAt1MustBeDeterred)! Must be the case! Since this is just the case when entry must be deterred by all incumbents.

```
nOfAllIncumbentsDeter = n /. %

2185.04

xOfAllIncumbentsDeter[n1_] := xOfAllIncumbentsDeter[n1] = {j, i, k} /. (FindRoot[
     {(Evaluate[redProfit4[2] /. {n -> n1, x[2] -> x2opt4firms[x[1], x[3], x[4]]}] /.
          {x[1] -> j, x[3] -> i, x[4] -> k})[[1]] == 0,
      (Evaluate[redProfit4[3] /. {n -> n1, x[3] -> x3opt4firms[x[1], x[2], x[4]]}] /.
          {x[1] -> j, x[2] -> i, x[4] -> k})[[1]] == 0,
      (Evaluate[redProfit4[4] /. {n -> n1, x[4] -> x4opt4firms[x[1], x[2], x[3]]}] /.
          {x[1] -> j, x[2] -> i, x[3] -> k})[[1]] == 0},
     {j, .1, .2}, {i, .48, .52}, {k, .85, .9}])

General::spell1 : Possible spelling error: new symbol
   name "xOfAllIncumbentsDeter" is similar to existing symbol "nOfAllIncumbentsDeter".

xOfAllIncumbentsDeter[2200]

{0.13427, 0.457643, 0.730112}

xOfAllIncumbentsDeter[2185.0369332221053`]

{0.13068, 0.456401, 0.729554}
```

Algorithm yields the right locations. The locations are continuous in market size.

```
xOfAllIncumbentsDeter[2500]
```

```
{0.198307, 0.480399, 0.74034}
```

Next I calculate the value of N at which first and second entrant have the same profit in the all incumbents deter scenario.

```
FindRoot[(test2[n] /. {x[1] ->
       xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
      x[3] -> xOfAllIncumbentsDeter[n][[3]]}) - (test3[n] /. {x[1] ->
       xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
      x[3] -> xOfAllIncumbentsDeter[n][[3]]}) == 0, {n, 2300, 2400}]
```

```
{n → 2420.91}
```

```
nOf1stEntrantSwitchesBackToCenter = n /. %
```

```
2420.91
```

The first entrant switches back to the center position at this market size.

Value of market size at which entry of a forth entrant can no longer be deterred according to Neven and EHM.

```
25 / .009
```

```
2777.78
```

```
xOfAllIncumbentsDeter[2777.777777777778`]
```

```
{0.246718, 0.498427, 0.748448}
```

```
1 - %
```

```
{0.753282, 0.501573, 0.251552}
```

```
n
n1
```

```
n
```

```
n1
```

In the following I calculate the market size at which entry of a forth entrant can no longer be deterred. First, I derive the optimal choice of the forth entrant which locates at 0. The respective function is x1opt4firms. This value must be calculated since it is strictly greater than 0.

```
redProfit4[1]
```

$$\left\{-25 - \left(n\,(x[1]-x[2])\,\left(-3\,x[2]^2\,x[3] + x[1]\,(x[2]-x[3])\,(3\,x[2]+x[3]-4\,x[4]) + 6\,x[1]^2\,(x[2]-x[4]) + x[3]\,(2\,x[3]\,(-1+x[4]) + x[4]\,(2+x[4])) - x[2]\,(3\,x[3]^2 + x[4]\,(2+x[4]) - 2\,x[3]\,(1+2\,x[4]))\right)^2\right) \Big/ \left(18\,\left(x[2]^2 + 2\,x[2]\,x[3] + x[3]\,(x[3]-4\,x[4]) - 4\,x[1]\,(x[2]-x[4])\right)^2\right)\right\}$$

```
Solve[D[redProfit4[1], x[1]] == 0, x[1]];
```

```
% /. {x[2] -> 0.23, x[3] -> .48, x[4] -> .76}
```

```
{{x[1] → 0.559878}, {x[1] → -0.412866}, {x[1] → 0.453952 - 0.18505 ⅈ},
 {x[1] → 0.0452992 - 1.21431×10^-17 ⅈ}, {x[1] → 0.453952 + 0.18505 ⅈ}}
```

```
x1opt4firms[i_, j_, k_] = x[1] /.
   (Solve[D[redProfit4[1], x[1]] == 0, x[1]][[4]] /. {x[2] -> i, x[3] -> j, x[4] -> k});
```

```
x1opt4firms[.23, .48, .76]
```

```
0.0452992 - 1.21431×10^-17 ⅈ
```

```
1 - x4opt4firms[1 - x[3], 1 - x[2], 1 - x[1]] /. {x[1] -> 0.23, x[2] -> .48, x[3] -> .76}
```

```
0.0452992
```

Above: Consistency check.

```
n = 2777.8;
```

```
x1opt4firms[x[2], x[3], x[4]] /. {x[2] ->
   xOfAllIncumbentsDeter[n][[1]],
  x[3] -> xOfAllIncumbentsDeter[n][[2]], x[4] -> xOfAllIncumbentsDeter[n][[3]]}
```

```
0.052969 - 3.81639×10^-17 ⅈ
```

The profit of the potential entrant at 0.

```
((redProfit4[1] /. x[1] -> x1opt4firms[x[2], x[3], x[4]]) /. {x[2] ->
     xOfAllIncumbentsDeter[n][[1]], x[3] -> xOfAllIncumbentsDeter[n][[2]],
    x[4] -> xOfAllIncumbentsDeter[n][[3]]})[[1]]
```

```
-0.821295 + 3.38717×10^-29 ⅈ
```

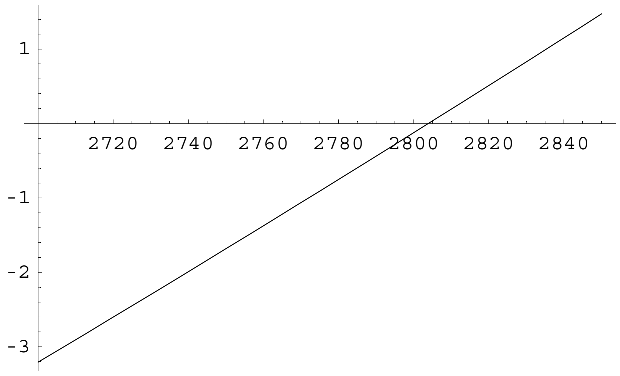
```
n = 2850;
```

```
((redProfit4[1] /. x[1] -> x1opt4firms[x[2], x[3], x[4]]) /. {x[2] ->
     xOfAllIncumbentsDeter[n][[1]], x[3] -> xOfAllIncumbentsDeter[n][[2]],
    x[4] -> xOfAllIncumbentsDeter[n][[3]]})[[1]]
```

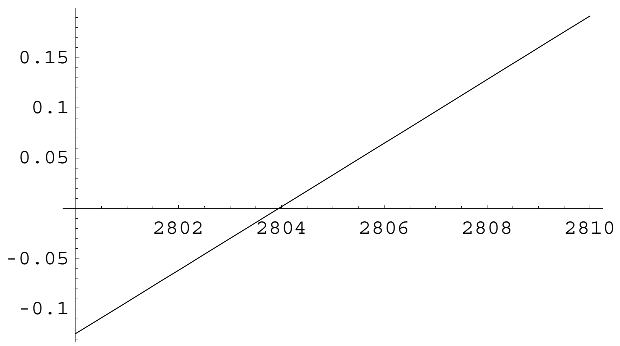
```
1.47013 - 5.01173×10^-29 ⅈ
```

```
n =.
```

```
Plot[((redProfit4[1] /. x[1] -> x1opt4firms[x[2], x[3], x[4]]) /. {x[2] ->
      xOfAllIncumbentsDeter[n][[1]], x[3] -> xOfAllIncumbentsDeter[n][[2]],
     x[4] -> xOfAllIncumbentsDeter[n][[3]]})[[1]], {n, 2700, 2850}]
```




```
- Graphics -

25 / 2805.

0.00891266

xOfAllIncumbentsDeter[2805]

{0.251009, 0.500063, 0.749185}

Plot[((redProfit4[1] /. x[1] -> x1opt4firms[x[2], x[3], x[4]]) /. {x[2] ->
      xOfAllIncumbentsDeter[n][[1]], x[3] -> xOfAllIncumbentsDeter[n][[2]],
     x[4] -> xOfAllIncumbentsDeter[n][[3]]})[[1]], {n, 2800, 2810}]
```




```
- Graphics -

xOfAllIncumbentsDeter[2804]

{0.250852, 0.500003, 0.749158}
```

The following three commands show that the value cannot easily be derived by employing the built-in functions of *Mathematica*. I therefore use a rather crude method and calcualte the profits for values market sizes. From the above diagram it is clear that the respective market size must be close to 2804.

```
FindRoot[(Evaluate[redProfit4[1] /. x[1] -> x1opt4firms[x[2], x[3], x[4]]] /. {x[2] ->
       xOfAllIncumbentsDeter[n][[1]], x[3] -> xOfAllIncumbentsDeter[n][[2]],
      x[4] -> xOfAllIncumbentsDeter[n][[3]]})[[1]] ==
   0, {n, 2750, 2820}, AccuracyGoal -> 7]
```

```
FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 15 iterations.

FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 15 iterations.

FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 15 iterations.

General::stop : Further output of FindRoot::frsec will be suppressed during this calculation.

$Aborted
```

```
FindRoot[(Evaluate[
      redProfit4[1] /. {n -> n1, x[1] -> x1opt4firms[x[2], x[3], x[4]]}] /. {x[2] ->
       xOfAllIncumbentsDeter[n1][[1]], x[3] -> xOfAllIncumbentsDeter[n1][[2]],
      x[4] -> xOfAllIncumbentsDeter[n1][[3]]})[[1]] == 0,
 {n1, 2790, 2810}, AccuracyGoal -> 10]
```

```
FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 15 iterations.

FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 15 iterations.

FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 15 iterations.

General::stop : Further output of FindRoot::frsec will be suppressed during this calculation.

$Aborted
```

```
FindRoot[((redProfit4[4] /. {x[4] -> x4opt4firms[x[1], x[2], x[3]]}) /. {x[1] ->
       1 - xOfAllIncumbentsDeter[n][[1]], x[2] -> 1 - xOfAllIncumbentsDeter[n][[2]],
      x[3] -> 1 - xOfAllIncumbentsDeter[n][[3]]})[[1]] == 0, {n, 2750, 2850}]
```

```
FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 15 iterations.

FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 15 iterations.

FindRoot::frsec :
 Secant method failed to converge to the prescribed accuracy after 15 iterations.

General::stop : Further output of FindRoot::frsec will be suppressed during this calculation.

{n → 1039.22}
```

```
Table[{n, ((redProfit4[1] /. x[1] -> x1opt4firms[x[2], x[3], x[4]]) /. {x[2] ->
       xOfAllIncumbentsDeter[n][[1]], x[3] -> xOfAllIncumbentsDeter[n][[2]],
      x[4] -> xOfAllIncumbentsDeter[n][[3]]})[[1]]}, {n, 2803.5, 2804.2, .1}]
```

$\{\{2803.5, -0.0140771 - 2.83497\times10^{-31}\,i\},$
$\{2803.6, -0.0109178 - 1.21041\times10^{-29}\,i\}, \{2803.7, -0.00775834 + 2.45286\times10^{-30}\,i\},$
$\{2803.8, -0.00459875 + 4.71591\times10^{-29}\,i\}, \{2803.9, -0.00143901 - 1.77001\times10^{-29}\,i\},$
$\{2804., 0.00172087 - 7.1244\times10^{-30}\,i\}, \{2804.1, 0.00488089 + 1.03661\times10^{-29}\,i\}\}$

```
Table[{n, ((redProfit4[1] /. x[1] -> x1opt4firms[x[2], x[3], x[4]]) /. {x[2] ->
       xOfAllIncumbentsDeter[n][[1]], x[3] -> xOfAllIncumbentsDeter[n][[2]],
      x[4] -> xOfAllIncumbentsDeter[n][[3]]})[[1]]}, {n, 2803.9, 2804., .01}]
```

{{2803.9, −0.00143901 − 1.77001 × 10^{-29} ⅈ}, {2803.91, −0.00112303 + 2.75608 × 10^{-29} ⅈ},
{2803.92, −0.000807044 − 1.74042 × 10^{-29} ⅈ}, {2803.93, −0.000491059 − 2.7191 × 10^{-29} ⅈ},
{2803.94, −0.000175074 + 2.46026 × 10^{-29} ⅈ}, {2803.95, 0.000140914 − 5.26565 × 10^{-29} ⅈ},
{2803.96, 0.000456902 − 2.67966 × 10^{-29} ⅈ}, {2803.97, 0.000772892 + 2.76348 × 10^{-29} ⅈ},
{2803.98, 0.00108888 + 2.32467 × 10^{-29} ⅈ}, {2803.99, 0.00140488 + 6.87788 × 10^{-30} ⅈ}}

```
Table[{n, ((redProfit4[1] /. x[1] -> x1opt4firms[x[2], x[3], x[4]]) /. {x[2] ->
       xOfAllIncumbentsDeter[n][[1]], x[3] -> xOfAllIncumbentsDeter[n][[2]],
      x[4] -> xOfAllIncumbentsDeter[n][[3]]})[[1]]}, {n, 2803.943, 2803.947, .001}]
```

{{2803.94, −0.0000802775 + 1.35585 × 10^{-29} ⅈ}, {2803.94, −0.0000486788 − 6.48345 × 10^{-30} ⅈ},
{2803.95, −0.0000170801 − 6.55741 × 10^{-30} ⅈ}, {2803.95, 0.0000145186 − 2.3863 × 10^{-29} ⅈ}}

```
Table[{n, ((redProfit4[1] /. x[1] -> x1opt4firms[x[2], x[3], x[4]]) /. {x[2] ->
       xOfAllIncumbentsDeter[n][[1]], x[3] -> xOfAllIncumbentsDeter[n][[2]],
      x[4] -> xOfAllIncumbentsDeter[n][[3]]})[[1]]},
  {n, 2803 + 9454 / 10000, 2803 + 9457 / 10000, 1 / 10000}]
```

{{$\frac{14019727}{5000}$, −4.44061 × 10^{-6} − 2.33947 × 10^{-29} ⅈ},
{$\frac{5607891}{2000}$, −1.28073 × 10^{-6} + 1.51609 × 10^{-30} ⅈ},
{$\frac{1752466}{625}$, 1.87914 × 10^{-6} − 2.45656 × 10^{-29} ⅈ},
{$\frac{28039457}{10000}$, 5.03901 × 10^{-6} + 3.52522 × 10^{-30} ⅈ}}

```
N[%, 15]
```

{{2803.95, −4.44061 × 10^{-6} − 2.33947 × 10^{-29} ⅈ}, {2803.95, −1.28073 × 10^{-6} + 1.51609 × 10^{-30} ⅈ},
{2803.95, 1.87914 × 10^{-6} − 2.45656 × 10^{-29} ⅈ}, {2803.95, 5.03901 × 10^{-6} + 3.52522 × 10^{-30} ⅈ}}

```
Precision[%]
```

16

```
InputForm[%%]
```

```
{{2803.9454, -4.440606740985231*^-6 -
   2.339465622046063*^-29*I},
 {2803.9455, -1.2807341249754245*^-6 +
   1.516092052221632*^-30*I},
 {2803.9456, 1.8791384768235275*^-6 -
   2.456562162664807*^-29*I},
 {2803.9457, 5.03901135928686*^-6 +
   3.5252221702063965*^-30*I}}
```

```
Table[{n, ((redProfit4[1] /. x[1] -> x1opt4firms[x[2], x[3], x[4]]) /. {x[2] ->
       xOfAllIncumbentsDeter[n][[1]], x[3] -> xOfAllIncumbentsDeter[n][[2]],
      x[4] -> xOfAllIncumbentsDeter[n][[3]]})[[1]]},
  {n, 2803 + 94553 / 100000, 2803 + 94555 / 100000, 1 / 100000}]
```

{{$\frac{280394553}{100000}$, −3.32772 × 10^{-7} + 2.88427 × 10^{-30} ⅈ},
{$\frac{140197277}{50000}$, −1.6785 × 10^{-8} + 5.42342 × 10^{-30} ⅈ},
{$\frac{56078911}{20000}$, 2.99202 × 10^{-7} − 4.16617 × 10^{-30} ⅈ}}

```
InputForm[N[%]]
```

```
{{2803.94553, -3.3277234834372393*^-7 +
   2.8842726847143244*^-30*I},
 {2803.94554, -1.6785016043741052*^-8 +
   5.423418723394456*^-30*I},
 {2803.94555, 2.9920219191126307*^-7 -
   4.1661716556984686*^-30*I}}
```

My approximation shows that at the below market size the profit is of size 10^{-8}, a sufficiently good approximation.

```
nOfEntryOf4thEntrantCannotBeDeterred = 2803.94554
```

```
2803.95
```

```
xOfAllIncumbentsDeter[2803.94554]
```

```
{0.250844, 0.5, 0.749156}
```

```
1 - %
```

```
{0.749156, 0.5, 0.250844}
```

```
25 / 2803.94554
```

```
0.00891601
```

```
prices3Firms /. {x[1] → 0.2508437430875839`, x[2] -> 0.5, x[3] -> 0.7491562568303687`}
```

```
{{p[1] → 0.145411, p[2] → 0.103745, p[3] → 0.145411}}
```

```
n
```

```
n
```

The derivation of the figures of the paper can be found at the end of this file.

■ Four active firms: Check of the equilibria derived by EHM and Neven.

Check whether the equilibrium values of EHM and Neven constitute an equilibrium.
Prices

```
equLocations4FirmsEconomides = {x[1] -> .09, x[2] -> .43, x[3] -> .71, x[4] -> .94}
```

```
{x[1] → 0.09, x[2] → 0.43, x[3] → 0.71, x[4] → 0.94}
```

```
equLocations4FirmsNeven = {x[1] -> .067, x[2] -> .295, x[3] -> .575, x[4] -> .876}
```

```
{x[1] → 0.067, x[2] → 0.295, x[3] → 0.575, x[4] → 0.876}
```

```
prices4Firms /. {x[1] -> .09, x[2] -> .43, x[3] -> .71, x[4] -> .94}
```

```
{{p[1] → 0.138255, p[2] → 0.0997096, p[3] → 0.07619, p[4] → 0.078345}}
```

```
prices4Firms /. {x[1] -> .067, x[2] -> .295, x[3] -> .575, x[4] -> .876}
```

```
{{p[1] → 0.0784016, p[2] → 0.0742673, p[3] → 0.0924227, p[4] → 0.128836}}
```

```
a[4, 5] =.
```

```
a[5, 6] = 1
```

1

```
foc[4]
```

$$n\,p[4]\left(\frac{1}{2\,(x[3]-x[4])}-\frac{1}{2\,(-x[4]+x[5])}\right)+ n\left(\frac{1}{2}\,(-x[3]-x[4])-\frac{p[3]-p[4]}{2\,(x[3]-x[4])}+\frac{-p[4]+p[5]}{2\,(-x[4]+x[5])}+\frac{1}{2}\,(x[4]+x[5])\right)$$

```
foc[5]
```

$$n\left(1+\frac{1}{2}\,(-x[4]-x[5])-\frac{p[4]-p[5]}{2\,(x[4]-x[5])}\right)+\frac{n\,p[5]}{2\,(x[4]-x[5])}$$

```
prices5Firms = Solve[{foc[1] == 0, foc[2] == 0, foc[3] == 0, foc[4] == 0, foc[5] == 0},
    {p[1], p[2], p[3], p[4], p[5]}] // FullSimplify
```

```
(prices5Firms /.
    {x[1] -> .051, x[2] -> .245, x[3] -> .51, x[4] -> .765, x[5] -> .955})[[1]] // Sort
```

$\{p[1]\to 0.0560474,\ p[2]\to 0.0546709,\ p[3]\to 0.0605056,\ p[4]\to 0.0522483,\ p[5]\to 0.0527241\}$

```
(prices5Firms /. {x[1] -> .04, x[2] -> .24, x[3] -> .5, x[4] -> .76, x[5] -> .96})[[1]] //
 Sort
```

$\{p[1]\to 0.0555072,\ p[2]\to 0.0550145,\ p[3]\to 0.0613072,\ p[4]\to 0.0550145,\ p[5]\to 0.0555072\}$

```
profit[i] /. prices5Firms
```

$$\left\{-25+n\,p[i]\left(\frac{1}{2}\,(-x[-1+i]-x[i])-\frac{p[-1+i]-p[i]}{2\,(x[-1+i]-x[i])}+\frac{-p[i]+p[1+i]}{2\,(-x[i]+x[1+i])}+\frac{1}{2}\,(x[i]+x[1+i])\right)\right\}$$

```
redProfit5[i_] := redProfit5[i] = Simplify[profit[i] /. prices5Firms][[1]]
```

The reduced profit function of the second firm (fifth entrant) in the 5 firms case.

```
redProfit5[2]
redProfit5[3]
redProfit5[4]
```

■ EHM

Entry would probably take place between the first and the second firm, which are the second and first entrant, resp. according to EHM (p.17)

```
equLocations4FirmsEconomides
```

$\{x[1]\to 0.09,\ x[2]\to 0.43,\ x[3]\to 0.71,\ x[4]\to 0.94\}$

```
redProfit5[2] /. {x[1] → 0.09`, x[3] → 0.43`, x[4] → 0.71`, x[5] → 0.94`}
```

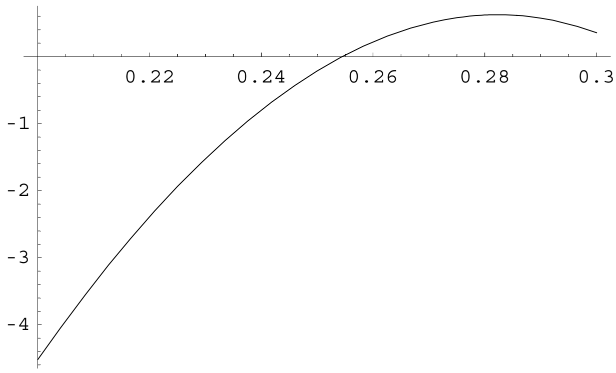
$$-25 - (n\,(0.09 - x[2])\,(-0.43 + x[2])\,(-1.42772 + 0.6579\,x[2] + 1.53\,x[2]^2 - 0.09\,(-1.1507 + 1.53\,x[2]))\,(-0.0081\,(-1.1507 + 1.53\,x[2]) + 0.43\,(1.42772 - 0.6579\,x[2] - 1.53\,x[2]^2) + 0.09\,(-1.92252 + 1.3158\,x[2] + 1.53\,x[2]^2)))\Big/\left(18\,(-0.494801 + 0.09\,(1.37 - 2.04\,x[2]) + 0.4386\,x[2] + 0.51\,x[2]^2)^2\right)$$

EHM claim, contrary to Neven, that a range in which entry is blockaded exists in the case of four active firms. The respective interval is f=[.0085,.0089]. Note that n is 1 in EHM. Since I assumed that fixed costs are 25, I increase n proportionally in order to check whether the entrant could break even. First, I take an intermediate value from the interval.

```
n = 25 / .0087
```

2873.56

```
Plot[redProfit5[2] /. {x[1] → 0.09`, x[3] → 0.43`, x[4] → 0.71`, x[5] → 0.94`},
 {x[2], .2, .3}]
```



- Graphics -

```
(redProfit5[2] /. {x[1] → 0.09`, x[3] → 0.43`, x[4] → 0.71`, x[5] → 0.94`}) /. x[2] -> .28
```

0.618568

Profit is clearly positive. Therefore entry is not blockaded. To derive profit with market size 1 I devide by the value of n I used.

```
% / n
```

0.000215262

```
% / .0087
```

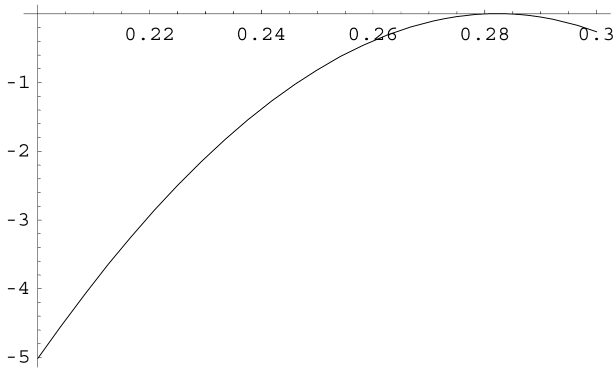
0.0247427

Pure profit amounts to 2.5 % of fixed costs. Next I take the value for which entry of a third entrant can no longer be deterred from my simulation.

```
n = 2803.94554
25 / n
Plot[redProfit5[2] /. {x[1] → 0.09`, x[3] → 0.43`, x[4] → 0.71`, x[5] → 0.94`},
 {x[2], .2, .3}]
```

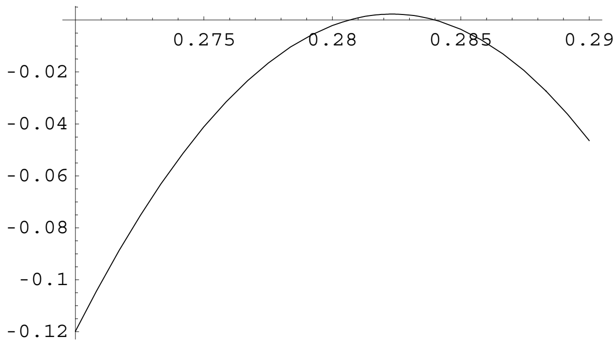
2803.95

0.00891601



- Graphics -

```
Plot[redProfit5[2] /. {x[1] → 0.09`, x[3] → 0.43`, x[4] → 0.71`, x[5] → 0.94`},
 {x[2], .27, .29}]
```



- Graphics -

```
(redProfit5[2] /. {x[1] → 0.09`, x[3] → 0.43`, x[4] → 0.71`, x[5] → 0.94`}) /. x[2] -> .283
```

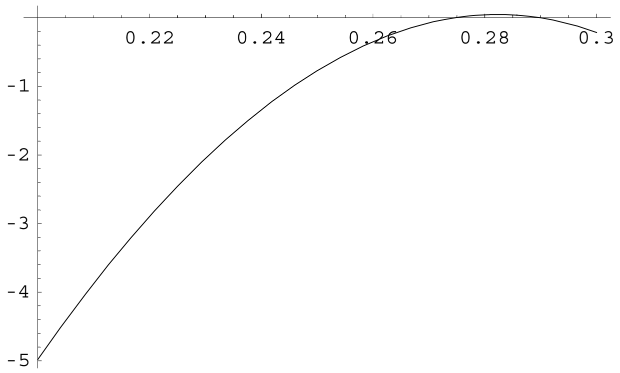
0.00194058

Entry must always be deterred in the case of for active firms!

```
n = 25 / .0089
```

2808.99

```
Plot[redProfit5[2] /. {x[1] → 0.09`, x[3] → 0.43`, x[4] → 0.71`, x[5] → 0.94`},
 {x[2], .2, .3}]
```




```
- Graphics -
```

```
(redProfit5[2] /. {x[1] → 0.09`, x[3] → 0.43`, x[4] → 0.71`, x[5] → 0.94`}) /. x[2] -> .28
```

```
0.0428696
```

■ Neven

Next I check Neven's locations(see Table 2, p.429). These locations are supposed to be equilibrium locations for f=0.007. The claim is that these locations deter entry. Here entry seems to be most profitable between firms 3 and 4. The fifth entrant is therefore firm 4 (in the 4 firm-case).

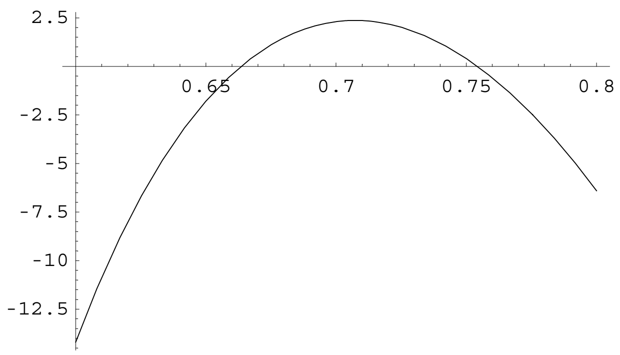
```
n = 25 / .007
```

```
3571.43
```

```
equLocations4FirmsNeven
```

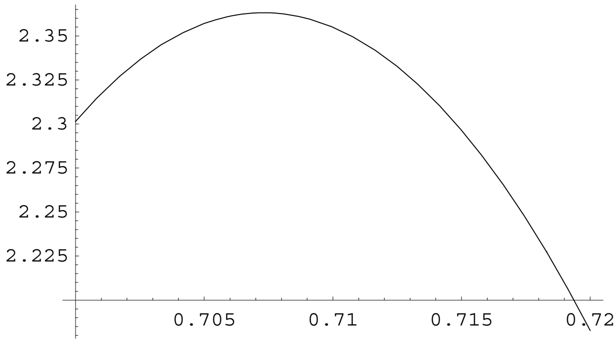
```
{x[1] → 0.067, x[2] → 0.295, x[3] → 0.575, x[4] → 0.876}
```

```
Plot[redProfit5[4] /. {x[1] → 0.067`, x[2] → 0.295`, x[3] → 0.575`, x[5] → 0.876`},
 {x[4], .6, .8}]
```




```
- Graphics -
```

```
Plot[redProfit5[4] /. {x[1] → 0.067`, x[2] → 0.295`, x[3] → 0.575`, x[5] → 0.876`},
 {x[4], .7, .72}]
```




```
- Graphics -
```

```
redProfit5[4] /. {x[1] → 0.067`, x[2] → 0.295`, x[3] → 0.575`, x[5] → 0.876`} /.
 x[4] -> .707
```

```
2.36303
```

```
% / n
```

```
0.000661649
```

```
% / .007
```

```
0.0945213
```

The entrant would earn pure profits of nearly 10% of her fixed costs.

To sum up the four firms case discussion in both Neven and Economides et al. The claimed equilibrium locations do not constitute an equilibrium. A fifth entrant could break even.

■ The derivation of Figures 4-6

Threshold values of n

```
In[493]:= n

Out[493]= n

In[494]:= nOfEntryBlockade3Firms

Out[494]= 1136.86

In[495]:= nOf2ndEntrantLowerProfitThan3rd

Out[495]= 1329.69

In[496]:= nOffirstEntrantDeterAtonehalf

Out[496]= 1359.64
```

In[497]:= **nOfEntryBetween1stAnd3rdEntrandMustBeDeterred**

Out[497]= 1556.19

In[498]:= **nOf1stEntrantSwitchesToEdge**

Out[498]= 1840.94

In[499]:= **nOfEntryAt1MustBeDeterred**

Out[499]= 2185.04

In[500]:= **nOfAllIncumbentsDeter**

Out[500]= 2185.04

In[501]:= **nOfAllIncumbentsDeter - nOfEntryAt1MustBeDeterred**

Out[501]= 4.91697×10^{-7}

In[502]:= **nOf1stEntrantSwitchesBackToCenter**

Out[502]= 2420.91

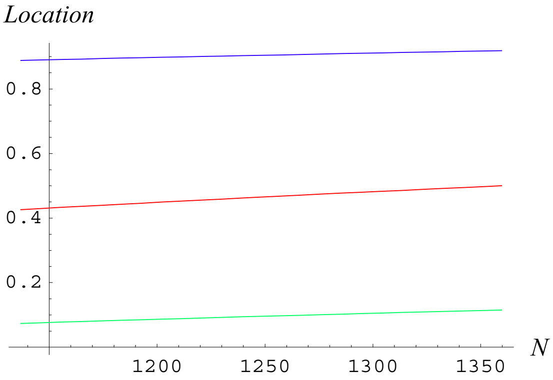
In[503]:= **nOfEntryOf4thEntrantCannotBeDeterred**

Out[503]= 2803.95

■ Locations

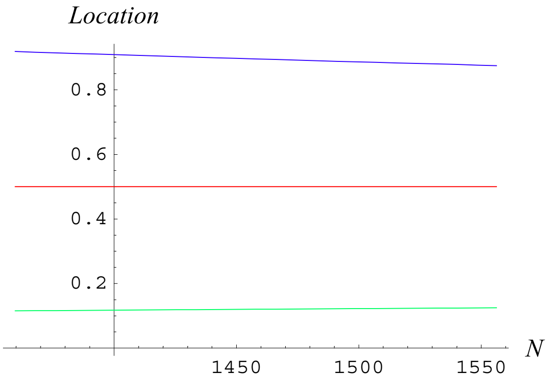
In the following figures red is used for the values of the first entrant, blue for the second and green for the third entrant.

In[504]:=
```
locations3Firms1 = Plot[{x[1] /.
    ((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
       {x[2] -> x1stEntrantDeters4thEntrant[n1]}]), x1stEntrantDeters4thEntrant[
    n1], (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])},
  {n1, nOfEntryBlockade3Firms, nOffirstEntrantDeterAtonehalf},
  PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
 {FontForm["N", {"Times-Italic", 12}], FontForm["Location",
 {"Times-Italic", 12}]}]
```

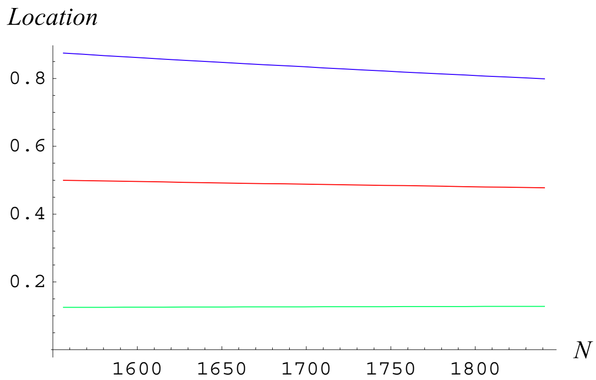


Out[504]= - Graphics -

```
In[505]:= locations3Firms2 = Plot[
    {x[1] /. (react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
         , x[2] -> .5}), .5, xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]},
    {n, nOffirstEntrantDeterAtonehalf,
  nOfEntryBetween1stAnd3rdEntrandMustBeDeterred},
    PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
  {FontForm["N", {"Times-Italic", 12}], FontForm["Location",
  {"Times-Italic", 12}]}]
```

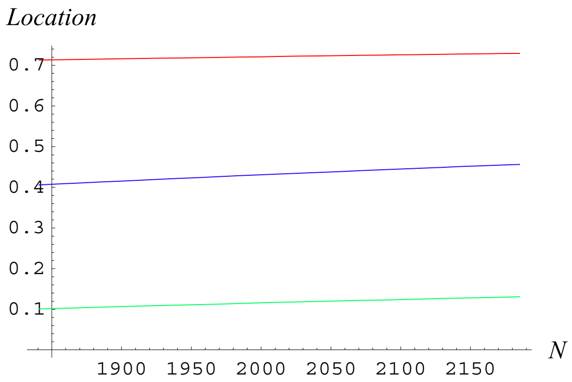



```
Out[505]= - Graphics -

In[506]:= locations3Firms3 =
   Plot[{x[1] /. (react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
         x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
           n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}),
     xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
     xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
      xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]},
    {n, nOfEntryBetween1stAnd3rdEntrandMustBeDeterred,
     nOf1stEntrantSwitchesToEdge},
    PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]},
    AxesLabel ->
  {FontForm["N", {"Times-Italic", 12}], FontForm["Location",
  {"Times-Italic", 12}]}]
```

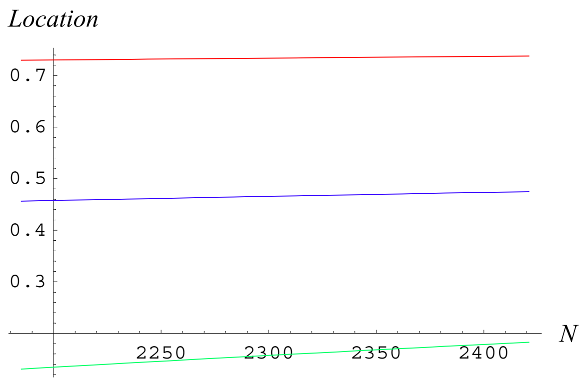



```
Out[506]= - Graphics -
```

```
In[507]:= locations3Firms3a = Plot[{
             xOptAt0[xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
              xOf1stAnd2ndEntrant1stAtEdge[n][[2]]],
             xOf1stAnd2ndEntrant1stAtEdge[n][[1]], xOf1stAnd2ndEntrant1stAtEdge[n][[2]]},
            {n, nOf1stEntrantSwitchesToEdge, nOfEntryAt1MustBeDeterred},
            PlotStyle -> {Hue[0.4], Hue[0.7], Hue[.0]}, AxesLabel ->
          {FontForm["N", {"Times-Italic", 12}], FontForm["Location",
          {"Times-Italic", 12}]}]

General::spell1 : Possible spelling error: new symbol
   name "locations3Firms3a" is similar to existing symbol "locations3Firms3".
```

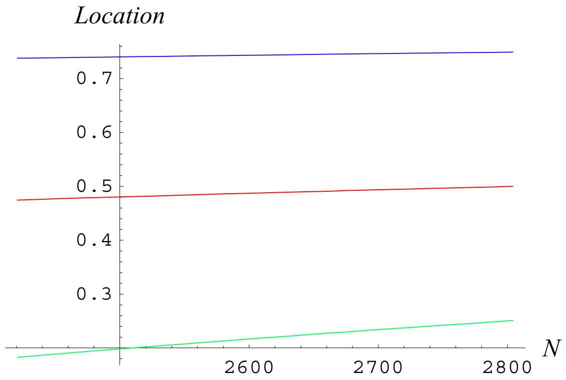



```
Out[507]= - Graphics -

In[508]:= locations3Firms4a = Plot[{
             xOfAllIncumbentsDeter[n][[1]],
             xOfAllIncumbentsDeter[n][[2]], xOfAllIncumbentsDeter[n][[3]]},
            {n, nOfEntryAt1MustBeDeterred, nOf1stEntrantSwitchesBackToCenter},
            PlotStyle -> {Hue[0.4], Hue[0.7], Hue[.0]}, AxesLabel ->
          {FontForm["N", {"Times-Italic", 12}], FontForm["Location",
          {"Times-Italic", 12}]}]
```

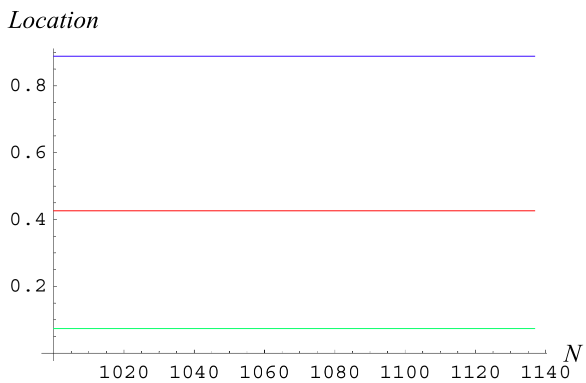



```
Out[508]= - Graphics -
```

```
In[509]:= locations3Firms4b = Plot[{
              xOfAllIncumbentsDeter[n][[1]],
              xOfAllIncumbentsDeter[n][[2]], xOfAllIncumbentsDeter[n][[3]]},
             {n, nOf1stEntrantSwitchesBackToCenter, nOfEntryOf4thEntrantCannotBeDeterred},
             PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
           {FontForm["N", {"Times-Italic", 12}], FontForm["Location",
           {"Times-Italic", 12}]}]

        General::spell1 : Possible spelling error: new symbol
           name "locations3Firms4b" is similar to existing symbol "locations3Firms4a".
```

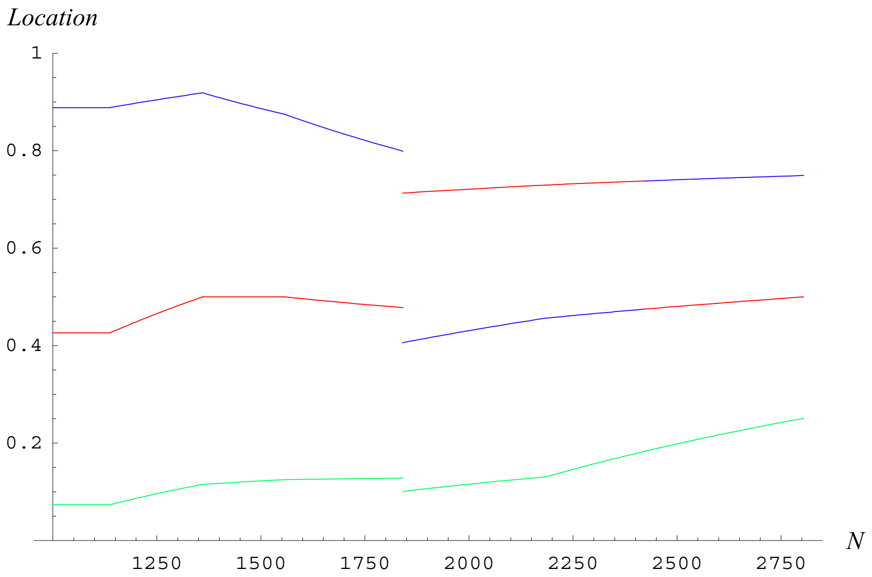



```
Out[509]= - Graphics -

In[510]:= locations3Firms0 = Plot[
             Evaluate[{x[1], x[2], x[3]} /. locationsSym], {n, 1000, nOfEntryBlockade3Firms},
             PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
           {FontForm["N", {"Times-Italic", 12}], FontForm["Location",
           {"Times-Italic", 12}]}]
```




```
Out[510]= - Graphics -
```

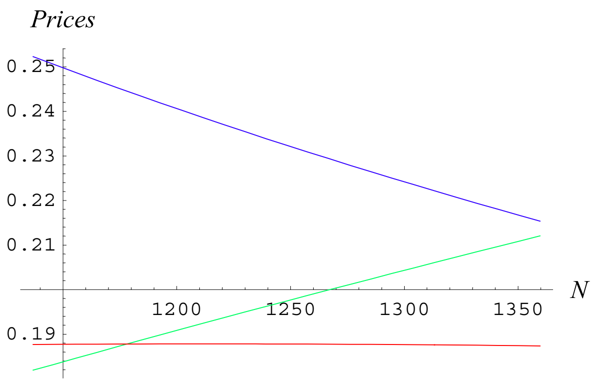
```
In[511]:= Show[locations3Firms0, locations3Firms1, locations3Firms2, locations3Firms3,
           locations3Firms3a, locations3Firms4a, locations3Firms4b, PlotRange → {0, 1}]
```




```
Out[511]= - Graphics -
```

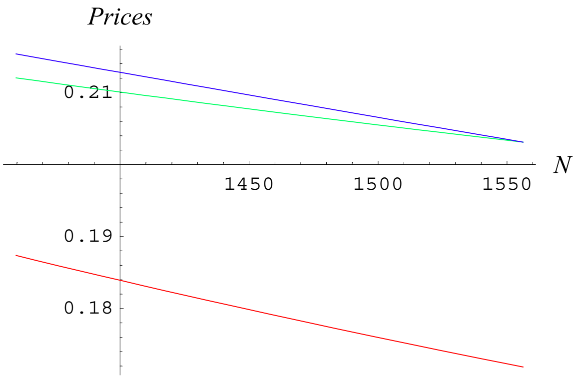
■ Prices

In[512]:=
```
prices3Firms1 = Plot[{(p[1] /. prices3Firms) [[1]] /.
     {((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
          {x[2] -> x1stEntrantDeters4thEntrant[n1]}]) [[1]],
      x[2] -> x1stEntrantDeters4thEntrant[n1],
      x[3] -> (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])},
    (p[2] /. prices3Firms) [[1]] /.
     {((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
          {x[2] -> x1stEntrantDeters4thEntrant[n1]}]) [[1]],
      x[2] -> x1stEntrantDeters4thEntrant[n1],
      x[3] -> (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])},
    (p[3] /. prices3Firms) [[1]] /.
     {((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
          {x[2] -> x1stEntrantDeters4thEntrant[n1]}]) [[1]],
      x[2] -> x1stEntrantDeters4thEntrant[n1],
      x[3] -> (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])}},
   {n1, nOfEntryBlockade3Firms, nOffirstEntrantDeterAtonehalf},
   PlotStyle ->
    {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
 {FontForm["N", {"Times-Italic", 12}], FontForm["Prices",
 {"Times-Italic", 12}]}]
```



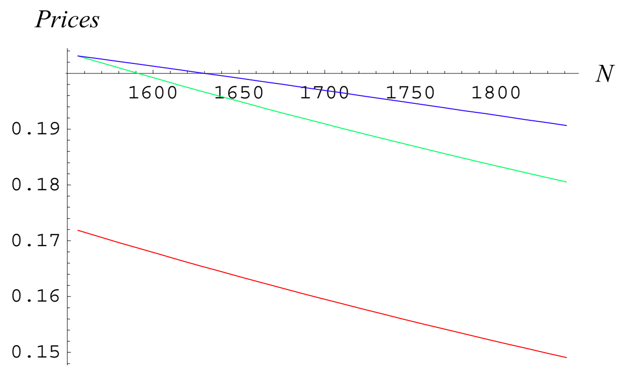
Out[512]= - Graphics -

In[513]:=
```
prices3Firms2 = Plot[{(p[1] /. prices3Firms)[[1]] /.
      {(react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
           , x[2] -> .5})[[1]], x[2] -> .5,
       x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]},
     (p[2] /. prices3Firms)[[1]] /.
      {(react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
           , x[2] -> .5})[[1]], x[2] -> .5,
       x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]},
     (p[3] /. prices3Firms)[[1]] /.
      {(react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
           , x[2] -> .5})[[1]], x[2] -> .5,
       x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]}},
   {n, nOffirstEntrantDeterAtonehalf,
 nOfEntryBetween1stAnd3rdEntrandMustBeDeterred},
   PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
 {FontForm["N", {"Times-Italic", 12}], FontForm["Prices",
 {"Times-Italic", 12}]}]
```



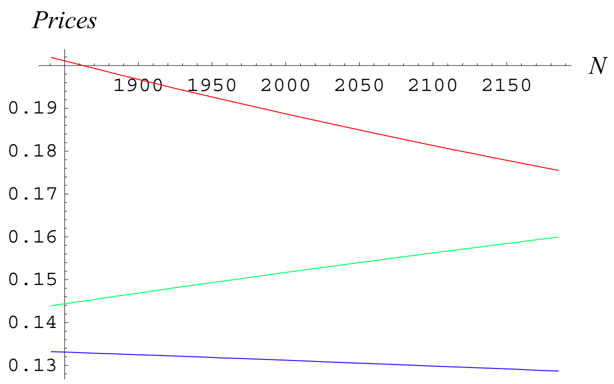
Out[513]= - Graphics -

In[514]:=
```
prices3Firms3 = Plot[{(p[1] /. prices3Firms)[[1]] /.
     {(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
          x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
            xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]})[[1]],
      x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
      x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
        n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]},
    (p[2] /. prices3Firms)[[1]] /.
     {(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
          x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
            xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]})[[1]],
      x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
      x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
        n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]},
    (p[3] /. prices3Firms)[[1]] /.
     {(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
          x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
            xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]})[[1]],
      x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
      x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
        n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}},
   {n, nOfEntryBetween1stAnd3rdEntrandMustBeDeterred,
    nOf1stEntrantSwitchesToEdge},
   PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]},
   AxesLabel ->
  {FontForm["N", {"Times-Italic", 12}], FontForm["Prices",
  {"Times-Italic", 12}]}]
```



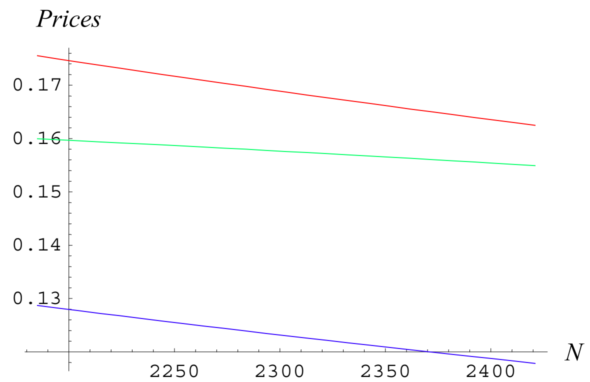
Out[514]= - Graphics -

```
In[515]:= prices3Firms3a = Plot[{(p[1] /. prices3Firms)[[1]] /. {x[1] ->
              xOptAt0[xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
               xOf1stAnd2ndEntrant1stAtEdge[n][[2]]],
             x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
             x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n][[2]]},
           (p[2] /. prices3Firms)[[1]] /. {x[1] ->
              xOptAt0[xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
               xOf1stAnd2ndEntrant1stAtEdge[n][[2]]],
             x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
             x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n][[2]]},
           (p[3] /. prices3Firms)[[1]] /. {x[1] ->
              xOptAt0[xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
               xOf1stAnd2ndEntrant1stAtEdge[n][[2]]],
             x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
             x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n][[2]]}},
          {n, nOf1stEntrantSwitchesToEdge, nOfEntryAt1MustBeDeterred},
          PlotStyle -> {Hue[0.4], Hue[0.7], Hue[.0]},
          AxesLabel ->
        {FontForm["N", {"Times-Italic", 12}], FontForm["Prices",
        {"Times-Italic", 12}]}]

       General::spell1 : Possible spelling error: new symbol
          name "prices3Firms3a" is similar to existing symbol "prices3Firms3".
```



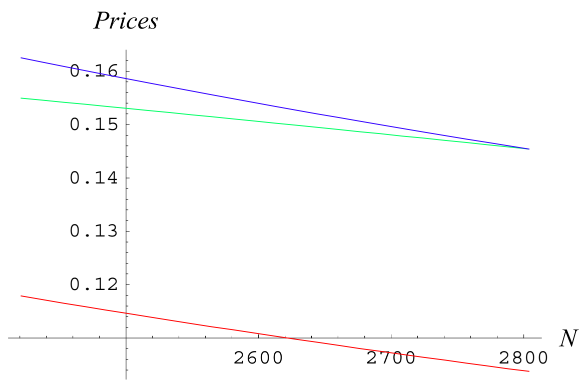

```
Out[515]= - Graphics -
```

```
In[516]:= prices3Firms4a = Plot[{(p[1] /. prices3Firms)[[1]] /. {x[1] ->
               xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]},
           (p[2] /. prices3Firms)[[1]] /. {x[1] ->
               xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]}, (p[3] /. prices3Firms)[[1]] /. {x[1] ->
               xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]}},
          {n, nOfEntryAt1MustBeDeterred, nOf1stEntrantSwitchesBackToCenter},
          PlotStyle -> {Hue[0.4], Hue[0.7], Hue[.0]}, AxesLabel ->
        {FontForm["N", {"Times-Italic", 12}], FontForm["Prices",
        {"Times-Italic", 12}]}]
```



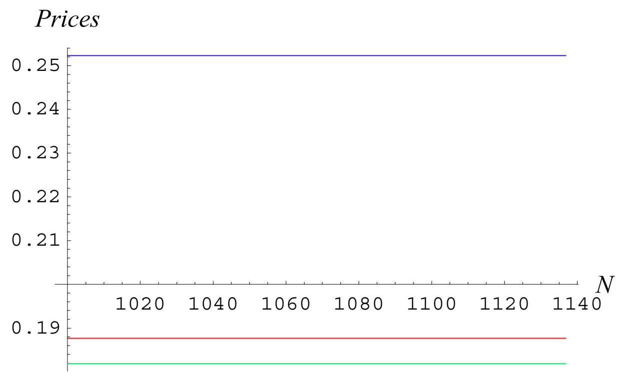

```
Out[516]= - Graphics -
```

```
In[517]:= prices3Firms4b = Plot[{(p[1] /. prices3Firms)[[1]] /. {x[1] ->
               xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]},
           (p[2] /. prices3Firms)[[1]] /. {x[1] ->
               xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]}, (p[3] /. prices3Firms)[[1]] /. {x[1] ->
               xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]}},
          {n, nOf1stEntrantSwitchesBackToCenter, nOfEntryOf4thEntrantCannotBeDeterred},
          PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
        {FontForm["N", {"Times-Italic", 12}], FontForm["Prices",
        {"Times-Italic", 12}]}]

      General::spell1 : Possible spelling error: new symbol
         name "prices3Firms4b" is similar to existing symbol "prices3Firms4a".
```



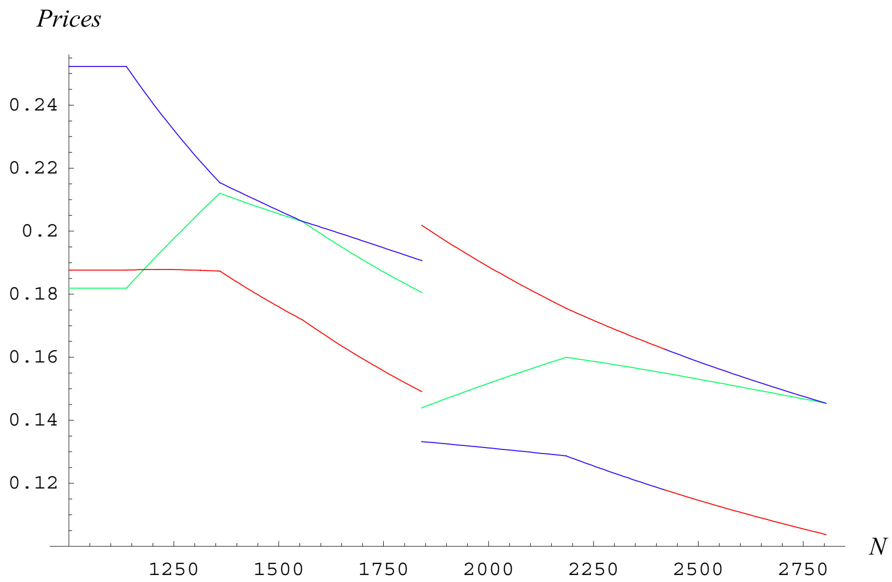

```
Out[517]= - Graphics -
```

```
In[518]:= prices3Firms0 =
           Plot[Evaluate[({p[1], p[2], p[3]} /. prices3Firms)[[1]] /. locationsSym],
            {n, 1000, nOfEntryBlockade3Firms},
            PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
          {FontForm["N", {"Times-Italic", 12}], FontForm["Prices",
          {"Times-Italic", 12}]}]
```




```
Out[518]= - Graphics -
```

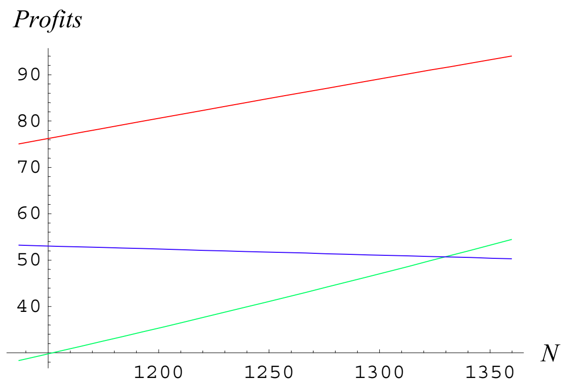
```
In[519]:= Show[prices3Firms0, prices3Firms1, prices3Firms2,
           prices3Firms3, prices3Firms3a, prices3Firms4a, prices3Firms4b]
```




```
Out[519]= - Graphics -
```

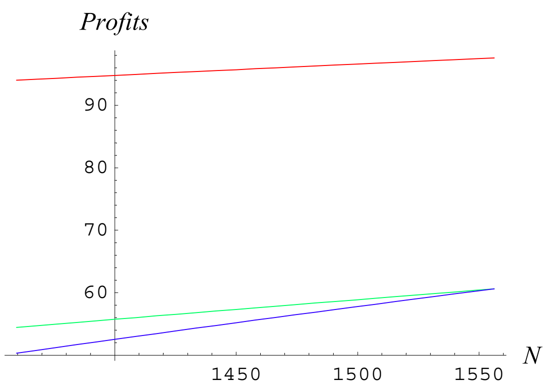
■ Profits

In[520]:=
```
profits3Firms1 = Plot[{(redProfitSym[1] /. n → n1) /.
    {((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
         {x[2] -> x1stEntrantDeters4thEntrant[n1]}])[[1]],
     x[2] -> x1stEntrantDeters4thEntrant[n1],
     x[3] -> (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])},
   (redProfitSym[2] /. n → n1) /.
    {((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
         {x[2] -> x1stEntrantDeters4thEntrant[n1]}])[[1]],
     x[2] -> x1stEntrantDeters4thEntrant[n1],
     x[3] -> (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])},
   (redProfitSym[3] /. n → n1) /.
    {((react1 /. {x[3] -> x3}) /. Join[reactNum3[x1stEntrantDeters4thEntrant[n1]],
         {x[2] -> x1stEntrantDeters4thEntrant[n1]}])[[1]],
     x[2] -> x1stEntrantDeters4thEntrant[n1],
     x[3] -> (x3 /. reactNum3[x1stEntrantDeters4thEntrant[n1]])}},
  {n1, nOfEntryBlockade3Firms, nOffirstEntrantDeterAtonehalf},
  PlotStyle ->
   {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
 {FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
 {"Times-Italic", 12}]}]
```



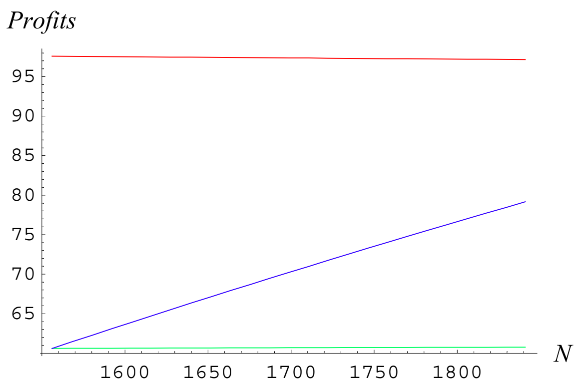
Out[520]= - Graphics -

In[521]:=
```
profits3Firms2 = Plot[{redProfitSym[1] /.
    {(react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
         , x[2] -> .5})[[1]], x[2] -> .5,
     x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]},
   redProfitSym[2] /.
    {(react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
         , x[2] -> .5})[[1]], x[2] -> .5, x[3] ->
      xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]}, redProfitSym[3] /.
    {(react1 /. {x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]
         , x[2] -> .5})[[1]], x[2] -> .5,
     x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n, .5]}},
  {n, nOffirstEntrantDeterAtonehalf,
nOfEntryBetween1stAnd3rdEntrandMustBeDeterred},
  PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
{"Times-Italic", 12}]}]
```



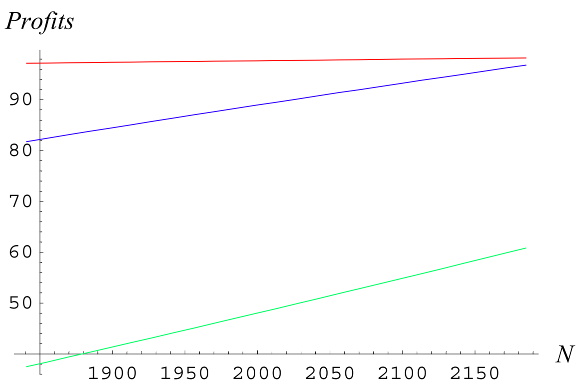
Out[521]= - Graphics -

```
In[522]:= profits3Firms3 = Plot[{redProfitSym[1] /.
       {(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
            x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
              xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]})[[1]],
        x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
        x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
          n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]},
      redProfitSym[2] /. {(react1 /. {x[2] ->
             xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
            x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
              xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]})[[1]],
        x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
        x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
          xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}, redProfitSym[3] /.
       {(react1 /. {x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
            x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[n,
              xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]})[[1]],
        x[2] -> xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n],
        x[3] -> xOf2ndEntrant1stAnd2ndEntrantDeter4thEntrant[
          n, xOf1stEntrant1stAnd2ndEntrantDeterTwoEntrants[n]]}},
     {n, nOfEntryBetween1stAnd3rdEntrandMustBeDeterred,
      nOf1stEntrantSwitchesToEdge},
     PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]},
     AxesLabel ->
    {FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
    {"Times-Italic", 12}]}]
```



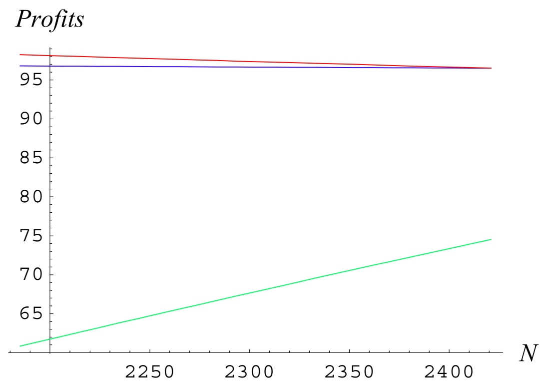

```
Out[522]= - Graphics -

In[523]:= n1 =.
```

```
In[524]:= profits3Firms3a = Plot[{redProfitSym[1] /. {x[1] ->
              xOptAt0[xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
               xOf1stAnd2ndEntrant1stAtEdge[n][[2]]],
             x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
             x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n][[2]]},
           redProfitSym[2] /. {x[1] ->
              xOptAt0[xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
               xOf1stAnd2ndEntrant1stAtEdge[n][[2]]],
             x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
             x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n][[2]]}, redProfitSym[3] /. {x[1] ->
              xOptAt0[xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
               xOf1stAnd2ndEntrant1stAtEdge[n][[2]]],
             x[2] -> xOf1stAnd2ndEntrant1stAtEdge[n][[1]],
             x[3] -> xOf1stAnd2ndEntrant1stAtEdge[n][[2]]}},
         {n, nOf1stEntrantSwitchesToEdge, nOfEntryAt1MustBeDeterred},
         PlotStyle -> {Hue[0.4], Hue[0.7], Hue[.0]},
         AxesLabel ->
       {FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
       {"Times-Italic", 12}]}]

        General::spell1 : Possible spelling error: new symbol
           name "profits3Firms3a" is similar to existing symbol "profits3Firms3".
```



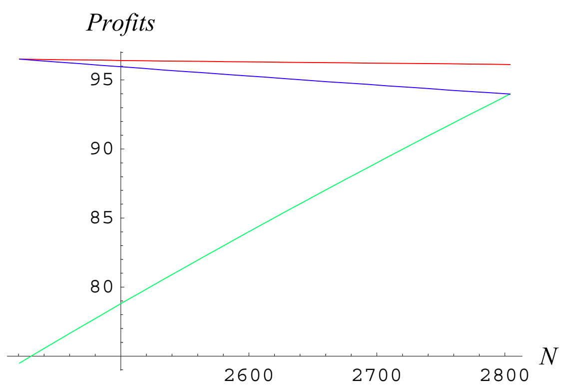

```
Out[524]= - Graphics -
```

```
In[525]:= profits3Firms4a = Plot[{redProfitSym[1] /. {x[1] ->
              xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]},
           redProfitSym[2] /. {x[1] ->
              xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]}, redProfitSym[3] /. {x[1] ->
              xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]}},
          {n, nOfEntryAt1MustBeDeterred, nOf1stEntrantSwitchesBackToCenter},
          PlotStyle -> {Hue[0.4], Hue[0.7], Hue[.0]}, AxesLabel ->
        {FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
        {"Times-Italic", 12}]}]
```



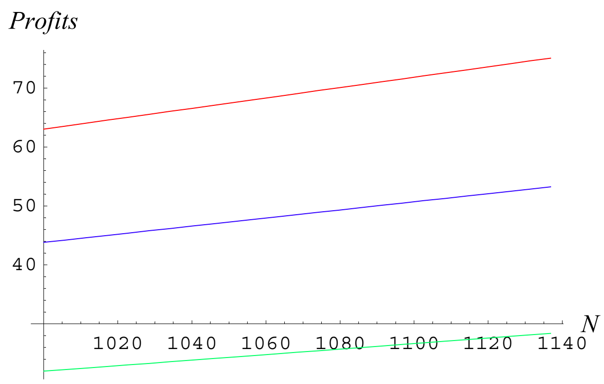

```
Out[525]= - Graphics -
```

```
In[526]:= profits3Firms4b = Plot[{redProfitSym[1] /. {x[1] ->
              xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]},
           redProfitSym[2] /. {x[1] ->
              xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]}, redProfitSym[3] /. {x[1] ->
              xOfAllIncumbentsDeter[n][[1]], x[2] -> xOfAllIncumbentsDeter[n][[2]],
             x[3] -> xOfAllIncumbentsDeter[n][[3]]}},
          {n, nOf1stEntrantSwitchesBackToCenter, nOfEntryOf4thEntrantCannotBeDeterred},
          PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
        {FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
        {"Times-Italic", 12}]}]

      General::spell1 : Possible spelling error: new symbol
         name "profits3Firms4b" is similar to existing symbol "profits3Firms4a".
```



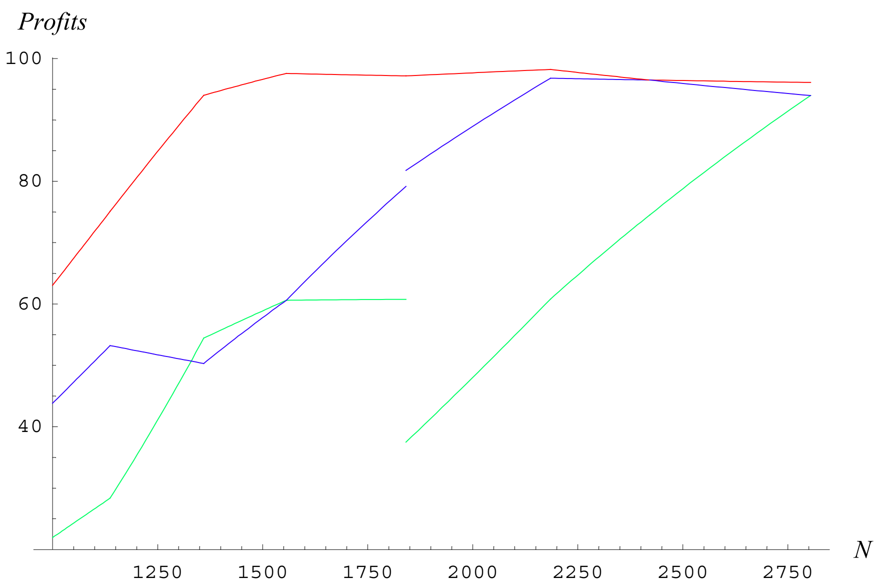

```
Out[526]= - Graphics -
```

In[527]:=
```
profits3Firms0 = Plot[
  Evaluate[({redProfitSym[1], redProfitSym[2], redProfitSym[3]}) /. locationsSym],
  {n, 1000, nOfEntryBlockade3Firms},
  PlotStyle -> {Hue[0.4], Hue[0.0], Hue[.7]}, AxesLabel ->
{FontForm["N", {"Times-Italic", 12}], FontForm["Profits",
{"Times-Italic", 12}]}]
```



Out[527]= - Graphics -

In[528]:=
```
Show[profits3Firms0, profits3Firms1, profits3Firms2,
 profits3Firms3, profits3Firms3a, profits3Firms4a, profits3Firms4b]
```



Out[528]= - Graphics -