A Principal-Agent Model for Product Specification and Production

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This paper develops and analyzes a principal-agent model for product specification and production motivated by “core buying” decisions at an automobile manufacturer. The model focuses on two important elements of the “core” buyer’s responsibility: (1) assessing the supplier’s capability, and (2) allocating some or all of a fixed level of some buyer-internal resource to help the supplier. Under the contracting scheme we model, the buyer (principal) delegates the majority of product specification and production activity to the supplier (agent), but retains the flexibility to commit a given, observable amount of an internally available, limited resource (e.g., engineering hours) to help the supplier. The supplier, in turn, allocates his resource (e.g., engineering hours) to produce the finished product. As in the motivating scenario, both the supplier’s resource allocation and capability are assumed to be hidden from the buyer. Hence, the principal’s problem is to determine a menu of (resource-commitment, transfer-price) contracts to minimize her total expected cost. Our analysis demonstrates that if buyer resource and supplier capability are substitutes, then the buyer’s second-best involvement in the supplier’s production process will be greater than first-best. The opposite is true if they are complements. Further, when the opportunity cost for the buyer’s resource is zero, then in the substitutes case the buyer will commit all of its resource, while in the complements case the buyer may withhold some resources to screen the supplier type. We describe two applications of the model—one in inventory management and one in pharmaceutical drug discovery—to illustrate its applicability and versatility. Finally, we use insights from the model to suggest hypotheses for empirical study.

Key words: production outsourcing; hidden information; adverse selection; contract menu; complements; substitutes

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1. Introduction

Times have changed since Henry Ford made the River Rouge complex in Dearborn, Michigan, into the ultimate in vertical integration, with iron ore going in at one end and shiny model A’s coming out the other. Now vertical dis-integration is the order of the day—in autos, in handheld computers, in pharmaceuticals, in ink-jet printers, in health care, in cameras….

(Forbes 2001a, p. 106)

Good-bye mergers and acquisitions. In a global market tied together by the Internet, corporate partnerships and alliances are proving a more productive way to keep companies growing.

(Forbes 2001b, p. 26)

Outsourcing and partnering are changing the nature of companies’ organizational charts and providing a new source for competitive advantage: the ability to form and manage effective buyer-supplier relationships. In the automobile industry, for example, Chrysler outsources over 80% of the parts it assembles; Ford, over 65%, and General Motors, over 55%. Cisco System partners provide final assembly for almost half of its switches and routers. Nearly 80% of Kodak’s reloadable cameras are sourced in Asia. Nearly all of Hewlett Packard’s printers are outsourced.

In this paper, we present and analyze a buyer-supplier model for product specification and production motivated by decisions made by “core” buyers at Ford Motor Company. Each of Ford’s approximately 350 core buyers is responsible for overseeing the relationship between Ford and one or more of its approximately 1,150 suppliers for the component parts or systems (e.g., brake pedals, seats, car audio systems) that are assembled into Ford products. More specifically, core buyers are responsible for assessing the capability of each supplier; negotiating specifications, prices, and quantities; and allocating Ford-internal product design and/or process-engineering resources to help the supplier. The amount of these resources...
available for a given vehicle development program, and hence, to any core buyer, is fixed.

Our model focuses on two important elements of the core buyer’s responsibilities: (1) assessing the capability of the supplier, and (2) allocating some or all of a fixed level of some given buyer-internal resource to help the supplier. As prescribed by our model, and as confirmed in practice, the associated decisions are linked. To illustrate, consider the Ford core buyer responsible for the Sony “head unit,” which is one component of a car audio system. The head unit fits into the dashboard and provides source selection and control, one or more sources (tuner, CD, cassette), and speaker control. Ford product designers will have prepared an initial set of physical, environmental (e.g., heat, vibration), and audio specifications. The joint goal of Ford and Sony is to transform the initial specifications into a finished head unit.

Buyers typically have several different types of resources to help suppliers in transforming specifications into finished components and systems. Our model considers a single resource, which we will arbitrarily label “engineering hours.” The Ford core buyer may deploy these engineering hours in many different ways, for example: (1) to improve the initial design, (2) to cooperate with the supplier in the transformation process, or (3) to work independently, but in parallel with the supplier.

Regardless of the nature of the buyer’s available engineering hours or how they are employed, the nature of the real-world buyer-supplier relationship is complex and interactive. We model this as a (necessarily simple) process as follows: The buyer commits herself to a given, observable number of engineering hours, \( x_1 \). Our analysis assumes that a fixed number of hours, \( R \), is available; i.e., \( x_1 \leq R \). The supplier allocates a portion of his own resource, \( x_2 \), to the transformation process. Our analysis takes an agency perspective, wherein the buyer is the principal who uses the supplier as an agent to produce the system according to the buyer’s specifications. Unlike the classical principal-agent model, however, the principal provides her own resources, \( x_1 \), to reduce total cost. Further, as in the motivating scenario, the supplier’s resource allocation, \( x_2 \), and \( \phi \), the supplier’s capability to perform the transformation given \( x_1 \) and \( x_2 \), are hidden from the buyer;¹ we refer to the buyer’s decision as “resource commitment” and to the supplier’s decision as “resource allocation.” The former is observable by both parties, while the latter is not.

¹ One has to be careful about the interpretation of capability. Our intended interpretation is that the supplier is capable of pursuing the task at hand, but at a cost that is not completely known to the buyer. That is, capability refers to the supplier’s ability to complete the desired task at low cost.

The buyer’s objective is to minimize her total cost, which includes the opportunity cost of her resources, the supplier’s cost of production, plus an agency cost (i.e., information rent) caused by the information asymmetry. Our analysis focuses on a resource-commitment screening contract in which the buyer proposes a menu of resource-commitment contracts and transfer prices, whereby each commitment-price pair in the menu is tailored to a supplier of certain capability. The supplier will reveal his capability by the choice he makes. Truthful revelation of the supplier’s capability is achieved by offering a price that includes an information rent that is increasing in the supplier’s capability. Our analysis shows that the buyer’s involvement should depend on whether her resource commitment and the supplier’s capability are complements or substitutes. Mathematically, complements (substitutes) assume that increasing the buyer’s resource commitment not only lowers the total cost, but it also increases (decreases) the cost advantage enjoyed by a more capable supplier relative to a less capable one. As we will demonstrate, in the complements case, the optimal buyer commitment is less than first-best, while in the substitutes case it exceeds first-best.

Note that \( x_1 \) can be any observable buyer resource (e.g., equipment, capital), \( x_2 \) can be any hidden supplier investment (e.g., dollars, time), and \( \phi \) can be any hidden supplier capability. For example, the model analyzed here could be used in interpreting the well-known Japanese practice of “shukko,” under which buyers temporarily transfer employees to work for their suppliers. Table 1 presents several other possible applications of our basic model resources.

**Literature Review.** We are unaware of any studies that examine “product specification and production”

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**Table 1 Possible Applications of Our Model**

<table>
<thead>
<tr>
<th>Industry/activity</th>
<th>Buyer resource commitment</th>
<th>Supplier resource allocation</th>
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<tbody>
<tr>
<td>Auto parts</td>
<td>Concept (Ford)</td>
<td>Design, manufacture (Johnson Controls)</td>
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<tr>
<td>Pharmaceuticals</td>
<td>Drug development (Merck, Pfizer)</td>
<td>Scale up and manufacturing (Catalytica)</td>
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<td>manufacturing</td>
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<td>Pharmaceuticals</td>
<td>Basic research (Merck, Pfizer)</td>
<td>Research, testing (Covance, Quintiles)</td>
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<tr>
<td>IT hardware</td>
<td>Concept (Cisco)</td>
<td>Subassembly, final assembly (Seoletron)</td>
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<tr>
<td>Food and beverage</td>
<td>Brand management (Coca Cola, Redox)</td>
<td>Bottling, distribution (Bottlers, Korex)</td>
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<tr>
<td>Financial services</td>
<td>Product, service design (PNC, First Union)</td>
<td>Transactions (MBNA)</td>
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<td>Dot.Com business</td>
<td>Business plan</td>
<td>Transactions processing and CRM</td>
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from a general, normative perspective. Nonetheless, our models and analysis are firmly based in the broad area of product development. For example, our model is related to many of those reviewed by Krishnan and Ulrich (2001). Although their review is restricted to development projects within a single firm, the perspective of Krishnan and Ulrich, like ours, is focused on decision making. Further, their four categories of decisions—“concept development,” “supply chain design,” “product design,” and “production ramp-up and launch”—are quite similar to our model’s two decision variables, if both concept development and supply chain design are viewed as part of product specification. An interesting backdrop for our model and its results is also provided by Clark’s study (1989) of parts strategy and supplier involvement in product development in the world auto industry. Indeed, we will argue in §5 that our analysis sheds some light on Clark’s empirical findings.

Game-Theoretic Models in Managing Buyer-Supplier Relationships. Liron (1999) examines a joint product-process development contract in a two-party supply chain similar to ours and provides insights by considering both a static and a dynamic model. His analysis focuses on the “free-rider” problem, but it does not consider information asymmetry or contractual limitations. Whang (1992) develops a game-theoretic model in which an outside contractor is hired to develop software for a buyer. That paper shares some common features with our model. Specifically, like the supplier in our model, the developer of the software in the Whang model has private information about the production cost. However, in the Whang model the buyer cannot influence the supplier’s cost by her actions. In fact, one can view our work as a hybrid that includes features of the Liron model (joint product-process development by two parties) and of the Whang model (better-informed supplier).

Reyniers and Tapiero (1995a) model a buyer and supplier as players in a zero-sum game, wherein the supplier can control the effort invested in the delivery of quality and the buyer may or may not inspect incoming materials. Reyniers and Tapiero (1995b) examine a similar relationship and model the effect of contract parameters such as price rebates and after-sales warranty cost on the choice of quality by the supplier.

Kim (2000) examines a buyer-supplier relationship in which the buyer subsidizes supplier effort to reduce cost, thereby increasing channel profit. Baiman et al. (2001) employ an agency model in a buyer-supplier relationship to examine contracting issues with respect to internal and external failures. Their results indicate that in choosing a product architecture (e.g., bill of material), the buyer should consider both manufacturability and its implications for contractibility and efficiency in managing the buyer-supplier supply chain.

Game-Theoretic Models in Supply Chain (Production and Inventory) Management. There is a large, rapidly increasing literature that applies game theory and contracting to the management of independently managed supply chains. A complete review of this literature is outside the bounds of this work. Cachon (1998) describes the fundamental issues, summarizes early results (e.g., Spengler, Pasternak), and describes several alternative contracting/coordinating mechanisms. Corbett and Tang (1998) provide a framework for contracting under asymmetric information. Tsay et al. (1998) review the modeling of supply chain contracts. More recently, the supply chain work most closely related to ours because it involves a supplier with private information about its marginal cost of production, includes Corbett (2001), Corbett and de Groot (2000), and Ha (2001). Cachon (2003) reviews the literature on the management of incentive conflicts in supply chains with contracts, while Chen (2003) summarizes the research body on information sharing and supply chain coordination.

Agency Models in Economics. The analytical underpinnings for our work are provided by the principal-agent paradigm. An introductory description of the basic model is given in Mas-Colell et al. (1995), where its two main variations are described: hidden action (moral hazard), where the agent’s actions are unobservable; and hidden information (adverse selection), where the agent has private information about the difficulty of the task he is to perform on behalf of the principal. The hidden-information scenario (which is the one we consider here) generates the so-called mechanism design problem: Design a mechanism that will induce the agent to reveal his private information. Chapter 7 of Fudenberg and Tirole (1991) analyzes the mechanism design problem and provides numerous references. A more advanced treatment of the topic is provided in Wilson (1997). Our basic model resembles models used to study the regulation of monopolists with unknown costs (Baron and Myerson 1982), or to study problems of control in firms with decentralized hidden information (Guesnerie and Laffont 1984).

The remainder of this paper is organized as follows. Section 2 presents the principal-agent model for product specification and production, and the analysis is presented in §3. Section 4 presents two applications of our model, one in supply chain management and the second one in pharmaceutical research and development. Section 5 presents the main managerial insights, and §6 provides the concluding remarks. Proofs for the main results are provided in the appendix.
2. The Principal-Agent Model

Our model assumes that the buyer (i.e., principal) will purchase a product from a supplier (i.e., agent). The parameters of the relationship include a technological component that describes the product specification, production process, and the production-cost function; and an economic component that specifies the interaction between the two parties.

The technological component assumes that the supplier’s production cost depends on the resource commitment made by the buyer. The buyer’s allocated resource $x_2$, and the supplier’s capability $\phi \in [\phi_1, \phi_2]$; where $\phi_1$ denotes the least (most) capable supplier. The resources provided by the two parties are not necessarily measured in the same units. The supplier’s production cost will be denoted by $V(x_1, x_2, \phi)$ and is assumed to be strictly decreasing in $x_1$ and $\phi$, and to be jointly strictly convex in $x_1$ and $x_2$. The buyer opportunity cost for her resource commitment is denoted $G(x_1)$, where $x_1$ is constrained such that

$$0 \leq x_1 \leq R.$$ (1)

$G(x_1)$ is assumed to be increasing and convex. Supplier resources may also be constrained, but this is not recognized explicitly. For concreteness, the reader can conceptualize $x_1$ as the buyer resources committed to product specification, and $x_2$ as the supplier resources allocated to complete product specification and to production.

The economic component specifies the sequence of events and the information asymmetry. The buyer is the leader and the supplier is the follower. The supplier’s capability is known by the supplier, but not by the buyer. The buyer’s beliefs about the supplier’s capability are reflected in her prior: $\pi(\phi)$. The sequence of events is as follows: (1) The buyer moves first and offers a contract. Because the model reflects an adverse-selection problem, the contract is a menu that consists of the buyer’s resource commitment and a corresponding transfer price. (2) The supplier chooses, among all options offered, the pair (resource commitment, price) that maximizes his total payoff. (3) The buyer provides the resource commitment chosen by the supplier. (4) The supplier observes the buyer’s resource commitment, chooses his resource allocation, incurs the cost, and delivers the final product to the buyer. (5) Finally, the buyer pays the supplier.

According to the revelation principle, it is sufficient to focus on contracts in which the supplier will reveal his capabilities by the option he selects. Following common practice, let $(x_1(\phi), t(\phi))$ denote the (resource commitment, transfer price) pair intended for a type-$\phi$ supplier.

The basic problem to be studied is: Determine a menu of contracts $(x_1(\phi), t(\phi))_{\phi \in [\phi_1, \phi_2]}$ to minimize the buyer’s total expected cost

$$\int_{\phi_1}^{\phi_2} \pi(\phi) [G(x_1(\phi)) + t(\phi)] d\phi,$$ (2)

subject to the incentive-compatibility constraint that a supplier of type $\phi$ will choose buyer resource-commitment level $x_1(\phi)$,

$$t(\phi) - \min_{x_2} V(x_1(\phi), x_2, \phi) \geq t(\hat{\phi}) - \min_{x_2} V(x_1(\hat{\phi}), x_2, \phi)$$

for all $\phi \in [\phi_1, \hat{\phi}]$, (3)

and the participation constraint that the supplier’s payoff will exceed the prevailing expected payoff in the marketplace

$$t(\phi) - \min_{x_2} V(x_1(\phi), x_2(\phi), \phi) \geq \bar{u}$$

for each $\phi \in [\phi_1, \hat{\phi}]$. (4)

For brevity, define the induced cost function $\tilde{V}(x_1, \phi)$

$$\tilde{V}(x_1, \phi) = \min_{x_2} V(x_1, x_2, \phi).$$

The monotonicity of $V(x_1, x_2, \phi)$ implies that $\tilde{V}(x_1, \phi)$ is decreasing in $x_1$, and its convexity implies that $\tilde{V}(x_1, \phi)$ is convex in $x_1$. Note that the incentive-compatibility constraint (3) and participation constraint (4) reflect the assumption that the supplier will choose his resource allocation $x_2$ only after he decides which pair $(x_1(\phi), t(\phi))$ to select.

Several economic assumptions are worth discussing. First, despite the assumed information asymmetry about the supplier’s capabilities, our analysis assumes that the buyer can determine the optimal level for the supplier’s resource allocation in the (hypothetical) case that the supplier’s capability is known. Implicitly, this assumes that the buyer is familiar with the technological aspects of the production process, but not with the exact parameters of the technology.

Similarly, our analysis assumes that the buyer’s resource commitment is observable by the supplier and occurs before the supplier determines his own resource allocation. In many instances, resource allocation and commitment decisions are done in parallel, and not sequentially as assumed in our model. However, our model still applies if the buyer’s resource commitment is contractible. This could happen if the buyer’s involvement is limited along a well-defined dimension while the supplier’s involvement is
much more elaborate and not easily measurable—for example, when the buyer provides physical resources that enhance the supplier’s resources but where the supplier provides trained and specialized personnel. While the nature of the physical resources is easily measurable, the training and skill level of the personnel cannot be specified in an enforceable contract.

It should also be noted that our model is not appropriate for a different sequence of events. For example, if the supplier invests in capability-dependent technology that is inflexible, the sequence of events changes and the model cannot be used. In this example, the buyer’s leadership position is reduced and the corresponding cost structure would be different from the one derived from our model.

A remark about quality is also appropriate. Specifically, the model assumes that the quality of the product is fixed and that buyer and supplier resources have no discernible effect on quality. However, there are various ways in which quality can be incorporated in the model. The most direct is to interpret buyer resource as being used, in part, to develop and implement quality specifications and that these specifications are associated with future costs (such as warranty costs) incurred by the supplier. A similar interpretation could be given to the supplier’s resource. Another alternative would be to explicitly model quality and its impact on the supplier’s cost and the buyer’s desirability for the product.

Finally, it should also be emphasized that although our model focuses on the adverse-selection problem, there is also a possible double moral-hazard problem in the buyer-supplier relationship. Both the buyer and the supplier may not “actually” dedicate the planned amount of resources. However, the double moral-hazard problem is avoided because the buyer’s input is assumed to occur before the supplier invests its own resources, and to be observed by the supplier. Therefore, she cannot deviate from the precommitted level of resource input.

### 3. Model Analysis and Theorems

The analysis proceeds in two steps. First, we consider the first-best case where the buyer knows the supplier’s capability and chooses both the buyer and supplier resource levels. This solution provides a benchmark. Next, we analyze the “screening” contract.

#### 3.1. First-Best Problem

Here the buyer knows \( \phi \), and chooses \( x_1 \) and \( x_2 \) to minimize total cost. The problem is as follows: For each \( \phi \in [\phi, \bar{\phi}] \), find \( x_1(\phi) \) and \( x_2(\phi) \) to minimize the total production cost plus the opportunity cost

\[
V(x_1(\phi), x_2(\phi), \phi) + G(x_1(\phi))
\]

subject to

\[
0 \leq x_1(\phi) \leq R.
\]

The optimal (first-best) choice of the two inputs for each type \( \phi \in [\phi, \bar{\phi}] \) is denoted \( x_1^*(\phi) \) and \( x_2^*(\phi) \), respectively. The first-best solution can be obtained using the Karush-Kuhn-Tucker conditions that are necessary and sufficient for the convex problem (5)-(6).

#### 3.2. Resource-Commitment Screening Contract

We now analyze a resource-commitment screening contract. For a supplier of type \( \phi \), this contract will specify a resource level \( x_1(\phi) \) and a price \( t(\phi) \). The intention is that a type-\( \phi \) supplier will find it to his best interest to choose the pair \( (x_1(\phi), t(\phi)) \).

The analysis will demonstrate that the nature of the optimal contract depends on whether the cross partials \( V_{x_1d}(x_1, \phi) \) are positive or negative. These are the well-known “single-crossing” conditions from mechanism design, but are reinterpreted in the context of our model. We consider two cases yielding different results. In the first case, we assume that \( V_{x_1d}(x_1, \phi) \leq 0 \) for all \( x_1 \) and \( \phi \). This implies that the buyer’s efforts and the supplier’s capabilities are complements: More buyer resources increase the magnitude of the supplier’s marginal cost-reduction capability (i.e., it decreases \( \partial V(1, \phi)/\partial \phi \)). Given complements, we will show that the optimal menu of contracts separates less capable suppliers by committing fewer resources compared to the resources that would be committed to a more capable supplier. In the second case, we assume that \( V_{x_1d}(x_1, \phi) \geq 0 \) for all \( x_1 \) and \( \phi \). This implies that the buyer’s efforts and the supplier’s capabilities are substitutes: More buyer resources reduce the supplier’s marginal cost-reduction capability (i.e., it increases \( \partial V(1, \phi)/\partial \phi \)). Given substitutes, we will show that the optimal menu of contracts may become a pooling contract with a full resource commitment and a fixed price. The examples in Figure 1 illustrate that, given complements, increases in buyer resources enlarge the gap between suppliers of different capabilities. By contrast, given substitutes, increases in buyer resources reduce this gap.

The analysis of the mechanism design problem (2)-(4), described in Fudenberg and Tirole (1991, Chapter 7) proceeds in two steps. First, for any given menu of resource-commitment options \( (x_1(\phi))_{\phi=\Phi} \), find the transfer price menu \( (t(\phi))_{\phi=\Phi} \) that will make it desirable for a type-\( \phi \) supplier to choose the pair \( (x_1(\phi), t(\phi)) \). Second, obtain the menu of resource-commitment options that, when implemented with the pricing mechanism from Step 1, minimizes the buyer’s total expected cost.

The main result from Step 1 is given below.
Figure 1  A Graphical Example of Complements and Substitutes

**PROPOSITION 1.** (1) Assume that $\tilde{V}_{x_1,\phi}(x_1, \phi) \leq 0$. Then, a (resource-commitment, price) menu $(x_1(\phi), t(\phi))_{\phi \in \Phi}$, where $x_1(\phi)$ is nondecreasing, satisfies (3) and (4) if $t(\phi)$ is given by

$$t(\phi) = u + \tilde{V}(x_1(\phi), \phi) - \int_{\phi}^{\hat{\phi}} \tilde{V}(x_1(\phi), \tilde{\phi}) d\tilde{\phi}. \quad (7)$$

(2) Assume that $\tilde{V}_{x_1,\phi}(x_1, \phi) \geq 0$. Then, a (resource-commitment, price) menu $(x_1(\phi), t(\phi))_{\phi \in \Phi}$, where $x_1(\phi)$ is nonincreasing, satisfies (3) and (4) if $t(\phi)$ is given by (7).

The nature of the nonlinear transfer payment system (7) provides an important insight. The total price for a supplier of type $\phi$ is equal to the reservation profit margin $u$ plus the total cost of production when the supplier chooses the desired resource commitment $x_1(\phi)$, plus an information rent. The information rent is increasing in the supplier’s capabilities, and hence makes it attractive for the supplier to reveal his capabilities by the choice he makes.

The second step of the analysis identifies the menu that will minimize the buyer’s expected total cost when implemented with the pricing mechanism (7). If we ignore the requirement that this mechanism implements policies that are monotone, then the problem becomes: Find a menu of resource-commitments $(x_1(\phi))_{\phi \in \Phi}$ to minimize

$$
\int_{\phi}^{\hat{\phi}} \left[ G(x_1(\phi)) + \tilde{V}(x_1(\phi), \phi) - \int_{\phi}^{\tilde{\phi}} \tilde{V}(x_1(\phi), \tilde{\phi}) d\tilde{\phi} \right] \pi(\phi) d\phi,
$$

where (8) is derived by substituting (7) into (2). A change in the order of integration implies that this problem is equivalent to minimizing

$$
\int_{\phi}^{\hat{\phi}} \left[ G(x_1(\phi)) + \tilde{V}(x_1(\phi), \phi) - \frac{1 - P(\phi)}{\pi(\phi)} \tilde{V}(x_1(\phi), \phi) \right] \pi(\phi) d\phi,
$$

where $P(\phi)$ is the cumulative density function for the supplier’s capabilities. Then, the second step is to obtain the menu $(x_1(\phi))_{\phi \in \Phi}$ that minimizes $(9)$, ignoring the constraints implied in Proposition 1, and then to verify that the constraints are satisfied.

Before we proceed with the analysis, it is worth noting that (9) highlights the trade-off that must be balanced by the optimal screening contract: The buyer wishes for the supplier to choose a resource level that minimizes total production cost, but also wants to extract some of the cost savings. To achieve this, she has to offer the supplier an information rent. Hence, the menu designed by the buyer proposes a resource commitment that may be different from first-best—one that may increase the direct product development cost and the buyer’s opportunity cost compared to first-best, but one with smaller information rents.

We are now in a position to present our main results. This requires three technical assumptions on $\tilde{V}(x, \phi)$ and $\pi(\phi)$. The first assumption ensures that the integrand in (9) is convex; the remaining two assumptions imply that the solution to the first-order condition satisfies the desired monotonicity conditions. The assumptions are: (a) $\tilde{V}_{x_1,\phi}(x_1, \phi) < 0$; (b) the hazard rate $(1 - P(\phi))/\pi(\phi)$ is nonincreasing; and (c) $\tilde{V}_{x_1,\phi}(x_1, \phi) < 0$ in the case of complements and $\tilde{V}_{x_1,\phi}(x_1, \phi) > 0$ in the case of substitutes. These assumptions are ad hoc and only necessary for analytical tractability. In their absence, one can still minimize (9), but care must be taken not to use the first-order conditions naively, and also to verify that the derived solution satisfies the relevant monotonicity conditions as articulated in Proposition 1.

**PROPOSITION 2.** (a) Let $x_1^*(\phi)$ denote the buyer’s second-best resource-commitment level. Then, in the case of complements

$$x_1^*(\phi) \leq x_1^*(\phi), \quad (10)$$

while in the case of substitutes

$$x_1^*(\phi) \geq x_1^*(\phi). \quad (11)$$

(b) If the buyer’s resource and supplier’s capability are complements, then the optimal menu of resource
The optimal resource allocation for a type-$\phi$ supplier, $x^*_\phi(\phi)$, chosen in response to the optimal screening contract, satisfies (14). The pricing system that implements the optimal contract is given by (15).

**Insights from the Propositions.** The main insight from our analysis is that the buyer’s optimal resource commitment depends on its interaction with supplier capability. When supplier capability and buyer resources are substitutes, increasing the buyer’s commitment reduces the supplier’s marginal capability, which drives down the information rents. It follows that it is optimal for the buyer to commit more resources than first-best even though this increases her opportunity cost. Furthermore, if the substitution effect is sufficiently strong (i.e., if $\bar{V}_{x_1}(R, \phi) \geq \pi(\phi)/(1 - P(\phi))[\bar{V}_{x_1}(R, \phi) + G_{x_1}(R)]$, then (17) implies that $\phi^*_U = \hat{\phi}$, then the optimal contract will become a pooling contract where the buyer commits all her resources ($x_1 = R$) and offers a fixed price.

In the case of complements, an increase in the buyer commitment enhances the supplier’s marginal capability and, hence, increases the information rents. Therefore, the buyer finds it advantageous to restrain her involvement to effectively price-discriminate: She provides contracts with limited commitment (low $x_1$) and contracts with a more extensive commitment (higher $x_1$), but at a price discount (lower $t$). Hence, the more capable supplier finds it advantageous to choose the more extensive involvement at a lower price because he can better integrate this involvement in his processes and achieve larger cost reductions. Therefore, even though the price paid to a more capable supplier is lower than the one paid to a less capable supplier, the former is left with a larger surplus (by taking advantage of the buyer’s involvement). As in the case of substitutes, a pooling contract becomes optimal when the complementation effect is sufficiently strong, but in this case the buyer will not commit any resources.

An important observation here is that the buyer exploits her involvement in the product development project to extract some of the supplier’s surplus. In fact, the supplier with a high level of capability retains more of the surplus than the less capable supplier. In the case of substitutes, the buyer’s ability to extract supplier surplus is constrained by resource availability. That is, while in the case of complements the second-best resource commitment is lower than the first-best one, and hence the upper bound on the resource constraint is less likely to be restrictive, in the case of substitutes the opposite is true. Therefore, the resource constraint is critical in determining the buyer’s ability to extract the surplus generated by the supplier’s capabilities through an effective use of screening contracts.
A Simple Case. To further illustrate the role of complements and substitutes, let us suppose the buyer’s marginal opportunity cost is zero, so that the first-best solution is to commit all buyer resources irrespective of supplier capabilities. Then, Proposition 2 implies that the optimal contract for the case of substitutes is a pooling contract with a full resource commitment offered for a fixed price, while for the case of complements a screening contract emerges.

In the next section, we present two applications of the model to operationalize the concepts of complements and substitutes and to illustrate the main insights extracted from the generic model.

4. Examples

We now present two examples: the first, an inventory-management example; and the second, a sequential-experimentation model in pharmaceutical drug discovery. Our goal is twofold: first, to demonstrate the versatility and applicability of the model; second, to add more structure to the general findings from §3. Both examples assume the buyer’s resources have zero opportunity cost, but they can easily be modified to reflect a positive cost.

4.1. Inventory Management: EOQ Analysis

The buyer requires a product from the supplier at a fixed rate $D$ per unit time. Demand is met from finished-goods inventory. Shortages are prohibited. The supplier incurs a production setup cost and an inventory-holding cost. The setup cost depends on the buyer’s specifications, which in turn depend on the buyer’s resource commitment, $x_1$, and on the supplier’s setup capability, $\phi$. Hence, the setup cost will be denoted by $K(x_1, \phi)$. The supplier’s decision is the production lot size $x_2$. The holding cost is $h$ per item per unit time. Then, the supplier’s cost function is

$$V(x_1, x_2, \phi) = \frac{DK(x_1, \phi)}{x_2} + \frac{h}{2} x_2.$$  

It is sufficient to consider the induced cost function $\tilde{V}(x_1, \phi) = \min_{x_2} V(x_1, x_2, \phi)$. It is easy to show that $\tilde{V}(x_1, \phi) = \sqrt{2DhK(x_1, \phi)}$. We assume that each setup consists of a number of steps, $N(x_1, \phi)$, that depend on both product specification and supplier capability, and that the cost of each step is capability dependent, and denoted by $s(\phi)$; i.e., $K(x_1, \phi) = N(x_1, \phi) \times s(\phi)$. This example is based on Porteus (1985), where a single decision maker without information asymmetry invests in setup cost reduction. The cost to decrease the setup cost is not modeled explicitly.

Next, we will describe circumstances under which the buyer resource and supplier capability are complements versus substitutes. First, we assume that the cost per step decreases for more capable suppliers (i.e., $ds(\phi)/d\phi < 0$), that the number of steps decreases as the buyer commits more resources (i.e., $\partial N(x_1, \phi)/\partial x_1 < 0$), and also that the number of steps is nonincreasing in the supplier’s capabilities (i.e., $\partial N(x_1, \phi)/\partial \phi \leq 0$). To obtain a cost function that satisfies the substitutes case, it is sufficient to have the number of steps be independent of $\phi$ (i.e., $\partial N(x_1, \phi)/\partial \phi = 0$). Simple algebra then shows that $\tilde{V}_{x_1}(x_1, \phi) \geq 0$. On the other hand, when $\partial N(x_1, \phi)/\partial \phi < 0$ and $\partial^2 N(x_1, \phi)/\partial x_1 \partial \phi < 0$, then it is possible to construct examples that satisfy the complements case; i.e., $\tilde{V}_{x_1}(x_1, \phi) < 0$ (note that one must use the functional form of $\tilde{V}$ to find regions where the condition for complements holds).

In summary, when the buyer’s resource reduces the number of steps and when the supplier’s capability only influences the cost of each step, but not the number of steps, then we have a case of substitutes. On the other hand, when the supplier’s capability influences both the cost of each step and the number of steps, it is possible to have either a case of complements or a case of substitutes.

A Numerical Example. For the case of substitutes, let $N(x_1, \phi) = (1 - (x_1/4))$ and $s(\phi) = (1 - \phi)$. Assume that $R = 2, \sqrt{2hD} = 1$, that supplier capability $\phi$ takes a value between 0 and 1, and is uniformly distributed between 0 and 1. The corresponding $\tilde{V}_{x_1}(x_1, \phi) = 1/16 \sqrt{2K(x_1, \phi)} > 0$. It can then be verified that it is optimal to set $x_1^*(\phi) = 2$ and thus offer a transfer payment of $t(\phi) = \frac{1}{\sqrt{2}} = 0.707$ to all suppliers regardless of capability. The corresponding surplus for a supplier with capability $\phi$ is $1/\sqrt{2[1 - \sqrt{1 - \phi}]}$, which is increasing in the supplier type. Figure 2A plots the supplier surplus as a function of capability. Note that the surplus is equal to zero for the least capable supplier (when $\phi = 0$) and the contract provides a surplus for all other suppliers. This corresponds to the results for the single resource-commitment contract.

For the case of complements, let $N(x_1, \phi) = (1 - (\phi x_1/4))$ and $s(\phi) = 1$; all other model components are the same as above. Note that the benefit from a given $x_1$ is greater for more capable suppliers, i.e., a complements relationship. The associated $\tilde{V} = \sqrt{2Dh x_1^2/4}$ and the cross partial $\partial^2 \tilde{V}/\partial \phi \partial \phi x_1 = (-8 + (\phi x_1))/[8(4 - (\phi x_1))]^{3/2}$; this is negative for all feasible values of $x_1$ (i.e., $x_1 \leq 2$).

It can also be verified that the optimal menu of contracts would set $x_1^*(\phi) = 0$ for $\phi < 0.5, x_1^*(\phi) = 8(2\phi - 1)/(\phi ((3\phi - 1))$ for $0.5 < \phi < 0.5425$, and $x_1^*(\phi) = 2$ for $\phi > 0.5425$. This contract (plotted in Figures 2B and 2C) shows that the buyer offers a contract that induces the least capable suppliers to choose no involvement by the buyer, while the most capable suppliers choose full use of buyer resources. (Note that the value 0.5425 was obtained by identifying the value of $\phi$ such that the $x_1^*(\phi)$ hits the value of $R = 2$.) The associated menu
of payments offered to suppliers is \( t(\phi) = 1 \) for \( \phi \leq 0.5 \), 
\( t(\phi) = 0.8601 \) for \( \phi > 0.5425 \), and 
\( t(\phi) = \tilde{V}(x_1^*(\phi), \phi) - \int_0^\phi \tilde{V}_\phi(x_1^*(\phi), \phi) \, d\phi \) for \( 0.5 < \phi \leq 0.5425 \) (see Figure 2C). Figure 2C shows that the payments offered to the less capable suppliers are higher than those for more capable suppliers; i.e., the payments decrease with capability. However, Figure 2D also shows that supplier surplus increases with capability, thus providing stronger incentives for the more capable suppliers. Because the more capable suppliers use buyer resources more efficiently, they see a greater cost reduction.

### 4.2. Pharmaceutical Drug Discovery

Our second example is drawn from the lead-identification step in drug discovery. In this step, thousands of molecules are tested using a biological assay that reflects a particular disease. Molecules that are found to block some of the disease-specific biological activities are called “leads” and progress to the second stage of development.

While several technological platforms are available for lead identification, one novel technology called “combichem” is now becoming dominant; see Thomke (2001). In this technology, several molecules are generated using combinatorial chemistry and then tested using automated methodologies referred to as “high-throughput screening.” In a typical scenario, the pharmaceutical company (buyer) enters into an agreement with the startup (supplier), in which the latter will use its proprietary technology to identify one or more leads.

Lead identification by the supplier can be modeled as a sequence of independent experiments, with the probability of success in each experiment denoted \( 1 - p \), where \( p = \exp(\phi) \); the parameter \( \phi (\phi \leq 0) \) is the supplier’s private information that reflects his proprietary technology. The supplier contracts with the buyer to identify a lead. The supplier’s decision \( x_3 \) is a stopping rule. We assume that he will stop (running experiments) when the first lead is identified. If the cost of each experiment is \( c \), then the supplier’s total expected cost of experimentation is \( c/(1 - p) \); for a review of alternative experimentation models, see Dahan (1999).

The buyer can share resources with the supplier that can be used to perform several parallel experiments. That is, if the buyer’s resources committed to lead identification are denoted \( x_1 \), then each experiment consists of \( x_1 + 1 \) independent experiments. In addition, there are two possibilities: Either the buyer’s
resources are used to perform experiments using the supplier’s technology or using the buyer’s own technology. In the former case, the probability of success in each round of experiments becomes 1 − \( p^x \phi \), where \( p \) denotes the probability of failure in experiments performed using the buyer’s technology. In addition, to reflect a scenario where the supplier has access to a novel technology while the buyer uses established technology, it is natural to assume that \( q \) is known to both parties, but \( p \) is only known to the supplier.

In the case where the buyer’s resources are used with the supplier’s technology, then the production cost function \( \tilde{V}(x, \phi) = c/[1 - \exp[\phi + \phi]] \). By contrast, when the buyer’s resources are used with the buyer’s own technology, then production cost function \( \tilde{V}(x, \phi) = c/[1 - \exp[\beta x + \phi]] \), where \( \beta = \ln(p) \). Then, straightforward algebra demonstrates that the former scenario reflects a case of complements, while the latter is a case of substitutes. One can use the general framework developed in §3 to obtain the optimal contracts. However, because the special structure of the cost functions violates some of the technical assumptions, one must be careful with the first-order conditions in (13) and must check the conditions on the boundaries of the feasible region for \( x \); the details will not be presented here.

This example demonstrates that when buyer resources will be used in conjunction with supplier technology, then the buyer may find it beneficial to offer a menu of resource-commitment options to elicit information about the supplier’s capabilities and minimize the information rents. If, on the other hand, buyer resources cannot be integrated with the supplier’s technology, but only with the buyer’s own technology, then the buyer is better off committing all her resources in the relationship.

Other Examples. For another example of substitutes, consider the printing industry where digital files are used by the printer to generate a product that satisfies buyer specifications. As the buyer increases the effort to provide a cleaner digital file to the printer, the cost to generate a prototype that satisfies buyer specifications not only decreases, but the gap between more and less capable printers (with the same equipment) also decreases. Thus, we see that less capable suppliers receive a greater absolute benefit than more capable suppliers—hence, a cost structure that resembles substitutes.

Finally, consider the use of computer-aided design (CAD) technology for transmitting buyer specifications, which permits the use of computer simulation to develop and test product specifications. As the buyer increases the role of computer simulation to adjust product specifications, the benefit to more capable suppliers (i.e., with more effective engineering talent, and, thus better ability to integrate these files to reduce production costs) is higher. Such a cost relationship represents “complements.”

5. Model Insights and Observations on Practice

In this section, we will highlight the insights our model provides for product specification and production contracting. In particular, we interpret anecdotal and empirical observations about buyer-supplier partnerships through the lens of our model and propose hypotheses for possible empirical tests. Given the complexity of most real-world buyer-supplier relationships, our interpretations and hypotheses must, of course, be conditioned by phrases such as “all other things being equal” or “holding all other influences constant.” Nonetheless, we believe these insights to be of value in interpreting buyer-supplier relationships and, correspondingly, that the appropriate empirical tests would validate our hypotheses.

The Value of Providing Buyer Resources to a Supplier Through Screening Contracts. Why would/should a buyer provide physical, financial, or intellectual resources to help a supplier? This question has haunted other manufacturers ever since learning that Honda, Toyota, and other Japanese manufacturing companies have elaborate programs to do just that. Our analysis provides one answer: that the Japanese contracting process—in particular, offers of help or exchanges of information and the associated—provides otherwise hidden information about the supplier’s capability. Further, it demonstrates that screening contracts specify a link between resources offered by the buyer and prices paid to suppliers, and thus offer the incentive to suppliers to choose a level of buyer resource allocation that best suits their capability. In addition, given that a buyer uses screening contracts, her resource level determines the space of cost reduction available to her. These findings suggest the following hypothesis:

**Hypothesis 1 (H1).** For companies with buyers that use screening contracts with suppliers, those buyers that have more potential resources available to assist suppliers and potential suppliers are likely to have a more accurate knowledge about supplier capability than those who provide fewer such resources.

To test this hypothesis empirically, one must identify a collection of relationships governed by screening contracts, and must also provide metrics for the so-called accuracy of the buyer’s knowledge about supplier capability. Screening contracts cannot be observed directly because any data will most likely be obtained from executed contracts and not from any
menu options used during contract negotiation. However, the nature of the executed contracts may suggest the presence of screening contracts: If the resources committed and prices paid to suppliers with similar identifiable characteristics are different, it follows that the buyer might have used screening contracts during the negotiations to obtain information about the supplier’s nonidentifiable characteristics. Similarly, measures of the accuracy of the buyers’ knowledge about their respective suppliers’ capabilities can be obtained using different dimensions of the supplier’s competencies. As an example, consider a supplier’s product design capability, which might be measured in terms of its skill levels in CAD, the number (or percentage) of advanced degrees or certificates in design among its designers, etc. Similarly, a supplier’s manufacturing capability might be measured in terms of its manufacturing cycle time/s, first-pass yield/s, new product ramp-up time/s, etc. Then, the buyer’s knowledge can be measured in terms of these dimensions.

The Importance of the Buyer-Supplier Relationship on Optimal Buyer Resource Commitment and Total Cost. Our analysis also demonstrates the importance of complementarity versus substitutability in determining the amount of resources that the buyer should commit. In particular, depending on the nature of the relationship—substitute or complement—and the supplier’s capability, the optimal level of buyer allocation either equals zero; the maximum possible, \( R_t \); or something strictly in-between. Further, if both complementarity and substitutability do exist, then their existence provides one important motive for manufacturers to “qualify” suppliers (see Ford 1996) and/or offer programs such as those described by Dyer and Nebeoka (2000) to align suppliers or potential suppliers.

The examples in §4 suggest that complementarity between the buyer’s resources and the supplier’s capabilities exists in joint ventures involving big pharmaceutical corporations (i.e., entities with drug-related sales of more than $1 billion) and biotechnology startups (i.e., entities with no marketable drug), in which the former use the relationship to access the startup’s technology (Arora and Gambardella 1990). Then, our model states that everything else being equal, the involvement of a pharmaceutical partner in startups will vary to reflect the utilization of screening contracts.

On the other hand, the setup cost example provides contexts in which we have substitutes and contexts in which we have complements. We can test the model empirically by identifying cases where the buyer’s resources reduce the number of setup steps but not the costs of each step (substitutes), and cases where the buyer’s resources reduce both the number of setup steps and their cost (complements). Then, in the former case we would expect little variation in buyer resources used in the contract with the supplier, whereas in the latter we expect variations. Hence, we empirically identify contexts that suggest complements versus substitutes, and test the hypothesis empirically by looking into resource variations.

We use the following definitions: If the buyer provides the same level of resources to all suppliers independent of capability, we call this type of resource provision “push.” However, if the level of resources provided by the buyer to the supplier depends on the supplier’s capability, we call such a type of resource provision “pull,” because the nature of the help and its timing is tuned to supplier requirements; i.e., it depends on the identification of resource demand signals from the supplier. The corresponding hypotheses are:

**Hypothesis 2 (H2).** Consider setup-related buyer-supplier relationships (as in §4.1) in which when the buyer knows the supplier capability, her optimal strategy is to offer all of its resources to the supplier. If buyer resources are of the substitutes type, then the buyer is likely to use a “push” system of resource provision. If buyer resources are of the complements type, then the buyer is likely to use a “pull” system of resource provision.

A closely related hypothesis involves mismatches that occur in practice; i.e., that buyers whose resources are similar to those of their suppliers provide “pull”-type resource provision, etc. If such mismatches occur, then our model suggests that the buyer’s ability to reduce supplier-delivered cost will be affected. The corresponding hypothesis is:

**Hypothesis 3 (H3).** Consider setup-related buyer-supplier relationships (as in §4.1) where, with knowledge of the supplier capability, it is best for the buyer to offer all of its resources to the supplier. Given a complements (substitutes) relationship, “pull” (push)-type resource provision is more likely to be observed than “push” (pull)-type resource provision.

An important challenge with these hypotheses is the verification of the “first-best” statement that when the buyer knows supplier capabilities, she uses all her resources. To verify this statement, one must choose the empirical context carefully. The focus must be on well-understood production technologies where the “first-best” strategy is unambiguous. The biopharmaceutical examples in §4.2 provide such a context.

Reflections on Japanese and U.S. Manufacturing Partnerships. There is, of course, considerable literature describing Japanese manufacturers’ use of partnerships as contrasted with U.S. manufacturers. For example, Lee and Ansari (1985, p. 7), in a comparative analysis of traditional U.S. and Japanese just-in-time purchasing practices, characterize the Japanese as providing “loose” specifications, wherein “the buyer
relies more on performance specification (i.e., concept specification) than on product design” (i.e., product specification). In contrast, they characterize traditional U.S. purchasing as providing “rigid” specifications, wherein the buyer relies more on the opposite. In fact, Clark (1989) found systematic differences between Japanese automakers and their U.S. and European counterparts with respect to what we model as buyer effort in product design. In particular, “black box” parts—“those parts whose functional specification is done by the assemblers (buyers), while detailed engineering is done by parts suppliers”—account for 62% of the Japanese automakers’ total procurement costs, but only 16% and 32% for the U.S. and European automakers, respectively. Correspondingly, “detail-controlled” parts—“those parts that are developed entirely by the assemblers (buyers), from functional specifications to detailed engineering drawings”—account for only 30% of Japanese purchases, compared to 81% and 54% for the U.S. and European automakers.

However, Liker and Wu (2000) describe partnering arrangements that Honda’s and Toyota’s transplanted U.S. assembly plants have with their U.S. suppliers, and describe how these companies use their relationships to manage the capability of these suppliers. Kamath and Liker (1994) observe that even with respect to their first-tier suppliers, Japanese manufacturers classify suppliers into four classes, which can be viewed as representing the spectrum of buyer involvement with suppliers. Dyer and Nebeoka (2000) describe Toyota’s “knowledge-sharing network” to manage supplier capability.

We now suggest how our model can be used to reconcile these different findings. For any given buyer-supplier relationship, any given product, and any given supplier capability (\(\phi\), consider a total effort pie chart. Initially, the pie is divided into two slices, representing \(x_1(\phi)\), the buyer’s resource allocation; and \(x_2(\phi)\), the supplier’s resource commitment. Now cut the \(x_1(\phi)\) slice into two pieces: buyer resources allocated for product design-related effort (\(x_{11}(\phi)\)), and the buyer resources allocated for process specification-related effort (\(x_{12}(\phi)\)). Similarly, consider such a split of supplier-committed resources (\(x_2(\phi)\)) into \(x_{21}(\phi)\) and \(x_{22}(\phi)\).

As noted above, Clark’s (1989) study generally focused on design-related efforts. His empirical results compare the sizes of the design slices. Specifically, Clark found that the \(x_{11}(\phi)\) slice for U.S. automakers (buyers) was larger than that for typical Japanese automakers. However, Kamath and Liker (1994) focus on the composition of \(x_{12}(\phi)\) and \(x_{22}(\phi)\) and suggest that the value of \(x_{12}(\phi)\) for Japanese automakers is larger than that for U.S. automakers. Liker and Wu (2000) can be interpreted as suggesting that Honda and Toyota use screening contracts to adjust the value of \(x_1(\phi)\) based on supplier capability. Similarly, Dyer and Nebeoka (2000) suggest the role of adjusting \(x_{12}(\phi)\) by Japanese automakers to manage their suppliers.

These findings can be interpreted in the context of the example in §4.1. For “product specification” or design, one can define setup time and associated steps as all design tasks undertaken by the supplier that are specific to this buyer and are necessary before routine design activities at the supplier. For example, setup time could refer to design meetings between supplier and buyer wherein the design parameters and product functionality are fleshed out. This activity can be separated from the design activity by the supplier where the tasks are carried out independent of the buyer. Then, Clark’s study suggests the following hypothesis:

Hypothesis 4 (H4). The resources allocated by U.S. automakers to suppliers for setup improvement for “product specification” are more likely to follow a substitutes relationship. The corresponding resources provided by Japanese automakers to their suppliers for setups are likely to follow a complements relationship.

For process specification, we refer to setups as the traditional operational process of batch setup for production. The corresponding hypothesis derived from the results in Kamath and Liker (1994) is:

Hypothesis 5 (H5). The resources allocated by U.S. automakers for setup improvement for “process specification” are more likely to follow a substitutes relationship. The corresponding resources allocated by Japanese automakers for supplier setup improvement are likely to follow a substitutes relationship.

6. Conclusions

We have developed a principal-agent model for product specification and production. This model enables us to suggest the optimal use of resources in a screen- ing contract. Our analysis concludes that this decision depends on the cost relationship between the buyer’s commitment of resources and its impact on the supplier’s ability to decrease costs (based on his capability). We introduce two cost structures—substitutes and complements—and provide the optimal screening contract. The complements cost structure suggests that it may be optimal for the buyer to use less than all her available resources to optimize the total cost—which consists of the supplier product cost, reservation profit, and information rent. We provide two applications of the model to illustrate its versatility—one considers inventory management in the context of an EOQ model and the other models product development in the pharmaceutical industry. The managerial implications of this model are explored to suggest links to observed choices made in the auto industry.
Appendix

Proof of Proposition 1. Suppose that the constraint is not binding for some \( \phi' \). Then, \( x_1(\phi') < R \). However, it is assumed that \( \partial V(x_1, x_2, \phi)/\partial x_1 < 0 \), hence one can decrease the total cost by increasing \( x_1(\phi') \) until \( x_1(\phi') = R \). Therefore, the resource constraint is binding for each \( \phi \). □

Proof of Proposition 2. The implementability of the menu of resource commitments follows from Theorem 7.3 in Fudenberg and Tirole (1991, p. 261). To derive the expression for the transfer price we proceed as follows (the method is described in Fudenberg and Tirole 1991 and is replicated here for completeness). First, define the indirect utility function

\[
V_1(\phi) = \max_{\tilde{\phi}} t(\tilde{\phi}) - \tilde{V}(x_1(\tilde{\phi}), \phi). \tag{A.1}
\]

Then, Theorem 7.3 of Fudenberg and Tirole (1991) implies that the incentive compatibility constraint under the conditions spelled out in the proposition can be restated as follows:

\[
\frac{dt(\phi)}{d\phi} - \tilde{V}_{x_1}(x_1(\phi), \phi) \frac{d\tilde{V}_1(\phi)}{d\phi} = 0, \tag{A.2}
\]

which is equivalent to the statement \( \phi = \max_{\tilde{\phi}} t(\tilde{\phi}) - \tilde{V}(x_1(\tilde{\phi}), \tilde{\phi}) \). Equation (A.2), together with an application of the envelope theorem (see Mas-Colell et al. 1995, p. 964) to (A.1), implies that

\[
\frac{dV_1(\phi)}{d\phi} = -\tilde{V}_\phi(x_1(\phi), \phi). \tag{A.3}
\]

It follows that

\[
V_1(\phi) = u - \int_{\phi}^{\phi} \tilde{V}_\phi(x_1(\phi), \phi) d\phi. \tag{A.4}
\]

Hence,

\[
t(\phi) = u + \tilde{V}(x_1(\phi), \phi) - \int_{\phi}^{\phi} \tilde{V}_\phi(x_1(\phi), \phi) d\phi. \tag{A.5}
\]

Parts (b) and (c) can be similarly proven. □

Proof of Proposition 3. We will first prove parts (b) and (c) of the proposition and then part (a).

Parts (b) and (c). First, we obtain the first-order conditions for (9) and provide a characterization for their solution. Then, verify that the first-order conditions are necessary and sufficient for optimality. We will do that for the case of complements; i.e., \( \tilde{V}_{\phi x_1}(x_1(\phi), \phi) < 0 \). Similar analysis applies to the case of substitutes.

1°. The first-order conditions for (9) can be written as

\[
G_{x_1}(x_1(\phi)) + \tilde{V}_{x_1}(x_1(\phi), \phi) - \eta(\phi) \tilde{V}_{\phi x_1}(x_1(\phi), \phi) = 0. \tag{A.6}
\]

Because \( \tilde{V}_{\phi x_1}(x_1(\phi), \phi) < 0 \), then the left-hand side of (A.6) is positive for \( \phi \) in the neighborhood of \( \phi \) (because \( \eta(\phi) \gg 1 \) in that neighborhood), and it is negative in the neighborhood of \( \phi \) (because \( \eta(\phi) \approx 0 \) in that neighborhood); its sign coincides with the sign of \( G_{x_1}(x_1(\phi)) + \tilde{V}_{x_1}(x_1(\phi), \phi) \). This implies that the solution of the first-order condition (A.6) is given by (13).

2°. Now, the second-order conditions are

\[
G_{x_1x_1}(x_1(\phi)) + \tilde{V}_{x_1x_1}(x_1(\phi), \phi) - \eta(\phi) \tilde{V}_{\phi x_1x_1}(x_1(\phi), \phi) - 2 \eta(\phi) \tilde{V}_{\phi x_1}(x_1(\phi), \phi) \tilde{V}_{x_1x_1}(x_1(\phi), \phi) \leq 0. \tag{A.7}
\]

the nonnegativity follows because \( G(x_1) \) is assumed to be convex in \( x_1 \), \( V(x_1, x_2, \phi) \) is assumed jointly convex in \( x_1 \) and \( x_2 \), and because \( \tilde{V}_{\phi x_1}(x_1(\phi), \phi) < 0 \). Next, it remains to confirm that \( x_1^*(\phi) \) is nondecreasing in \( \phi \). Totally differentiating (A.6) with respect to \( \phi \) implies

\[
\frac{dx_1^*(\phi)}{d\phi} = \frac{\partial G_{x_1}(x_1(\phi))}{\partial \phi} + \eta(\phi) \tilde{V}_{\phi x_1}(x_1(\phi), \phi) - \eta(\phi) \tilde{V}_{\phi x_1x_1}(x_1(\phi), \phi), \tag{A.8}
\]

because both \( \partial G_{x_1}(x_1(\phi)) / \partial \phi \) and \( \tilde{V}_{\phi x_1}(x_1(\phi), \phi) \) are nonpositive, \( \tilde{V}_{\phi x_1}(x_1(\phi), \phi) \geq 0 \), and the denominator is the second-order condition (A.7), which is nonnegative. Finally, because of the joint convexity of \( V(x_1, x_2, \phi) \), it follows that the supplier’s optimal resource allocation will satisfy the first-order condition (14). The results for the case of substitutes can be similarly proven.

Next, to prove part (a) we proceed as follows (again we present the results for the case \( \tilde{V}_{\phi x_1}(x_1(\phi), \phi) \leq 0 \); the case of \( \tilde{V}_{\phi x_1}(x_1(\phi), \phi) > 0 \) is similar). For a given \( \phi \), let us assume that the first-best solution \( x^*_1(\phi) \) is in the interior of the feasible region \([0, R]\). That is, \( G_{x_1}(x^*_1(\phi)) + \tilde{V}_{x_1}(x^*_1(\phi), \phi) = 0 \). Now, use \( x^*_1(\phi) \) as a trial solution for the first-order condition (A.6). Because \( \tilde{V}_{\phi x_1}(x_1(\phi), \phi) \leq 0 \), it follows that

\[
G_{x_1}(x^*_1(\phi)) + \tilde{V}_{x_1}(x^*_1(\phi), \phi) - \eta(\phi) \tilde{V}_{\phi x_1}(x^*_1(\phi), \phi) \leq 0. \tag{A.9}
\]

Furthermore, the expression \( G_{x_1}(x^*_1(\phi)) + \tilde{V}_{x_1}(x^*_1(\phi), \phi) - \eta(\phi) \tilde{V}_{\phi x_1}(x^*_1(\phi), \phi) \) is nondecreasing in \( x \) (because \( G_{x_1}(x) \geq 0 \), \( \tilde{V}_{x_1}(x, \phi) \geq 0 \), and \( \tilde{V}_{\phi x_1}(x, \phi) \geq 0 \). Therefore, it follows that the solution to the first-order condition (A.6) is strictly less than \( x^*_1(\phi) \). Now, if strict convexity is replaced by convexity, then the strict inequality in (10) is replaced by a weak inequality. □

References


