THE JOURNAL OF FINANCE · VOL. XXX, NO. 3 · JUNE 1975

THE WEIGHTED AVERAGE COST OF CAPITAL: SOME QUESTIONS ON ITS DEFINITION, INTERPRETATION, AND USE: COMMENT*

JOHN J. MCCONNELL AND CARL M. SANDBERG**

IN HIS PAPER on the "weighted average cost of capital" Arditti [1] concludes, among other things, that ". . . the components of the weighted average after-tax cost of capital have been incorrectly specified," and that ". . . the capital structure that minimizes the weighted average after-tax cost of capital is a non-optimal one." He arrives at these conclusions via a two-step procedure: First, he derives a relationship that he claims is the correct weighted average after-tax cost of capital. Second, he shows that the capital structure that minimizes his weighted average after-tax cost of capital is inconsistent with maximization of the total market value of the firm.

The primary purpose of this comment is to reconcile the generally accepted definition of the after-tax weighted average cost of capital (wacc) with the definition proposed by Arditti. In accomplishing this objective we will derive three definitions of the wacc, and will show that the capital structure that minimizes two of the three definitions (within Arditti's framework) is an optimal one.

The traditional wacc is derived from the relationship between the market value of a levered firm, V_L, and its unlevered counterpart, V_U, both of which are expected to earn the perpetual cash flow $\bar{X}$ before interest and taxes.[1] Specifically,

$$V_L = V_U + tD \tag{1}$$

$$= \frac{\bar{X}(1-t)}{r_t} + tD \tag{1a}$$

where

t = marginal tax rate applicable to corporate earnings.
D = total market value of corporate debt.
r_t = rate at which the market capitalizes the expected returns net of taxes of an unlevered firm.

By differentiating (1) with respect to investment I, we obtain the investment acceptability criterion

$$\frac{dV_L}{dI} = \frac{(1-t)}{r_t} \cdot \frac{d\bar{X}}{dI} + t \cdot \frac{dD}{dI} \geqslant 1$$

and

$$\frac{d\bar{X}}{dI} = \left(1 - t \cdot \frac{dD}{dI}\right) \frac{r_t}{(1-t)} \tag{2}$$

* The ideas expressed herein are those of the authors alone and do not reflect the opinions of their respective institutions.

** Ohio State University and Morgan Guaranty Trust Company, respectively.

1. This definition of the earnings stream implies non-depreciable assets and zero growth in earnings.

From (2) we can derive three definitions of the wacc, each of which is appropriate for discounting different earnings flows, and all of which yield equivalent firm values. If we assume a constant debt/total value ratio, so that $dD/dI = D/V_L$, let S be the total market value of common stock, k_t be the after-tax required return on equity, i be the expected return on debt, and w_1 be the before-tax wacc, then we can derive the relationship that is most commonly labeled the before-tax wacc

$$w_1 = \frac{k_t}{(1 - t)}\left(\frac{S}{V_L}\right) + i\left(\frac{D}{V_L}\right) \tag{3}$$

Note that this relationship is equivalent to Arditti's (1a), which he also calls the before-tax wacc, only if his r is interpreted as $k_t/(1 - t)$. It appears that it is his failure to distinguish clearly between r (equity return in a world of no taxes) and k_t (equity return in a world with taxes) that leads to his disagreement with received cost of capital doctrine. The cost of capital defined in (3) is properly used to discount the earnings flow $\bar{X}$, so that

$$V_L = \frac{\bar{X}}{w_1}. \tag{4}$$

Multiplication of (4) by $(1 - t)$ yields a second definition of the wacc,

$$w_2 = \frac{\bar{X}(1 - t)}{V_L} \tag{5}$$

$$= k_t\left(\frac{S}{V_L}\right) + i(1 - t)\frac{D}{V_L} \tag{5a}$$

or

$$V_L = \frac{\bar{X}(1 - t)}{w_2}. \tag{6}$$

It is w_2 that is defined as the after-tax wacc by those authors with whom Arditti takes issue.[2] According to this definition of the wacc, the total market value of the firm equals the expected *pure equity* after-tax cash flow discounted at w_2. The same conclusion is reached by recognizing that

$$\begin{aligned} \bar{X}^t &= (\bar{X} - iD)(1 - t) + iD \\ &= \bar{X}(1 - t) + tiD \end{aligned} \tag{7}$$

and from (5) and (7)

$$\begin{aligned} w_2 &= \frac{\bar{X}^t}{V_L} - ti\left(\frac{D}{V_L}\right) \\ &= \frac{(\bar{X} - iD)(1 - t)}{V_L} + i\left(\frac{D}{V_L}\right) - ti\left(\frac{D}{V_L}\right) \end{aligned}$$

Since $k_t = \dfrac{(\bar{X} - iD)(1 - t)}{S}$ we have

$$w_2 = k_t\left(\frac{S}{V_L}\right) + i(1 - t)\left(\frac{D}{V_L}\right).$$

2. These include [2] pp. 123-124 and 173, [4] pp. 154 and 252-257, and [5] pp. 152 and 318-320.

which again is the after-tax wacc used to discount the earnings $\bar{X}(1 - t)$. Arditti's equations (4b) and (5) suggest, however, that the earnings $\bar{X}^t = (\bar{X} - iD)(1 - t) + iD$ are discounted at the rate

$$w_3 = r(1 - t)\left(\frac{S}{S + D}\right) + i\left(\frac{D}{S + D}\right).$$

If we again interpret $r = k_t/(1 - t)$, this becomes

$$w_3 = k_t\left(\frac{S}{S + D}\right) + i\left(\frac{D}{S + D}\right).$$

It should be reasonably obvious that it is appropriate to discount different definitions of the earnings stream (i.e., $\bar{X}(1 - t)$ vs. $\bar{X}^t$) at different "cost of capital" (i.e., w_2 vs. w_3).

We will now show that when specified and used correctly, both rates yield identical results. The results are equivalent if

$$\frac{\bar{X}(1 - t)}{w_2} = \frac{\bar{X}^t}{w_3}$$

which implies that

$$\begin{aligned} w_3 &= \frac{\bar{X}^t w_2}{\bar{X}(1 - t)} \\ &= w_2\left[1 + \frac{tiD}{\bar{X}(1 - t)}\right]. \end{aligned}$$

Substituting (5a) for w_2 gives

$$w_3 = k_t\left(\frac{S}{V_L}\right) + i(1 - t)\left(\frac{D}{V_L}\right) + tiD\left[\frac{k_tS + i(1 - t)D}{V_L\bar{X}(1 - t)}\right].$$

But, we know that

$$k_t(S) = \bar{X}(1 - t) - iD(1 - t),$$

hence

$$\begin{aligned} w_3 &= k_t\left(\frac{S}{V_L}\right) + i(1 - t)\left(\frac{D}{V_L}\right) + tiD\left[\frac{\bar{X}(1 - t)}{V_L\bar{X}(1 - t)}\right] \\ &= k_t\left(\frac{S}{V_L}\right) + i(1 - t)\left(\frac{D}{V_L}\right) + ti\left(\frac{D}{V_L}\right) \\ &= k_t\left(\frac{S}{V_L}\right) + i\left(\frac{D}{V_L}\right) \end{aligned}$$

which corresponds to our original definition of w_3 and to Arditti's (6a) when $k_t = r(1 - t)$.

Thus, we have three costs of capital and three equivalent specifications of total market value—(4), (6), and

$$V_L = \frac{\bar{X}^t}{w_3}$$

$$= \frac{(\bar{X} - iD)(1 - t) + iD}{k_t\left(\frac{S}{V_L}\right) + i\left(\frac{D}{V_L}\right)} \tag{8}$$

Since the numerators of market value specifications (4) and (6) are constants independent of corporate capital structure, minimization of w_1 and w_2 will, in fact, maximize V_L. Further, since w_1 differs from w_2 only by the scale factor $(1 - t)$, both "costs of capital" reach minima at the same point. However, the same is not true for market value specification (8). In (8) the numerator is not independent of corporate capital structure; the capital structure that minimizes w_3 will not, in general, maximize V_L.[3]

REFERENCES

1. Fred D. Arditti. "The Weighted Average Cost of Capital: Some Questions on Its Definition, Interpretation, and Use," *Journal of Finance*, September 1973, pp. 1001-1007.
2. H. Bierman, Jr. and S. Smidt. *The Capital Budgeting Decision*, MacMillan and Company, New York, 1971.
3. Charles W. Haley and Lawrence D. Schall. *The Theory of Financial Decisions*, McGraw-Hill Book Company, New York, 1973.
4. Robert W. Johnson. *Financial Management*, Allyn and Bacon, Boston, 1971.
5. F. Weston and E. Brigham. *Managerial Finance*, Holt, Rinehart, and Winston, 1972.

3. Although we were unaware of the reference when we originally submitted this comment, we direct the reader's attention to [3], chap. 13, for an excellent discussion of this point.