Oligopoly Limit Pricing: Strategic Substitutes, Strategic Complements | Appendix

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Abstract

Contents

1. Quantity-setting firms

1.1. Duopoly

Recall the notation

$$u_{ij} = \text{firm } i\text{'s prior probability that firm } j\text{'s costs are high;}$$

$$v_{ij} = \text{firm } i\text{'s prior probability that firm } j\text{'s costs are low;}$$

for $i, j = 1, 2, 3$ and $i \neq j$.

With the linear demand curve

$$p = a - Q. \tag{1.1}$$

firm 1L's expected profit is

$$\pi_{1L} = (a_L - q_{1L} - u_{12}q_{2H} - v_{12}q_{2L})q_{1L}, \tag{1.2}$$

where $a_L = a - c_L$. Maximizing (1.2) with respect to q_{1L} gives the equation of firm 1L's reaction function,

$$2q_{1L} + u_{12}q_{2H} + v_{12}q_{2L} = a_L. \tag{1.3}$$

Note that (1.2) and (1.3) together imply that anywhere along 1L's reaction function, and in particular in equilibrium,

$$\pi_{1L} = q_{1L}^2 \tag{1.4}$$

In like manner, one obtains the equations of the other 3 reaction functions,

$$2q_{1H} + u_{12}q_{2H} + v_{12}q_{2L} = a_H. \tag{1.5}$$

$$u_{21}q_{1H} + v_{21}q_{1L} + 2q_{2L} = a_L. \tag{1.6}$$

$$u_{21}q_{1H} + v_{21}q_{1L} + 2q_{2H} = a_H. \tag{1.7}$$

(where $a_H = a - c_H$).

Duopoly equilibrium for a single-period game can be illustrated graphically. Define the output each firm expects to see from the other as

$$q_1(u_{21}) = u_{21}q_{1H} + v_{21}q_{1L} \tag{1.8}$$

$$q_2(u_{12}) = u_{12}q_{2H} + v_{12}q_{2L} \tag{1.9}$$

Now multiply (1.5) by u_{21}, (1.3) by v_{21}, and add to obtain

$$2q_1(u_{21}) + q_2(u_{12}) = u_{21}a_H + v_{21}a_L = a(u_{21}) \tag{1.10}$$

In like manner, from (1.6) and (1.7) one obtains

$$q_1(u_{21}) + 2q_2(u_{12}) = u_{12}a_H + v_{12}a_L = a(u_{12}) \tag{1.11}$$

(1.10) and (1.11) have the form of conventional quantity reaction functions for the linear model. They can be graphed, as in Figure A1, to illustrate equilibrium expected outputs $q_1^*(u_{21})$ and $q_2^*(u_{12})$.

Insert Figure A1 about here

Now graph (1.10) and the reaction functions of firms 1L (1.3) and 1H (1.5) together, as in Figure A2. The reaction function with equation (1.10) is bounded below by the reaction function of firm 1H, and above by the reaction function of firm 1L. The equilibrium outputs of firms 1H and 1L can be read off the graph at the intersection of the respective reaction functions with the equilibrium expected output of firm 2, $q_2^*(u_{12})$. In like manner, the equilibrium outputs of firms 2H and 2L are found at the intersections of their reaction functions with the vertical line showing $q_1^*(u_{21})$. Figure A3 shows duopoly equilibrium outputs for all four notional players.

Insert Figure A2 about here
Insert Figure A3 about here

For the pooling equilibrium discussed in the text, in the first period incumbents produce their equilibrium outputs from the one-period game. For the separating equilibrium discussed in the text, in the first period incumbents produce the outputs given by the intersection of the L-firm reaction functions.

To find an explicit solution to the system of equations of the reaction functions, subtract (1.5) from (1.3) to obtain

$$q_{1L} = q_{1H} + \frac{1}{2}(a_L - a_H) = q_{1H} + \frac{1}{2}(c_H - c_L) \tag{1.12}$$

Similarly, from (1.6) and (1.7),

$$q_{2L} = q_{2H} + \frac{1}{2}(a_L - a_H) = q_{2H} + \frac{1}{2}(c_H - c_L) \tag{1.13}$$

Substitute from (1.13) into (1.3) to eliminate q_{2H} and from (1.12) into (1.6) to eliminate q_{1H}. The result is a system of two equations in q_{1L} and q_{2L},

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} q_{1L} \\ q_{2L} \end{pmatrix} = a_L \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} u_{12} \\ u_{21} \end{pmatrix} (c_H - c_L); \tag{1.14}$$

which yields expressions for the equilibrium outputs of the low-cost firms,

$$q_{1L} = \frac{1}{3}\left(a_L + \frac{2u_{12} - u_{21}}{2}(c_H - c_L)\right) \tag{1.15}$$

$$q_{2L} = \frac{1}{3}\left(a_L + \frac{2u_{21} - u_{12}}{2}(c_H - c_L)\right) \tag{1.16}$$

In like manner, one obtains expressions for the equilibrium outputs of high-cost firms,

$$q_{1H} = \frac{1}{3}\left(a_H - \frac{2v_{12} - v_{21}}{2}(c_H - c_L)\right) \tag{1.17}$$

$$q_{2H} = \frac{1}{3}\left(a_H - \frac{2v_{21} - v_{12}}{2}(c_H - c_L)\right) \tag{1.18}$$

Here and throughout, I restrict the discussion to the case in which equilibrium outputs are nonnegative.

Using (1.4) and the corresponding relationships for other players, equilibrium payoffs are

$$\pi_{1L}(u_{12}, u_{21}) = \frac{1}{9}\left(a_L + \frac{2u_{12} - u_{21}}{2}(c_H - c_L)\right)^2 \tag{1.19}$$

$$\pi_{2L}(u_{12}, u_{21}) = \frac{1}{9}\left(a_L + \frac{2u_{21} - u_{12}}{2}(c_H - c_L)\right)^2 \tag{1.20}$$

$$\pi_{1H}(u_{12}, u_{21}) = \frac{1}{9}\left(a_H - \frac{2v_{12} - v_{21}}{2}(c_H - c_L)\right)^2 \tag{1.21}$$

$$\pi_{2H}(u_{12}, u_{21}) = \frac{1}{9}\left(a_H - \frac{2v_{21} - v_{12}}{2}(c_H - c_L)\right)^2 \tag{1.22}$$

1.2. Triopoly

A game of three players, each with its cost type known only to itself, must be analyzed to determine the limits of case 1, case 2, and case 3. Following the same procedure as in the duopoly case, the equations of reaction functions if there are 3 firms are

$$2q_{1L} + u_{12}q_{2H} + v_{12}q_{2L} + u_{13}q_{3H} + v_{13}q_{3L} = a_L. \tag{1.23}$$

$$2q_{1H} + u_{12}q_{2H} + v_{12}q_{2L} + u_{13}q_{3H} + v_{13}q_{3L} = a_H. \tag{1.24}$$

$$2q_{2L} + u_{21}q_{1H} + v_{21}q_{1L} + u_{23}q_{3H} + v_{23}q_{3L} = a_L. \tag{1.25}$$

$$2q_{2H} + u_{21}q_{1H} + v_{21}q_{1L} + u_{23}q_{3H} + v_{23}q_{3L} = a_H. \tag{1.26}$$

$$2q_{3L} + u_{31}q_{1H} + v_{31}q_{1L} + u_{32}q_{2H} + v_{32}q_{2L} = a_L. \tag{1.27}$$

$$2q_{3H} + u_{31}q_{1H} + v_{31}q_{1L} + u_{32}q_{2H} + v_{32}q_{2L} = a_H. \tag{1.28}$$

(1.12) and (1.13) continue to hold; in addition, from (1.27) and (1.28) one obtains

$$q_{3L} = q_{3H} + \frac{1}{2}(a_L - a_H) = q_{1H} + \frac{1}{2}(c_H - c_L) \tag{1.29}$$

Using (1.12), (1.13), and (1.29) to eliminate the outputs of high-cost firms from (1.23), (1.25), and (1.27) yields a system of 3 equations in q_{1L}, q_{2L}, and q_{3L}:

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} q_{1L} \\ q_{2L} \\ q_{3L} \end{pmatrix} = a_L \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} u_{12} + u_{13} \\ u_{21} + u_{23} \\ u_{31} + u_{32} \end{pmatrix} (c_H - c_L) \tag{1.30}$$

Write the coefficient matrix on the left as $I_3 + J_3 J_3'$, where I is the identity matrix, J is a column vector of ones, and subscripts indicate dimension. Then using

$$(I_3 + J_3 J_3')^{-1} = I_3 - \frac{1}{4} J_3 J_3'; \tag{1.31}$$

one obtains equilibrium outputs of low-cost firms,

$$q_{1L} = \frac{1}{4}\left[a_L + \frac{3(u_{12} + u_{13}) - (u_{21} + u_{23} + u_{31} + u_{32})}{2}(c_H - c_L) \right] \tag{1.32}$$

$$q_{2L} = \frac{1}{4}\left[a_L + \frac{3(u_{21} + u_{23}) - (u_{12} + u_{13} + u_{31} + u_{32})}{2}(c_H - c_L) \right] \tag{1.33}$$

$$q_{3L} = \frac{1}{4}\left[a_L + \frac{3(u_{31} + u_{32}) - (u_{12} + u_{13} + u_{21} + u_{23})}{2}(c_H - c_L) \right] \tag{1.34}$$

In like manner, equilibrium outputs of high-cost firms are

$$q_{1H} = \frac{1}{4}\left[a_H - \frac{3(v_{12} + v_{13}) - (v_{21} + v_{23} + v_{31} + v_{32})}{2}(c_H - c_L) \right] \tag{1.35}$$

$$q_{2H} = \frac{1}{4}\left[a_H - \frac{3(v_{21} + v_{23}) - (v_{12} + v_{13} + v_{31} + v_{32})}{2}(c_H - c_L) \right] \tag{1.36}$$

$$q_{3H} = \frac{1}{4}\left[a_H - \frac{3(v_{31} + v_{32}) - (v_{12} + v_{13} + v_{21} + v_{23})}{2}(c_H - c_L) \right] \tag{1.37}$$

Equilibrium payoffs (for firm 3, subtracting entry cost) are the squares of equilibrium outputs.

1.3. Pooling on q_L

1.3.1. Case 2

If

$$\pi_{3H}(u_{12}, u_{13}, u_{21}, u_{23}, u_{31}, u_{32}) - K = \frac{1}{16}\left[a_H - \frac{3(v_{31} + v_{32}) - (v_{12} + v_{13} + v_{21} + v_{23})}{2}(c_H - c_L)\right]^2 - K \tag{1.38}$$

is positive, pooling cannot limit entry. If (1.38) is nonpositive but

$$\pi_{3L}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) - K = \frac{1}{16}\left[a_L + \frac{3(u_{31} + u_{32}) - (u_{12} + u_{21})}{2}(c_H - c_L)\right]^2 - K \tag{1.39}$$

is positive, then a low-cost entrant would come into the market after first-period pooling, while a high-cost entrant would stay out. This is case 2. If (1.39) is nonpositive, the game falls in case 3; incumbents can limit all entry by pooling in the first period.

If entry costs place the game in case 2 and a high-cost incumbent defects from the pooling equilibrium, revealing its cost type, then a high-cost entrant's payoff would be

$$\pi_{3H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32}) - K = \frac{1}{16}\left[a_H - \frac{3v_{32} - (v_{12} + v_{13} + v_{23})}{2}(c_H - c_L)\right]^2 - K. \tag{1.40}$$

If (1.40) is positive, the game is in case 2(a); otherwise, it is in case 2(b).

As noted in the text, low-cost firms will always adhere to the pooling strategy.

Entrants Case 2(b) is defined by the inequalities

$$\begin{aligned} q_{3L}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) &= \frac{1}{4}\left[a_L + \frac{3(u_{31} + u_{32}) - (u_{12} + u_{21})}{2}(c_H - c_L)\right] \\ &> \sqrt{K} > \\ \frac{1}{4}\left[a_H - \frac{3v_{32} - (v_{12} + v_{13} + v_{23})}{2}(c_H - c_L)\right] &= q_{3H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32}). \end{aligned} \tag{1.41}$$

If incumbents pool in case 2(a) and entry occurs, in equilibrium the incumbents believe that the entrant has low cost. If the entrant has high cost, its payoff (before allowing for sunk entry cost) is the square of

$$q_{3H}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) = \qquad (1.42)$$
$$\frac{1}{4}\left[a_H - \frac{3(v_{31}+v_{32}) - (v_{12}+1+v_{21}+1)}{2}(c_H - c_L)\right]$$

The beliefs indicate that the entrant does not know the cost types of the incumbents, that neither incumbent knows the cost type of the other incumbent, and that both incumbents incorrectly believe that the entrant has low cost.

First compare the output of a high-cost entrant that mimics a low-cost entrant with the output level that defines the upper limit of Case 2:

$$4[q_{3L}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) - q_{3H}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32})] = \qquad (1.43)$$
$$a_L + \frac{3(u_{31}+u_{32}) - (u_{12}+u_{21})}{2}(c_H - c_L)$$
$$- a_H + \frac{3(v_{31}+v_{32}) - (v_{12}+1+v_{21}+1)}{2}(c_H - c_L)$$
$$= \left[1 + \frac{3(u_{31}+u_{32}) - (u_{12}+u_{21}) + 3(v_{31}+v_{32}) - (v_{12}+1+v_{21}+1)}{2}\right](c_H - c_L)$$
$$= \left(1 + \frac{6-4}{2}\right)(c_H - c_L) = 2(c_H - c_L) > 0.$$

It follows that there is always an upper segment of the range that defines Case 2(b) in which a high-cost entrant would not wish to mimic a low-cost entrant.

Now compare the output of a high-cost entrant that mimics a low-cost entrant with the output level that defines the boundary between Case 2(b) and Case 2(a):

$$\frac{8}{c_H - c_L}[q_{3H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32}) - q_{3H}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32})]$$
$$= -3v_{32} + (v_{12}+v_{13}+v_{23}) + 3(v_{31}+v_{32}) - (v_{12}+1+v_{21}+1) \qquad (1.44)$$
$$= (v_{13}+v_{23}) + 3v_{31} - (1+v_{21}+1) = 3v_{31} - v_{21} - u_{13} - u_{23}.$$

This can be positive or negative. The range within which a high-cost entrant will wish to mimic a low-cost entrant may or may not extend from Case 2(a) into Case 2(b).

Now compare the output of a high-cost entrant that pretends to have low cost with the output that defines the border between Case 2 and Case 1.

$$\frac{8}{c_H - c_L}[q_{3H}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) - q_{3H}(u_{12}, u_{13}, u_{21}, u_{23}, u_{31}, u_{32})] \qquad (1.45)$$
$$= -3(v_{31}+v_{32}) + (v_{12}+1+v_{21}+1) + 3(v_{31}+v_{32}) - (v_{12}+v_{13}+v_{21}+v_{23})$$
$$= v_{12}+1+v_{21}+1 - (v_{12}+v_{13}+v_{21}+v_{23}) = u_{13}+u_{23} > 0.$$

There is always a range in Case 2(a) in which a high-cost entrant will wish to mimic a low-cost entrant.

High-cost incumbents In case 2(a), firm 1H will pool if

$$
\begin{aligned}
& u_{13}[\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(u_{12}, 1)] && (1.46)\\
& + v_{13}[\pi_{1H}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) - \pi_{1H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32})] \\
& + u_{13}[\pi_{1H}(u_{12}, 1) - \pi_{1H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32})] \geq \pi^{br}_{1H}(q_L) - \pi_{1H}(q_L)
\end{aligned}
$$

This is equation (8) of the text.

In case 2(b), firm 1H will pool if

$$
\begin{aligned}
& u_{13}[\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(u_{12}, 1)] && (1.47)\\
& + v_{13}[\pi_{1H}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) - \pi_{1H}(u_{12}, 0, 1, 0, 1, u_{32})] \geq \pi^{br}_{1H}(q_L) - \pi_{1H}(q_L).
\end{aligned}
$$

This is equation (12) of the text.

Let us begin by evaluating the first-period gain from defection,

$$\pi^{br}_{1H}(q_L) - \pi_{1H}(q_L), \tag{1.48}$$

for the linear model.

Firm 1H's best response payoff is

$$\pi^{br}_{1H}(q_L) = [a_H - q^{br}_{1H} - q_L]q^{br}_{1H} = \left(q^{br}_{1H}\right)^2, \tag{1.49}$$

using $a - q^{br}_{1H} - q_L = q^{br}_{1H}$ from the equation of the reaction function.

Then firm 1H's first-period pooling payoff can be written

$$
\begin{aligned}
\pi_{1H}(q_L) &= [a_H - q_L - q_L]q_L && (1.50)\\
&= [a_H - q^{br}_{1H} - q_L - (q_L - q^{br}_{1H})][q^{br}_{1H} + (q_L - q^{br}_{1H})] \\
&= [q^{br}_{1H} - (q_L - q^{br}_{1H})][q^{br}_{1H} + (q_L - q^{br}_{1H})] \\
&= (q^{br}_{1H})^2 - (q_L - q^{br}_{1H})^2 \\
&= \pi^{br}_{1H}(q_L) - (q_L - q^{br}_{1H})^2.
\end{aligned}
$$

Hence

$$\pi^{br}_{1H}(q_L) - \pi_{1H}(q_L) = (q_L - q^{br}_{1H})^2. \tag{1.51}$$

Finally, from the equation of firm 1H's reaction function, $q^{br}_{1H} = (1/2)(a_H - q_L)$, and recalling $q_L = a_L/3$,

$$q_L - q^{br}_{1H} = \frac{3}{2}q_L - \frac{1}{2}a_H = \frac{1}{2}(a_L - a_H) = \frac{1}{2}(c_H - c_L). \tag{1.52}$$

Now turn to the impact of cost type revelation on the defecting firm's profits, terms which appear in both (1.46) and (1.47). Using (1.17) or equivalently (1.21),

$$\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(u_{12}, 1) = q^2_{1H}(u_{12}, u_{21}) - q^2_{1H}(u_{12}, 1) \tag{1.53}$$
$$= [q_{1H}(u_{12}, u_{21}) + q_{1H}(u_{12}, 1)][q_{1H}(u_{12}, u_{21}) - q_{1H}(u_{12}, 1)]$$
$$= \frac{v_{21}}{9}\left[a_H - \frac{4v_{12} - v_{21}}{4}(c_H - c_L)\right](c_H - c_L) \geq 0.$$

In like manner, using (1.35)

$$\pi_{1H}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) - \pi_{1H}(u_{12}, 0, 1, 0, 1, u_{32}) \tag{1.54}$$
$$= q^2_{1H}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) - q^2_{1H}(u_{12}, 0, 1, 0, 1, u_{32})$$
$$= \frac{v_{21} + v_{31}}{16}\left[a_H - \frac{4 + 6v_{12} - (v_{21} + v_{31} + 2v_{32})}{4}(c_H - c_L)\right](c_H - c_L) \geq 0.$$

When quantity-setting firms produce strategic substitutes, a firm's rivals expand second-period output if it becomes known that the firm has high cost, compared with the situation in which there is some probability that the firm has low cost. This cost revelation effect reduces the defecting high-cost firm's second-period payoffs.

Similarly

$$\pi_{1H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32}) - \pi_{1H}(u_{12}, 0, 1, 0, 1, u_{32}) \tag{1.55}$$
$$= q^2_{1H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32}) - q^2_{1H}(u_{12}, 0, 1, 0, 1, u_{32})$$
$$= \frac{3u_{13} - u_{23}}{16}\left[a_H - \frac{2 + 6v_{12} + 3v_{13} - (v_{23} + 2v_{32})}{4}(c_H - c_L)\right](c_H - c_L) \geq 0.$$

The entry effect, which appears only in case 2(a), is

$$\pi_{1H}(u_{12}, 1) - \pi_{1H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32}) \tag{1.56}$$
$$= [q_{1H}(u_{12}, 1) + q_{1H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32})][q_{1H}(u_{12}, 1) - q_{1H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32})].$$

This shows the change in profit of a known high-cost firm 1 if an entrant of unknown cost type comes into the market. Examining the terms in the second brackets on the right,

$$q_{1H}(u_{12}, 1) = \frac{1}{3}[a_H - v_{12}(c_H - c_L)] \tag{1.57}$$
$$q_{1H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32}) = \frac{1}{4}\left[a_H - \frac{3v_{12} + 3v_{13} - (v_{23} + v_{32})}{2}(c_H - c_L)\right].$$

The normal case is that

$$q_{1H}(u_{12}, 1) - q_{1H}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32}) \tag{1.58}$$
$$= \frac{1}{12}a_H + \left[\frac{v_{12}}{24} + \frac{3v_{13} - (v_{23} + v_{32})}{8}\right](c_H - c_L)$$

is positive: we expect entry to reduce incumbents' equilibrium outputs and profits. But this is not a necessary result. If $u_{12} = u_{13} = 1$ and $u_{23} = u_{32} = 0$, then firm 1 is known to have high cost, firm 1 believes both rivals have high cost, and firms 2 and 3 each believe the other has low cost. Then (1.58) may be positive. In particular

$$q_{1H}(1;1) = \frac{1}{3}a_H = \frac{1}{3}(a - c_H); \tag{1.59}$$

$$q_{1H}(1;1;1;0;1;0) = \frac{1}{4}a_L = \frac{1}{4}(a - c_L);$$

and if $a_L/4 > a_H/3$, firm 1H would expect greater output and profit with entry than without entry. With downward-sloping quantity reaction functions, and believing its rivals to have high cost, firm 1H would expand output. Firms 2 and 3, each believing the other to have low cost, will reduce output.

1.3.2. Case 3

If entry costs place the game in case 3 and a high-cost incumbent defects from the pooling equilibrium, revealing its cost type, a low-cost entrant's payoff is

$$\pi_{3L}(u_{12};0;1;0;1;u_{32}) = \frac{1}{16}\left[a_L + \frac{2 + 3u_{32} - u_{12}}{2}(c_H - c_L)\right]^2 - K \tag{1.60}$$

If this is positive, then the game is in case 3(a): defection by a high-cost incumbent will induce entry by a low-cost potential entrant. If (1.60) is nonpositive, then a high-cost incumbent can defect without inducing entry; this is case 3(b).

Entrants The high-output pooling strategy requires an entrant to come into the market if it expects a positive post-entry profit and to stay out otherwise. If incumbents pool in the first period, even a low-cost entrant expects a negative post-entry profit, and will maximize its payoff by staying out of the market. A high-cost entrant would make a lower profit than a low-cost entrant, even if incumbents mistakenly believe that it has low cost, and would therefore stay out of the market as well.

Low-cost incumbent If a low-cost incumbent defects, it reduces its first-period payoff and rivals infer that it has high cost. This leads the other incumbent to expand second-period output. It may (in case 3(a)) induce entry. Both consequences have the effect of reducing the defecting low-cost firm's second-period payoff. A low-cost incumbent would therefore be willing to pool on q_L in the first period.

High-cost incumbent Now turn to the incentives of a high-cost incumbent. If firm 1H pools on q_L in the first period, its expected equilibrium payoff is

$$\pi_{1H}(q_L) + \pi_{1H}(u_{12};u_{21}): \tag{1.61}$$

If firm 1H defects from the pooling equilibrium, its expected payoff is

$$\pi^{br}_{1H}(q_L) + u_{13}\pi_{1H}(u_{12}, 1) + v_{13}\pi_{1H}(u_{12}, 0, 1, 0, 1, u_{32}) \tag{1.62}$$

if the game is in case 3(a) and

$$\pi^{br}_{1H}(q_L) + \pi_{1H}(u_{12}, 1) \tag{1.63}$$

if the game is in case 3(b).

Comparing pooling and defection payoffs, in case 3(a) the incumbent will pool if

$$\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(u_{12}, 1) + v_{13}[\pi_{1H}(u_{12}, 1) - \pi_{1H}(u_{12}, 0, 1, 0, 1, u_{32})]$$
$$\geq \pi^{br}_{1H}(q_L) - \pi_{1H}(q_L). \tag{1.64}$$

In case 3(b), the incumbent will pool if

$$\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(u_{12}, 1) \geq \pi^{br}_{1H}(q_L) - \pi_{1H}(q_L). \tag{1.65}$$

The right-hand side of (1.65) is the first-period gain from defection. The left-hand side is the second-period lost profit due to cost type revelation. The additional term that appears on the left-hand side in (1.64) is the lost second-period profit if entry occurs, weighted by the prior probability that the entrant has low cost.

For the linear example, the first-period gain from defection is given by (1.51) and (1.52). The revelation effect is given by (1.53). In contrast to the result for case 2(a), in case 3(a) entry always results in lower output and profit for the defecting high-cost firm:

$$\pi_{1H}(u_{12}, 1) - \pi_{1H}(u_{12}, 0, 1, 0, 1, u_{32}) \tag{1.66}$$
$$= [q_{1H}(u_{12}, 1) + q_{1H}(u_{12}, 0, 1, 0, 1, u_{32})]\,[q_{1H}(u_{12}, 1) - q_{1H}(u_{12}, 0, 1, 0, 1, u_{32})].$$

$q_{1H}(u_{12}, 1)$ is given by (1.57);

$$q_{1H}(u_{12}, 0, 1, 0, 1, u_{32}) = \frac{1}{4}\left[a_H - \frac{3v_{12} + 1 + u_{32}}{2}(c_H - c_L)\right]. \tag{1.67}$$

Then

$$q_{1H}(u_{12}, 1) - q_{1H}(u_{12}, 0, 1, 0, 1, u_{32}) = \frac{a_H}{12} + \left(\frac{v_{12}}{24} + \frac{1 + u_{32}}{8}\right)(c_H - c_L) > 0. \tag{1.68}$$

In case 3(a), if entry occurs at all, it is known that the entrant has low cost. With downward sloping reaction functions, this induces incumbents to restrict output.

1.4. Separating equilibrium: payoffs from entry

To determine incumbents' separating and defection payoffs, we need to specify the relationship between entry costs and expected payoffs for entrants of different cost types. Entrants' payoffs can be ranked as follows:

$$\pi_{3L}(1;0;1;0;1;1) > \pi_{3L}(0;0;1;0;1;0) > \pi_{3L}(0;0;0;0;0;0) \tag{1.69}$$

$$\geq \pi_{3H}(1;u_{13};1;u_{23};1;1) > \pi_{3H}(0;u_{13};1;u_{23};1;0) > \pi_{3H}(0;u_{13};0;u_{23};0;0):$$

To establish these inequalities for quantity-setting firms, it is sufficient to show that the corresponding relationships hold for outputs. Evaluating (1.34) and (1.37) for alternative beliefs,

$$q_{3L}(1;0;1;0;1;1) = \frac{1}{4}[a_L + 2(c_H - c_L)] \tag{1.70}$$

$$q_{3L}(0;0;1;0;1;0) = \frac{1}{4}[a_L + (c_H - c_L)] \tag{1.71}$$

$$q_{3L}(0;0;0;0;0;0) = \frac{1}{4}a_L \tag{1.72}$$

$$q_{3H}(1;u_{13};1;u_{23};1;1) = \frac{1}{4}\left[a_H - \frac{u_{13}+u_{23}-2}{2}(c_H - c_L)\right] \tag{1.73}$$

$$q_{3H}(0;u_{13};1;u_{23};1;0) = \frac{1}{4}\left[a_H - \frac{u_{13}+u_{23}}{2}(c_H - c_L)\right] \tag{1.74}$$

$$q_{3H}(0;u_{13};0;u_{23};0;0) = \frac{1}{4}\left[a_H - \frac{u_{23}+u_{13}+2}{2}(c_H - c_L)\right] \tag{1.75}$$

The only pairwise comparison that is not immediate is between (1.72) and (1.73); for this, by subtraction,

$$q_{3L}(0;0;0;0;0;0) - q_{3H}(1;u_{13};1;u_{23};1;1) = \frac{u_{13}+u_{23}}{8}(c_H - c_L) \geq 0; \tag{1.76}$$

with equality holding only if $u_{13} = u_{23} = 0$.

1.5. Separation by L-firm output expansion

It can be a sequential equilibrium for low-cost incumbents to separate by producing outputs so great that their (notional) high-cost counterparts would not find it profitable to imitate. Consider the following collection of strategies and beliefs:

(a) firms 1L and 2L produce outputs $q_{1*} > q_{1L}$ and $q_{2*} > q_{2L}$, respectively, in period 1;

(b) firms 1H and 2H produce their best-response outputs q_{1h} and q_{2h}, respectively, in period 1;

(c) incumbents' cost types are revealed by observing first-period outputs;

(d) the potential entrant comes into the market in the second period if it expects a positive second-period profit, net of entry cost;

(e) if the fact of entry does not reveal the entrant's cost type, incumbents carry forward prior beliefs from the first period to the second; all firms maximize expected profit in the second period;

If firm 1 produces any output other than q_{1*} (firm 2 produces any output other than q_{2*}), rivals infer that it has high cost.

The low-cost entrant maximizes its expected payoff by behaving as indicated, given that other players behave as indicated. An additional condition is that the high-cost entrant must not wish to mimic a low-cost entrant; see the discussion in the section of separation on Nash duopoly outputs.

1.5.1. High-cost incumbents

The inequalities (1.69) define the seven ranges of entry cost that must be considered to examine incumbents' incentives to separate. Firm 1H's separation and defection payoffs for the seven ranges, and the conditions for firm 1H to separate, are:

Case S1 $K > \pi_{3L}(1;0;1;0;1;1)$

Separation payoff:

$$\pi_{1H}(q_{1h}) + u_{12}\pi_{1H}(1;1) + v_{12}\pi_{1H}(0;1): \quad (1.77)$$

Defection payoff:

$$\pi_{1H}(q_{1*}) + u_{12}\pi_{1H}(1;0) + v_{12}\pi_{1H}(0;0): \quad (1.78)$$

Condition for 1H to separate:

$$\pi_{1H}(q_{1h}) - \pi_{1H}(q_{1*}) \geq u_{12}[\pi_{1H}(1;0) - \pi_{1H}(1;1)] + v_{12}[\pi_{1H}(0;0) - \pi_{1H}(0;1)]: \quad (1.79)$$

Case S2 $\pi_{3L}(1;0;1;0;1;1) > K \geq \pi_{3L}(0;0;1;0;1;0)$

Separation payoff:

$$\pi_{1H}(q_{1h}) + u_{12}[u_{13}\pi_{1H}(1;1) + v_{13}\pi_{1H}(1;0;1;0;1;1)] + v_{12}\pi_{1H}(0;1): \tag{1.80}$$

Defection payoff:

$$\pi_{1H}(q_{1\lambda}) + u_{12}\pi_{1H}(1;0) + v_{12}\pi_{1H}(0;0): \tag{1.81}$$

Condition for 1H to separate:

$$\begin{aligned} \pi_{1H}(q_{1h}) - \pi_{1H}(q_{1\lambda}) &\geq u_{12}u_{13}[\pi_{1H}(1;0) - \pi_{1H}(1;1)] \\ + v_{12}[\pi_{1H}(0;0) - \pi_{1H}(0;1)] &+ u_{12}v_{13}[\pi_{1H}(1;0) - \pi_{1H}(1;0;1;0;1;1)]: \end{aligned} \tag{1.82}$$

Case S3 $\pi_{3L}(0;0;1;0;1;0) > K \geq \pi_{3L}(0;0;0;0;0;0)$

Separation payoff:

$$\begin{aligned} \pi_{1H}(q_{1h}) + u_{12}[u_{13}\pi_{1H}(1;1) + v_{13}\pi_{1H}(1;0;1;0;1;1)] \\ + v_{12}[u_{13}\pi_{1H}(0;1) + v_{13}\pi_{1H}(0;0;1;0;1;0)]: \end{aligned} \tag{1.83}$$

Defection payoff:

$$\pi_{1H}(q_{1\lambda}) + u_{12}[u_{13}\pi_{1H}(1;0) + v_{13}\pi_{1H}(1;0;0;0;0;1)] + v_{12}\pi_{1H}(0;0): \tag{1.84}$$

Condition for 1H to separate:

$$\pi_{1H}(q_{1h}) - \pi_{1H}(q_{1\lambda}) \geq \tag{1.85}$$

$$\begin{aligned} + u_{12}u_{13}[\pi_{1H}(1;0) - \pi_{1H}(1;1)] + u_{12}v_{13}[\pi_{1H}(1;0;0;0;0;1) - \pi_{1H}(1;0;1;0;1;1)] \\ + v_{12}u_{13}[\pi_{1H}(0;0) - \pi_{1H}(0;1)] + v_{12}v_{13}[\pi_{1H}(0;0) - \pi_{1H}(0;0;1;0;1;0)] \end{aligned}$$

Case S4 $\pi_{3L}(0;0;0;0;0;0) > K \geq \pi_{3H}(1;u_{13};1;u_{23};1;1)$

Separation payoff:

$$\begin{aligned} \pi_{1H}(q_{1h}) + u_{12}[u_{13}\pi_{1H}(1;1) + v_{13}\pi_{1H}(1;0;1;0;1;1)] \\ + v_{12}[u_{13}\pi_{1H}(0;1) + v_{13}\pi_{1H}(0;0;1;0;1;0)]: \end{aligned} \tag{1.86}$$

Defection payoff:

$$\begin{aligned} \pi_{1H}(q_{1\lambda}) + u_{12}[u_{13}\pi_{1H}(1;0) + v_{13}\pi_{1H}(1;0;0;0;0;1)] \\ + v_{12}[u_{13}\pi_{1H}(0;0) + v_{13}\pi_{1H}(0;0;0;0;0;0)]: \end{aligned} \tag{1.87}$$

Condition for 1H to separate:

$$\pi_{1H}(q_{1h}) - \pi_{1H}(q_{1\lambda}) \geq \tag{1.88}$$

$$\begin{aligned} u_{12}u_{13}[\pi_{1H}(1;0) - \pi_{1H}(1;1)] + u_{12}v_{13}[\pi_{1H}(1;0;0;0;0;1) - \pi_{1H}(1;0;1;0;1;1)] \\ + v_{12}u_{13}[\pi_{1H}(0;0) - \pi_{1H}(0;1)] + v_{12}v_{13}[\pi_{1H}(0;0;0;0;0;0) - \pi_{1H}(0;0;1;0;1;0)]: \end{aligned}$$

Case S5 $\pi_{3H}(1, u_{13}, 1, u_{23}, 1, 1) > K \geq \pi_{3H}(0, u_{13}, 1, u_{23}, 1, 0)$

Separation payoff:

$$\pi_{1H}(q_{1h}) + u_{12}\pi_{1H}(1, u_{13}, 1, u_{23}, 1, 1) + v_{12}[u_{13}\pi_{1H}(0, 1) + v_{13}\pi_{1H}(0, 0, 1, 0, 1, 0)]. \quad (1.89)$$

Defection payoff:

$$\pi_{1H}(q_{1\lambda}) + u_{12}[u_{13}\pi_{1H}(1, 0) + v_{13}\pi_{1H}(1, 0, 0, 0, 0, 1)] + v_{12}[u_{13}\pi_{1H}(0, 0) + v_{13}\pi_{1H}(0, 0, 0, 0, 0, 0)]. \quad (1.90)$$

Condition for 1H to separate:

$$\pi_{1H}(q_{1h}) - \pi_{1H}(q_{1\lambda}) \geq u_{12}u_{13}[\pi_{1H}(1, 0) - \pi_{1H}(1, u_{13}, 1, u_{23}, 1, 1)] + u_{12}v_{13}[\pi_{1H}(1, 0, 0, 0, 0, 1) - \pi_{1H}(1, u_{13}, 1, u_{23}, 1, 1)] + v_{12}u_{13}[\pi_{1H}(0, 0) - \pi_{1H}(0, 1)] + v_{12}v_{13}[\pi_{1H}(0, 0, 0, 0, 0, 0) - \pi_{1H}(0, 0, 1, 0, 1, 0)]. \quad (1.91)$$

Case S6 $\pi_{3H}(0, u_{13}, 1, u_{23}, 1, 0) > K \geq \pi_{3H}(0, u_{13}, 0, u_{23}, 0, 0)$

Separation payoff:

$$\pi_{1H}(q_{1h}) + u_{12}\pi_{1H}(1, u_{13}, 1, u_{23}, 1, 1) + v_{12}\pi_{12}(0, u_{13}, 1, u_{23}, 1, 0). \quad (1.92)$$

Defection payoff:

$$\pi_{1H}(q_{1\lambda}) + u_{12}\pi_{1H}(1, u_{13}, 0, u_{23}, 0, 1) + v_{12}[u_{13}\pi_{1H}(0, 0) + v_{13}\pi_{1H}(0, 0, 0, 0, 0, 0)]. \quad (1.93)$$

Condition for 1H to separate:

$$\pi_{1H}(q_{1h}) - \pi_{1H}(q_{1\lambda}) \geq u_{12}[\pi_{1H}(1, u_{13}, 0, u_{23}, 0, 1) - \pi_{1H}(1, u_{13}, 1, u_{23}, 1, 1)] + v_{12}u_{13}[\pi_{1H}(0, 0) - \pi_{1H}(0, u_{13}, 1, u_{23}, 1, 0)] + v_{12}v_{13}[\pi_{1H}(0, 0, 0, 0, 0, 0) - \pi_{12}(0, u_{13}, 1, u_{23}, 1, 0)]. \quad (1.94)$$

Case S7 $\pi_{3H}(0, u_{13}, 0, u_{23}, 0, 0) > K$

Separation payoff:

$$\pi_{1H}(q_{1h}) + u_{12}\pi_{1H}(1, u_{13}, 1, u_{23}, 1, 1) + v_{12}\pi_{1H}(0, u_{13}, 1, u_{23}, 1, 0). \quad (1.95)$$

Defection payoff:

$$\pi_{1H}(q_{1\lambda}) + u_{12}\pi_{1H}(1, u_{13}, 0, u_{23}, 0, 1) + v_{12}\pi_{1H}(0, u_{13}, 0, u_{23}, 0, 0). \quad (1.96)$$

Condition for 1H to separate:

$$\pi_{1H}(q_{1h}) - \pi_{1H}(q_{1\lambda}) \geq u_{12}[\pi_{1H}(1, u_{13}, 0, u_{23}, 0, 1) - \pi_{1H}(1, u_{13}, 1, u_{23}, 1, 1)] + v_{12}[\pi_{1H}(0, u_{13}, 0, u_{23}, 0, 0) - \pi_{1H}(0, u_{13}, 1, u_{23}, 1, 0)]. \quad (1.97)$$

In cases S1, S4, and S7, the potential entrant's decision is not affected by action of the high-cost incumbent, with the result that the possibility of entry or entry deterrence does not affect the incentives of the high-cost incumbent. In the remaining cases, the possibility of deterring entry is an incentive to defect from the separating strategy. For high-cost incumbents, separation is not entry-limiting behavior.

1.5.2. Low-cost incumbents

Case S1 $K > \pi_{3L}(1,0,1,0,1,1)$

Separation payoff:

$$\pi_{1L}(q_{1*}, q_{2*}) + u_{12}\pi_{1L}(1,0) + v_{12}\pi_{1L}(0,0). \tag{1.98}$$

Defection payoff:

$$\pi^{br}_{1L}(q_{1*}, q_{2*}) + u_{12}\pi_{1L}(1,1) + v_{12}\pi_{1L}(0,1). \tag{1.99}$$

Condition for 1L to separate:

$$u_{12}[\pi_{1L}(1,0) - \pi_{1L}(1,1)] + v_{12}[\pi_{1L}(0,0) - \pi_{1L}(0,1)] \geq \pi^{br}_{1L}(q_{1*}, q_{2*}) - \pi_{1L}(q_{1*}, q_{2*}) \tag{1.100}$$

Case S2 $\pi_{3L}(1,0,1,0,1,1) > K \geq \pi_{3L}(0,0,1,0,1,0)$

Separation payoff:

$$\pi_{1L}(q_{1*}, q_{2*}) + u_{12}\pi_{1L}(1,0) + v_{12}\pi_{1L}(0,0). \tag{1.101}$$

Defection payoff:

$$\pi^{br}_{1L}(q_{1*}, q_{2*}) + u_{12}[u_{13}\pi_{1L}(1,1) + v_{13}\pi_{1L}(1,0,1,0,1,1)] + v_{12}\pi_{1L}(0,1). \tag{1.102}$$

Condition for 1L to separate:

$$u_{12}u_{13}[\pi_{1L}(1,0) - \pi_{1L}(1,1)] + u_{12}v_{13}[\pi_{1L}(1,0) - \pi_{1L}(1,0,1,0,1,1)] + v_{12}[\pi_{1L}(0,0) - \pi_{1L}(0,1)] \geq \pi^{br}_{1L}(q_{1*}, q_{2*}) - \pi_{1L}(q_{1*}, q_{2*}). \tag{1.103}$$

Case S3 $\pi_{3L}(0,0,1,0,1,0) > K \geq \pi_{3L}(0,0,0,0,0,0)$

Separation payoff:

$$\pi_{1L}(q_{1*}, q_{2*}) + u_{12}[u_{13}\pi_{1L}(1,0) + v_{13}\pi_{1L}(1,0,0,0,0,1)] + v_{12}\pi_{1L}(0,0). \tag{1.104}$$

Defection payoff:

$$\pi^{br}_{1L}(q_{1*}, q_{2*}) + u_{12}[u_{13}\pi_{1L}(1,1) + v_{13}\pi_{1L}(1,0,1,0,1,1)] + v_{12}[u_{13}\pi_{1L}(0,1) + v_{13}\pi_{1L}(0,0,1,0,1,0)]. \tag{1.105}$$

Condition for 1L to separate:

$$u_{12}u_{13}[\pi_{1L}(1,0) - \pi_{1L}(1,1)] + u_{12}v_{13}[\pi_{1L}(1,0,0,0,0,1) - \pi_{1L}(1,0,1,0,1,1)] + v_{12}u_{13}[\pi_{1L}(0,0) - \pi_{1L}(0,1)] + v_{12}v_{13}[\pi_{1L}(0,0) - \pi_{1L}(0,0,1,0,1,0)] \geq \pi^{br}_{1L}(q_{1*}, q_{2*}) - \pi_{1L}(q_{1*}, q_{2*}). \tag{1.106}$$

Case S4 $\pi_{3L}(0;0;0;0;0;0) > K \ge \pi_{3H}(1;u_{13};1;u_{23};1;1)$

Separation payoff:

$$\pi_{1L}(q_{1\lambda};q_{2\lambda}) + u_{12}[u_{13}\pi_{1L}(1;0) + v_{13}\pi_{1L}(1;0;0;0;0;1)] \tag{1.107}$$

$$+ v_{12}[u_{13}\pi_{1L}(0;0) + v_{13}\pi_{1L}(0;0;0;0;0;0)]:$$

Defection payoff:

$$\pi_{1L}^{br}(q_{1\lambda};q_{2\lambda}) + u_{12}[u_{13}\pi_{1L}(1;1) + v_{13}\pi_{1L}(1;0;1;0;1;1)] \tag{1.108}$$

$$+ v_{12}[u_{13}\pi_{1L}(0;1) + v_{13}\pi_{1L}(0;0;1;0;1;0)]:$$

Condition for 1L to separate:

$$u_{12}u_{13}[\pi_{1L}(1;0) - \pi_{1L}(1;1)] + u_{12}v_{13}[\pi_{1L}(1;0;0;0;0;1) - \pi_{1L}(1;0;1;01;1)] \tag{1.109}$$

$$+ v_{12}u_{13}[\pi_{1L}(0;0) - \pi_{1L}(0;1)] + v_{12}v_{13}[\pi_{1L}(0;0;0;0;0;0) - \pi_{1L}(0;0;1;0;1;0)]$$

$$\ge \pi_{1L}^{br}(q_{1\lambda};q_{2\lambda}) - \pi_{1L}(q_{1\lambda};q_{2\lambda}):$$

Case S5 $\pi_{3H}(1;u_{13};1;u_{23};1;1) > K \ge \pi_{3H}(0;u_{13};1;u_{23};1;0)$

Separation payoff:

$$\pi_{1L}(q_{1\lambda};q_{2\lambda}) + u_{12}[u_{13}\pi_{1L}(1;0) + v_{13}\pi_{1L}(1;0;0;0;0;1)] \tag{1.110}$$

$$+ v_{12}[u_{13}\pi_{1L}(0;0) + v_{13}\pi_{1L}(0;0;0;0;0;0)]:$$

Defection payoff:

$$\pi_{1L}^{br}(q_{1\lambda};q_{2\lambda}) + u_{12}\pi_{1L}(1;u_{13};1;u_{23};1;1)] \tag{1.111}$$

$$+ v_{12}[u_{13}\pi_{1L}(0;1) + v_{13}\pi_{1L}(0;0;1;0;1;0)]:$$

The condition for firm 1L to pool is

$$u_{12}u_{13}[\pi_{1L}(1;0) - \pi_{1L}(1;u_{13};1;u_{23};1;1)] \tag{1.112}$$

$$+ u_{12}v_{13}[\pi_{1L}(1;0;0;0;0;1) - \pi_{1L}(1;u_{13};1;u_{23};1;1)]$$

$$+ v_{12}u_{13}[\pi_{1L}(0;0) - \pi_{1L}(0;1)] + v_{12}v_{13}[\pi_{1L}(0;0;0;0;0;0) - \pi_{1L}(0;0;1;0;1;0)]$$

$$\ge \pi_{1L}^{br}(q_{1\lambda};q_{2\lambda}) - \pi_{1L}(q_{1\lambda};q_{2\lambda}):$$

Case S6 $\pi_{3H}(0, u_{13}, 1, u_{23}, 1, 0) > K \geq \pi_{3H}(0, u_{13}, 0, u_{23}, 0, 0)$

Separation payoff:

$$\pi_{1L}(q_{1\lambda}, q_{2\lambda}) + u_{12}\pi_{1L}(1, u_{13}, 0, u_{23}, 0, 1) \tag{1.113}$$

$$+ v_{12}[u_{13}\pi_{1L}(0, 0) + v_{13}\pi_{1L}(0, 0, 0, 0, 0, 0)].$$

Defection payoff:

$$\pi^{br}_{1L}(q_{1\lambda}, q_{2\lambda}) + u_{12}\pi_{1L}(1, u_{13}, 1, u_{23}, 1, 1) + v_{12}\pi_{1L}(0, u_{13}, 1, u_{23}, 1, 0). \tag{1.114}$$

Condition for 1L to separate:

$$u_{12}[\pi_{1L}(1, u_{13}, 0, u_{23}, 0, 1) - \pi_{1L}(1, u_{13}, 1, u_{23}, 1, 1)] \tag{1.115}$$

$$+ v_{12}u_{13}[\pi_{1L}(0, 0) - \pi_{1L}(0, u_{13}, 1, u_{23}, 1, 0)]$$

$$+ v_{12}v_{13}[\pi_{1L}(0, 0, 0, 0, 0, 0) - \pi_{1L}(0, u_{13}, 1, u_{23}, 1, 0)] \geq \pi^{br}_{1L}(q_{1\lambda}, q_{2\lambda}) - \pi_{1L}(q_{1\lambda}, q_{2\lambda}).$$

Case S7 $\pi_{3H}(0, u_{13}, 0, u_{23}, 0, 0) > K$

Separation payoff:

$$\pi_{1L}(q_{1\lambda}, q_{2\lambda}) + u_{12}\pi_{1L}(1, u_{13}, 0, u_{23}, 0, 1) + v_{12}\pi_{1L}(0, u_{13}, 0, u_{23}, 0, 0) \tag{1.116}$$

Defection payoff:

$$\pi^{br}_{1L}(q_{1\lambda}, q_{2\lambda}) + u_{12}\pi_{1L}(1, u_{13}, 1, u_{23}, 1, 1) + v_{12}\pi_{1L}(0, u_{13}, 1, u_{13}, 1, 0). \tag{1.117}$$

Condition for 1L to separate:

$$u_{12}[\pi_{1L}(1, u_{13}, 0, u_{23}, 0, 1) - \pi_{1L}(1, u_{13}, 1, u_{23}, 1, 1)] \tag{1.118}$$

$$+ v_{12}[\pi_{1L}(0, u_{13}, 0, u_{23}, 0, 0) - \pi_{1L}(0, u_{13}, 1, u_{13}, 1, 0)] \geq \pi^{br}_{1L}(q_{1\lambda}, q_{2\lambda}) - \pi_{1L}(q_{1\lambda}, q_{2\lambda}).$$

Separation by L-firm output expansion: resumé In cases S1, S4, and S7, the potential entrant's decision is not affected by action of the low-cost incumbent, with the result that the possibility of entry or entry deterrence does not affect the incentives of the low-cost incumbent. In the remaining cases, the possibility of deterring entry is an incentive to separate. For low-cost incumbents, separation by output expansion is entry-limiting behavior in four of the possible seven cases.

1.5.3. Linear example

I consider only case S2 here. If firms 1L and 2L expand output so that the high-cost firm separation conditions hold with equality (to expand output even more would lower own profit without altering rivals' behaviors), the high-cost incumbent separation conditions are (compare (1.82))

$$\pi_{1H}(q_{1h}) - \pi_{1H}(q_{1\lambda}) = u_{12}u_{13}[\pi_{1H}(1;0) - \pi_{1H}(1;1)] \tag{1.119}$$

$$+\ v_{12}[\pi_{1H}(0;0) - \pi_{1H}(0;1)] \ +\ u_{12}v_{13}[\pi_{1H}(1;0) - \pi_{1H}(1;0;1;0;1;1)] = RHS^2_{1H};$$

$$\pi_{2H}(q_{2h}) - \pi_{2H}(q_{2\lambda}) = u_{21}u_{23}[\pi_{2H}(0;1) - \pi_{2H}(1;1)] \tag{1.120}$$

$$+\ v_{21}[\pi_{1H}(0;0) - \pi_{2H}(1;0)] \ +\ u_{21}v_{23}[\pi_{2H}(0;1) - \pi_{2H}(1;0;1;0;1;1)] = RHS^2_{2H};$$

where the terms RHS_{1H} and RHS_{2H} are defined for notational convenience.

To evaluate $\pi_{1H}(q_{1h}) - \pi_{1H}(q_{1\lambda})$, write

$$\pi_{1H}(q_{1\lambda}) = [a_H - q_{1\lambda} - u_{12}q_{2h} - v_{12}q_{2\lambda}]q_{1\lambda} \tag{1.121}$$

$$= [a_H - q_{1h} - u_{12}q_{2h} - v_{12}q_{2\lambda} - (q_{1\lambda} - q_{1h})][q_{1h} + (q_{1\lambda} - q_{1h})]$$

$$= [q_{1h} - (q_{1\lambda} - q_{1h})][q_{1h} + (q_{1\lambda} - q_{1h})]$$

(using the equation of 1H's reaction function)

$$= q^2_{1h} - (q_{1\lambda} - q_{1h})^2$$

$$= \pi_{1H}(q_{1h}) - (q_{1\lambda} - q_{1h})^2:$$

Hence

$$\pi_{1H}(q_{1h}) - \pi_{1H}(q_{1\lambda}) = (q_{1\lambda} - q_{1h})^2; \tag{1.122}$$

and in like manner

$$\pi_{2H}(q_{2h}) - \pi_{2H}(q_{2\lambda}) = (q_{2\lambda} - q_{2h})^2: \tag{1.123}$$

The high-cost firm separation conditions (1.119) and (1.120) become

$$q_{1\lambda} - q_{1h} = RHS_{1H} \tag{1.124}$$

and

$$q_{2\lambda} - q_{2h} = RHS_{2H} \tag{1.125}$$

respectively.

q_{1h} and q_{2h} are best reply outputs. From the equations of the firm 1H, 2H reaction functions, therefore, q_{1h} and q_{2h} are related to $q_{1\lambda}$ and $q_{2\lambda}$ via the system of equations

$$\begin{pmatrix} 2 & u_{12} \\ u_{21} & 2 \end{pmatrix}\begin{pmatrix} q_{1h} \\ q_{2h} \end{pmatrix} = a_H\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 & v_{12} \\ v_{21} & 0 \end{pmatrix}\begin{pmatrix} q_{1\lambda} \\ q_{2\lambda} \end{pmatrix} \tag{1.126}$$

By subtraction,

$$\begin{pmatrix} 2 & u_{12} \\ u_{21} & 2 \end{pmatrix} \begin{pmatrix} q_{1h} - q_{2*} \\ q_{2h} - q_{2*} \end{pmatrix} = a_H \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} q_{1*} \\ q_{2*} \end{pmatrix}. \tag{1.127}$$

Now substituting from (1.124) and (1.125), we obtain a system of two equations that determines the smallest values of q_{1*} and q_{2*} that high-cost incumbents will not be willing to mimic:

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} q_{1*} \\ q_{2*} \end{pmatrix} = a_H \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 & u_{12} \\ u_{21} & 2 \end{pmatrix} \begin{pmatrix} RHS_{1H} \\ RHS_{2H} \end{pmatrix}. \tag{1.128}$$

$$\begin{pmatrix} q_{1*} \\ q_{2*} \end{pmatrix} = \frac{a_H}{3} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 4 - u_{21} & -2v_{12} \\ -2v_{21} & 4 - u_{12} \end{pmatrix} \begin{pmatrix} RHS_{1H} \\ RHS_{2H} \end{pmatrix}. \tag{1.129}$$

It remains to determine whether the 1L separation condition, (1.103), and the corresponding condition for 2L, are satisfied.

1.6. Separation on Nash duopoly outputs

This is the separating equilibrium discussed in the text.

1.6.1. High-cost entrants

In cases S5, S6, and S7 entry does not reveal the entrant's cost type, so the possibility of the high-cost entrant masquerading as a low-cost firm does not arise.

In the remaining cases, the most favorable situation for the entrant occurs if both incumbents are revealed as having high cost. In this occurs and entry costs place the game in case S4, if a high-cost entrant comes into the market and incumbents believe that it has low cost, its profit-maximizing output is

$$q_{3H}(1;0;1;0;1;1) =$$

$$\frac{1}{4}\left[a_H - \frac{3(0+0) - (0+1+0+1)}{2}(c_H - c_L)\right] \tag{1.130}$$

$$= \frac{1}{4}(a_H + c_H - c_L) = \frac{1}{4}a_L = q_{3L}(0;0;0;0;0;0);$$

and its gross payoff is the square of its output. By definition of S4, this will yield a positive payoff after allowing for entry cost. In case S4, a high-cost entrant would always wish to mimic a low-cost entrant if both incumbents had revealed themselves as having high cost.

In cases S3, S2, and S1, entry cost is so large that it is not profitable for a firm producing equilibrium output $a_L/4$ to come into the market. A high-cost entrant would therefore not masquerade as a low-cost entrant in these cases.

1.6.2. High-cost incumbents

The inequalities (1.69) define the seven ranges of entry cost that must be considered to examine incumbents' incentives to separate. Firm 1H's separation and defection payoffs for the seven ranges, and the conditions for firm 1H to separate, are:

Case S1 $K > \pi_{3L}(1,0,1,0,1,1)$

Separation payoff:

$$\pi_{1H}(u_{12}, u_{21}) + u_{12}\pi_{1H}(1,1) + v_{12}\pi_{1H}(0,1). \tag{1.131}$$

Defection payoff:

$$\pi_{1H}(q_{1L}) + u_{12}\pi_{1H}(1,0) + v_{12}\pi_{1H}(0,0). \tag{1.132}$$

Condition for 1H to separate:

$$\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(q_{1L}) \geq u_{12}[\pi_{1H}(1,0) - \pi_{1H}(1,1)] + v_{12}[\pi_{1H}(0,0) - \pi_{1H}(0,1)]. \tag{1.133}$$

Case S2 $\pi_{3L}(1,0,1,0,1,1) > K \geq \pi_{3L}(0,0,1,0,1,0)$

Separation payoff:

$$\pi_{1H}(u_{12}, u_{21}) + u_{12}[u_{13}\pi_{1H}(1,1) + v_{13}\pi_{1H}(1,0,1,0,1,1)] + v_{12}\pi_{1H}(0,1) \tag{1.134}$$

Defection payoff:

$$\pi_{1H}(q_{1L}) + u_{12}\pi_{1H}(1,0) + v_{12}\pi_{1H}(0,0). \tag{1.135}$$

Condition for 1H to separate:

$$\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(q_{1L}) \geq u_{12}u_{13}[\pi_{1H}(1,0) - \pi_{1H}(1,1)] + v_{12}[\pi_{1H}(0,0) - \pi_{1H}(0,1)] + u_{12}v_{13}[\pi_{1H}(1,0) - \pi_{1H}(1,0,1,0,1,1)]. \tag{1.136}$$

Case S3 $\pi_{3L}(0,0,1,0,1,0) > K \geq \pi_{3L}(0,0,0,0,0,0)$

Separation payoff:

$$\pi_{1H}(u_{12}, u_{21}) + u_{12}[u_{13}\pi_{1H}(1,1) + v_{13}\pi_{1H}(1,0,1,0,1,1)] + v_{12}[u_{13}\pi_{1H}(0,1) + v_{13}\pi_{1H}(0,0,1,0,1,0)] \tag{1.137}$$

Defection payoff:

$$\pi_{1H}(q_{1L}) + u_{12}[u_{13}\pi_{1H}(1,0) + v_{13}\pi_{1H}(1,0,0,0,0,1)] + v_{12}\pi_{1H}(0,0). \tag{1.138}$$

Condition for 1H to separate:

$$\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(q_{1L}) \geq + u_{12}u_{13}[\pi_{1H}(1,0) - \pi_{1H}(1,1)] + u_{12}v_{13}[\pi_{1H}(1,0,0,0,0,1) - \pi_{1H}(1,0,1,0,1,1)]. + v_{12}u_{13}[\pi_{1H}(0,0) - \pi_{1H}(0,1)] + v_{12}v_{13}[\pi_{1H}(0,0) - \pi_{1H}(0,0,1,0,1,0)] \tag{1.139}$$

Case S4 $\pi_{3L}(0;0;0;0;0;0) > K \geq \pi_{3H}(1;u_{13};1;u_{23};1;1)$

Separation payoff:

$$\pi_{1H}(u_{12};u_{21}) + u_{12}[u_{13}\pi_{1H}(1;1) + v_{13}\pi_{1H}(1;0;1;0;1;1)] \tag{1.140}$$
$$+ v_{12}[u_{13}\pi_{1H}(0;1) + v_{13}\pi_{1H}(0;0;1;0;1;0)]:$$

Defection payoff:

$$\pi_{1H}(q_{1L}) + u_{12}[u_{13}\pi_{1H}(1;0) + v_{13}\pi_{1H}(1;0;0;0;0;1)] \tag{1.141}$$
$$+ v_{12}[u_{13}\pi_{1H}^{br}(0;0) + v_{13}\pi_{1H}^{br}(0;0;0;0;0;0)]:$$

Condition for 1H to separate:

$$\pi_{1H}(u_{12};u_{21}) - \pi_{1H}(q_{1L}) \geq \tag{1.142}$$
$$u_{12}u_{13}[\pi_{1H}(1;0) - \pi_{1H}(1;1)] + u_{12}v_{13}[\pi_{1H}(1;0;0;0;0;1) - \pi_{1H}(1;0;1;0;1;1)]$$
$$+ v_{12}u_{13}[\pi_{1H}(0;0) - \pi_{1H}(0;1)] + v_{12}v_{13}[\pi_{1H}(0;0;0;0;0;0) - \pi_{1H}(0;0;1;0;1;0)]:$$

Case S5 $\pi_{3H}(1;u_{13};1;u_{23};1;1) > K \geq \pi_{3H}(0;u_{13};1;u_{23};1;0)$

Separation payoff:

$$\pi_{1H}(u_{12};u_{21}) + u_{12}\pi_{1H}(1;u_{13};1;u_{23};1;1) \tag{1.143}$$
$$+ v_{12}[u_{13}\pi_{1H}(0;1) + v_{13}\pi_{1H}(0;0;1;0;1;0)]:$$

Defection payoff:

$$\pi_{1H}(q_{1L}) + u_{12}[u_{13}\pi_{1H}(1;0) + v_{13}\pi_{1H}(1;0;0;0;0;1)] \tag{1.144}$$
$$+ v_{12}[u_{13}\pi_{1H}(0;0) + v_{13}\pi_{1H}(0;0;0;0;0;0)]:$$

Condition for 1H to separate:

$$\pi_{1H}(u_{12};u_{21}) - \pi_{1H}(q_{1L}) \geq u_{12}u_{13}[\pi_{1H}(1;0) - \pi_{1H}(1;u_{13};1;u_{23};1;1)] \tag{1.145}$$
$$+ u_{12}v_{13}[\pi_{1H}(1;0;0;0;0;1) - \pi_{1H}(1;u_{13};1;u_{23};1;1)]$$
$$+ v_{12}u_{13}[\pi_{1H}(0;0) - \pi_{1H}(0;1)] + v_{12}v_{13}[\pi_{1H}(0;0;0;0;0;0) - \pi_{1H}(0;0;1;0;1;0)]:$$

Case S6 $\pi_{3H}(0;u_{13};1;u_{23};1;0) > K \geq \pi_{3H}(0;u_{13};0;u_{23};0;0)$

Separation payoff:

$$\pi_{1H}(u_{12};u_{21}) + u_{12}\pi_{1H}(1;u_{13};1;u_{23};1;1) + v_{12}\pi_{12}(0;u_{13};1;u_{23};1;0): \tag{1.146}$$

Defection payoff:

$$\pi_{1H}(q_{1L}) + u_{12}\pi_{1H}(1;u_{13};0;u_{23};0;1) + v_{12}[u_{13}\pi_{1H}(0;0) + v_{13}\pi_{1H}(0;0;0;0;0;0)] \tag{1.147}$$

Condition for 1H to separate:

$$\pi_{1H}(u_{12};u_{21}) - \pi_{1H}(q_{1L}) \geq u_{12}[\pi_{1H}(1;u_{13};0;u_{23};0;1) - \pi_{1H}(1;u_{13};1;u_{23};1;1)] \tag{1.148}$$
$$+ v_{12}u_{13}[\pi_{1H}(0;0) - \pi_{1H}(0;u_{13};1;u_{23};1;0)]$$
$$+ v_{12}v_{13}[\pi_{1H}(0;0;0;0;0;0) - \pi_{12}(0;u_{13};1;u_{23};1;0)]:$$

Case S7 $\pi_{3H}(0,u_{13},0,u_{23},0,0) > K$

Separation payoff:

$$\pi_{1H}(u_{12},u_{21}) + u_{12}\pi_{1H}(1,u_{13},1,u_{23},1,1) + v_{12}\pi_{1H}(0,u_{13},1,u_{23},1,0). \tag{1.149}$$

Defection payoff:

$$\pi_{1H}(q_{1L}) + u_{12}\pi_{1H}(1,u_{13},0,u_{23},0,1) + v_{12}\pi_{1H}(0,u_{13},0,u_{23},0,0) \tag{1.150}$$

Condition for 1H to separate:

$$\pi_{1H}(u_{12},u_{21}) - \pi_{1H}(q_{1L}) \geq u_{12}[\pi_{1H}(1,u_{13},0,u_{23},0,1) - \pi_{1H}(1,u_{13},1,u_{23},1,1)] \tag{1.151}$$

$$+ v_{12}[\pi_{1H}(0,u_{13},0,u_{23},0,0) - \pi_{1H}(0,u_{13},1,u_{23},1,0)].$$

In cases S1, S4, and S7, the potential entrant's decision is not affected by action of the high-cost incumbent, with the result that the possibility of entry or entry deterrence does not affect the incentives of the high-cost incumbent. In the remaining cases, the possibility of deterring entry is an incentive to defect from the intuitive separating strategy. For high-cost incumbents, separation on equilibrium outputs from the one-period game is not entry-limiting behavior.

2. Price-setting firms

2.1. Duopoly

Inverting the equations of the inverse demand curves

$$p_1 - c_1 = a - c_1 - \theta q_2 - q_1 \tag{2.1}$$

$$p_2 - c_2 = a - c_2 - \theta q_1 - q_2, \tag{2.2}$$

one obtains the equations of the demand curves

$$q_1 = \frac{(a - c_1) - \theta(a - c_2) + \theta(p_2 - c_2) - (p_1 - c_1)}{1 - \theta^2} \tag{2.3}$$

$$q_2 = \frac{(a - c_2) - \theta(a - c_1) + \theta(p_1 - c_1) - (p_2 - c_2)}{1 - \theta^2} \tag{2.4}$$

The equations of the inverse demand curves are valid for nonnegative prices; the equations of the demand curves are valid for nonnegative quantities.

Firm 1L's expected profit is

$$\pi_{1L} = \bar{p}_{1L}\left[u_{12}\left(\frac{a_L - \theta a_H + \theta\bar{p}_{2H} - \bar{p}_{1L}}{1-\theta^2}\right) + v_{12}\left(\frac{a_L - \theta a_L + \theta\bar{p}_{2L} - \bar{p}_{1L}}{1-\theta^2}\right)\right] \tag{2.5}$$

$$= \bar{p}_{1L}\left[\frac{(1-\theta)a_L + \theta u_{12}(a_L - a_H) + u_{12}\theta\bar{p}_{2H} + \theta v_{12}\bar{p}_{2L} - \bar{p}_{1L}}{1-\theta^2}\right];$$

writing $\bar{p}_{1L} = p_{1L} - c_L$, $\bar{p}_{1H} = p_{1H} - c_H$; $\bar{p}_{2L} = p_{2L} - c_L$, $\bar{p}_{2H} = p_{2H} - c_H$; and $a_L = a - c_L$, and $a_H = a - c_H$.

Maximizing (2.5) with respect to $\bar{p}_{1L}$ gives the equation of firm 1L's price reaction function,

$$2\bar{p}_{1L} - u_{12}\theta\bar{p}_{2H} - \theta v_{12}\bar{p}_{2L} = (1-\theta)a_L + \theta u_{12}(a_L - a_H). \tag{2.6}$$

Observe that (2.6) implies

$$(1-\theta)a_L + \theta u_{12}(a_L - a_H) + u_{12}\theta\bar{p}_{2H} + \theta v_{12}\bar{p}_{2L} - \bar{p}_{1L} = \bar{p}_{1L}. \tag{2.7}$$

It follows that anywhere along firm 1L's reaction function, and in particular in equilibrium,

$$\pi_{1L} = \frac{1}{1-\theta^2}\left(\bar{p}_{1L}\right)^2. \tag{2.8}$$

Proceeding in the same way, one obtains the equations of the reaction functions of firms 1H, 2L, and 2H:

$$2\bar{p}_{1H} - u_{12}\theta\bar{p}_{2H} - \theta v_{12}\bar{p}_{2L} = (1-\theta)a_H - \theta v_{12}(a_L - a_H) \tag{2.9}$$

$$2\bar{p}_{2L} - u_{21}\theta\bar{p}_{1H} - \theta v_{21}\bar{p}_{1L} = (1-\theta)a_L + \theta u_{21}(a_L - a_H) \tag{2.10}$$

$$2\bar{p}_{2H} - u_{21}\theta\bar{p}_{1H} - \theta v_{21}\bar{p}_{1L} = (1-\theta)a_H - \theta v_{21}(a_L - a_H) \tag{2.11}$$

The duopoly equilibrium can be illustrated graphically, in a way that parallels the quantity-setting case. Weighting (2.6) and (2.9) by v_{21} and u_{21} respectively and adding gives the equation of firm 1's expected (by firm 2) reaction function:

$$2\bar{p}_1(u_{21}) - \theta\bar{p}_2(u_{12}) = (1-\theta)a_L - (u_{21} - \theta u_{12})(a_L - a_H) \tag{2.12}$$

In like manner, one obtains the equation of firm 2's expected (by firm 1) reaction function,

$$-\theta\bar{p}_1(u_{21}) + 2\bar{p}_2(u_{12}) = (1-\theta)a_L - (u_{12} - \theta u_{21})(a_L - a_H)$$

Insert Figure 4 about here

Graphing the two expected reaction functions, as in Figure 4, gives equilibrium expected prices. Matching the firm 1L and 1H reaction functions with firm 2's equilibrium expected net price, as in Figure 5, illustrates $\bar{p}_{1L}$ and $\bar{p}_{1H}$.

Insert Figure 5 about here

Figure 6 shows all four equilibrium net prices.

Insert Figure 6 about here

Comparing equilibrium prices net of marginal cost, note that $\bar{p}_{1L} > \bar{p}_{1H}$ and $\bar{p}_{2L} > \bar{p}_{2H}$, although the relative magnitudes of prices is the reverse. Low-cost firms charge lower prices and earn greater margins than the corresponding high-cost firms.

To obtain explicit expressions for equilibrium net prices, subtract (2.9) from (2.6) and (2.11) from (2.10) to obtain

$$\bar{p}_{1L} = \bar{p}_{1H} + \frac{c_H - c_L}{2}; \quad (2.13)$$

and

$$\bar{p}_{2L} = \bar{p}_{2H} + \frac{c_H - c_L}{2}: \quad (2.14)$$

respectively.

Using (2.13) and (2.14), the system of four equations in four unknowns can be reduced to two subsystems each of two equations in two unknowns,

$$\begin{pmatrix} 2 & -\mu \\ -\mu & 2 \end{pmatrix} \begin{pmatrix} \bar{p}_{1L} \\ \bar{p}_{2L} \end{pmatrix} = (1 - \mu)a_L + \frac{1}{2}\mu \begin{pmatrix} u_{12} \\ u_{21} \end{pmatrix} (c_H - c_L) \quad (2.15)$$

and

$$\begin{pmatrix} 2 & -\mu \\ -\mu & 2 \end{pmatrix} \begin{pmatrix} \bar{p}_{1H} \\ \bar{p}_{2H} \end{pmatrix} = (1 - \mu)a_H - \frac{1}{2}\mu \begin{pmatrix} v_{12} \\ v_{21} \end{pmatrix} (c_H - c_L): \quad (2.16)$$

Noncooperative equilibrium prices are

$$\bar{p}_{1L} = \frac{1 - \mu}{2 - \mu}a_L + \frac{\mu}{2}\left(\frac{2u_{12} + \mu u_{21}}{4 - \mu^2}\right)(c_H - c_L) \quad (2.17)$$

$$\bar{p}_{2L} = \frac{1 - \mu}{2 - \mu}a_L + \frac{\mu}{2}\left(\frac{\mu u_{12} + 2u_{21}}{4 - \mu^2}\right)(c_H - c_L) \quad (2.18)$$

$$\bar{p}_{1H} = \frac{1 - \mu}{2 - \mu}a_H - \frac{\mu}{2}\left(\frac{2v_{12} + \mu v_{21}}{4 - \mu^2}\right)(c_H - c_L) \quad (2.19)$$

$$\bar{p}_{2H} = \frac{1 - \mu}{2 - \mu}a_H - \frac{\mu}{2}\left(\frac{\mu v_{12} + 2v_{21}}{4 - \mu^2}\right)(c_H - c_L) \quad (2.20)$$

Here and throughout I restrict attention to the case in which net prices are nonnegative (prices are not less than marginal costs).

Having found equilibrium prices, equilibrium payoffs are obtain using (2.8) and the corresponding relationships.

2.2. Triopoly

Inverting the equations of the inverse demand curves

$$p_1 - c_1 = a - c_1 - q_1 - \theta q_2 - \theta q_3 \tag{2.21}$$

$$p_2 - c_2 = a - c_2 - \theta q_1 - q_2 - \theta q_3 \tag{2.22}$$

$$p_3 - c_3 = a - c_3 - \theta q_1 - \theta q_2 - q_3; \tag{2.23}$$

one obtains the equations of the demand curves

$$q_1 = \frac{(1+\theta)a_1 - \theta(a_2 + a_3) - (1+\theta)\bar p_1 + \theta(\bar p_2 + \bar p_3)}{(1-\theta)(1+2\theta)} \tag{2.24}$$

$$q_2 = \frac{(1+\theta)a_2 - \theta(a_1 + a_3) - (1+\theta)\bar p_2 + \theta(\bar p_1 + \bar p_3)}{(1-\theta)(1+2\theta)} \tag{2.25}$$

$$q_3 = \frac{(1+\theta)a_3 - \theta(a_1 + a_2) - (1+\theta)\bar p_3 + \theta(\bar p_1 + \bar p_2)}{(1-\theta)(1+2\theta)} \tag{2.26}$$

Firm 1L's expected profit satisfies

$$(1-\theta)(1+2\theta)\pi_{1L} = \bar p_{1L}\,\{(1-\theta)a_L + \theta(u_{12}+u_{13})(c_H - c_L) + \theta[u_{12}\bar p_{2H} + v_{12}\bar p_{2L} + u_{13}\bar p_{3H} + v_{13}\bar p_{3L}] - (1+\theta)\bar p_{1L}\} \tag{2.27}$$

Maximizing (2.27) with respect to $\bar p_{1L}$ gives the equation of firm 1L's price reaction function,

$$2(1+\theta)\bar p_{1L} - \theta u_{12}\bar p_{2H} - \theta v_{12}\bar p_{2L} - \theta u_{13}\bar p_{3H} - \theta v_{13}\bar p_{3L} = (1-\theta)a_L + \theta(u_{12}+u_{13})(c_H - c_L) \tag{2.28}$$

In like manner, one obtains the equations of the reaction functions of the other players:

$$2(1+\theta)\bar p_{1H} - \theta u_{12}\bar p_{2H} - \theta v_{12}\bar p_{2L} - \theta u_{13}\bar p_{3H} - \theta v_{13}\bar p_{3L} = (1-\theta)a_H - \theta(v_{12}+v_{13})(c_H - c_L) \tag{2.29}$$

$$2(1+\theta)\bar p_{2L} - \theta u_{21}\bar p_{1H} - \theta v_{21}\bar p_{1L} - \theta u_{23}\bar p_{3H} - \theta v_{23}\bar p_{3L} = (1-\theta)a_L + \theta(u_{21}+u_{23})(c_H - c_L) \tag{2.30}$$

$$2(1+\theta)\bar p_{2H} - \theta u_{21}\bar p_{1H} - \theta v_{21}\bar p_{1L} - \theta u_{23}\bar p_{3H} - \theta v_{23}\bar p_{3L} = (1-\theta)a_H - \theta(v_{21}+v_{23})(c_H - c_L) \tag{2.31}$$

$$2(1+\theta)\bar p_{3L} - \theta u_{31}\bar p_{1H} - \theta v_{31}\bar p_{1L} - \theta u_{32}\bar p_{2H} - \theta v_{32}\bar p_{2L} = (1-\theta)a_L + \theta(u_{31}+u_{31})(c_H - c_L) \tag{2.32}$$

$$2(1+\theta)\bar p_{3H} - \theta u_{31}\bar p_{1H} - \theta v_{31}\bar p - \theta u_{32}\bar p_{2H} - \theta v_{32}\bar p_{2L} = (1-\theta)a_H - \theta(v_{31}+v_{31})(c_H - c_L) \tag{2.33}$$

Observe that (2.27) and (2.28) yield an expression for firm 1L's profit anywhere along its reaction function, and in particular in equilibrium,

$$\pi_{1L} = \frac{1+\mu}{(1-\mu)(1+2\mu)}(\bar{p}_{1L})^2. \qquad (2.34)$$

Corresponding relationships hold for the payoffs of other firms (for the entrant, subtracting off entry cost K).

(2.13) and (2.14) continue to hold; in addition, from (2.32) and (2.33), one obtains

$$\bar{p}_{3L} = \bar{p}_{3H} + \frac{c_H - c_L}{2}. \qquad (2.35)$$

Using (2.13), (2.14) , and (2.35), reduce the system of equations of the reaction functions to a system of three equations in the low-cost firm prices,

$$[(2+3\mu)I_3 - \mu J_3 J_3'] \begin{pmatrix} \bar{p}_{1L} \\ \bar{p}_{2L} \\ \bar{p}_{3L} \end{pmatrix} = (1-\mu)a_L J_3 + \frac{\mu}{2} \begin{pmatrix} u_{12}+u_{13} \\ u_{21}+u_{23} \\ u_{31}+u_{32} \end{pmatrix} (c_H - c_L). \qquad (2.36)$$

In like manner, obtain a system of three equations in the high-cost firm prices,

$$[(2+3\mu)I_3 - \mu J_3 J_3'] \begin{pmatrix} \bar{p}_{1H} \\ \bar{p}_{2H} \\ \bar{p}_{3H} \end{pmatrix} = (1-\mu)a_H J_3 - \frac{\mu}{2} \begin{pmatrix} v_{12}+v_{13} \\ v_{21}+v_{23} \\ v_{31}+v_{32} \end{pmatrix} (c_H - c_L). \qquad (2.37)$$

Using the inverse

$$[(2+3\mu)I_3 - \mu J_3 J_3']^{-1} = \frac{1}{2+3\mu}\left(I_3 + \frac{\mu}{2} J_3 J_3' \right); \qquad (2.38)$$

solve the two subsystems of equations to obtain expressions for equilibrium prices

$$\bar{p}_{1L} = \frac{1}{2}(1-\mu)a_L + \frac{1}{2}\frac{\mu}{2+3\mu}\left(u_{12}+u_{13}+\frac{\mu}{2}U \right)(c_H - c_L) \qquad (2.39)$$

$$\bar{p}_{2L} = \frac{1}{2}(1-\mu)a_L + \frac{1}{2}\frac{\mu}{2+3\mu}\left(u_{21}+u_{23}+\frac{\mu}{2}U \right)(c_H - c_L) \qquad (2.40)$$

$$\bar{p}_{3L} = \frac{1}{2}(1-\mu)a_L + \frac{1}{2}\frac{\mu}{2+3\mu}\left(u_{31}+u_{32}+\frac{\mu}{2}U \right)(c_H - c_L) \qquad (2.41)$$

$$\bar{p}_{1H} = \frac{1}{2}(1-\mu)a_H - \frac{1}{2}\frac{\mu}{2+3\mu}\left(v_{12}+v_{13}+\frac{\mu}{2}V \right)(c_H - c_L) \qquad (2.42)$$

$$\bar{p}_{2H} = \frac{1}{2}(1-\mu)a_H - \frac{1}{2}\frac{\mu}{2+3\mu}\left(v_{21}+v_{23}+\frac{\mu}{2}V \right)(c_H - c_L) \qquad (2.43)$$

$$\bar{p}_{3H} = \frac{1}{2}(1-\mu)a_H - \frac{1}{2}\frac{\mu}{2+3\mu}\left(v_{31}+v_{32}+\frac{\mu}{2}V \right)(c_H - c_L); \qquad (2.44)$$

writing $U = u_{12}+u_{13}+u_{21}+u_{23}+u_{31}+u_{32}$ and $V = v_{12}+v_{13}+v_{21}+v_{23}+v_{31}+v_{32}$ for notational compactness.

2.3. Pooling on p_L

2.3.1. Boundaries of Case 2

The boundaries of case 2 are determined by the inequalities

$$\frac{1+µ}{(1 ¡ µ)(1+2µ)}[\overline{p}_{3L}(u_{12}; 0; u_{21}; 0; u_{31}; u_{32})]^2 ¸ K \quad (2.45)$$

$$¸ \frac{1+µ}{(1 ¡ µ)(1+2µ)}[\overline{p}_{3H}(u_{12}; u_{13}; u_{21}; u_{23}; u_{31}; u_{32})]^2:$$

If the game is in case 2, a high-cost incumbent defects from the pooling equilibrium, revealing its cost type, and a high-cost entrant comes into the market, the high-cost entrant's payo® is

$$¼_{3H}(u_{12}; u_{13}; 1; u_{23}; 1; u_{32}) = \frac{1+µ}{(1 ¡ µ)(1+2µ)}[\overline{p}_{3H}(u_{12}; u_{13}; 1; u_{23}; 1; u_{32})]^2 ¡ K: \quad (2.46)$$

If (2.46) is positive, the game is in case 2(a); otherwise, it is in case 2(b).

2.3.2. Case 2

Case 2(a) $¼_{3H}(u_{12}; u_{13}; 1; u_{23}; 1; u_{32}) > K ¸ ¼_{3H}(u_{12}; u_{13}; u_{21}; u_{23}; u_{31}; u_{32})$ 1H pooling payo®:

$$¼_{1H}(p_L) + u_{13}¼_{1H}(u_{12}; u_{21}) + v_{13}¼_{1H}(u_{12}; 0; u_{21}; 0; u_{31}; u_{32}) \quad (2.47)$$

1H defection payo®:

$$¼^{br}_{1H}(p_L) + ¼_{1H}(u_{12}; u_{13}; 1; u_{23}; 1; u_{32}) \quad (2.48)$$

Condition for 1H pooling:

$$u_{13}[¼_{1H}(u_{12}; 1) ¡ ¼_{1H}(u_{12}; u_{13}; 1; u_{23}; 1; u_{32})] ¸ \quad (2.49)$$

$$¼^{br}_{1H}(p_L) ¡ ¼_{1H}(p_L) + u_{13}[¼_{1H}(u_{12}; 1) ¡ ¼_{1H}(u_{12}; u_{21})]$$

$$+ v_{13}[¼_{1H}(u_{12}; u_{13}; 1; u_{23}; 1; u_{32}) ¡ ¼_{1H}(u_{12}; 0; u_{21}; 0; u_{31}; u_{32})]$$

1L pooling payo®:

$$¼_{1L}(p_L) + u_{13}¼_{1L}(u_{12}; u_{21}) + v_{13}¼_{1L}(u_{12}; 0; u_{21}; 0; u_{31}; u_{32}): \quad (2.50)$$

1L defection payo®:

$$¼_{1L}(p_L) + ¼_{1L}(u_{12}; u_{13}; 1; u_{23}; 1; u_{32}): \quad (2.51)$$

Condition for 1L pooling:

$$u_{13}[¼_{1L}(u_{12}; 1) ¡ ¼_{1L}(u_{12}; u_{13}; 1; u_{23}; 1; u_{32})] ¸ u_{13}[¼_{1L}(u_{12}; 1) ¡ ¼_{1L}(u_{12}; u_{21})] \quad (2.52)$$

$$+ v_{13}[¼_{1L}(u_{12}; u_{13}; 1; u_{23}; 1; u_{32}) ¡ ¼_{1L}(u_{12}; 0; u_{21}; 0; u_{31}; u_{32})]$$

Case 2(b) $\pi_{3H}(u_{12};0;u_{21};0;u_{31};u_{32}) > K \geq \pi_{3H}(u_{12};u_{13};1;u_{23};1;u_{32})$ 1H pooling payoff:

$$\pi_{1H}(p_L) + u_{13}\pi_{1H}(u_{12};u_{21}) + v_{13}\pi_{1H}(u_{12};0;u_{21};0;u_{31};u_{32}) \tag{2.53}$$

1H defection payoff:

$$\pi_{1H}^{br}(p_L) + u_{13}\pi_{1H}(u_{12};1) + v_{13}\pi_{1H}(u_{12};0;1;0;1;u_{32}) \tag{2.54}$$

Condition for 1H pooling: 1H will always wish to defect.

2.3.3. Linear example, Case 2(a)

High-cost incumbents

$$\pi_{1H}(p_L) = [\bar{p}_L - (c_H - c_L)]\frac{(1-\theta)a_L + \theta\bar{p}_L - \bar{p}_L}{1-\theta^2} \tag{2.55}$$

$$= [\bar{p}_L - (c_H - c_L)]\frac{a_H - \theta a_L + \theta\bar{p}_L - \bar{p}_{1H}^{br} + \bar{p}_{1H}^{br} - \bar{p}_L + (a_L - a_H)}{1-\theta^2}$$

$$= [\bar{p}_{1H}^{br} - (\bar{p}_{1H}^{br} - \bar{p}_L + c_H - c_L)]\frac{\bar{p}_{1H}^{br} + [\bar{p}_{1H}^{br} - \bar{p}_L + c_H - c_L)}{1-\theta^2}$$

(using $a_H - \theta a_L + \theta\bar{p}_L - \bar{p}_{1H}^{br} = \bar{p}_{1H}^{br}$ from the equation of 1H's reaction function)

$$= \frac{1}{1-\theta^2}\left(\bar{p}_{1H}^{br}\right)^2 - \frac{1}{1-\theta^2}\left(\bar{p}_{1H}^{br} - \bar{p}_L + c_H - c_L\right)^2$$

$$= \pi_{1H}^{br}(p_L) - \frac{1}{4}\frac{1}{1-\theta^2}(c_H - c_L)^2$$

(using the definition of $\bar{p}_L$ to evaluate the final term). Hence the first-period gain from defection is

$$\pi_{1H}^{br}(p_L) - \pi_{1H}(p_L) = \frac{1}{4}\frac{1}{1-\theta^2}(c_H - c_L)^2 \tag{2.56}$$

Firm 1H's second-period gain from revealing its cost type is positively proportional to the difference between its prices if it has and has not revealed its cost type. This is

$$p_{1H}(u_{12};1) - p_{1H}(u_{12};u_{21}) = \frac{\theta^2}{2}\frac{v_{21}}{4-\theta^2}(c_H - c_L) \tag{2.57}$$

if the entrant has high cost and

$$p_{1H}(u_{12};u_{13};1;u_{23};1;u_{32}) - p_{1H}(u_{12};0;u_{21};0;u_{31};u_{32}) = \tag{2.58}$$

$$\frac{1}{2}\frac{\theta}{2+3\theta}\left[u_{13} + \frac{\theta}{2}(u_{13} + v_{21} + u_{23} + v_{31})\right](c_H - c_L) \geq 0:$$

if the entrant has low cost.

Explicit evaluation of the entry deterrence term, the left-hand side of (2.49), is unrevealing.

Low-cost incumbents For firm 1L, the second-period cost revelation effects are positively proportional to the difference between prices in the different regimes. These are

$$p_{1L}(u_{12}, 1) - p_{1L}(u_{12}, u_{21}) = \frac{\theta^2}{2}\frac{v_{21}}{4 - \theta^2}(c_H - c_L) \geq 0 \tag{2.59}$$

and

$$p_{1L}(u_{12}, u_{13}, 1, u_{23}, 1, u_{32}) - p_{1L}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) = \tag{2.60}$$

$$\frac{1}{2}\frac{\theta}{2 + 3\theta}\left[u_{13} + \frac{\theta}{2}(u_{13} + u_{23} + v_{21} + v_{31})\right](c_H - c_L) \geq 0.$$

Explicit evaluation of the entry deterrence term, the left-hand side of (2.52), is unrevealing.

2.3.4. Case 3

Case 3(a) $\pi_{3L}(u_{12}, 0, 1, 0, 1, u_{32}) > K > \pi_{3L}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32})$

1H pooling payoff:

$$\pi_{1H}(p_L) + \pi_{1H}(u_{12}, u_{21}) \tag{2.61}$$

1H defection payoff:

$$\pi_{1H}^{br}(p_L) + u_{13}\pi_{1H}(u_{12}, 1) + v_{13}\pi_{1H}(u_{12}, 0, 1, 0, 1, u_{32}) \tag{2.62}$$

Condition for 1H pooling:

$$\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(u_{12}, 1) + v_{13}[\pi_{1H}(u_{12}, 1) - \pi_{1H}(u_{12}, 0, 1, 0, 1, u_{32})] \geq \tag{2.63}$$

$$\pi_{1H}^{br}(p_L) - \pi_{1H}(p_L)$$

1L second-period pooling payoff:

$$\pi_{1L}(u_{12}, u_{21}). \tag{2.64}$$

1L second-period defection payoff:

$$u_{13}\pi_{1L}(u_{12}, 1) + v_{13}\pi_{1L}(u_{12}, 0, 1, 0, 1, u_{32}) \tag{2.65}$$

Condition for 1L pooling:

$$v_{13}[\pi_{1L}^{br}(p_L) - \pi_{1L}(u_{12}, 0, 1, 0, 1, u_{32})] \geq \pi_{1L}(u_{12}, 1) - \pi_{1L}(u_{12}, u_{21}) \tag{2.66}$$

Case 3(b) $K > \pi_{3L}(u_{12}, 0, 1, 0, 1, u_{32})$

1H pooling payoff:

$$\pi_{1H}(p_L) + \pi_{1H}(u_{12}, u_{21}) \tag{2.67}$$

1H defection payoff:

$$\pi_{1H}^{br}(p_L) + \pi_{1H}(u_{12}, 1) \tag{2.68}$$

Condition for 1H pooling: 1H will always wish to defect.

2.4. Pooling on p_H

It can be a sequential equilibrium for price-setting firms to pool on high prices. In a certain sense, this equilibrium is dual to the high-output pooling equilibrium described for quantity-setting firms. But if pooling on a high price is an equilibrium, it is not a limit-pricing equilibrium: the profit to be gained by discouraging entry encourages firms to defect from high-price pooling behavior.

Let p_H be the first-period Nash equilibrium price if both incumbents are known to have high cost. Suppose an alleged equilibrium strategy calls for firms to pool on p_H, and let out-of-equilibrium beliefs be such that if the entrant sees any price other than p_H, it infers that the defector has low cost.

For concreteness, let the game be in case 2 (if incumbents pool, a high-cost entrant will stay out of the market but a low-cost entrant will come into the market). Conditions for high-price pooling in the other cases can be analyzed in a similar manner.

To evaluate defection payoffs, case 2 must be divided into two subcases. In what I will call case 2(c), entry cost K is relatively small,

$$\pi_{3L}(u_{12}, 0, 0, 0, 0, u_{32}) > K \geq \pi_{3H}(u_{12}, u_{13}, u_{21}, u_{23}, u_{31}, u_{32}), \tag{2.69}$$

and a low-cost entrant will come into the market in the second period if one incumbent has defected and revealed itself as having low cost. For larger values of K, in case 2(d),

$$\pi_{3L}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) > K \geq \pi_{3L}(u_{12}, 0, 0, 0, 0, u_{32}); \tag{2.70}$$

a low-cost potential entrant will stay out of the market if it is known that once incumbent has low cost.

If incumbent $1H$ pools on p_H, it earns its best-response payoff in the first period, and has expected second-period payoff

$$u_{13}\pi_{1H}(u_{12}, u_{21}) + v_{13}\pi_{1H}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}). \tag{2.71}$$

If the game is in case 2(c), firm $1H$ would never defect. To do so would reduce its first-period payoff. It would not alter the probability of entry. By convincing rivals that it had low cost, firm 1 would cause rivals to set lower prices in the second period, reducing its expected second-period payoff.

Now suppose the game falls in case 2(d). By defecting slightly from the high-price pooling equilibrium, firm $1H$ slightly reduces its first-period payoff. Defection convinces rivals that it has low cost and causes a low-cost entrant to stay out of the market. Firm $1H$'s expected second-period defection payoff is

$$\pi_{1H}(u_{12}, 0); \tag{2.72}$$

its best-response payoff if incumbent 2 believes incumbent 1 has low cost while incumbent 1 does not know incumbent 2's cost type.

Comparing (2.71) and (2.72), in case 2(d) a high-cost incumbent 1 will pool on high prices if

$$\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(u_{12}, 0) \geq v_{13}[\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32})]. \quad (2.73)$$

The term on the left, lost profit due to pretending to have low cost, is a pure masquerade effect. The term on the right is the gain in profit if a low-cost entrant stays out of the market, weighted by the prior probability that the entrant has low cost. If the profit lost by entry is great enough, and if the high-cost incumbent thinks it very unlikely that the potential entrant has low cost, then it will defect from the high-price pooling strategy.

Now consider the incentives of a low-cost firm 1 to pool on p_H. Its pooling payoff is

$$\pi_{1L}(p_H) + u_{13}\pi_{1L}(u_{12}, u_{21}) + v_{13}\pi_{1L}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}). \quad (2.74)$$

Any defection by firm 1L reveals it as a low-cost firm. This being the case, if 1L defects, it will set a price that maximizes its first-period payoff. If the game is in case 2(c), defection does not affect the likelihood of entry. Firm 1L's defection payoff is

$$\pi_{1L}^{br}(p_H) + u_{13}\pi_{1L}(u_{12}, 0) + v_{13}\pi_{1L}(u_{12}, 0, 0, 0, 0, u_{32}). \quad (2.75)$$

The second term is firm 1L's payoff if the entrant has high cost and stays out of the market; the third term is firm 1L's payoff if the entrant has low cost and comes into the market. Each payoff is weighted by the appropriate prior probability, and each payoff reflects the fact that after defection firm 1 is known to have low cost.

Comparing (2.74) and (2.75), in case 2(c) a low-cost firm 1L will pool on high prices if

$$u_{13}[\pi_{1L}(u_{12}, u_{21}) - \pi_{1L}(u_{12}, 0)] \quad (2.76)$$

$$+ v_{13}[\pi_{1L}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32}) - \pi_{1L}(u_{12}, 0, 0, 0, 0, u_{32})] \geq \pi_{1L}^{br}(p_H) - \pi_{1L}(p_H).$$

The terms on the left are firm 1L's profit lost by revealing its cost type, in the event that entry does not and does occur, weighted by the appropriate prior probabilities. The term on the right is the first-period gain from defection. Firm 1L will pool on high prices if cost revelation is sufficiently costly. But in case 2(c), entry limitation is not a factor in the low-cost incumbent's decision.

In case 2(d), on the other hand, a consequence of firm 1L's defection is that a low-cost entrant will stay out of the market. Firm 1L's expected defection payoff is

$$\pi_{1L}^{br}(p_H) + \pi_{1L}(u_{12}, 0). \quad (2.77)$$

Comparing (2.74) and (2.77), in case 2(d) firm 1L will pool on high prices if

$$u_{13}[\pi_{1L}(u_{12}, u_{21}) - \pi_{1L}(u_{12}, 0)] \geq \quad (2.78)$$

$$\pi_{1L}^{br}(p_H) - \pi_{1L}(p_H) + v_{13}[\pi_{1L}(u_{12}, 0) - \pi_{1L}(u_{12}, 0, u_{21}, 0, u_{31}, u_{32})].$$

The term on the left is firm 1L's expected second-period lost profit due to cost-type revelation if the entrant has high cost and would have stayed out of the market in any event. The terms on the right are the first-period gain from defection and the expected second period gain from defection if the entrant has low cost and stays out of the market because it is known that firm 1 has low cost.

Condition (2.78) will fail if v_{13} is sufficiently large — if firm 1L thinks it very likely that the incumbent has low cost. The possibility of entry limitation is an incentive to defect from the high-price pooling equilibrium.

2.5. Separating equilibrium: payoffs from entry

For price-setting firms, the inequalities (1.69)

$$\pi_{3L}(1;0;1;0;1;1) > \pi_{3L}(0;0;1;0;1;0) > \pi_{3L}(0;0;0;0;0;0)$$

$$\lambda \quad \pi_{3H}(1;u_{13};1;u_{23};1;1) > \pi_{3H}(0;u_{13};1;u_{23};1;0) > \pi_{3H}(0;u_{13};0;u_{23};0;0)$$

can be verified by showing that the corresponding relationships hold for net prices. Evaluating (2.41) and (2.44) for alternative beliefs,

$$\overline{p}_{3L}(1;0;1;0;1;1) = \frac{1}{2}(1-\theta)a_L + \frac{1+2\theta}{2+3\theta}(c_H - c_L) \tag{2.79}$$

$$\overline{p}_{3L}(0;0;1;0;1;0) = \frac{1}{2}(1-\theta)a_L + \frac{1}{2}\frac{\theta}{2+3\theta}(1+\theta)(c_H - c_L) \tag{2.80}$$

$$\overline{p}_{3L}(0;0;0;0;0;0) = \frac{1}{2}(1-\theta)a_L \tag{2.81}$$

$$\overline{p}_{3H}(1;u_{13};1;u_{23};1;1) = \frac{1}{2}(1-\theta)a_H - \frac{1}{4}\frac{\theta^2}{2+3\theta}(v_{13}+v_{23})(c_H - c_L) \tag{2.82}$$

$$\overline{p}_{3H}(0;u_{13};1;u_{23};1;0) = \frac{1}{2}(1-\theta)a_H - \frac{1}{4}\frac{\theta}{2+3\theta}[2+(2+v_{13}+v_{23})\theta](c_H - c_L) \tag{2.83}$$

$$\overline{p}_{3H}(0;u_{13};0;u_{23};0;0) = \frac{1}{2}(1-\theta)a_H - \frac{1}{4}\frac{\theta}{2+3\theta}[4+(4+v_{13}+v_{23})\theta](c_H - c_L) \tag{2.84}$$

The inequalities (1.69) are satisfied.

2.6. Separation by L-firm price reduction

Consider the follow collection of actions and beliefs:

(a) firms 1L and 2L set prices $p_{1\lambda} > p_{1L}$ and $p_{2\lambda} > p_{2L}$, respectively, in period 1;

(b) firms 1H and 2H set their best-response prices p_{1h} and p_{2h}, respectively, in period 1;

(c) incumbents' cost types are revealed by observing first-period prices;

(d) the potential entrant comes into the market in the second period if it expects a positive second-period profit, net of entry cost;

(e) if the fact of entry does not reveal the entrant's cost type, incumbents carry forward prior beliefs from the first period to the second; all firms maximize expected profit in the second period;

If firm 1 sets any price other than $p_{1\lambda}$ (firm 2 sets any price other than $p_{2\lambda}$), rivals infer that it has high cost.

It is clear that the entrant maximizes its expected payoff by behaving as indicated, given that other players behave as indicated.

2.6.1. High-cost incumbents

The inequalities (1.69) define the seven ranges of entry cost that must be considered to examine incumbents' incentives to separate. Firm 1H's separation and defection payoffs for the seven ranges, and the conditions for firm 1H to separate, are:

Case S1 $K > \pi_{3L}(1;0;1;0;1;1)$

Separation payoff:

$$\pi_{1H}(p_{1h}) + u_{12}\pi_{1H}(1;1) + v_{12}\pi_{1H}(0;1) \tag{2.85}$$

Defection payoff:

$$\pi_{1H}(p_{1\lambda}) + u_{12}\pi_{1H}(1;0) + v_{12}\pi_{1H}(0;0) \tag{2.86}$$

Condition for 1H to separate:

By imitating a low-cost firm, firm 1H reduces its first-period payoff. By convincing rivals that it has low cost, it induces them to set lower prices in the second period. This reduces firm 1H's second-period payoff. It follows that firm 1H would always wish to separate.

Case S2 $\pi_{3L}(1;0;1;0;1;1) > K \geq \pi_{3L}(0;0;1;0;1;0)$

Separation payoff:

$$\pi_{1H}(p_{1h}) + u_{12}[u_{13}\pi_{1H}(1;1) + v_{13}\pi_{1H}(1;0;1;0;1;1)] + v_{12}\pi_{1H}(0;1) \tag{2.87}$$

Defection payoff:

$$\pi_{1H}(p_{1\lambda}) + u_{12}\pi_{1H}(1;0) + v_{12}\pi_{1H}(0;0) \tag{2.88}$$

Condition for 1H to separate:

$$\pi_{1H}(p_{1h}) - \pi_{1H}(p_{1\lambda}) + u_{12}u_{13}[\pi_{1H}(1;1) - \pi_{1H}(1;0)] + v_{12}[\pi_{1H}(0;1) - \pi_{1H}(0;0)] \geq u_{12}v_{13}[\pi_{1H}(1;0) - \pi_{1H}(1;0;1;0;1;1)] \tag{2.89}$$

Case S3 $\pi_{3L}(0,0,1,0,1,0) > K \geq \pi_{3L}(0,0,0,0,0,0)$

Separation payoff:

$$\pi_{1H}(p_{1h}) + u_{12}[u_{13}\pi_{1H}(1,1) + v_{13}\pi_{1H}(1,0,1,0,1,1)] + v_{12}[u_{13}\pi_{1H}(0,1) + v_{13}\pi_{1H}(0,0,1,0,1,0)] \quad (2.90)$$

Defection payoff:

$$\pi_{1H}(p_{1\lambda}) + u_{12}[u_{13}\pi_{1H}(1,0) + v_{13}\pi_{1H}(1,0,0,0,0,1)] + v_{12}\pi_{1H}(0,0) \quad (2.91)$$

Condition for 1H to separate:

$$\pi_{1H}(p_{1h}) - \pi_{1H}(p_{1\lambda}) + u_{12}u_{13}[\pi_{1H}(1,1) - \pi_{1H}(1,0)] + u_{12}v_{13}[\pi_{1H}(1,0,1,0,1,1) - \pi_{1H}(1,0,0,0,0,1)] + v_{12}u_{13}[\pi_{1H}(0,1) - \pi_{1H}(0,0)] \geq v_{12}v_{13}[\pi_{1H}(0,0) - \pi_{1H}(0,0,1,0,1,0)] \quad (2.92)$$

Case S4 $\pi_{3L}(0,0,0,0,0,0) > K \geq \pi_{3H}(1,u_{13},1,u_{23},1,1)$

Separation payoff:

$$\pi_{1H}(p_{1h}) + u_{12}[u_{13}\pi_{1H}(1,1) + v_{13}\pi_{1H}(1,0,1,0,1,1)] + v_{12}[u_{13}\pi_{1H}(0,1) + v_{13}\pi_{1H}(0,0,1,0,1,0)] \quad (2.93)$$

Defection payoff:

$$\pi_{1H}(p_{1\lambda}) + u_{12}[u_{13}\pi_{1H}(1,0) + v_{13}\pi_{1H}(1,0,0,0,0,1)] + v_{12}[u_{13}\pi_{1H}(0,0) + v_{13}\pi_{1H}(0,0,0,0,0,0)] \quad (2.94)$$

Condition for 1H to separate: by the same argument that was used for case S1, firm 1H will always wish to separate.

Case S5 $\pi_{3H}(1,u_{13},1,u_{23},1,1) > K \geq \pi_{3H}(0,u_{13},1,u_{23},1,0)$

Separation payoff:

$$\pi_{1H}(p_{1h}) + u_{12}\pi_{1H}(1,u_{13},1,u_{23},1,1) + v_{12}[u_{13}\pi_{1H}(0,1) + v_{13}\pi_{1H}(0,0,1,0,1,0)] \quad (2.95)$$

Defection payoff:

$$\pi_{1H}(p_{1\lambda}) + u_{12}[u_{13}\pi_{1H}(1,0) + v_{13}\pi_{1H}(1,0,0,0,0,1)] + v_{12}[u_{13}\pi_{1H}(0,0) + v_{13}\pi_{1H}(0,0,0,0,0,0)] \quad (2.96)$$

Condition for 1H to separate:

$$\pi_{1H}(p_{1h}) - \pi_{1H}(p_{1\lambda}) + u_{12}v_{13}[\pi_{1H}(1,u_{13},1,u_{23},1,1) - \pi_{1H}(1,0,0,0,0,1)] + v_{12}u_{13}[\pi_{1H}(0,1) - \pi_{1H}(0,0)] + v_{12}v_{13}[\pi_{1H}(0,0,1,0,1,0) - \pi_{1H}(0,0,0,0,0,0)] \geq u_{12}u_{13}[\pi_{1H}(1,0) - \pi_{1H}(1,u_{13},1,u_{23},1,1)] \quad (2.97)$$

Case S6 $\pi_{3H}(0; u_{13}; 1; u_{23}; 1; 0) > K \geq \pi_{3H}(0; u_{13}; 0; u_{23}; 0; 0)$
Separation payoff:

$$\pi_{1H}(p_{1h}) + u_{12}\pi_{1H}(1; u_{13}; 1; u_{23}; 1; 1) + v_{12}\pi_{1H}(0; u_{13}; 1; u_{23}; 1; 0) \tag{2.98}$$

Defection payoff:

$$\pi_{1H}(p_{1\lambda}) + u_{12}\pi_{1H}(1; u_{13}; 0; u_{23}; 0; 1) + v_{12}[u_{13}\pi_{1H}(0; 0) + v_{13}\pi_{1H}(0; 0; 0; 0; 0; 0)] \tag{2.99}$$

Condition for 1H to separate:

$$\begin{aligned} &\pi_{1H}(p_{1h}) - \pi_{1H}(p_{1\lambda}) + \\ &u_{12}[\pi_{1H}(1; u_{13}; 1; u_{23}; 1; 1) - \pi_{1H}(1; u_{13}; 0; u_{23}; 0; 1)] \\ &+ v_{12}v_{13}[\pi_{1H}(0; u_{13}; 1; u_{23}; 1; 0) - \pi_{1H}(0; 0; 0; 0; 0; 0)] \\ &\geq v_{12}u_{13}[\pi_{1H}(0; 0) - \pi_{1H}(0; u_{13}; 1; u_{23}; 1; 0)] \end{aligned} \tag{2.100}$$

Case S7 $\pi_{3H}(0; u_{13}; 0; u_{23}; 0; 0) > K$
Separation payoff:

$$\pi_{1H}(p_{1h}) + u_{12}\pi_{1H}(1; u_{13}; 1; u_{23}; 1; 1) + v_{12}\pi_{1H}(0; u_{13}; 1; u_{23}; 1; 0) \tag{2.101}$$

Defection payoff:

$$\pi_{1H}(p_{1\lambda}) + u_{12}\pi_{1H}(1; u_{13}; 0; u_{23}; 0; 1) + v_{12}\pi_{1H}(0; u_{13}; 0; u_{23}; 0; 0) \tag{2.102}$$

Condition for 1H to separate: by the same argument that was used for case S1, firm 1H will always wish to separate.

In cases S1, S4, and S7, the entrant's decision is not affected by a high-cost incumbent's first-period action, with the result that the possibility of entry deterrence does not affect the high-cost incumbent's incentives to separate. In the remaining cases, the possibility of deterring entry is an incentive for the high-cost incumbent to defect from separating behavior.

2.6.2. Low-cost incumbents

Case S1 $K > \pi_{3L}(1; 0; 1; 0; 1; 1)$
Separation payoff:

$$\pi_{1L}(p_{1\lambda}; p_{2\lambda}) + u_{12}\pi_{1L}(1; 0) + v_{12}\pi_{1L}(0; 0) \tag{2.103}$$

Defection payoff:

$$\pi^{br}_{1L}(p_{1\lambda}; p_{2\lambda}) + u_{12}\pi_{1L}(1; 1) + v_{12}\pi_{1L}(0; 1) \tag{2.104}$$

If firm 1L defects, it sets its best-response price in the first period, and leads firm 2 to believe that it has high cost. This leads firm 2 to set a higher price in the second period, which allows firm 1L to set a higher price and earn a greater profit. Firm 1L earns a greater profit in both periods if it defects if the game is in case S1, and will never be willing to separate.

Case S2 $\pi_{3L}(1;0;1;0;1;1) > K \geq \pi_{3L}(0;0;1;0;1;0)$

Separation payoff:

$$\pi_{1L}(p_{1\lambda};p_{2\lambda}) + u_{12}\pi_{1L}(1;0) + v_{12}\pi_{1L}(0;0) \tag{2.105}$$

Defection payoff:

$$\pi_{1L}^{br}(p_{1\lambda};p_{2\lambda}) + u_{12}[u_{13}\pi_{1L}(1;1) + v_{13}\pi_{1L}(1;0;1;0;1;1)] + v_{12}\pi_{1L}(0;1) \tag{2.106}$$

Condition for 1L to separate:

$$\begin{aligned} u_{12}v_{13}[\pi_{1L}(1;0) - \pi_{1L}(1;0;1;0;1;1)] \geq \pi_{1L}^{br}(p_{1\lambda};p_{2\lambda}) - \pi_{1L}(p_{1\lambda};p_{2\lambda}) \\ + u_{12}u_{13}[\pi_{1L}(1;1) - \pi_{1L}(1;0)] + v_{12}[\pi_{1L}(0;1) - \pi_{1L}(0;0)] \end{aligned} \tag{2.107}$$

Case S3 $\pi_{3L}(0;0;1;0;1;0) > K \geq \pi_{3L}(0;0;0;0;0;0)$

Separation payoff:

$$\pi_{1L}(p_{1\lambda};p_{2\lambda}) + u_{12}[u_{13}\pi_{1L}(1;0) + v_{13}\pi_{1L}(1;0;0;0;0;1)] + v_{12}\pi_{1L}(0;0) \tag{2.108}$$

Defection payoff:

$$\begin{aligned} \pi_{1L}^{br}(p_{1\lambda};p_{2\lambda}) + u_{12}[u_{13}\pi_{1L}(1;1) + v_{13}\pi_{1L}(1;0;1;0;1;1)] \\ + v_{12}[u_{13}\pi_{1L}(0;1) + v_{13}\pi_{1L}(0;0;1;0;1;0)] \end{aligned} \tag{2.109}$$

Condition for 1L to separate:

$$\begin{aligned} v_{12}v_{13}[\pi_{1L}(0;0) - \pi_{1L}(0;0;1;0;1;0)] \geq \pi_{1L}(p_{1\lambda};p_{2\lambda}) - \pi_{1L}(p_{1\lambda};p_{2\lambda}) \\ + u_{12}u_{13}[\pi_{1L}(1;1) - \pi_{1L}(1;0)] + u_{12}v_{13}[\pi_{1L}(1;0;1;0;1;1) - \pi_{1L}(1;0;0;0;0;1)] \\ + v_{12}u_{13}[\pi_{1L}(0;1) - \pi_{1L}(0;0)] \end{aligned} \tag{2.110}$$

Case S4 $\pi_{3L}(0;0;0;0;0;0) > K \geq \pi_{3H}(1;u_{13};1;u_{23};1;1)$

Separation payoff:

$$\begin{aligned} \pi_{1L}(p_{1\lambda};p_{2\lambda}) + u_{12}[u_{13}\pi_{1L}(1;0) + v_{13}\pi_{1L}(1;0;0;0;0;1)] \\ + v_{12}[u_{13}\pi_{1L}(0;0) + v_{13}\pi_{1L}(0;0;0;0;0;0)] \end{aligned} \tag{2.111}$$

Defection payoff:

$$\begin{aligned} \pi_{1L}(p_{1\lambda};p_{2\lambda}) + u_{12}[u_{13}\pi_{1L}(1;1) + v_{13}\pi_{1L}(1;0;1;0;1;1)] \\ + v_{12}[u_{13}\pi_{1L}(0;1) + v_{13}\pi_{1L}(0;0;1;0;1;0)] \end{aligned} \tag{2.112}$$

Condition for 1L to separate: by the same argument that was used for case S1, firm 1L will always wish to defect from separating behavior.

Case S5 $\pi_{3H}(1, u_{13}, 1, u_{23}, 1, 1) > K \geq \pi_{3H}(0, u_{13}, 1, u_{23}, 1, 0)$

Separation payoff:

$$\pi_{1L}(p_{1\lambda}, p_{2\lambda}) + u_{12}[u_{13}\pi_{1L}(1, 0) + v_{13}\pi_{1L}(1, 0, 0, 0, 0, 1)] \tag{2.113}$$
$$+ v_{12}[u_{13}\pi_{1L}(0, 0) + v_{13}\pi_{1L}(0, 0, 0, 0, 0, 0)]$$

Defection payoff:

$$\pi_{1L}^{br}(p_{1\lambda}, p_{2\lambda}) + u_{12}\pi_{1L}(1, u_{13}, 1, u_{23}, 1, 1) \tag{2.114}$$
$$+ v_{12}[u_{13}\pi_{1L}(0, 1) + v_{13}\pi_{1L}(0, 0, 1, 0, 1, 0)]$$

Condition for 1L to separate:

$$u_{12}u_{13}[\pi_{1L}(1, 0) - \pi_{1L}(1, u_{13}, 1, u_{23}, 1, 1)] \geq \pi_{1L}^{br}(p_{1\lambda}, p_{2\lambda}) - \pi_{1L}(p_{1\lambda}, p_{2\lambda}) \tag{2.115}$$
$$+ u_{12}v_{13}[\pi_{1L}(1, u_{13}, 1, u_{23}, 1, 1) - \pi_{1L}(1, 0, 0, 0, 0, 1)]$$
$$+ v_{12}u_{13}[\pi_{1L}(0, 1) - \pi_{1L}(0, 0)] + v_{12}v_{13}[\pi_{1L}(0, 0, 1, 0, 1, 0) - \pi_{1L}(0, 0, 0, 0, 0, 0)]$$

Case S6 $\pi_{3H}(0, u_{13}, 1, u_{23}, 1, 0) > K \geq \pi_{3H}(0, u_{13}, 0, u_{23}, 0, 0)$

Separation payoff:

$$\pi_{1L}(p_{1\lambda}, p_{2\lambda}) + u_{12}\pi_{1L}(1, u_{13}, 0, u_{23}, 0, 1) \tag{2.116}$$
$$+ v_{12}[u_{13}\pi_{1L}(0, 0) + v_{13}\pi_{1L}(0, 0, 0, 0, 0, 0)]$$

Defection payoff:

$$\pi_{1L}^{br}(p_{1\lambda}, p_{2\lambda}) + u_{12}\pi_{1L}(1, u_{13}, 1, u_{23}, 1, 1) + v_{12}\pi_{1L}(0, u_{13}, 1, u_{23}, 1, 0) \tag{2.117}$$

Condition for 1L to separate:

$$v_{12}u_{13}[\pi_{1L}(0, 0) - \pi_{1L}(0, u_{13}, 1, u_{23}, 1, 0)] \geq \tag{2.118}$$
$$\pi_{1L}^{br}(p_{1\lambda}, p_{2\lambda}) - \pi_{1L}(p_{1\lambda}, p_{2\lambda}) + u_{12}[\pi_{1L}(1, u_{13}, 1, u_{23}, 1, 1) - \pi_{1L}(1, u_{13}, 0, u_{23}, 0, 1)]$$
$$+ v_{12}v_{13}[\pi_{1L}(0, u_{13}, 1, u_{23}, 1, 0) - \pi_{1L}(0, 0, 0, 0, 0, 0)]$$

Case S7 $\pi_{3H}(0, u_{13}, 0, u_{23}, 0, 0) > K$

Separation payoff:

$$\pi_{1L}(p_{1\lambda}, p_{2\lambda}) + u_{12}\pi_{1L}(1, u_{13}, 0, u_{23}, 0, 1) + v_{12}\pi_{1L}(0, u_{13}, 0, u_{23}, 0, 0) \tag{2.119}$$

Defection payoff:

$$\pi_{1L}^{br}(p_{1\lambda}, p_{2\lambda}) + u_{12}\pi_{1L}(1, u_{13}, 1, u_{23}, 1, 1) + v_{12}\pi_{1L}(0, u_{13}, 1, u_{23}, 1, 0) \tag{2.120}$$

Condition for 1L to separate: by the same argument that was used for case S1, firm 1L will always wish to defect from separating behavior.

Separation by L-firm price reduction: resumé

A low-cost incumbent will never be willing to separate if entry costs and prior beliefs place the game in case S1, S4, or S7. A low-cost incumbent may be willing to separate in remaining cases. If it is willing to separate, it is because the expected lost profit from the entry that would following defection (pretending to have high cost) outweighs expected gains from inducing rivals to set higher prices in the second period. If low-cost incumbents are willing to separate, their motivation is to limit entry. For high-cost incumbents, however, the expected profit to be gained by deterring entry is an incentive to defect from the separating strategy.

2.7. Separation on Nash duopoly prices

This is the case considered in the text.

2.7.1. High-cost incumbents

Case S1 $K > \pi_{3L}(1,0,1,0,1,1)$

Separation payoff:

$$\pi_{1H}(u_{12}, u_{21}) + u_{12}\pi_{1H}(1,1) + v_{12}\pi_{1H}(0,1) \tag{2.121}$$

Defection payoff:

$$\pi_{1H}(p_{1L}) + u_{12}\pi_{1H}(1,0) + v_{12}\pi_{1H}(0,0) \tag{2.122}$$

By imitating a low-cost firm, firm 1H reduces its first-period payoff. By convincing rivals that it has low cost, it induces them to set lower prices in the second period. This reduces firm 1H's second-period payoff. It follows that firm 1H would always wish to separate.

Case S2 $\pi_{3L}(1,0,1,0,1,1) > K \geq \pi_{3L}(0,0,1,0,1,0)$

Separation payoff:

$$\pi_{1H}(u_{12}, u_{21}) + u_{12}[u_{13}\pi_{1H}(1,1) + v_{13}\pi_{1H}(1,0,1,0,1,1)] + v_{12}\pi_{1H}(0,1) \tag{2.123}$$

Defection payoff:

$$\pi_{1H}(p_{1L}) + u_{12}\pi_{1H}(1,0) + v_{12}\pi_{1H}(0,0) \tag{2.124}$$

Condition for 1H to separate:

$$\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(p_{1L}) + u_{12}u_{13}[\pi_{1H}(1,1) - \pi_{1H}(1,0)] \tag{2.125}$$

$$+ v_{12}[\pi_{1H}(0,1) - \pi_{1H}(0,0)] \geq u_{12}v_{13}[\pi_{1H}(1,0) - \pi_{1H}(1,0,1,0,1,1)]$$

Case S3 $\pi_{3L}(0;0;1;0;1;0) > K \geq \pi_{3L}(0;0;0;0;0;0)$

Separation payoff:

$$\pi_{1H}(u_{12};u_{21}) + u_{12}[u_{13}\pi_{1H}(1;1) + v_{13}\pi_{1H}(1;0;1;0;1;1)] \tag{2.126}$$
$$+ v_{12}[u_{13}\pi_{1H}(0;1) + v_{13}\pi_{1H}(0;0;1;0;1;0)]$$

Defection payoff:

$$\pi_{1H}(p_{1L}) + u_{12}[u_{13}\pi_{1H}(1;0) + v_{13}\pi_{1H}(1;0;0;0;0;1)] + v_{12}\pi_{1H}(0;0) \tag{2.127}$$

Condition for 1H to separate:

$$\pi_{1H}(u_{12};u_{21}) - \pi_{1H}(p_{1L}) + u_{12}u_{13}[\pi_{1H}(1;1) - \pi_{1H}(1;0)] \tag{2.128}$$
$$+ u_{12}v_{13}[\pi_{1H}(1;0;1;0;1;1) - \pi_{1H}(1;0;0;0;0;1)] + v_{12}u_{13}[\pi_{1H}(0;1) - \pi_{1H}(0;0)]$$
$$\geq v_{12}v_{13}[\pi_{1H}(0;0) - \pi_{1H}(0;0;1;0;1;0)]$$

Case S4 $\pi_{3L}(0;0;0;0;0;0) > K \geq \pi_{3H}(1;u_{13};1;u_{23};1;1)$

Separation payoff:

$$\pi_{1H}(u_{12};u_{21}) + u_{12}[u_{13}\pi_{1H}(1;1) + v_{13}\pi_{1H}(1;0;1;0;1;1)] \tag{2.129}$$
$$+ v_{12}[u_{13}\pi_{1H}(0;1) + v_{13}\pi_{1H}(0;0;1;0;1;0)]$$

Defection payoff:

$$\pi_{1H}(p_{1L}) + u_{12}[u_{13}\pi_{1H}(1;0) + v_{13}\pi_{1H}(1;0;0;0;0;1)] \tag{2.130}$$
$$+ v_{12}[u_{13}\pi_{1H}(0;0) + v_{13}\pi_{1H}(0;0;0;0;0;0)]$$

Condition for 1H to separate: by the same argument that was used for case S1, firm 1H will always wish to separate.

Case S5 $\pi_{3H}(1;u_{13};1;u_{23};1;1) > K \geq \pi_{3H}(0;u_{13};1;u_{23};1;0)$

Separation payoff:

$$\pi_{1H}(u_{12};u_{21}) + u_{12}\pi_{1H}(1;u_{13};1;u_{23};1;1) \tag{2.131}$$
$$+ v_{12}[u_{13}\pi_{1H}(0;1) + v_{13}\pi_{1H}(0;0;1;0;1;0)]$$

Defection payoff:

$$\pi_{1H}(p_{1L}) + u_{12}[u_{13}\pi_{1H}(1;0) + v_{13}\pi_{1H}(1;0;0;0;0;1)] \tag{2.132}$$
$$+ v_{12}[u_{13}\pi_{1H}(0;0) + v_{13}\pi_{1H}(0;0;0;0;0;0)]$$

Condition for 1H to separate:

$$\pi_{1H}(u_{12};u_{21}) - \pi_{1H}(p_{1L}) + u_{12}v_{13}[\pi_{1H}(1;u_{13};1;u_{23};1;1) - \pi_{1H}(1;0;0;0;0;1)] \tag{2.133}$$
$$+ v_{12}u_{13}[\pi_{1H}(0;1) - \pi_{1H}(0;0)] + v_{12}v_{13}[\pi_{1H}(0;0;1;0;1;0) - \pi_{1H}(0;0;0;0;0;0)]$$
$$\geq u_{12}u_{13}[\pi_{1H}(1;0) - \pi_{1H}(1;u_{13};1;u_{23};1;1)]$$

Case S6 $\pi_{3H}(0,u_{13},1,u_{23},1,0) > K \geq \pi_{3H}(0,u_{13},0,u_{23},0,0)$

Separation payoff:

$$\pi_{1H}(u_{12},u_{21}) + u_{12}\pi_{1H}(1,u_{13},1,u_{23},1,1) + v_{12}\pi_{1H}(0,u_{13},1,u_{23},1,0) \tag{2.134}$$

Defection payoff:

$$\pi_{1H}(p_{1L}) + u_{12}\pi_{1H}(1,u_{13},0,u_{23},0,1) + v_{12}[u_{13}\pi_{1H}(0,0) + v_{13}\pi_{1H}(0,0,0,0,0,0)] \tag{2.135}$$

Condition for 1H to separate:

$$\begin{aligned} &\pi_{1H}(u_{12},u_{21}) - \pi_{1H}(p_{1L}) + \\ &u_{12}[\pi_{1H}(1,u_{13},1,u_{23},1,1) - \pi_{1H}(1,u_{13},0,u_{23},0,1)] \\ &+ v_{12}v_{13}[\pi_{1H}(0,u_{13},1,u_{23},1,0) - \pi_{1H}(0,0,0,0,0,0)] \\ &\geq v_{12}u_{13}[\pi_{1H}(0,0) - \pi_{1H}(0,u_{13},1,u_{23},1,0)] \end{aligned} \tag{2.136}$$

Case S7 $\pi_{3H}(0,u_{13},0,u_{23},0,0) > K$

Separation payoff:

$$\pi_{1H}(u_{12},u_{21}) + u_{12}\pi_{1H}(1,u_{13},1,u_{23},1,1) + v_{12}\pi_{1H}(0,u_{13},1,u_{23},1,0) \tag{2.137}$$

Defection payoff:

$$\pi_{1H}(p_{1L}) + u_{12}\pi_{1H}(1,u_{13},0,u_{23},0,1) + v_{12}\pi_{1H}(0,u_{13},0,u_{23},0,0) \tag{2.138}$$

Condition for 1H to separate: by the same argument that was used for case S1, firm 1H will always wish to separate.

2.7.2. Low-cost incumbents

Given the assumed nature of beliefs about disequilibrium actions, a minimal deviation from p_{1L} convinces rivals that the offending firm has high cost. Incentives to defect essentially depend on a comparison of second-period payoffs.

Case S1 $K > \pi_{3L}(1,0,1,0,1,1)$

Separation payoff:

$$u_{12}\pi_{1L}(1,0) + v_{12}\pi_{1L}(0,0) \tag{2.139}$$

Defection payoff:

$$u_{12}\pi_{1L}(1,1) + v_{12}\pi_{1L}(0,1) \tag{2.140}$$

If firm 1L defects, it leads firm 2 to believe that it has high cost. This leads firm 2 to set a higher price in the second period, which allows firm 1L to set a higher price and earn a greater profit. Firm 1L will not wish to separate in case S1.

Case S2 $¼_{3L}(1;0;1;0;1;1) > K \; ¸ \; ¼_{3L}(0;0;1;0;1;0)$
Separation payo®:

$$u_{12}¼_{1L}(1;0) + v_{12}¼_{1L}(0;0) \tag{2.141}$$

Defection payo®:

$$u_{12}[u_{13}¼_{1L}(1;1) + v_{13}¼_{1L}(1;0;1;0;1;1)] + v_{12}¼_{1L}(0;1) \tag{2.142}$$

Condition for 1L to separate:

$$u_{12}v_{13}[¼_{1L}(1;0) \; ¡ \; ¼_{1L}(1;0;1;0;1;1)] \; ¸ \tag{2.143}$$

$$+ \; u_{12}u_{13}[¼_{1L}(1;1) \; ¡ \; ¼_{1L}(1;0)] + v_{12}[¼_{1L}(0;1) \; ¡ \; ¼_{1L}(0;0)]$$

Case S3 $¼_{3L}(0;0;1;0;1;0) > K \; ¸ \; ¼_{3L}(0;0;0;0;0;0)$
Separation payo®:

$$u_{12}[u_{13}¼_{1L}(1;0) + v_{13}¼_{1L}(1;0;0;0;0;1)] + v_{12}¼_{1L}(0;0) \tag{2.144}$$

Defection payo®:

$$u_{12}[u_{13}¼_{1L}(1;1) + v_{13}¼_{1L}(1;0;1;0;1;1)] \tag{2.145}$$

$$+ \; v_{12}[u_{13}¼_{1L}(0;1) + v_{13}¼_{1L}(0;0;1;0;1;0)]$$

Condition for 1L to separate:

$$v_{12}v_{13}[¼_{1L}(0;0) \; ¡ \; ¼_{1L}(0;0;1;0;1;0)] \; ¸ \tag{2.146}$$

$$+ \; u_{12}u_{13}[¼_{1L}(1;1) \; ¡ \; ¼_{1L}(1;0)] + u_{12}v_{13}[¼_{1L}(1;0;1;0;1;1) \; ¡ \; ¼_{1L}(1;0;0;0;0;1)]$$

$$+ \; v_{12}u_{13}[¼_{1L}(0;1) \; ¡ \; ¼_{1L}(0;0)]$$

Case S4 $¼_{3L}(0;0;0;0;0;0) > K \; ¸ \; ¼_{3H}(1;u_{13};1;u_{23};1;1)$
Separation payo®:

$$u_{12}[u_{13}¼_{1L}(1;0) + v_{13}¼_{1L}(1;0;0;0;0;1)] \tag{2.147}$$

$$+ \; v_{12}[u_{13}¼_{1L}(0;0) + v_{13}¼_{1L}(0;0;0;0;0;0)]$$

Defection payo®:

$$u_{12}[u_{13}¼_{1L}(1;1) + v_{13}¼_{1L}(1;0;1;0;1;1)] \tag{2.148}$$

$$+ \; v_{12}[u_{13}¼_{1L}(0;1) + v_{13}¼_{1L}(0;0;1;0;1;0)]$$

By the same argument that was used for case S1, ¯rm 1L will not wish to separate in case S4.

Case S5 $\pi_{3H}(1; u_{13}; 1; u_{23}; 1; 1) > K \geq \pi_{3H}(0; u_{13}; 1; u_{23}; 1; 0)$
Separation payoff:

$$u_{12}[u_{13}\pi_{1L}(1; 0) + v_{13}\pi_{1L}(1; 0; 0; 0; 0; 1)] \tag{2.149}$$
$$+ v_{12}[u_{13}\pi_{1L}(0; 0) + v_{13}\pi_{1L}(0; 0; 0; 0; 0; 0)]$$

Defection payoff:

$$u_{12}\pi_{1L}(1; u_{13}; 1; u_{23}; 1; 1) + v_{12}[u_{13}\pi_{1L}(0; 1) + v_{13}\pi_{1L}(0; 0; 1; 0; 1; 0)] \tag{2.150}$$

Condition for 1L to separate:

$$u_{12}u_{13}[\pi_{1L}(1; 0) - \pi_{1L}(1; u_{13}; 1; u_{23}; 1; 1)] \geq \tag{2.151}$$
$$+ u_{12}v_{13}[\pi_{1L}(1; u_{13}; 1; u_{23}; 1; 1) - \pi_{1L}(1; 0; 0; 0; 0; 1)]$$
$$+ v_{12}u_{13}[\pi_{1L}(0; 1) - \pi_{1L}(0; 0)] + v_{12}v_{13}[\pi_{1L}(0; 0; 1; 0; 1; 0) - \pi_{1L}(0; 0; 0; 0; 0; 0)]$$

Case S6 $\pi_{3H}(0; u_{13}; 1; u_{23}; 1; 0) > K \geq \pi_{3H}(0; u_{13}; 0; u_{23}; 0; 0)$
Separation payoff:

$$u_{12}\pi_{1L}(1; u_{13}; 0; u_{23}; 0; 1) + v_{12}[u_{13}\pi_{1L}(0; 0) + v_{13}\pi_{1L}(0; 0; 0; 0; 0; 0)] \tag{2.152}$$

Defection payoff:

$$u_{12}\pi_{1L}(1; u_{13}; 1; u_{23}; 1; 1) + v_{12}\pi_{1L}(0; u_{13}; 1; u_{23}; 1; 0) \tag{2.153}$$

Condition for 1L to separate:

$$v_{12}u_{13}[\pi_{1L}(0; 0) - \pi_{1L}(0; u_{13}; 1; u_{23}; 1; 0)] \geq \tag{2.154}$$
$$u_{12}[\pi_{1L}(1; u_{13}; 1; u_{23}; 1; 1) - \pi_{1L}(1; u_{13}; 0; u_{23}; 0; 1)]$$
$$+ v_{12}v_{13}[\pi_{1L}(0; u_{13}; 1; u_{23}; 1; 0) - \pi_{1L}(0; 0; 0; 0; 0; 0)]$$

Case S7 $\pi_{3H}(0; u_{13}; 0; u_{23}; 0; 0) > K$
Separation payoff:

$$\pi_{1L}(p_{1\lambda}; p_{2\lambda}) + u_{12}\pi_{1L}(1; u_{13}; 0; u_{23}; 0; 1) + v_{12}\pi_{1L}(0; u_{13}; 0; u_{23}; 0; 0) \tag{2.155}$$

Defection payoff:

$$\pi^{br}_{1L}(p_{1\lambda}; p_{2\lambda}) + u_{12}\pi_{1L}(1; u_{13}; 1; u_{23}; 1; 1) + v_{12}\pi_{1L}(0; u_{13}; 1; u_{23}; 1; 0) \tag{2.156}$$

By the same argument that was used for case S1, firm 1L will not wish to separate in case S7.

Separation on Nash duopoly prices: resumé A low-cost incumbent will not be willing to separate if entry costs and prior beliefs place the game in case S1, S4, or S7. A low-cost incumbent may be willing to separate in remaining cases. If it is willing to separate, it is because the expected lost profit from the entry that would following defection (pretending to have high cost) outweighs expected gains from inducing rivals to set higher prices in the second period. If low-cost incumbents are willing to separate, their motivation is to limit entry.

For high-cost incumbents, however, the expected profit to be gained by deterring entry is an incentive to defect from the separating strategy.

2.7.3. Linear example, case S2

The condition for firm 1H to separate, reproduced here for convenience, is

$$\pi_{1H}(u_{12},u_{21}) - \pi_{1H}(p_{1L}) + u_{12}u_{13}[\pi_{1H}(1,1) - \pi_{1H}(1,0)]$$
$$+ v_{12}[\pi_{1H}(0,1) - \pi_{1H}(0,0)] \geq u_{12}v_{13}[\pi_{1H}(1,0) - \pi_{1H}(1,0,1,0,1,1)].$$

Firm 1H's payoff in the one-period game with only own cost type known is

$$\pi_{1H}(u_{12},u_{21}) = \frac{1}{1-\theta^2}(\bar{p}_{1H})^2. \tag{2.157}$$

If instead of separating firm 1H masquerades as firm 1L, its payoff is

$$\pi_{1H}(p_{1L}) = (\bar{p}_{1L} + c_L - c_H)\frac{a_L - u_{12}\theta a_H - v_{12}\theta a_L + u_{12}\theta\bar{p}_{2H} + v_{12}\theta\bar{p}_{2L} - \bar{p}_{1L}}{1-\theta^2} \tag{2.158}$$

$$= \left[\bar{p}_{1H} + \frac{c_H - c_L}{2} - (c_H - c_L)\right]\frac{a_L - u_{12}\theta a_H - v_{12}\theta a_L + u_{12}\theta\bar{p}_{2H} + v_{12}\theta\bar{p}_{2L} - \bar{p}_{1L}}{1-\theta^2}$$

(making use of (2.13))

$$= \left(\bar{p}_{1H} - \frac{c_H - c_L}{2}\right)\times$$

$$\frac{c_H - c_L + \bar{p}_{1H} - \bar{p}_{1L} + [(1-\theta)a_H - v_{12}\theta(a_L - a_H) + u_{12}\theta\bar{p}_{2H} + v_{12}\theta\bar{p}_{2L} - \bar{p}_{1L}]}{1-\theta^2}$$

$$= \left(\bar{p}_{1H} - \frac{c_H - c_L}{2}\right)\frac{c_H - c_L + \bar{p}_{1H} - \bar{p}_{1L} + \bar{p}_{1H}}{1-\theta^2}$$

(using the equation of firm 1H's reaction function)

$$= \frac{1}{1-\theta^2}\left(\bar{p}_{1H} - \frac{c_H - c_L}{2}\right)\left(\bar{p}_{1H} + \frac{c_H - c_L}{2}\right)$$

$$= \frac{1}{1-\theta^2}(\bar{p}_{1H})^2 - \frac{1}{4}\frac{1}{1-\theta^2}(c_H - c_L)^2$$

$$= \pi_{1H}(u_{12},u_{21}) - \frac{1}{4}\frac{1}{1-\theta^2}(c_H - c_L)^2.$$

Hence

$$\pi_{1H}(u_{12}, u_{21}) - \pi_{1H}(p_{1L}) = \frac{1}{4}\frac{1}{1-\theta^2}(c_H - c_L)^2. \quad (2.159)$$

Expressions for the other elements of the stability condition are

$$\pi_{1H}(1,1) - \pi_{1H}(1,0) = \frac{1}{1-\theta^2}\left\{[\overline{p}_{1H}(1,1)]^2 - [\overline{p}_{1H}(1,0)]^2\right\} \quad (2.160)$$

$$= \frac{\theta^2}{(1-\theta^2)(4-\theta^2)}\left[\frac{1-\theta}{2-\theta}a_H - \frac{1}{4}\frac{\theta^2}{4-\theta^2}(c_H - c_L)\right](c_H - c_L)$$

$$\pi_{1H}(0,1) - \pi_{1H}(0,0) = \frac{1}{1-\theta^2}\left\{[\overline{p}_{1H}(0,1)]^2 - [\overline{p}_{1H}(0,0)]^2\right\} \quad (2.161)$$

$$= \frac{\theta^2}{(1-\theta^2)(4-\theta^2)}\left[\frac{1-\theta}{2-\theta}a_H - \frac{\theta}{4}\frac{4+\theta}{4-\theta^2}(c_H - c_L)\right](c_H - c_L)$$

$$\pi_{1H}(1,0) - \pi_{1H}(1,0,1,0,1,1) \quad (2.162)$$

$$= \frac{1}{1-\theta^2}[\overline{p}_{1H}(0,1)]^2 - \frac{1+\theta}{(1-\theta)(1+2\theta)}[\overline{p}_{1H}(1,0,1,0,1,1)]^2.$$

This can be evaluated using (2.19) and (2.42); the result is not informative.