11 Resource allocation by a competition authority

Stephen Martin

And when we deal with questions relating to principles of law and their applications, we do not suddenly rise into a stratosphere of icy certainty.

(Charles Evans Hughes)

Introduction

The workings of the law are uncertain. This is true in general and it is true for antitrust or competition law.

If firms are found to have violated competition policy, they open themselves up to the possibility of fines\(^1\) – if the offending conduct is detected, if the enforcement agency decides to levy a fine or challenge the conduct in court, if a trial court sustains the enforcement agency, and, if all of these things come to pass, if the trial court is not overturned in the event of appeal to higher courts.

In some cases it is clear that a certain course of action violates competition policy. If a manufacturer has distributors in several EU member states, and forbids those distributors to sell across national boundaries, the manufacturer knows it is violating EU competition policy. Uncertainty is then limited to whether or not the conduct will be detected and the sizes of legal expenses and the eventual fine.

In other cases, whether a course of action violates competition law is far from clear in advance. For example, behavior that is found to constitute illegal collusion in one instance is often difficult to distinguish from behavior that is found to be legal conscious parallelism in another.\(^2\)

Conscious parallelism decisions in both the European Union and the United States suggest that independent decisions independently arrived at will not be found to violate competition policy, even if those independent decisions lead to outcomes that generate economic profits over the long run. Some kinds of conduct will shift behavior from the legal category of ‘independent’ to the legal category of ‘collusive’, but exactly what kinds of conduct will have this effect sometimes seems idiosyncratic.\(^3,4\)

In like manner, it seems fair to say that at this writing (September 1998), it is not clear whether the marketing practices that have been used by Microsoft Corporation for its operating systems and network browsers violate US antitrust law.
In this chapter, I develop a model of a competition authority that administers a deterrence-based competition policy regulating firm conduct within a legal system that harbors an irreducible element of uncertainty. Firms and the competition authority take this uncertainty into account in deciding their actions.

The basic model, presented in the second section, considers a competition authority and two monopolized industries; this is used to examine the implications of changes in the competition authority's budget, in investigation cost, and in market size for enforcement decisions. Later sections consider oligopoly markets and discuss the impact of a deterrence-based competition policy on noncooperative collusion in repeated games.

**The firm's problem**

**Demand**

There are two monopolized industries. The markets serve final consumer demand, and the products of the two industries are independent in demand.\(^6\) The demand curve for industry \(i\) is

\[
p_i = p_i(q_i) + \varepsilon_i, \tag{1}\]

with \(p'_i < 0, p''_i \geq 0\). \(\varepsilon_i\) is a random element of demand, with well-behaved density function \(f_i(\varepsilon_i)\), zero mean, and defined over the interval

\[
\varepsilon \leq \varepsilon_i \leq \bar{\varepsilon} \leq \infty. \tag{2}\]

The lower limit \(\varepsilon\) is such that price is not less than marginal cost if the quantity supplied is sufficiently small; that is,

\[
p_i(0) + \varepsilon \geq c_i, \tag{3}\]

where \(c_i\) is the firm's constant marginal cost of production. There is a random element to demand, but the random element is not so negative that price would be less than marginal cost even at very low output levels.

**The enforcement mechanism**

In the real world, a competition authority will receive and monitor many signals about an industry's market performance. Such signals will include complaints from customers, from rivals, and information from trade sources about many aspects of market performance—prices, inventories, delivery times, and others.

In the model developed here, I collapse all these sources into a single signal, the realized price. I suppose that the competition agency sets a threshold price \(g_i\) for industry \(i\). If the observed price rises above \(g_i\), the agency investigates the industry. After investigation, it makes a decision about further action.
The probability that the realized price is above \( g_i \), triggering an investigation, is

\[
\tau_i = \Pr \left[ p_i(q_i) + \epsilon_i \geq g_i \right] = \Pr \left[ \epsilon_i \geq g_i - p_i(q_i) \right]
\]  

(4)

If the firm is investigated, it is prosecuted and convicted with probability \( \gamma_i \), with \( 0 < \gamma_i < 1 \). If convicted, it pays a fine \( F_i \).

The probability of prosecution and conviction \( \gamma_i \) is less than one. First, the competition authority may investigate the industry, conclude that the firm is not violating competition law, and close the matter without further action.

Second, an initial decision by the competition authority to prosecute or fine the firm may not survive eventual appeal to the courts. The legal system has its own mores, which are only imperfectly controlled by the legislature (and only indirectly influenced by economic considerations).

This is particularly true in common law systems; see Priest (1977) and Rubin (1977). The elaboration of the concepts of ‘antitrust injury’ and ‘standing’ in US antitrust law, which limit the ability of private plaintiffs to pursue private antitrust actions, are examples of endogenous judicial development of competition law.

For simplicity, I treat \( \gamma_i \) and \( F_i \) as parameters, with values that are known to firms and to the competition authority. It would be possible to examine an extended model in which the probability of conviction \( \gamma_i \) is positively influenced by spending of the competition authority to prosecute that case and negatively influenced by spending of the challenged firm in its own defense. Similarly, \( F_i \) might modelled as depending on the nature of the offense.

**Firm behavior**

Firm \( i \) maximizes its expected profit,

\[
E(\pi_i) = \left[ p_i(q_i) - c_i \right] q_i - \tau_i \gamma_i F_i
\]  

(5)

The first-order condition is

\[
\frac{\partial E(\pi_i)}{\partial q_i} = p_i - c_i + q_i \frac{dp_i}{dq_i} - \gamma_i F_i \frac{\partial \tau_i}{\partial q_i} = 0.
\]  

(6)

This implies

\[
p_i + q_i \frac{dp_i}{dq_i} = c_i + \gamma_i F_i \frac{\partial \tau_i}{\partial q_i} < c_i,
\]  

(7)

so that when the firm maximizes expected profit marginal revenue is less than marginal production cost. To reduce expected fines, the firm expands output
above the level that makes marginal revenue equal to marginal production cost. This is the deterrence effect of competition policy.

Theorem 1, which is proven in the Appendix, gives some of the properties of equilibrium firm behavior.

**Theorem 1**

(a) The firm’s profit-maximizing price is below the threshold price,

$$ g_i - p_i > 0, \quad \text{(8)} $$

provided $\gamma_i F_i$ is sufficiently large;

(b) a lower threshold price increases equilibrium output, all else equal:

$$ \frac{\partial q_i}{\partial g_i} < 0; \quad \text{(9)} $$

(c) greater expected fines increase equilibrium output, all else equal:

$$ \frac{\partial q_i}{\partial (\gamma_i F_i)} > 0. \quad \text{(10)} $$

By (b), lowering the threshold price expands output and lowers expected price. To rule out destabilizing overreactions by the firm to changes in the threshold price, I assume that lowering the threshold price lowers the equilibrium threshold-price gap:

$$ \frac{\partial (g_i - p_i)}{\partial g_i} = 1 - \frac{dp_i}{dq_i} \frac{\partial q_i}{\partial g_i} > 0. \quad \text{(11)} $$

If the government lowers $g$, the firm reacts by producing more, and price falls. But price does not fall so much that $g - p$ rises.\(^8\)

Condition (11) means that when $g$ falls, $g - p$ falls and the probability of investigation rises:

$$ \frac{d\tau_i}{dg_i} = -f_i(g_i - p_i) \left( 1 - \frac{dp_i}{dq_i} \right) = -f_i(g_i - p_i) \left( 1 - \frac{dp_i}{dq_i} \frac{\partial q_i}{\partial g_i} \right) < 0 \quad \text{(13)} $$

I also assume that

$$ \frac{d^2\tau_i}{dg_i^2} > 0, \quad \text{(14)} $$

implying that from the point of view of the competition authority, there are positive but decreasing returns to deterrence.

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Table 1

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<th>65</th>
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*Notes: $p$ = monopoly consumer-
Example

Consider a market with linear inverse demand curve

\[ p = 101 - \frac{1}{10} Q + \varepsilon \]  \hspace{1cm} (15)

Let marginal cost be constant, 1 per unit, and suppose there are no fixed costs. If the industry were perfectly competitive, long-run equilibrium price would be 1.

Let the density of the random part of demand be exponential,

\[ f(\varepsilon) = \frac{1}{10} \exp\left(\frac{-\varepsilon + 10}{10}\right) \]  \hspace{1cm} (16)

This has range \((-10, \infty)\) (see Figure 11.1). \(\varepsilon\) has mean 0 and variance \(\sigma^2 = 100\). For this density function, it is more likely that \(\varepsilon\) will fall in a range of modestly negative values than in a higher range of identical length.\(^9\)

Table 11.1 reports the main characteristics of monopoly equilibrium for this example for threshold prices ranging from 70 to 10. The row labeled 'no CP' gives values for equilibrium without competition policy; the expected monopoly price is 51.\(^10\) An investigation threshold \(g = 70\) is almost two standard deviations above this; it results in a probability of investigation of 4.4 per cent, and an output of expansion of the same proportion. Profit falls, while consumers' surplus and the sum of profit and consumers' surplus rise.

As the investigation threshold falls, output and the probability of investigation rise. Profit falls continuously, and consumers' surplus rises continuously, as the

<table>
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<tr>
<th>(g)</th>
<th>(q_m)</th>
<th>(p_m)</th>
<th>(\tau)</th>
<th>(\pi_m)</th>
<th>(CS)</th>
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Notes: \(p = 101 - (1/10)q_m\), \(c = 1\), \(\gamma = 1/2\), \(F = 20000\), \(\sigma = 10\); \(p_m\) = expected monopoly price, \(q_m\) = monopoly output, \(\tau\) = probability of investigation, \(\pi_m\) = expected monopoly profit, \(CS\) = expected consumers' surplus, \(\tau\ gamma\) = expected antitrust fines.
investigation threshold falls. The sum of profit and consumers' surplus rises until the investigation threshold reaches a low level, $g = 15$, and then begins to fall. For $g = 15$, the improvement in profit plus consumers' surplus compared with the no competition policy case is about 12 per cent.

For very low threshold levels, expected fines become so great that private sector welfare falls with the investigation threshold. As shown in Figure 11.2, overall welfare - profit plus consumers' surplus plus expected fines - rises continuously as the investigation threshold falls.

For comparison purposes, Table 11.2 gives the duopoly outcomes for the parameters of Table 11.1.¹¹ The results for monopoly and duopoly are qualitatively similar. Profit plus consumers' surplus rises as the investigation threshold falls to 45, then declines. The improvement in profit plus consumers' surplus for $g = 45$, compared with the no-competition policy case, is negligible, about 1/10th of 1 per cent; this is also apparent from Figure11.3.

To put this in perspective, duopoly welfare for $g = 45$ is about 11 per cent greater than monopoly welfare for $g = 45$. If markets converge on the noncooperative equilibrium of a one-shot game, actual competition yields greater static welfare gains than competition policy.¹²

**The competition authority's problem**

I model a competition authority that deals with two industries subject to a binding budget constraint.

Let $I_i$ = cost of investigating industry $i$, for $i = 1, 2$.

I assume that the cost of investigation is industry specific, but do not model
Figure 11.2 Monopoly investigation threshold–market performance relationship
Notes: $a = 101, c = 1, b = 1/10, \gamma = 1/2, F = 20000, \sigma = 10$

Figure 11.3 Duopoly investigation threshold–market performance relationship
Notes: $a = 101, c = 1, b = 1/10, \gamma = 1/2, F = 20000, \sigma = 10$
Table 11.2 Static duopoly market performance: alternative investigation thresholds

<table>
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<th>$p_N$</th>
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Notes: $p = 101 - (1/10) Q_N$, $c = 1$, $\gamma = 1/2$, $F = 20000$, $q = 10$; $p_N$ = expected Nash equilibrium duopoly price, $Q_N$ = expected duopoly equilibrium industry output, $\tau$ = probability of investigation, $\pi_N$ = expected Nash equilibrium profit per firm, CS = expected consumers' surplus, $\tau$ $\gamma F$ = expected antitrust fines.

the determinants of investigation cost. If the competition authority has had extensive experience with an industry, it may cost relatively little to investigate, even if the industry is large or complex in some objective sense. On the other hand, a small industry may be relatively costly to investigate if it is one with which the competition authority has not had prior contact. These considerations rule out any easy specification of the determinants of $I_\tau$, such as a positive relationship with market size.

The expected fine that follows investigation is $\gamma_i F_i$; the probability of investigation is $\tau_i$. Overall, the expected fine is $\tau_i \gamma_i F_i$. This is an expected transfer from the firm to the government, and drops out of the expression for net social welfare.

The budget constraint is

$$\tau_1 I_1 + \tau_2 I_2 \leq B,$$

(17)

where $B$ is the budget allocated by the legislature to the competition authority.\(^{13}\)

I assume the budget constraint is binding: the competition authority does not have enough resources to investigate both industries with 100 per cent probability. Such an assumption is realistic,\(^{14}\) and necessary to render the problem interesting.

This way of formulating the budget constraint implies that the competition authority has sufficient reserves to cover realized investigation costs. It may happen that both industries are investigated, and that realized investigation costs exceed $B$.

Net social welfare generated in industry $i$ is the sum of consumers' and producers' surplus,
\[ W_i(q_i) = \int_0^q \left[ p_i(x_i) - c_i \right] dx_i. \]  

(18)

The competition authority's problem to maximize social welfare, net of enforcement cost,\(^{15}\)

\[ \max_{g_1, g_2} W_1(q_1) + W_2(q_2) - \tau_1 I_1 - \tau_2 I_2 \]  

(19)

subject to the budget constraint (17). Variations might weight consumers' and producers' surplus differently; for an example in the context of a model of competition policy, see Besanko and Spulber (1993).

To the extent that competition authorities act to maximize other goals, the analysis that follows from the assumption that (19) is the objective function is normative rather than positive.

A formal solution of the competition authority's problem is given in the Appendix. From an analytical point of view, the problem is like the standard analysis of consumer utility maximization. The equilibrium choice of investigation thresholds can be illustrated graphically in terms of a tangency in threshold- or \((g_1, g_2)\)-space between an isobudget curve that represents the competition authority's budget constraint and an isowelfare curve indicating the maximum attainable social welfare.

**Isobudget curves**

An isobudget curve shows all combinations of \(g_1\) and \(g_2\) that yield a given total expected investigation cost. The equation of the isobudget curve for budget level \(B\) is

\[ \tau_1 (g_1 - p_1) I_1 + \tau_2 (g_2 - p_2) I_2 \equiv B. \]  

(20)

In this expression, prices are functions of the respective quantities supplied, via the demand curves. The quantities supplied are in turn functions of the investigation thresholds, functions that are defined by first-order conditions for the firms' expected profit maximization problems.

The slope of an isobudget curve is

\[ \frac{dg_2}{dg_1} \bigg|_{B} = - \frac{I_1}{I_2} \frac{d \tau_1 / dg_1}{d \tau_2 / dg_2} < 0. \]  

(21)

Greater values of \(B\) allow the agency to set lower values of one or both investigation thresholds. The map of budget curves therefore consists of downward-sloping lines, with values closer to the origin corresponding to higher budget levels (see Figure 11.1).

As shown in the Appendix, isobudget curves are convex. If \(I_1\) rises, isobudget curves become steeper, all else equal; if \(I_2\) rises, isobudget curves become flatter, all else equal.
**Isowelfare curves**

Isowelfare curves have equations of the form

\[ W_i(q_1) + W_2(q_2) = \bar{W} \]  

\[ \int_0^{q_1} [p_1(x_1) - c_i] \, dx_1 + \int_0^{q_2} [p_2(x_2) - c_2] \, dx_2 = \bar{W} \]

(22)

(23)

where once again \( p_i \) is directly a function of \( q_i \) and indirectly a function of \( g_i \).

The slope of an isowelfare curve is

\[ \left. \frac{dg_2}{dg_1} \right|_{\bar{W}} = -\frac{dW_1 / dg_1}{dW_2 / dg_2} = -\frac{(p_1 - c_1) dq_1}{(p_2 - c_2) dq_2} < 0. \]

(24)

Isowelfare curves are downward sloping and convex (see Appendix), and isowelfare curves closer to the origin correspond to higher levels of welfare.

**Tangency**

It is shown in the Appendix that the second-order condition for the constrained optimization problem implies an equilibrium of the kind shown in Figure 11.4.

Isobudget curves and isowelfare curves are both downward sloping and convex. Isobudget curves are more sharply curved than isowelfare curves. The equilibrium investigation thresholds chosen by the competition authority are those at the point of tangency between the isobudget curve determined by the available budget, \( B \), and the isowelfare curve that is closest to the origin, shown as \( E_A \) in 11.4. At the tangency point, the marginal increase in welfare per marginal increase in investigation cost is the same for both industries.

**Comparative statics I**

The comparative static behavior of optimal investigation thresholds with respect to changes in budget \( B \) and investigation costs \( I_1, I_2 \) is much the same as the comparative static behavior of individual consumption with respect to changes in income and in the prices of goods.

**Comparative statics with respect to \( B \)**

The 'normal markets' case is one in which an increase in the investigation budget induces lower investigation thresholds for both industries, is shown in Figure 11.5.
Just as there can be inferior goods, the consumption of which falls as income rises, so there can be 'inferior markets', which receive less attention from a competition authority as the competition authority's budget rises. We assume that markets are not inferior in this sense.

As shown in the Appendix, the condition for investigation thresholds to fall when the investigation budget increases is

\[
\frac{1}{\partial W_i / \partial g_i} \frac{\partial^2 W_i}{\partial g_i^2} - \frac{1}{\partial \tau_i / \partial g_i} \frac{\partial^2 \tau_i}{\partial g_i^2} > 0
\]

(25)

for all \(i\).

Intuitively, an increase in the budget leads the competition authority to reduce investigation thresholds, which tends to lower \(g_i - p_i\) and increase the investigation probability \(\tau_i\). Firms react to this by expanding output, which mitigates the direct effects of a lower threshold price. Condition (25) implies that the indirect effects of firms’ reactions to lower threshold prices are not strong enough to reverse the direct effects of lower threshold prices.
Figure 11.5 Comparative statics, budget increase

Notes: $B_B > B_A$, $W_B > W_A$

Example

Figure 11.6 shows the investigation threshold-budget relationship when the competition authority monitors two identical monopolized industries of the kind shown in Figure 11.2. Investigation costs are relatively low, 1000 per industry.

For this example, social welfare is maximized for a budget of 1,910, which makes the shadow cost of an additional dollar of investigation funds equal to one. Welfare (profit plus consumers’ surplus minus the budget) is 49,020 per industry, compared with 37,500 in the no competition policy case (see Table 11.1). This is a welfare gain of nearly 31 per cent.

If the government as a whole allocates resources to equalize the marginal social value of funds devoted to alternative uses, the size of the budget allocated to enforcement of competition law would depend on the shadow value of funds to the government as a whole (and might therefore fall short of the level that would make the shadow value of funds allocated to enforcement of competition policy equal to one).
Figure 11.6 Budget--investigation threshold relationship

Notes: Two identical industries: \( p = 101 - q_m, \ c = 1, \ b = 1/10, \ \gamma = 1/2, \ F = 20000, \ \sigma = 10. \)

Comparative statics with respect to investigation costs

An increase in (say) \( I_1 \) has effects on the competition authority’s problem that are like the effects of an increase in price on consumer behavior.

When \( I_1 \) increases, isobudget curves become steeper and the feasible set of thresholds \( (g_1, g_2) \) retreats away from the origin. If one compensates for this real budget effect by increasing the nominal budget so that the competition authority can remain on the original isowelfare curve, the increase in \( I_1 \) induces a pure substitution effect: \( g_1 \) rises, \( g_2 \) falls, the competition authority gives less attention to the industry that has become more expensive to investigate and more attention to the other industry. This is shown as the move from \( E_A \) to \( E_B \) in Figure 11.7.

The full effect of the investigation cost increase combines the substitution effect with a pure budget effect, leading from point \( E_B \) to point \( E_C \). The case that is shown has a higher investigation threshold for the industry that has become more costly to investigate and a lower threshold for the other industry. If the budget effect is sufficiently strong, the investigation threshold for the other industry may rise as well.

It is shown in the Appendix that the condition for the comparative statics effects of an increase in \( I_1 \) to have the effects shown in Figure 11.7 is

\[
\frac{1}{\partial W_1 / \partial g_1} \frac{d^2 W_1}{d g_1^2} - \frac{1}{\frac{\tau_1'}{\tau_1} \frac{d g_1}{\tau_1}} > 0.
\]

(26)
Figure 11.7 Comparative statics, increase in $I_1$

Similarly to (25), the implication of (26) is that the direct effects of the competition authority’s reaction to an increase in $I_1$ are not reversed by the indirect effects of firm’s reactions.

One can show that (26) implies (25). If the consequences of changes in investigation costs for investigation thresholds are what one would intuitively expect, then so are the consequences of an increase in the budget.

Example

Figure 11.8 shows the investigation threshold-budget relationships for the industries of the previous example, if the investigation cost for industry 2, $I_2$, is increased from 1000 to 2000. The dashed line in Figure 11.8 reproduces, from Figure 11.6, the threshold-budget relationship when the two industries have investigation costs equal to 1000.

For $B \leq 500$, the entire budget is devoted to investigating industry 1, the low-investigation-cost industry. For $B > 500$, $g_2$ is always greater than $g_1$; the investigation threshold is lower for the industry that costs less to investigate, all else equal. Comparing this example and the previous one, the investigation threshold for industry 2 when $I = 2000$ is always greater than the investigation threshold for industry 2 when $I = 1000$. When $I_2$ doubles, $g_1$ is less for $B$ less than approximately 1333 (this is the kind of reaction shown in Figure 11.7), but is greater thereafter.
Figure 11.8 Budget-investigation threshold relationship 2

Notes: \( n_i = 1, a_i = 101, c_i = 1, b_i = 1/10, \gamma_i = 1/2, F_i = 20000, \sigma_i = 10, I_1 = 1000, I_2 = 2000 \)

Comparative statics with respect to market size

There are several alternative ways to measure market size, for example competitive equilibrium output or competitive equilibrium consumers' surplus. For a linear inverse demand curve,

\[
p_i = a_i - b_i q_i,
\]

(27)

market size in both these senses increases if \( a_i \), the price-axis intercept, increases, holding the slope of the demand curve constant, or if \( b_i \), the absolute value of the slope of the demand curve, declines, holding the price-axis intercept constant.

The effect of changes in \( a_i \) and \( b_i \) on equilibrium investigation thresholds is theoretically ambiguous. To give an indication of the nature of the results, I use the previous numerical examples to illustrate the effect of changes in the price axis intercept \( a_i \) on investigation thresholds. The impact of changes in \( b_i \) are qualitatively similar.

Figure 11.9 shows the equilibrium investigation thresholds as the intercept of the demand curve for good 1 varies from 51 to 191, for an investigation budget 1,100. The range for \( a_1 \) is chosen to ensure an interior solution for both industries.\(^{16}\) All other parameter values are as in the basic example. Figure 11.10 shows the equilibrium threshold-expected price margin over the same range.

As the price-axis intercept increases, the maximum amount consumers are willing to pay for every unit of output goes up by the same amount. All else equal, equilibrium price rises as the demand curve shifts upward.
Figure 11.9 Threshold–a₁ relationship
Notes: \( a_2 = 101, c_1 = c_2 = 1, b_1 = b_2 = 1 / 10, \gamma_1 F_1 = \gamma_2 F_2 = 20000, \sigma_1 = \sigma_2 = 10, n_1 = n_2 = 1, I_1 = I_2 = 1000, B = 1100 \)

Figure 11.10 g–p relationship
Notes: \( a_2 = 101, c_1 = c_2 = 1, b_1 = b_2 = 1 / 10, \gamma_1 F_1 = \gamma_2 F_2 = 10000, \sigma_1 = \sigma_2 = 10, n_1 = n_2 = 1, I_1 = I_2 = 1000, B = 1100 \)
When \( a_i \) is smaller than \( a_2 \), \( g_1 \) is greater than \( g_2 \); a lower threshold is set for the larger industry. As \( a_1 \) rises, \( g_1 \) and \( g_2 \) both rise. As industry 1 becomes larger, competition policy is tightened for industry 1: \( g_1 - p_1 \) falls (Figure 11.12), and as resources are transferred (in an expected value sense) from industry 2 to industry 1, \( g_2 - p_2 \) rises. There is thus a general equilibrium effect as the greater potential consumers’ surplus in industry 1 induces a reallocation of resources between the two industries.

The relationships shown in Figures 11.9 and 11.10 are typical. If both \( a_1 \) and \( B \) are small, there is a region in \((B, a_1)\)-space over which \( g_1 \) falls as \( a_1 \) increases.

**Summary**

The results of this section may be summarized as

**Theorem 2:**

(a) An increase in the investigation budget leads the competition authority to reduce investigation thresholds, all else equal, if

\[
\frac{1}{\partial W_1 / \partial g_i} \frac{\partial^2 W_1}{\partial g_i^2} - \frac{1}{\partial \tau_i / \partial g_i} \frac{\partial^2 \tau_i}{\partial g_i^2} > 0, \tag{28}
\]

for \( i = 1, 2 \);

(b) an increase in investigation cost \( I_i \) leads the competition authority to increase \( g_i \) and lower \( g_j, j \neq i \), all else equal, if

\[
\frac{1}{\partial W_1 / \partial g_i} \frac{d^2 W_1}{dg_i^2} - \frac{1}{\tau_i} \frac{d}{\tau_i} \left( \frac{\tau_i'}{\tau_i} \right) > 0, \tag{29}
\]

(and (29) implies (28));

(c) for linear inverse demand curves, an increase in market size tends to lower \( g - p \) for the larger industry and increase the investigation threshold for the other industry.

**Comparative statics II: number of firms (Nash equilibrium)**

To extend the model of competition policy from monopoly to oligopoly, suppose that there are \( n_i \) identical\(^{17} \) firms in industry \( i \), and that the expected profit of firm \( k \) in industry \( i \) is

\[
\pi_{ik} = \left[p_i(Q_i) - c_i\right] q_{ik} - \frac{\gamma_i F_i}{n_i} \int_{\varepsilon_i = p_i(Q_i)} f_i(\varepsilon_i) d\varepsilon_i. \tag{30}
\]

\( q_{ik} \) is the output of firm \( ik \), \( Q_i \) is industry output, \( c_i \) is unit cost for firms in industry \( i \), and other notation is unchanged from the monopoly case.
If there is a successful prosecution, each firm expects to pay a fraction $1/n_i$ of the resulting fine. This specification is appropriate for joint offenses against competition policy, such as collusion to raise price or deter entry. It would not be appropriate for single-firm violations of competition policy, such as abuse of a dominant position.

The qualitative impact of competition policy on noncooperative equilibrium firm behavior is as in the monopoly case. In particular, a lower investigation threshold increases equilibrium output.

If the number of firms in industry 2 increases, market performance in industry 2 improves. This reduces the marginal social value of funds in the competition authority’s budget. The improvement in market 2’s performance permits the competition authority to toughen competition policy for industry 1: as $n_2$ rises, $g_1$ falls. The direct effect of an increase in $n_2$ is to increase output and lower price in industry 2; this permits the

competition authority to lower the investigation threshold without

increasing the expected probability of investigation of industry 2. But the reduction in $g_1$ and the consequent transfer of investigation resources to industry 1 works in the opposite direction, tending to induce the competition authority to raise $g_2$. The net impact of an increase in $n_4$ on $g_1$ is ambiguous. These results, which are derived in the Appendix, are summarized in

Theorem 3:

If tougher competition policy and more intense competition are substitutes in the sense that an increase in the number of firms reduces the magnitude of $\partial W_i/\partial g_i < 0$ and a reduction in the investigation threshold reduces $\partial W_i/\partial n_i > 0$, then

(a) an increase in $n_i$ reduces the shadow value of funds in the competition authority’s budget;

(b) an increase in $n_i$ has an ambiguous effect on $g_i$;

(c) when there are two industries, an increase in $n_i$ leads the competition authority to lower the investigation threshold for industry $j$ (for $j \neq i$).

Example

Figure 11.11 shows the investigation thresholds for the basic example if $n_2$ is increased from 1 to 2, holding all other parameters fixed. The dashed line in Figure 11.8 reproduces the threshold-budget relationship from Figure 11.6, where both industries have a single supplier. For $B \leq 555.56$, all investigation resources are devoted to industry 1. For larger investigation budgets, $g_1$ is always lower than for $n_2 = 1$; $g_2$ is always greater than for $n_2 = 1$. 
Tacit collusion

Long before the emergence of game theory as the formal theoretical framework of industrial economics, Chamberlin (1933, p. 48) argued that if the number of firms in an industry were sufficiently small, suppliers would converge on the monopoly outcome simply through recognition of their own self-interest. The modern version of this argument is that the threat of a variety of punishment schemes can make tacit collusion on the joint-profit-maximizing output an equilibrium outcome of a repeated game, provided rates of time preference are sufficiently low.

For example (Friedman, 1971), a trigger strategy threatening reversion to Cournot outputs forever will make adherence to a joint-profit-maximizing output strategy an equilibrium if

\[
\frac{1}{r} \geq R = \frac{\pi_D - \pi_m}{\pi_m - \pi_N}, \text{ or } r \leq \frac{\pi_m - \pi_N}{\pi_D - \pi_m} = \frac{1}{R}
\]

(31)

where \(r\) is the interest rate used to discount future income, \(\pi_N\) is Cournot equilibrium profit per firm, \(\pi_m\) is one firm’s share of joint-profit-maximizing output, and \(\pi_D\) is a firm’s single-period profit if it simply maximizes its own profit.

Each of the elements of the fraction on the right in (31) depends on the investigation threshold. Competition policy affects Cournot, joint-profit-maximizing, and defection payoffs, hence affects whether or not it will be an equilibrium for firms to follow a joint-profit-maximizing strategy.

The sign of

\[
\frac{\partial}{\partial g} \left( \frac{\pi_D - \pi_m}{\pi_m - \pi_N} \right)
\]

(32)
is in general ambiguous. For the case of linear demand, constant marginal cost, and exponential demand uncertainty, reductions in $g$ from a high level at first reduce, then increase, the right-hand side of (31). This is shown in Table 11.3. Initial reductions in the investigation threshold make joint profit maximization more likely; further reductions make joint profit maximization less likely.

We know from p. 168 that reductions in the investigation threshold increase monopoly output. Over a certain range, at least, reductions in the investigation threshold can make tacit collusion more likely, although reducing the social cost of such behavior if it does emerge.

**Conclusion**

The research presented here is normative: it presents results about the way resources should be allocated to administer a deterrence-based competition policy that monitors joint exercise of market power if decisions are made to equalize the marginal benefit of competition policy across target industries when there is inherent uncertainty about the workings of the judicial system.

A deterrence-based competition policy improves market performance because it leads firms to expand output, thus reducing expected fines.

In a formal sense, the problem of resource allocation by a competition authority to maximize net social welfare has much in common with the problem of consumer welfare maximization. A greater enforcement budget allows a competition authority to monitor industries more intensely; if it is more costly to monitor a specific industry, that industry should normally be monitored less closely, others more closely, all else equal.

Competition policy significantly improves market performance if industries are monopolized or if firms tacitly collude on monopoly output. However, competition policy may make it more likely that such tacit collusion is an equilibrium.

**Table 11.3** Critical value for tacit duopoly collusion on joint profit-maximizing output

<table>
<thead>
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<th>$g$</th>
<th>$\pi_N$</th>
<th>$\pi_J$</th>
<th>$\pi_D$</th>
<th>$R$</th>
<th>$i/R$</th>
</tr>
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<tr>
<td>No CP</td>
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<td>12500</td>
<td>14063</td>
<td>1.1250</td>
<td>0.8889</td>
</tr>
<tr>
<td>70</td>
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<td>12255</td>
<td>13579</td>
<td>1.0584</td>
<td>0.9448</td>
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<tr>
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<td>12119</td>
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<td>1.0329</td>
<td>0.9681</td>
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<tr>
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<tr>
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</tr>
<tr>
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<td>11270</td>
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<td>1.0466</td>
</tr>
<tr>
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<td>1.0642</td>
</tr>
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<td>10676</td>
<td>0.9314</td>
<td>1.0737</td>
</tr>
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<td>9390.9</td>
<td>9789.5</td>
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<td>1.0752</td>
</tr>
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<td>8763.9</td>
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<td>1.0707</td>
</tr>
<tr>
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<td>6231.9</td>
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<td>1.0514</td>
</tr>
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<td>4662.9</td>
<td>4756.8</td>
<td>0.9609</td>
<td>1.0407</td>
</tr>
<tr>
<td>10</td>
<td>2968.8</td>
<td>3025.9</td>
<td>3081.4</td>
<td>0.9711</td>
<td>1.0298</td>
</tr>
</tbody>
</table>

*Notes: $p = 101 - Q$, $c = 1$, $b = 1/10$, $\gamma = 1/2$, $F = 20000$, $\sigma = 10$.**
strategy. Competition policy has minimal impact on oligopoly market performance if markets settle on the noncooperative equilibrium of a one-shot game.

The models outlined here focus on the consequences of uncertainty about the working of the judicial system. They could be extended to explicitly allow for uncertainty on the part of the competition authority about elements of market structure (for example, market size) and firm structure (unit cost). They could also be extended to allow for the possibility of entry, mergers, and strategic behavior by single firms. These topics are the subject of ongoing research.

Appendix

**Proof of Theorem 1**

The first-order condition for the firm's profit-maximization is

\[
\frac{\partial E(\pi_i)}{\partial q_i} = p_i - c_i + q_i \frac{dp_i}{dq_i} - \gamma_i F_i \frac{\partial \tau_i}{\partial q_i} = p_i - c_i + (q_i - \gamma_i F_i f_i) \frac{dp_i}{dq_i} = 0.
\]  

(33)

This implies

\[
q_i - \gamma_i F_i f_i > 0.
\]

(34)

The second-order condition is

\[
\frac{\partial^2 E(\pi_i)}{\partial q_i^2} = 2 \frac{dp_i}{dq_i} + q_i \frac{d^2 p_i}{dq_i^2} - \gamma_i F_i \frac{\partial \tau_i}{\partial q_i} < 0
\]

(35)

or equivalently

\[
\frac{\partial^2 E(\pi_i)}{\partial q_i^2} = 2 \frac{dp_i}{dq_i} + \gamma_i F_i f_i' \left( \frac{dp_i}{dq_i} \right)^2 + (q_i - \gamma_i F_i f_i) \frac{d^2 p_i}{dq_i^2} < 0.
\]

(36)

(a) Substituting \( p_i = g_i - \varepsilon_i \), rewrite the first-order condition as

\[
\frac{1}{\gamma_i F_i} \left( \frac{g_i - c_i - \varepsilon_i}{dp_i/dq_i} + q_i \right) = f_i(\varepsilon_i).
\]

(37)

In the denominator on the left, \( q_i \) is evaluated at \( g_i - \varepsilon_i^* \); \( dp_i/dq_i < 0 \) is evaluated at \( q_i (g_i - \varepsilon_i^*) \).

Fix \( g_i \) and consider the left-hand and right-hand sides as separate functions of \( \varepsilon_i \). The functions can be graphed, as in Figure 11.12, the intersection of the two functions gives the equilibrium value \( \varepsilon_i^* \).
Figure 11.12 Equilibrium $g - \varepsilon$

Given that $f_i(\varepsilon_i)$ falls to the right of $\varepsilon_i = 0$, the first condition for $\varepsilon_i^* = g_i - p_p^* > 0$ is that $\gamma_i F_i$ be sufficiently great that

$$\frac{1}{\gamma_i F_i} \left( \frac{g_i - c_i}{dp_i} + q_i \right) < f_i(0);$$

(38)

then the left-hand side of (37) is below the right-hand side at $\varepsilon_i = 0$.

The second requirement for $\varepsilon_i^* = g_i - p_p^* > 0$ is that the function on the left slope upward. The slope of this function is

$$-\frac{1}{\gamma_i F_i} \left[ 2 \frac{dq_i}{dp_i} + \left( p_i - c_i \right) \frac{d^2 q_i}{dp_i^2} \right].$$

(39)

The second condition for $\varepsilon_i^* = g_i - p_p^* > 0$ is that the term in brackets be negative; this is satisfied for linear demand, and is assumed to hold.

(b) Differentiating the first-order condition with respect to $g_i$ yields

$$\frac{\partial q_i}{\partial g_i} = \frac{1}{\partial^2 E(\pi_i)} \left[ \frac{\partial^2 E(\pi_i)}{\partial g_i \partial q_i} \right]$$

(40)

The second-order condition implies that the term in brackets on the right is positive.
Differentiating the first-order condition gives

\[ \frac{\partial^2 E(\pi_i)}{\partial g \partial q_i} = -\gamma_i f_i' \frac{dp_i}{dq_i} < 0, \]  

(41)

where the sign depends on \( \varepsilon_i^* = g_i^{-1}p_p^* > 0 \) and the assumption that \( f_i' < 0 \) for \( \varepsilon > 0 \).

(c) Note that

\[ \frac{\partial^2 E(\pi_i)}{\partial (\gamma, F_i) \partial q_i} = -f_i \frac{dp_i}{dq_i} > 0; \]  

then

\[ \frac{\partial q_i}{\partial (\gamma, F_i)} = \left[ -\frac{\partial^2 E(\pi_i)}{\partial \gamma \partial F_i} \right] \frac{\partial^2 E(\pi_i)}{\partial ^2 q_i} > 0. \]  

(42)

The competition authority’s problem: constrained optimization

We confine our attention to cases in which the budget constraint is binding: the competition authority does not have enough resources to investigate all industries for all realized prices. This allows us to reformulate the competition authority’s problem as

\[ \max_{g_1, g_2} W_1(q_1) + W_2(q_2) - B \]  

(43)

such that

\[ \tau_1 I_1 + \tau_2 I_2 \leq B. \]  

(44)

The first-order conditions for the solution to the competition authority’s constrained optimization problem come from the Lagrangian

\[ L = W_1(q_1) + W_2(q_2) - B + \lambda \left[ B - \tau_1 I_1 - \tau_2 I_2 \right]. \]  

(45)

First-order conditions

The first-order conditions are

\[ \frac{\partial L}{\partial \lambda} = B - \tau_1 I_1 - \tau_2 I_2 \equiv 0, \]  

(46)

\[ \frac{\partial L}{\partial g_1} = \frac{dW_1}{dg_1} - \lambda \frac{d\tau_1}{dg_1} \equiv 0, \]  

(47)
and

$$\frac{\partial L}{\partial g_2} = \frac{dW_2}{dg_2} - \lambda I_2 \frac{d\tau_2}{dg_2} \equiv 0,$$  \hspace{1cm} (48)

(47) and (48) then imply

$$\lambda = \frac{W_i'}{I_i\tau_i'} = \frac{W'_2}{I_2\tau_2'}$$  \hspace{1cm} (49)

(where $W_i' = \partial W_i/\partial g_i$).

Consider the expression for $\lambda$ in terms of market 1 variables. The numerator

$$\frac{dW_1}{dg_1}$$  \hspace{1cm} (50)

is negative: in absolute value, it gives the increase in welfare in market 1 if the enforcement agency lowers $g_1$ slightly.

The denominator,

$$I_1 \frac{d\tau_1}{dg_1}$$  \hspace{1cm} (51)

is, in absolute value, the increase in expected investigation cost if $g_1$ is lowered slightly. The Lagrangian multiplier $\lambda$, the shadow value of the marginal budget allocation to the competition agency, is seen to be the marginal increase in welfare in market 1 per marginal increase in spending on enforcement in market 1. The enforcement agency’s choice of threshold prices is optimal when this ratio is the same for all markets.

If (47) and (48) are rewritten breaking the total derivatives into their constituent parts, they become

$$\frac{\partial L}{\partial g_1} = \frac{\partial W_1}{\partial q_1} \frac{\partial q_1}{\partial g_1} - \lambda I_1 \tau_1' \left(1 - \frac{dp_1}{dq_1} \frac{\partial q_1}{\partial g_1}\right) \equiv 0$$  \hspace{1cm} (52)

and

$$\frac{\partial L}{\partial g_2} = \frac{\partial W_2}{\partial q_2} \frac{\partial q_2}{\partial g_2} - \lambda I_2 \tau_2' \left(1 - \frac{dp_2}{dq_2} \frac{\partial q_2}{\partial g_2}\right) \equiv 0$$  \hspace{1cm} (53)

respectively. (49) becomes
\[
\lambda = \frac{\frac{\partial W_1}{\partial q_1} \frac{\partial q_1}{\partial g_1}}{I_1 \tau_1' \left(1 - \frac{dp_1}{dq_1} \frac{\partial q_1}{\partial g_1}\right)} = \frac{\frac{\partial W_2}{\partial q_2} \frac{\partial q_2}{\partial g_2}}{I_2 \tau_2' \left(1 - \frac{dp_2}{dq_2} \frac{\partial q_2}{\partial g_2}\right)} \quad (54)
\]

This implies that where the first-order conditions are satisfied
\[
- \frac{\frac{\partial W_1}{\partial q_1} \frac{\partial q_1}{\partial g_1}}{\frac{\partial W_2}{\partial q_2} \frac{\partial q_2}{\partial g_2}} = \frac{I_1 \tau_1' \left(1 - \frac{dp_1}{dq_1} \frac{\partial q_1}{\partial g_1}\right)}{I_2 \tau_2' \left(1 - \frac{dp_2}{dq_2} \frac{\partial q_2}{\partial g_2}\right)} < 0 \quad (55)
\]

The expression on the left is the slope of an isowelfare curve; the expression on the right is the slope of an isobudget curve. The first-order conditions imply that the solution to the competition authority's problem occurs at the tangency of an isowelfare curve and the isobudget curve.

**Second-order conditions**

The second-order condition for a maximum requires that the determinant of the bordered Hessian

\[
\begin{pmatrix}
0 & I_1 \tau_1' & I_2 \tau_2' \\
I_1 \tau_1' & L_{11} & 0 \\
I_2 \tau_2' & 0 & L_{22}
\end{pmatrix}
\]

be positive, where \( L_{ii} = \frac{\partial^2 L}{\partial g_i^2} \). When written out, this condition is

\[-\left((I_1 \tau_1')^2 L_{22} + (I_2 \tau_2')^2 L_{11}\right) > 0. \quad (57)\]

Sufficient conditions for this to be met are that \( L_{ii} < 0 \) for \( i = 1, 2 \). Using (47) to evaluate \( L_{11} \),

\[
L_{11} = W_1^{'''} - \lambda I_1 \tau_1' = W_1' \left( \frac{W_1^{'''} - \tau_1''}{W_1'} \right). \quad (58)
\]

Since \( W_1' < 0 \), \( L_{11} < 0 \) if the proportional impact of a change in \( g_1 \) on \( \tau_1' \) is greater than its impact on a change in \( W_1' \)

\[-\frac{\tau_1''}{\tau_1'} > -\frac{W_1^{'''}}{W_1'} > 0. \quad (59)\]

We will henceforth assume \( L_{ii} < 0 \) for \( i = 1, 2 \).
We now show that the second-order condition implies the slope relationships (72).

Substitute
\[ L_i = W_i'' - \lambda I_1 \tau_i'' \]  
(60)
in
\[ (I_1 \tau_1')^2 L_{22} + (I_2 \tau_2')^2 L_{11} < 0. \]  
(61)
to obtain
\[ (I_1 \tau_1')^2 (W_2'' - \lambda \tau_2'') + (I_2 \tau_2')^2 (W_1'' - \lambda \tau_1'') < 0. \]  
(62)
Collect terms in \( \lambda \):
\[ (I_1 \tau_1')^2 W_2'' + (I_2 \tau_2')^2 W_1'' < \lambda [(I_1 \tau_1')^2 \tau_2'' + (I_2 \tau_2')^2 \tau_1''] \]  
(63)
The first-order conditions imply
\[ I_1 \tau_1' = \frac{1}{\lambda} W_i'. \]  
(64)
Substitute these expressions on the left in (63):
\[ \frac{1}{\lambda^2} \left[(W_i')^2 W_2'' + (W_2')^2 W_1''\right] < \lambda \left[(I_1 \tau_1')^2 \tau_2'' + (I_2 \tau_2')^2 \tau_1''\right] \]  
(65)
\( \lambda \) is positive; dividing both sides of (65) by \( \lambda \) does not change the direction of the inequality:
\[ \frac{1}{\lambda^3} \left[(W_i')^2 W_2'' + (W_2')^2 W_1''\right] < \left(I_1 \frac{d \tau_1}{d g_1}\right)^2 \frac{d^2 \tau_2}{d g_2^2} + \left(I_2 \frac{d \tau_2}{d g_2}\right)^2 \frac{d^2 \tau_1}{d g_1^2} \]  
(66)
Now substitute
\[ \lambda = \frac{dq_2 dq_2}{I \frac{d \tau_2}{d g_2}} \]  
(67)
to eliminate \( \lambda \), and rearrange terms slightly to obtain
\[ 0 \leq \frac{(W_i')^2 W_2'' + (W_2')^2 W_1''}{(W_i')^3} < \frac{(I_1 \tau_1')^2 I_2 \tau_2'' + (I_2 \tau_2')^2 I_1 \tau_1''}{(I_2 \tau_2')^3}, \]  
(68)
Differentiating (21), the second derivative of an isobudget curve is

\[
\frac{d^2 g_2}{dg_1^2} \bigg|_B = \frac{(I_1 \tau_1')^2 I_2 \tau_2'' + (I_2 \tau_2')^2 I_1 \tau_1''}{(I_2 \tau_2')^3} > 0.
\]  

(69)

Differentiating (24), the second derivative of an isowellfare curve is

\[
\frac{d^2 g_2}{dg_1^2} \bigg|_W = \frac{(W_1')^2 W_2'' + (W_2')^2 W_1''}{(W_2')^3} > 0,
\]  

(70)

where the sign depends on the assumption that there are decreasing returns to increasing welfare by lowering the investigation threshold,

\[
W_i'' > 0.
\]  

(71)

Using (69) and (70), the transformed second-order condition (68) can be written

\[
0 < \frac{d^2 g_2}{dg_1^2} \bigg|_W < \frac{d^2 g_2}{dg_1^2} \bigg|_B
\]  

(72)

This justifies the configuration shown in Figure 11.4.

**Comparative statics with respect to B**

Differentiate the first-order conditions (46), (47), and (48) with respect to B to obtain

\[
-I_1 \tau_1' \frac{\partial g_1}{\partial B} - I_2 \tau_2' \frac{\partial g_2}{\partial B} = -1
\]  

(73)

\[
L_{11} \frac{\partial g_1}{\partial B} - I_1 \tau_1' \frac{\partial \lambda}{\partial B} = 0,
\]  

(74)

and

\[
L_{22} \frac{\partial g_2}{\partial B} - I_2 \tau_2' \frac{\partial \lambda}{\partial B} = 0,
\]  

(75)

where
\[ L_{ii} = W_{ii}'' - \lambda L_{ii}'. \]

Write the system of equations in matrix form as

\[
\begin{pmatrix}
0 & -L_{11}' & -L_{22}' \\
-L_{11}' & L_{11} & 0 \\
-L_{22}' & 0 & L_{22}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \lambda}{\partial B} \\
\frac{\partial g_1}{\partial B} \\
\frac{\partial g_2}{\partial B}
\end{pmatrix}
= \begin{pmatrix}
-1 \\
0 \\
0
\end{pmatrix}.
\]

The second-order condition for a maximum, which we assume to be met, requires that the determinant of the bordered Hessian on the left be positive. Henceforth this determinant will be denoted as \( DET \).

Inverting the matrix on the left,

\[
\begin{pmatrix}
\frac{\partial \lambda}{\partial B} \\
\frac{\partial g_1}{\partial B} \\
\frac{\partial g_2}{\partial B}
\end{pmatrix}
= \frac{1}{DET}
\begin{pmatrix}
L_{11}L_{22} & L_{11}'L_{22} & L_{11}'L_{22}' \\
L_{11}'L_{22} & -(L_{22}')^2 & L_{11}'L_{22}' \\
-(L_{22}L_{11}) & -(L_{22}L_{11})' & -(L_{22}L_{11})'
\end{pmatrix}
\begin{pmatrix}
-1 \\
0 \\
0
\end{pmatrix}.
\]

\[
= -\frac{1}{DET}
\begin{pmatrix}
L_{11}L_{22} \\
L_{11}'L_{22} \\
L_{22}'L_{11}
\end{pmatrix}
\]

so

\[
\frac{\partial \lambda}{\partial B} = -\frac{L_{11}L_{22}}{DET} < 0
\]

\[
\frac{\partial g_1}{\partial B} = -\frac{L_{11}'L_{22}}{DET} < 0
\]

\[
\frac{\partial g_2}{\partial B} = -\frac{L_{22}'L_{11}}{DET} < 0.
\]

\( DET > 0 \) and \( \tau_1' < \tau_2' < 0 \). The signs given for (81) and (82) hold for the normal market case, and imply \( L_{11} < 0, L_{22} < 0 \). These in turn imply the sign given in (80). For these cases, as \( B \) increases, \( g_1 \) and \( g_2 \) move down a positively-sloped ‘budget expansion curve’, and the shadow value of budget funds falls.
Comparative statics with respect to $I_1$

Differentiating the first-order conditions (46), (47), and (48) with respect to $I_1$ gives

$$
\begin{pmatrix}
0 & -I_1\tau'_1 & -I_2\tau'_2 \\
-I_1\tau'_1 & L_1 & 0 \\
-I_2\tau'_2 & 0 & L_2
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \lambda}{\partial I_1} \\
\frac{\partial \lambda}{\partial g_1} \\
\frac{\partial \lambda}{\partial g_2}
\end{pmatrix}
= \begin{pmatrix}
\tau_1 \\
\frac{d\tau_1}{dg_1} \\
0
\end{pmatrix},
$$

from which

$$
\begin{pmatrix}
\frac{\partial \lambda}{\partial I_1} \\
\frac{\partial \lambda}{\partial g_1} \\
\frac{\partial \lambda}{\partial g_2}
\end{pmatrix}
= \frac{1}{\text{DET}}
\begin{pmatrix}
L_1 L_2 & I_1\tau'_1L_2 & I_2\tau'_2L_1 \\
I_1\tau'_1L_2 & -(I_2\tau'_2)^2 & I_1\tau'_1L_2 \\
I_2\tau'_2L_1 & I_1\tau'_1L_2 & -(I_1\tau'_1)^2
\end{pmatrix}
\begin{pmatrix}
\tau_1 \\
\frac{d\tau_1}{dg_1} \\
0
\end{pmatrix}.
$$

This leads to

$$
\frac{\partial \lambda}{\partial I_1} = \frac{L_2}{\text{DET}}[L_1\tau_1 + \lambda I_1(\tau'_1)^2]
$$

$$
\frac{\partial g_1}{\partial I_1} = \frac{I_2\tau'_2 L_1 -(I_2\tau'_2)^2 \lambda \tau'_1}{\text{DET}} > 0
$$

$$
\frac{\partial g_2}{\partial I_1} = \frac{I_2\tau'_2}{\text{DET}}[L_1\tau_1 + \lambda I_1(\tau'_1)^2]
$$

$\frac{\partial g_1}{\partial I_1} > 0$: if the cost of investigating industry 1 rises, the competition authority increases $g_i$.

The changes in $g_1$ and $g_2$ if $I_1$ rises can be broken down into a ‘budget-compensated’ change – a movement along a given isowelfare curve as the isobudget curve becomes steeper – and an outward move along a budget expansion curve.

To interpret the expression for $d\lambda/dI_1$, first differentiate

$$
I_2\lambda = \frac{W'_2}{\tau'_2}
$$

with respect to $I_1$ to obtain
\[ I_2 \frac{\partial \lambda}{\partial l_1} = \frac{W''_i}{\tau'_i} \left( \frac{W''_i}{W'_i} - \frac{\tau''}{\tau'_i} \right) \frac{\partial g_2}{\partial l_1}. \]  
(89)

If the condition for \( L_{11} < 0 \), (59), is satisfied, the term in parentheses on the right is positive and \( \frac{\partial \lambda}{\partial l_1} \) and \( \frac{\partial \lambda}{\partial l_i} \) have the same sign. This is also apparent directly from (85) and (87).

Second, differentiate the other expression for \( \lambda \),

\[ \lambda = \frac{W'_i}{\tau'_i}, \]  
(90)

with respect to \( l_1 \) to obtain

\[ \frac{\tau'_i l_2^2}{W'_i} \frac{\partial \lambda}{\partial l_1} = \frac{I_1}{W'_i/\tau'_i} \frac{d}{dl_1} \frac{\partial}{\partial l_1} \left( \frac{W'_i}{\tau'_i} \right) \frac{dg_1}{dl_1} - 1 \]  
(91)

The coefficient of \( \frac{\partial \lambda}{\partial l_1} \) on the left is positive. The first term on the right,

\[ \frac{I_1}{W'_i/\tau'_i} \frac{d}{dl_1} \frac{\partial}{\partial l_1} \left( \frac{W'_i}{\tau'_i} \right) \frac{dg_1}{dl_1} = \frac{I_1}{(\tau'_i)} \left( \frac{W''_i}{W'_i} - \frac{\tau''}{\tau'_i} \right) \frac{dg_1}{dl_1} \]  
(92)

is positive, so long as \( L_{11} < 0 \).

It follows that \( \frac{\partial \lambda}{\partial l_i} \) and \( \frac{\partial \lambda}{\partial l_i} \) are negative if the direct effect of an increase in \( I_1 \), the denominator of (90), in reducing \( \lambda \), is greater in magnitude than the indirect effect of an increase in \( I_1 \), which is to increase \( W'_i/\tau'_i \), the numerator of (90).

Directly from (85) and (87), if

\[ L_1 \tau_1 + \lambda I_1 (\tau'_i)^2 < 0, \]  
(93)

then \( \frac{\partial \lambda}{\partial l_1} \) and \( \frac{\partial \lambda}{\partial l_i} \) are negative. Evaluating (93), the condition for (93) is (compare (59))

\[-\frac{1}{\tau'_i/\tau_i} \frac{d}{dl_1} \left( \frac{\tau'_i}{\tau_i} \right) > -\frac{W''_i}{W'_i} > 0. \]  
(94)

Like (59), (94) is a condition which ensures that the direct effect of changes in investigation costs prevail over indirect effects. By expressing (59) and (94) in terms of their definitions, one can show that (94) implies (59).

If (94) is met, or equivalently if the right-hand side of (91) is positive, an increase in \( I_1 \) leads to a decrease in \( g_2 \), a decrease in the shadow value of budget funds \( \lambda \), and, as follows from (86), an increase in \( g_1 \).
**Comparative statics with respect to \( n_i \)**

**Industry output**

Differentiate (30), the expression for firm \( ik \)'s profit, with respect to \( q_{ik} \) to obtain firm \( ik \)'s first-order condition:

\[
\frac{\partial \pi_{ik}}{\partial q_{ik}} = p_i(Q) - c_i + q_{ik} \frac{dp_i}{dQ} - \frac{\gamma_i F_i}{n_i} f_i[g_i - p_i(Q)] \frac{dp_i}{dQ} = 0 \tag{95}
\]

Adding (95) over all \( j \) gives

\[
p_i(Q) - c_i + \frac{Q_i}{n_i} \frac{dp_i}{dQ} - \frac{\gamma_i F_i}{n_i} f_i[g_i - p_i(Q)] \frac{dp_i}{dQ} = 0. \tag{96}
\]

As usual for Cournot oligopoly with constant marginal cost, industry output depends on unit cost and the number of firms.

Differentiating (96) with respect to \( g_i \) gives the comparative static derivative

\[
\frac{\partial Q_i}{\partial g_i} = \frac{1}{D_i} \gamma_i F_i f'_i \frac{dp_i}{dQ} < 0, \tag{97}
\]

where stability conditions imply that

\[
D_i = (n_i - 1) \frac{dp_i}{dQ} + \frac{\gamma_i F_i f'_i}{n_i} \left( \frac{dp_i}{dQ} \right)^2 + \frac{1}{n_i} \frac{d^2 p_i}{dQ^2} (Q_i - \gamma_i F_i f_i) < 0. \tag{98}
\]

A lower investigation threshold increases industry output.

Differentiating (96) with respect to \( n_i \) gives

\[
\frac{\partial Q_i}{\partial n_i} = \frac{Q_i - \gamma_i F_i f_i}{n_i D_i} \frac{dp_i}{dQ} > 0. \tag{99}
\]

Industry output rises with the number of firms in the industry.

**The competition authority's problem**

The investigation thresholds selected by the competition authority maximize

\[
L = W_1(Q_1) + W_2(Q_2) - B + \lambda \left[ B - \tau_1 I_1 - \tau_2 I_2 \right] \tag{100}
\]

where

\[
W_i(Q_i) = \int_0^{Q_i} [p(x_i) - c_i] \, dx_i \tag{101}
\]
Comparative statics with respect to \( n_i \)

Industry output

Differentiate (30), the expression for firm \( ik \)'s profit, with respect to \( q_{ik} \) to obtain firm \( ik \)'s first-order condition:

\[
\frac{\partial \pi_{ik}}{\partial q_{ik}} = p_i(Q_i) - c_i + q_{ik} \frac{dp_i}{dQ_i} - \frac{\gamma_i F_i}{n_i} f_i \left[ q_i - p_i(Q_i) \right] \frac{dp_i}{dQ_i} = 0
\]  
(95)

Adding (95) over all \( j \) gives

\[
p_i(Q_i) - c_i + \frac{Q_i}{n_i} \frac{dp_i}{dQ_i} - \frac{\gamma_i F_i}{n_i} f_i \left[ q_i - p_i(Q_i) \right] \frac{dp_i}{dQ_i} = 0.
\]  
(96)

As usual for Cournot oligopoly with constant marginal cost, industry output depends on unit cost and the number of firms.

Differentiating (96) with respect to \( g_i \) gives the comparative static derivative

\[
\frac{\partial Q_i}{\partial g_i} = \frac{1}{D_i} \gamma_i F_i f_i' \frac{dp_i}{dQ_i} < 0,
\]  
(97)

where stability conditions imply that

\[
D_i = (n_i - 1) \frac{dp_i}{dQ_i} + \gamma_i F_i f_i' \left( \frac{dp_i}{dQ_i} \right)^2 + \frac{1}{n_i} \frac{d^2 p_i}{dQ_i^2} (Q_i - \gamma_i F_i f_i) < 0.
\]  
(98)

A lower investigation threshold increases industry output.

Differentiating (96) with respect to \( n_i \) gives

\[
\frac{\partial Q_i}{\partial n_i} = \frac{Q_i - \gamma_i F_i f_i}{n_i D_i} \frac{dp_i}{dQ_i} > 0.
\]  
(99)

Industry output rises with the number of firms in the industry.

The competition authority's problem

The investigation thresholds selected by the competition authority maximize

\[
L = W_1(Q_1) + W_2(Q_2) - B + \lambda \left[ B - \tau_1 I_1 - \tau_2 I_2 \right]
\]  
(100)

where

\[
W_i(Q_i) = \int_0^{Q_i} \left[ p(x_i) - c_i \right] dx_i
\]  
(101)
\[
\tau_i = \int_{\theta_{i-n+1}}^{\theta_i} f_{i}(\varepsilon_i) d\varepsilon_i.
\] (102)

The first-order conditions have the same form as (46)–(48). Differentiating the first-order conditions with respect to \( n_1 \) gives

\[
\begin{pmatrix}
0 & -I_1\tau'_{1} & -I_2\tau'_{2} \\
-I_1\tau'_{1} & L_{11} & 0 \\
-I_2\tau'_{2} & 0 & L_{22}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \lambda}{\partial n_1} \\
\frac{\partial g_1}{\partial n_1} \\
\frac{\partial g_2}{\partial n_1}
\end{pmatrix}
= \begin{pmatrix}
-I_1\tau'_{1} \frac{dp_1}{dQ_i} \frac{\partial Q_1}{\partial n_1} \\
-I_1\tau'_{1} \frac{dQ_1}{\partial n_1} \\
- \frac{\partial^2 L}{\partial n_1 \partial g_1} \\
0
\end{pmatrix},
\] (103)

Note that

\[
\frac{\partial^2 L}{\partial n_1 \partial g_1} = \frac{\partial^2 W_1}{\partial n_1 \partial g_1} - \lambda_1 \frac{\partial^2 \tau_1}{\partial n_1 \partial g_1}
\] (104)

\[
= \frac{\partial W_1}{\partial g_1} \left( \frac{1}{\partial W_1 / \partial g_1} \frac{\partial^2 w_1}{\partial n_1 \partial g_1} - \frac{1}{\partial \tau_1 / \partial g_1} \frac{\partial^2 \tau_1}{\partial n_1 \partial g_1} \right)
\] (105)

To interpret the term in parentheses in (105), note that welfare in market 1 rises as the investigation threshold falls, holding the number of firms constant

\[
\frac{\partial W_1}{\partial g_1} = \frac{\partial W_1}{\partial Q_1} \frac{\partial Q_1}{\partial g_1} < 0
\] (106)

and as the number of firms increases, holding the investigation threshold constant:

\[
\frac{\partial W_1}{\partial n_1} = \frac{\partial W_1}{\partial Q_1} \frac{\partial Q_1}{\partial n_1} > 0.
\] (107)

If

\[
\frac{\partial^2 W_1}{\partial n_1 \partial g_1} > 0,
\] (108)

this means that from the point of view of market performance tougher competition policy and more intense competition are substitutes in the sense that an increase in the number of firms reduces the magnitude of \( \partial W_1 / \partial g_1 \) < 0 and that a
reduction in the investigation threshold reduces \( \frac{\partial W}{\partial n_1} > 0 \).

If the impacts of lower \( g_1 \) and greater \( n_1 \) on \( W \) are related in this way, then

\[
\frac{\partial^2 L}{\partial n_1 \partial g_1} = \frac{\partial W}{\partial g_1} \left( \frac{1}{\partial W / \partial g_1} \frac{\partial^2 W}{\partial n_1 \partial g_1} - \frac{1}{\partial \tau_1 / \partial g_1} \frac{\partial^2 \tau_1}{\partial n_1 \partial g_1} \right) > 0
\]  

(109)

Returning to and solving (103) gives

\[
\frac{\partial \lambda}{\partial n_1} = -\frac{I_1 \tau_1 L_{22}}{DET} \left( \frac{L_{11}}{dQ_1} \frac{\partial Q_1}{\partial n_1} + \frac{\partial^2 L}{\partial n_1 \partial g_1} \right) < 0
\]  

(110)

\[
\frac{\partial g_1}{\partial n_1} = -\frac{1}{DET} \left[ (I_2 \tau_2')^2 \frac{\partial^2 L}{\partial n_1 \partial g_1} - (I_1 \tau_1')^2 L_{22} \frac{dP_1}{dQ_1} \frac{\partial Q_1}{\partial n_1} \right]
\]  

(111)

\[
\frac{\partial g_2}{\partial n_1} = \frac{I_1 \tau_1' I_2 \tau_2'}{DET} \left( \frac{L_{11}}{dQ_1} \frac{\partial Q_1}{\partial n_1} + \frac{\partial^2 L}{\partial n_1 \partial g_1} \right) < 0
\]  

(112)

An increase in \( n_1 \) improves market performance in market 1, all else equal, and reduces the shadow value of funds in the investigation budget. The improvement in market performance in market 1 leads the competition authority to lower the investigation threshold for industry 2.

The direct effect of an increase in \( n_1 \) is to raise output \( Q_1 \), permitting the competition authority to lower the investigation threshold for industry 1. The increase in \( g_2 \), which draws investigation resources away from industry 1, mitigates this. \( \frac{\partial g_1}{\partial n_1} \) is of ambiguous sign, although \( g_1 \) will fall if the direct effect of an increase in the number of firms dominates.¹⁹

Acknowledgement

I am grateful for comments received at Universitat Jaume I and the Copenhagen Business School. Responsibility for errors is my own.

Notes

1 In some legal regimes, employees of firms that violate antitrust law may be subject to criminal penalties. Such penalties are rare, and are not considered here.

2 See U.S. v. Chas. Pfizer & Co., Inc. 367 F. Supp. 91 (S.D.N.Y. 1973), where two firms were charged with collusion and conspiracy to monopolize following an agreement to settle certain patent disputes that was reached after discussions by the presidents of the two companies, and found not guilty when the court accepted their argument that the settlement reached after the discussions reflected the exercise of independent business judgment.

3 Competition policy typically frowns on the systematic exchange of price and/or quantity data referring to individual firms or transactions. Such a policy has a sound
basis in economic theory — detailed price reporting chills price rivalry by exposing price cutters to rapid retaliation — but detailed price reporting acts to enforce a ‘collusive’ set of strategies, once reached, not to facilitate initial convergence on such a set of strategies.

Nor is economic theory likely to be of much help in reducing this uncertainty. From the point of view of economic theory, all decisions by independent firms are independent if it is not possible for the firms involved to make binding commitments. If contracts to restrict output and raise price are not enforceable, all conduct that leads to outcomes in which firms exercise more market power than they would in the non-cooperative equilibrium of a one-shot game is non-cooperative, and in that sense independently arrived at.

Analysis of the administration of competition policy that regulates market structure is the subject of ongoing research.

Discussion of competition policy toward industries producing products related in demand would complicate expressions for net social welfare without changing fundamental aspects of the model. Discussion of competition policy toward industries producing intermediate goods (such as software) would require explicit analysis of input–output relationships throughout the economy.

This would be particularly important in a model including a separate stage of the game at which the firm would decide whether or not to settle and avoid litigation.

The condition is satisfied for a linear demand curve. It means that the output response to a change in the investigation threshold is less in magnitude than the slope of the demand curve,

\[
\frac{dq_i}{dp_i} > -\frac{\partial q_i}{\partial g_i} > 0.
\]  

The exponential distribution is convenient for examples because it always leads to interior solutions of the competition authority’s problem. A uniform distribution would involve the possibility of corner solutions (for example, in which the probability of investigation is so high that the firm treats fines as a fixed cost and produces the no-competition policy monopoly output; see Martin (1998)).

In what follows, I will omit the adjective ‘expected’ unless clarity requires it.

In what follows, 11.2 assumes that firms expect to evenly divide any fines; see p. 182.

In Martin (2000), I show that lower investigation thresholds generally stimulate private investment in cost-saving innovation. In this sense, stricter product-market competition policy promotes good dynamic market performance.

This formulation implies that expected fines are receipts to the government in general, not to the competition authority. In the contrary case, the budget constraint would be

\[
\tau_1 I_1 + \tau_2 I_2 \leq B + \tau_1 \gamma_1 F_1 + \tau_2 \gamma_2 F_2.
\]

In May 1998 the U.S. Department of Justice, prosecuting Microsoft and faced with the need to monitor a wave of mergers, requested an increase of ‘several million dollars’ in the Antitrust Division’s annual budget of $95 million (Associated Press, 20 May 1998).

Variations might weight consumers’ and producers’ surplus differently; for an example in the context of a model of competition policy, see Besanko and Spulber (1993).

For any value of \( B \), as the size of market 1 increases, holding the size of market 2 constant, it eventually becomes optimal to devote all investigation resources to industry 1.

The assumption that firms are identical can be relaxed without much difficulty. If unit costs are constant for each firm but differ across firms in the industry, then industry
output depends on the number of firms and unweighted average output. To examine the impact of an additional firm on equilibrium investigation thresholds, one must specify the unit cost of the additional firm.

18 Figure 11.12 is drawn for an exponential density and linear demand.

19 Differentiating the budget constraint

\[ I_2 \int_{q_1}^{q_2} f_1(e_1) \, de_1 + I_2 \int_{q_2}^{q_3} f_2(e_2) \, de_2 = B \]

with respect to \( n_1 \) gives

\[ I_1 f_1 \left( \frac{\partial g_1}{\partial n_1} - \frac{dp_1}{dQ_1} \frac{\partial Q_1}{\partial n_1} \right) + I_2 f_2 \frac{\partial g_2}{\partial n_1} = 0 \]

\( I_1 f_1 \) and \( I_2 f_2 \) are both positive. By (112), \( \frac{\partial g_2}{\partial n_1} < 0 \). It follows that

\[ \frac{\partial g_1}{\partial n_1} - \frac{dp_1}{dQ_1} \frac{\partial Q_1}{\partial n_1} > 0 \]

This is certainly satisfied if \( \frac{\partial g_1}{\partial n_1} > 0 \). It may be satisfied if \( \frac{\partial g_1}{\partial n_1} \) is negative, if the second term on the left, which is negative, is sufficiently large in magnitude.

References


Competition Policy Analysis

Edited by
Einar Hope