Notes on the Rosse-Panzar statistic

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December 2001

Abstract

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Contents

1. Monopoly

Revenue function:

$$R(q) = p(q)\, q.$$

Cost function:

$$C(q, w, \rho).$$

where

- $q =$ the firm's output
- $w =$ the wage rate
- $\rho =$ the rental rate of capital services

The revenue and cost functions may also be a functions of exogenous shift variables (for example, the income tax rate, or the level of a social insurance tax). Such relationships are suppressed for notational simplicity.

The cost function is derived as the solution to the constrained optimization problem

$$C(q, w, \rho) = \min_{w,\rho} wL + \rho K \ni f(K, L) \geq q,$$

where $f(K, L)$ is a neoclassical production function that is assumed to have all desirable properties.

The cost function is homogeneous of degree 1 in input prices.

Profit

$$\pi(q, w, \rho) = R(q) - C(q, w, \rho).$$

The profit function is homogeneous of degree 1 in output price and input prices.

Let

$$q(h) = \arg\max_{q} \pi[q, (1+h)\, w, (1+h)\, \rho]$$

for $h \geq 0$, and with some abuse of notation

$$R(h) = R[q(h)].$$

Since $q(h)$ depends on factor prices $[(1+h)\, w, (1+h)\, \rho]$, so does $R(h)$.

Then

$$R(h) - C[q(h), (1+h)\, w, (1+h)\, \rho] \geq R(0) - C[q(0), (1+h)\, w, (1+h)\, \rho].$$

Because the cost function is homogeneous of degree 1 in input prices, this can also be written

$$R(h) - (1+h)\, C[q(h), w, \rho] \geq R(0) - (1+h)\, C[q(0), w, \rho].$$

It is also the case that

$$R(0) - C[q(0), w, \rho] \geq R(h) - C[q(h), w, \rho],$$

so that

$$(1+h)R(0) - (1+h)C[q(0), w, \rho] \geq (1+h)R(h) - (1+h)C[q(h), w, \rho].$$

Add the two inequalities:

$$R(h) - (1+h)C[q(h), w, \rho] + (1+h)R(0) - (1+h)C[q(0), w, \rho] \geq$$

$$R(0) - (1+h)C[q(0), w, \rho] + (1+h)R(h) - (1+h)C[q(h), w, \rho]$$

$$R(h) + (1+h)R(0) \geq R(0) + (1+h)R(h)$$

$$-h[R(h) - R(0)] \geq 0$$

$$\frac{R(h) - R(0)}{h} \leq 0$$

$$\frac{R[(1+h)w, (1+h)\rho] - R(w, \rho)}{h} \leq 0.$$

Taking the limit as $h \to 0$, we obtain ψ, the Rosse-Panzar statistic:

$$\frac{\partial R(h)}{\partial h} = w\frac{\partial R[(1+h)w, (1+h)\rho]}{\partial w} + \rho\frac{\partial R[(1+h)w, (1+h)\rho]}{\partial \rho} \leq 0$$

$$\left.\frac{\partial R(h)}{\partial h}\right|_{h=0} = w\frac{\partial R(w, \rho)}{\partial w} + \rho\frac{\partial R(w, \rho)}{\partial \rho} \leq 0$$

$$\psi \equiv \frac{1}{R(0)}\left.\frac{\partial R(h)}{\partial h}\right|_{h=0} = \frac{w}{R(w, \rho)}\frac{\partial R(w, \rho)}{\partial w} + \frac{\rho}{R(w, \rho)}\frac{\partial R(w, \rho)}{\partial \rho} \leq 0.$$

This is Theorem 1 of Panzar and Rosse (1987): the elasticity the sum of the factor price elasticities of a (profit-maximizing) monopolist's reduced-form revenue function is nonpositive.

Define

$$k = 1 + h \qquad \widetilde{R}(k) = R(k-1);$$

then

$$\psi \equiv \frac{1}{R(0)}\left.\frac{\partial R(h)}{\partial h}\right|_{h=0} = \frac{1}{\widetilde{R}(1)}\left.\frac{\partial \widetilde{R}(k)}{\partial k}\right|_{k=1}$$

and ψ is also the elasticity of revenue with respect to a scalar increase in factor prices, holding relative factor prices constant.

- ψ can, in principle, be estimated with data on revenue and factor prices.
  - in contrast, estimation of an oligopoly Lerner-index equation requires data on revenue, expenditures on variable factors, and the value of the capital stock.
  - measuring the firm's rental cost of capital services is problematic (as is measurement of the economic value of capital stock in the Lerner-index approach).
- an estimated $\psi > 0$ permits rejection of the hypothesis that the firm is a monopolist.

2. Monopolistic competition

Assume

- there are n identical firms (more on this below);
- the (perceived) inverse demand function facing a single firm is $p(q, n)$, with

$$\frac{\partial p}{\partial q} < 0 \qquad \frac{\partial p}{\partial n} < 0 \qquad \frac{\partial}{\partial n}\left(-\frac{p}{q\frac{\partial p}{\partial q}}\right) \geq 0.$$

  The second assumption implies that $\frac{\partial R}{\partial n} = \frac{\partial p}{\partial n} q < 0$. The last assumption is that the price elasticity of demand facing a single firm does not fall as the number of firms rises.

- the number of firms adjusts until profit per firm equals zero.

The equations defining industry equilibrium are

$$\frac{\partial R(q, n)}{\partial q} - \frac{\partial C(q, w, \rho)}{\partial q} \equiv 0$$

$$R(q, n) - C(q, w, \rho) \equiv 0.$$

Differentiate with respect to w:

(1)

$$\frac{\partial^2 R(q, n)}{\partial q^2}\frac{\partial q}{\partial w} + \frac{\partial^2 R(q, n)}{\partial q \partial n}\frac{\partial n}{\partial w} - \left[\frac{\partial^2 C(q, w, \rho)}{\partial q^2}\frac{\partial q}{\partial w} + \frac{\partial^2 C(q, w, \rho)}{\partial q \partial w}\right] \equiv 0$$

$$\frac{\partial R(q,n)}{\partial q}\frac{\partial q}{\partial w}+\frac{\partial R(q,n)}{\partial n}\frac{\partial n}{\partial w}-\left[\frac{\partial C(q,w,\rho)}{\partial q}\frac{\partial q}{\partial w}+\frac{\partial C(q,w,\rho)}{\partial w}\right]\equiv 0.$$

(2)

$$\left[\frac{\partial^2 R(q,n)}{\partial q^2}-\frac{\partial^2 C(q,w,\rho)}{\partial q^2}\right]\frac{\partial q}{\partial w}+\frac{\partial^2 R(q,n)}{\partial q\partial n}\frac{\partial n}{\partial w}=\frac{\partial^2 C(q,w,\rho)}{\partial q\partial w}$$

$$\left[\frac{\partial R(q,n)}{\partial q}-\frac{\partial C(q,w,\rho)}{\partial q}\right]\frac{\partial q}{\partial w}+\frac{\partial R(q,n)}{\partial n}\frac{\partial n}{\partial w}=\frac{\partial C(q,w,\rho)}{\partial w}.$$

In the first equation, the coefficient of $\frac{\partial q}{\partial w}$ is negative by the second-order conditions for the firm's profit-maximization problem.

In the second equation, the coefficient of $\frac{\partial q}{\partial w}$ is zero by the first equilibrium condition.

(3)

$$\begin{pmatrix} \frac{\partial^2 R(q,n)}{\partial q^2}-\frac{\partial^2 C(q,w,\rho)}{\partial q^2} & \frac{\partial^2 R(q,n)}{\partial q\partial n} \\ 0 & \frac{\partial R(q,n)}{\partial n}\end{pmatrix}\begin{pmatrix}\frac{\partial q}{\partial w}\\ \frac{\partial n}{\partial w}\end{pmatrix}=\begin{pmatrix}\frac{\partial^2 C(q,w,\rho)}{\partial q\partial w}\\ \frac{\partial C(q,w,\rho)}{\partial w}\end{pmatrix}$$

Let

$$D=\left[\frac{\partial^2 R(q,n)}{\partial q^2}-\frac{\partial^2 C(q,w,\rho)}{\partial q^2}\right]\frac{\partial R(q,n)}{\partial n}>0$$

be the determinant of the coefficient matrix on the left. Then

$$D\begin{pmatrix}\frac{\partial q}{\partial w}\\ \frac{\partial n}{\partial w}\end{pmatrix}=\begin{pmatrix}\frac{\partial R(q,n)}{\partial n} & -\frac{\partial^2 R(q,n)}{\partial q\partial n}\\ 0 & \frac{\partial^2 R(q,n)}{\partial q^2}-\frac{\partial^2 C(q,w,\rho)}{\partial q^2}\end{pmatrix}\begin{pmatrix}\frac{\partial^2 C(q,w,\rho)}{\partial q\partial w}\\ \frac{\partial C(q,w,\rho)}{\partial w}\end{pmatrix}$$

$$\frac{\partial q}{\partial w}=\frac{1}{D}\left[\frac{\partial R(q,n)}{\partial n}\frac{\partial^2 C(q,w,\rho)}{\partial q\partial w}-\frac{\partial C(q,w,\rho)}{\partial w}\frac{\partial^2 R(q,n)}{\partial q\partial n}\right].$$

By Shepherd's Lemma,

$$L=\frac{\partial C(q,w,\rho)}{\partial w};$$

hence

$$\frac{\partial q}{\partial w}=\frac{1}{D}\left[\frac{\partial R(q,n)}{\partial n}\frac{\partial L}{\partial q}-L\frac{\partial^2 R(q,n)}{\partial q\partial n}\right].$$

Differentiate the second equilibrium condition

$$R(q,n)-C(q,w,\rho)\equiv 0$$

with respect to w:

$$\frac{\partial R}{\partial w}=\frac{\partial C}{\partial q}\frac{\partial q}{\partial w}+\frac{\partial C}{\partial w}=\frac{\partial C}{\partial q}\frac{\partial q}{\partial w}+L.$$

In the same way

$$\frac{\partial q}{\partial \rho}=\frac{1}{D}\left[\frac{\partial R\left(q,n\right)}{\partial n}\frac{\partial K}{\partial q}-K\frac{\partial^{2}R\left(q,n\right)}{\partial q\partial n}\right].$$

$$\frac{\partial R}{\partial \rho}=\frac{\partial C}{\partial q}\frac{\partial q}{\partial \rho}+\frac{\partial C}{\partial \rho}=\frac{\partial C}{\partial q}\frac{\partial q}{\partial \rho}+K.$$

Calculate the monopolistic competition value of ψ,

$$\psi\equiv\frac{w}{R}\frac{\partial R}{\partial w}+\frac{\rho}{R}\frac{\partial R}{\partial \rho}$$

$$=\frac{w}{R}\left[\frac{\partial C}{\partial q}\frac{\partial q}{\partial w}+L\right]+\frac{\rho}{R}\left[\frac{\partial C}{\partial q}\frac{\partial q}{\partial \rho}+K\right]$$

$$=\frac{wL+\rho K}{R}+\frac{1}{R}\frac{\partial C}{\partial q}\left(w\frac{\partial q}{\partial w}+\rho\frac{\partial q}{\partial \rho}\right)$$

$$=\frac{C}{R}+\frac{1}{R}\frac{\partial C}{\partial q}\left(w\frac{\partial q}{\partial w}+\rho\frac{\partial q}{\partial \rho}\right)$$

$$=1+\frac{1}{R}\frac{\partial C}{\partial q}\left(w\frac{\partial q}{\partial w}+\rho\frac{\partial q}{\partial \rho}\right)$$

(by the second equilibrium condition)

$$=1+$$

$$\frac{1}{RD}\frac{\partial C}{\partial q}\left\{w\left[\frac{\partial R\left(q,n\right)}{\partial n}\frac{\partial L}{\partial q}-L\frac{\partial^{2}R\left(q,n\right)}{\partial q\partial n}\right]+\rho\left[\frac{\partial R\left(q,n\right)}{\partial n}\frac{\partial K}{\partial q}-K\frac{\partial^{2}R\left(q,n\right)}{\partial q\partial n}\right]\right\}$$

$$=1+\frac{1}{RD}\frac{\partial C}{\partial q}\left\{\frac{\partial R\left(q,n\right)}{\partial n}\left[w\frac{\partial L}{\partial q}+\rho\frac{\partial K}{\partial q}\right]-\left(wL+\rho K\right)\frac{\partial^{2}R\left(q,n\right)}{\partial q\partial n}\right\}$$

$$=1+\frac{1}{RD}\frac{\partial C}{\partial q}\left\{\frac{\partial R\left(q,n\right)}{\partial n}\frac{\partial C}{\partial q}-C\frac{\partial^{2}R\left(q,n\right)}{\partial q\partial n}\right\}$$

$$=1+\frac{1}{RD}\frac{\partial R}{\partial q}\left[\frac{\partial R\left(q,n\right)}{\partial n}\frac{\partial R}{\partial q}-R\frac{\partial^{2}R\left(q,n\right)}{\partial q\partial n}\right]$$

(by the firm profit maximization condition and the zero profit condition.)

Now

$$R=pq$$

$$\frac{\partial R}{\partial n}=q\frac{\partial p}{\partial n}$$

$$\frac{\partial R}{\partial q} = p + q\frac{\partial p}{\partial q}$$

$$\frac{\partial^2 R}{\partial q \partial n} = \frac{\partial p}{\partial n} + q\frac{\partial^2 p}{\partial q \partial n}$$

$$\frac{\partial R}{\partial n}\frac{\partial R}{\partial q} - R\frac{\partial^2 R}{\partial q \partial n} = q\frac{\partial p}{\partial n}\left(p + q\frac{\partial p}{\partial q}\right) - pq\left(\frac{\partial p}{\partial n} + q\frac{\partial^2 p}{\partial q \partial n}\right) =$$

$$q^2\left(\frac{\partial p}{\partial q}\frac{\partial p}{\partial n} - p\frac{\partial^2 p}{\partial q \partial n}\right).$$

Then

$$\psi = 1 + \frac{q^2}{RD}\frac{\partial R}{\partial q}\left(\frac{\partial p}{\partial q}\frac{\partial p}{\partial n} - p\frac{\partial^2 p}{\partial q \partial n}\right).$$

The price elasticity of demand is

$$e = -\frac{p}{q\frac{\partial p}{\partial q}}$$

and

$$\frac{\partial e}{\partial n} = -\left[\frac{q\frac{\partial p}{\partial q}\frac{\partial p}{\partial n} - pq\frac{\partial^2 p}{\partial q \partial n}}{\left(q\frac{\partial p}{\partial q}\right)^2}\right] = -\frac{\frac{\partial p}{\partial q}\frac{\partial p}{\partial n} - p\frac{\partial^2 p}{\partial q \partial n}}{q\left(\frac{\partial p}{\partial q}\right)^2},$$

which we have assumed to be nonnegative. Then

$$\frac{\partial p}{\partial q}\frac{\partial p}{\partial n} - p\frac{\partial^2 p}{\partial q \partial n} = -q\left(\frac{\partial p}{\partial q}\right)^2\frac{\partial e}{\partial n} \geq 0$$

and

$$\psi = 1 + \frac{q^2}{RD}\frac{\partial R}{\partial q}\left(\frac{\partial p}{\partial q}\frac{\partial p}{\partial n} - p\frac{\partial^2 p}{\partial q \partial n}\right)$$

$$= 1 - \frac{q^3}{RD}\frac{\partial R}{\partial q}\left(\frac{\partial p}{\partial q}\right)^2\frac{\partial e}{\partial n} \leq 1.$$

Going through a similar analysis, Panzar and Rosse show that $\psi = 1$ in long-run equilibrium of a perfectly competitive industry. They discuss symmetric conjectural variations oligopoly, but are not able to obtain comparable results.

> *Remark*: it is common to assume in theoretical work that firms are identical, so that equilibrium is symmetric. This is not acceptable in theoretical models that are to be the basis of empirical work, since it is known that real-world firms are not identical. Normally, it is possible to generalize analyses that assume identical firms to the case of different firms, at the expense of an increase in the algebraic complexity of the problem but without increasing the level of technical difficulty.

3. The R-P statistic and the Lerner index

The Rosse-Panzar statistic is

$$\psi = \frac{w}{R(w,\rho)}\frac{\partial R(w,\rho)}{\partial w} + \frac{\rho}{R(w,\rho)}\frac{\partial R(w,\rho)}{\partial \rho}.$$

Now interpret q as the output of a single firm in an oligopoly.

$$\frac{\partial R}{\partial w} = \frac{\partial R}{\partial q}\frac{\partial q}{\partial w} \qquad \frac{\partial R}{\partial \rho} = \frac{\partial R}{\partial q}\frac{\partial q}{\partial \rho}.$$

From the first-order condition for profit maximization

$$\frac{\partial \pi(q)}{\partial q} = \frac{\partial R(q)}{\partial q} - \frac{\partial C(q,w,\rho)}{\partial q} \equiv 0,$$

we have

$$\frac{\partial^2 \pi(q)}{\partial q^2} = \frac{\partial^2 R(q)}{\partial q^2} - \frac{\partial^2 C(q,w,\rho)}{\partial q^2} < 0$$

(by the second-order condition for profit maximization) and

$$\frac{\partial^2 R(q)}{\partial q^2}\frac{\partial q}{\partial w} - \left[\frac{\partial^2 C(q,w,\rho)}{\partial q^2}\frac{\partial q}{\partial w} + \frac{\partial^2 C(q,w,\rho)}{\partial q \partial w}\right] = 0$$

$$\left[\frac{\partial^2 R(q)}{\partial q^2} - \frac{\partial^2 C(q,w,\rho)}{\partial q^2}\right]\frac{\partial q}{\partial w} = \frac{\partial^2 C(q,w,\rho)}{\partial q \partial w}$$

$$\frac{\partial^2 \pi(q)}{\partial q^2}\frac{\partial q}{\partial w} = \frac{\partial^2 C(q,w,\rho)}{\partial q \partial w}$$

$$\frac{\partial q}{\partial w} = \frac{\frac{\partial^2 C(q,w,\rho)}{\partial q \partial w}}{\frac{\partial^2 \pi(q)}{\partial q^2}} = \frac{C_{qw}}{\pi_{qq}}.$$

In the same way

$$\frac{\partial q}{\partial \rho} = \frac{\frac{\partial^2 C(q,w,\rho)}{\partial q \partial \rho}}{\frac{\partial^2 \pi(q)}{\partial q^2}} = \frac{C_{q\rho}}{\pi_{qq}}.$$

Substitute in the expression for the Rosse-Panzar statistic:

$$\psi = R_q \frac{w}{R}\frac{C_{qw}}{\pi_{qq}} + R_q \frac{\rho}{R}\frac{C_{q\rho}}{\pi_{qq}} = \frac{R_q}{R\pi_{qq}}\left(wC_{qw} + \rho C_{q\rho}\right).$$

The cost function is homogeneous of degree 1 in input prices; so, therefore, is the derivative of the cost function with respect to output:[1]

$$C_q(q, \lambda w, \lambda \rho) = \lim_{h \to 0} \frac{C(q+h, \lambda w, \lambda \rho) - C(q, \lambda w, \lambda \rho)}{h}$$

$$= \lambda \lim_{h \to 0} \frac{C(q+h, w, \rho) - C(q, w, \rho)}{h} = \lambda C_q(q, w, \rho)$$

Differentiate

$$C_q(q, \lambda w, \lambda \rho) = \lambda C_q(q, w, \rho)$$

with respect to λ:

$$w C_{qw}(q, \lambda w, \lambda \rho) + \rho C_{q\rho}(q, \lambda w, \lambda \rho) = C_q(q, w, \rho).$$

Set $\lambda = 1$:

$$w C_{qw}(q, w, \rho) + \rho C_{q\rho}(q, w, \rho) = C_q(q, w, \rho).$$

Substitute in the expression for the Rosse-Panzar statistic:

$$\psi = \frac{R_q}{R\pi_{qq}}(w C_{qw} + \rho C_{q\rho}) = \frac{R_q C_q}{R\pi_{qq}} = \frac{(R_q)^2}{R\pi_{qq}}$$

(using the first-order condition for profit-maximization).

We have used e to denote the industry price elasticity of demand. Let ε denote the firm's price elasticity of demand. In (for example) a conjectural elasticity model of a market producing a homogeneous product,

$$\varepsilon = \frac{e}{\alpha + (1-\alpha)s},$$

where s is the firm's market share.

Following Shaffer (1982), the firm's marginal revenue is

$$R_q = p + q\frac{\partial p}{\partial q} = p\left(1 + \frac{q}{p}\frac{\partial p}{\partial q}\right) = p\left(1 - \frac{1}{\varepsilon}\right).$$

$$p\left(1 - \frac{1}{\varepsilon}\right) = C_q$$

$$m = \frac{p - C_q}{p} = \frac{1}{\varepsilon}$$

[1]See Rosse and Panzar (1977, p. 7). I am grateful to Sherrill Shaffer for discussion for this proof.

$$\varepsilon = \frac{1}{m}$$

$$R_q = p\left(1 - \frac{1}{\varepsilon}\right) = p\left(1 - m\right).$$

$$\psi = \frac{R_q C_q}{R\pi_{qq}} = \frac{p^2}{R\pi_{qq}}\left(1 - m\right)^2$$

$$= \frac{p^2}{pq\pi_{qq}}\left(1 - m\right)^2$$

$$= \frac{p}{q\pi_{qq}}\left(1 - m\right)^2.$$

Differentiate

$$\pi_q = p\left(1 - \frac{1}{\varepsilon}\right) - C_q \equiv 0.$$

with respect to q:

$$\pi_{qq} = p\left[-\left(-\frac{1}{\varepsilon^2}\right)\frac{\partial \varepsilon}{\partial q}\right] + \left(1 - \frac{1}{\varepsilon}\right)\frac{\partial p}{\partial q} - C_{qq}.$$

$$= \left(1 - \frac{1}{\varepsilon}\right)\frac{\partial p}{\partial q} + \frac{p}{\varepsilon^2}\frac{\partial \varepsilon}{\partial q} - C_{qq}.$$

$$= \left(1 - \frac{1}{\varepsilon}\right)\frac{p}{q}\left(-\frac{1}{\varepsilon}\right) + \frac{p}{\varepsilon^2}\frac{\partial \varepsilon}{\partial q} - C_{qq}.$$

$$= \left(\frac{1 - \varepsilon}{\varepsilon^2}\right)\frac{p}{q} + \frac{p}{\varepsilon^2}\frac{\partial \varepsilon}{\partial q} - C_{qq}$$

$$= \left[\frac{1 - \frac{1}{m}}{\left(\frac{1}{m}\right)^2}\right]\frac{p}{q} + \frac{p}{\varepsilon^2}\frac{\partial \varepsilon}{\partial q} - C_{qq}.$$

$$= m\left(m - 1\right)\frac{p}{q} + \frac{p}{\varepsilon^2}\frac{\partial \varepsilon}{\partial q} - C_{qq}.$$

Substitute in the expression for ψ:

$$\psi = \frac{p}{q\pi_{qq}}\left(1 - m\right)^2$$

$$= \frac{p}{q\left[m\left(m - 1\right)\frac{p}{q} + \frac{p}{\varepsilon^2}\frac{\partial \varepsilon}{\partial q} - C_{qq}\right]}\left(1 - m\right)^2$$

$$= \frac{\left(m - 1\right)^2}{m\left(m - 1\right) + \frac{q}{\varepsilon^2}\frac{\partial \varepsilon}{\partial q} - \frac{q}{p}C_{qq}}.$$

As noted by Shaffer (1983),[2] if the price elasticity of demand is constant and cost is linear in output, this reduces to

$$\psi = 1 - \frac{1}{m}.$$

In oligopoly, the firm's price elasticity of demand will not, in general, be constant. For example, if industry price elasticity of demand e is constant, the firm's price elasticity of demand

$$\varepsilon = \frac{e}{\alpha + (1-\alpha)\, s}$$

varies with the firm's output $(s = q/Q)$.

Alternatively, note that for the case of linear inverse demand

$$p = a - bQ$$

the industry price elasticity of demand is

$$e = -\frac{p}{Q}\frac{dQ}{dp} = \frac{a - bQ}{bQ} = \frac{a}{bQ} - 1$$

and this falls as industry output rises:

$$\frac{\partial e}{\partial Q} < 0.$$

By analogy, assume that the firm's price elasticity of demand falls as firm output rises:[3]

$$\frac{\partial \varepsilon}{\partial q} < 0.$$

If in addition cost is linear in output, we then have

$$\psi = \frac{(m-1)^2}{m\,(m-1) + \frac{q}{\varepsilon^2}\frac{\partial \varepsilon}{\partial q}} > \frac{(m-1)^2}{m\,(m-1)} = 1 - \frac{1}{m}$$

and

$$\frac{1}{m} > 1 - \psi$$

$$m < \frac{1}{1-\psi}.$$

[2]Also noted by Panzar and Rosse (1987), in the context of a constant price elasticity of demand, Cobb-Douglas technology example.

[3]In a conjectural model, an expression for $\frac{\partial \varepsilon}{\partial q}$ could be explicitly calculated; see below.

Estimation of ψ then implies a straightforward upper bound on the firm-specific Lerner index.

In the context of specific oligopoly models, something more can be done. Suppose cost is linear in output ($C_{qq} = 0$); then

$$\psi = \frac{(m-1)^2}{m(m-1) + \frac{q}{\varepsilon^2}\frac{\partial \varepsilon}{\partial q}}.$$

$$\varepsilon = \frac{e}{\alpha + (1-\alpha)s}.$$

If conjectures are constant,

$$\frac{\partial \varepsilon}{\partial q} = \frac{[\alpha + (1-\alpha)s]\frac{\partial e}{\partial q} - e(1-\alpha)\frac{\partial s}{\partial q}}{[\alpha + (1-\alpha)s]^2}.$$

$$s = \frac{q}{Q}$$

$$\frac{\partial s}{\partial q} = \frac{Q - q\frac{\partial Q}{\partial q}}{Q^2}$$

$$= \frac{Q - q\frac{q+\alpha Q_{-1}}{q}}{Q^2}$$

(using $\frac{\partial Q}{\partial q} = 1 + \frac{\partial Q_{-1}}{\partial q} = \frac{q+\alpha Q_{-1}}{q}$)

$$= \frac{Q - q - \alpha Q_{-1}}{Q^2}$$

$$= \frac{(1-\alpha)Q_{-1}}{Q^2}$$

$$= \frac{(1-\alpha)(1-s)}{Q}$$

$$\frac{\partial \varepsilon}{\partial q} = \frac{[\alpha + (1-\alpha)s]\frac{\partial e}{\partial q} - e\frac{(1-\alpha)^2(1-s)}{Q}}{[\alpha + (1-\alpha)s]^2}.$$

If the industry price elasticity of demand is constant,

$$\frac{\partial \varepsilon}{\partial q} = \frac{-e}{[\alpha + (1-\alpha)s]^2}\frac{(1-\alpha)^2(1-s)}{Q} < 0.$$

$$\psi = \frac{(m-1)^2}{m(m-1) + \frac{q}{\varepsilon^2}\frac{\partial \varepsilon}{\partial q}}.$$

$$\psi = \frac{(m-1)^2}{m(m-1) - \frac{q}{\varepsilon^2}\frac{e}{[\alpha+(1-\alpha)s]^2}\frac{(1-\alpha)^2(1-s)}{Q}}.$$

$$\psi = \frac{(m-1)^2}{m(m-1) - \frac{1}{\varepsilon^2}\frac{e}{[\alpha+(1-\alpha)s]^2}(1-\alpha)^2 s(1-s)}.$$

Substitute

$$\varepsilon = \frac{e}{\alpha+(1-\alpha)s},$$

$$\psi = \frac{(m-1)^2}{m(m-1) - \frac{1}{\varepsilon^2}\frac{\varepsilon}{[\alpha+(1-\alpha)s]}(1-\alpha)^2 s(1-s)}.$$

$$= \frac{(m-1)^2}{m(m-1) - \frac{1}{\varepsilon}\frac{(1-\alpha)^2 s(1-s)}{\alpha+(1-\alpha)s}}$$

and substitute $m = 1/\varepsilon$

$$= \frac{(m-1)^2}{m(m-1) - m\frac{(1-\alpha)^2 s(1-s)}{\alpha+(1-\alpha)s}}$$

$$= \frac{(m-1)^2}{m\left[m-1-\frac{(1-\alpha)^2 s(1-s)}{\alpha+(1-\alpha)s}\right]}$$

$$= \frac{1-\frac{1}{m}}{1-\frac{(1-\alpha)^2 s(1-s)}{(m-1)[\alpha+(1-\alpha)s]}}$$

$$= \frac{1-\frac{1}{m}}{1-\frac{(1-\alpha)^2 s(1-s)}{(m-1)me}}.$$

In the case of matching conjectures, we have once again

$$\psi = 1 - \frac{1}{m}.$$

Matching conjectures imply that oligopolists as a group mimic the behavior of a monopolist. Since $m \leq 1$, $1/m \geq 1$ and with matching conjectures and a constant industry price-elasticity of demand

$$\psi = 1 - \frac{1}{m} \leq 0,$$

consistent with Panzer and Rosse's (1987) Theorem 1: the monopoly Rosse-Panzer statistic is nonpositive.

More generally, from

$$\psi = \frac{1 - \frac{1}{m}}{1 - \frac{(1-\alpha)^2 s(1-s)}{(m-1)me}}$$

we obtain

$$m = \frac{1}{2(1-\psi)} \left[\sqrt{\psi^2 + 4\psi^2 z - 4\psi z} - (\psi - 2) \right],$$

where

$$z = \frac{(1-\alpha)^2 s(1-s)}{e}.$$

Once again, estimation of ψ implies a value of the Lerner index m.

4. Postscript

To estimate the Rosse-Panzar statistic it is not necessary to have data on output or input levels. It is necessary to have data on the firm-specific rental cost of capital services, and in a qualitative sense this involves difficulties of the same kind that arise in measuring the economic value of a firm's capital stock.

In contrast to received analyses based on the Lerner index, the Rosse-Panzar statistic does not give a clear indication whether firms or (on average) an industry exercises market power, nor does it make it possible to relate the extent to which firms exercise market power in terms of the market structure or firm conduct.

It might prove interesting to extend the analysis that leads to the Rosse-Panzar statistic to allow for asymmetric firms (for example, different unit costs).

5. References

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