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Monopolistic Competition and Optimal Product
Diversity with Heterogeneous Firms
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Abstract

I extend the Dixit-Stiglitz (1977) model of monopolistic competition by relaxing the assumption of a single technology. The main objective is to study whether the equivalence of the monopolistically competitive equilibrium to the social optimum still holds in the extended model. I show that the answer crucially depends on the nature of the technological heterogeneity. If firms are free to choose technology, and free entry/exit condition holds to ensure a zero profit equilibrium, then the Dixit-Stiglitz result is robust to the existence of heterogeneous firms. However, if firm-level productivity is drawn from a heterogeneous distribution so that some infra-marginal firms earn positive profits then the market equilibrium no longer has the same desirable properties.

JEL Classification: D43, L11, L1

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In their (1977) seminal paper Avinash Dixit and Josef Stiglitz (henceforth DS) contributed to the disputes on the optimality of product diversification by developing a model in which the monopolistically competitive equilibrium coincides with the social optimum¹. Despite some lack of generality (see e.g. Oliver D. Hart 1985; Jean-Pascal Benassy 1996; Ottaviano Gianmarco I. P. and Jacques-François F. Thisse 1999) the model has become extremely popular as a basic framework in the fields of industrial organization, international economics, macroeconomics, development and growth (see ISI Web of Science for over 700 citations).

Following DS (1977), most of these papers assume that all firms use the same technology. However, empirical work makes it clear that technology differs substantially across firms (Lucia Foster, et. al. 1998 summarize this research). Guided by the empirical evidence several authors have begun to study the impact of heterogeneity in areas such as international trade (Anthony J. Venables 1993, Mark Melitz 2002, Stephen Ross Yeaple 2003), growth and development (Paul M. Romer 1994), and the efficiency of fiscal policy (Hassan Molana and Catia Montagna 2000). These papers, however, have not systematically examined how heterogeneity affects the basic question analyzed by DS, that is, whether market equilibrium approaches social optimum.

This paper explores whether the result derived by DS (1977) is affected by relaxing the assumption of a single technology. I show that the answer crucially depends on the choice between two alternatives of introducing heterogeneity into the model. If all firms have equal access to existing technologies, then the set of market equilibria coincides with the set of social optima. However, if access to technologies is asymmetric across firms, so that some firms are able to earn positive profit, the market equilibrium is

¹ DS call it a “constrained” social optimum. For more details please see the end of Section II of this paper.

no longer socially optimal. In particular the welfare can be improved by lowering markup of all firms accompanied with the lump sum transfers from the more efficient to the less efficient firms.

The first alternative is in spirit of traditional definition of a “large group” Chamberlinian monopolistic competition, according to which all firms earn zero profit in equilibrium (see Edward Hastings Chamberlin, 1933). However, starting with Chamberlin (1933) economists are aware of the diversity of conditions surrounding each producer, which is responsible for asymmetry in the size and profitability across producers (e.g. managerial abilities, location, patents). This became the basis for modeling Ricardian heterogeneity with fixed gaps in the productivity and profit levels across firms.

The rest of the paper is organized in the following way: Section I contains the setup and some results of the DS (1977) model; Section II extends the DS model to the multiple technologies case, but allowing all firms to have equal access to existing technologies; Section III shows that if the heterogeneity is of the Ricardian nature, then the social optimum is strictly better than the market equilibrium; Section IV concludes.

I. The Original DS (1977) Single-Technology Model

We briefly recall the DS model. Suppose we are considering an economy in which there is a large number of possible products, some number n of which are produced. A representative consumer chooses a consumption plan $x \in \mathfrak{R}_+^{n+1}$ so as to maximize the utility function:

$$(1) \quad u = U \left(x_0, \left\{ \sum_i x_i^{(\sigma-1)/\sigma} \right\}^{\sigma/(\sigma-1)} \right) \quad \sigma > 1$$

where U is a homothetic, strictly quasi-concave, increasing function; σ is the constant elasticity of substitution inside the group of monopolistic goods $i = 1, \dots, n$; and commodity 0 is a numeraire good. Utility is maximized subject to the budget constraint

$$(2) \quad x_0 + \sum_{i=1}^n p_i x_i = I$$

where p_i are prices of goods being produced, and I is income in terms of the numeraire.

The authors apply a two stage budgeting procedure. In particular, they define dual price and quantity indices:

$$(3) \quad y = \left\{ \sum_{i=1}^n x_i^{(\sigma-1)/\sigma} \right\}^{\sigma/(\sigma-1)} \quad q = \left\{ \sum_{i=1}^n p_i^{-(\sigma-1)} \right\}^{-1/(\sigma-1)}$$

and in the first stage they solve for the optimal values of y and x_0 :

$$(4) \quad y = I \frac{s(q)}{q} \quad x_0 = I(1 - s(q))$$

where function s depends on the form of U . Turning to the second stage of the problem the authors show that for each i

$$(5) \quad x_i = y \left[\frac{q}{p_i} \right]^\sigma$$

In their original (1977) paper DS let the total cost of producing product i be $\alpha + cx_i$, where α is the fixed cost and c is the constant marginal cost. However, in their more recent work DS (1993) stated that their results hold for a more general cost function

$C(x_i)$. The minor requirement which the authors impose on $C(x_i)$ is the existence and

relevance of the second order conditions. Let us define $C(x_i)$ as:

$$(6) \quad C(0) > 0 \quad C'(x_i) > 0 \quad C''(x_i) \geq 0$$

where $C(0)$ is a fixed cost of an acting firm, and $C'(x_i)$ is its marginal cost. Then it is easy to show that since $C''(x_i) = 0$ guarantees the existence and relevance of the second-order conditions, $C''(x_i) \geq 0$ will guarantee it as well.

Each of the n products faces an identical demand curve $p(x_i)$, given that prices of other products remain fixed. In the market equilibrium each firm solves a profit maximizing problem, which, given the constant elasticity of substitution σ , results in the symmetric prices for all firms:

$$(7) \quad p_e = \frac{\sigma}{\sigma-1} C(x_e)$$

Knowing that all firms earn zero profits due to the free entry we can derive the supply per firm which turns out to be symmetric too:

$$(8) \quad x_e = \frac{C(0)}{C'(x_e)} (\sigma - 1)$$

Finally, given that the demand for each of the differentiated goods is more elastic than the demand for the differentiated goods as a group, DS (1977) prove the existence and uniqueness of the monopolistically competitive equilibrium.

While discussing the possible improvement over the market solutions, DS (1977) show that the market equilibrium compares unfavorably with the ‘unconstrained’ optimum. However, achieving the unconstrained optimum requires taking a lump sum

subsidy from the numeraire sector and using it to cover the fixed costs of the firms in the differentiated sector. As noted by DS “the conceptual and practical difficulties of doing so are clearly formidable”². That is why they concentrate on the ‘constrained’ optimum in which the lump sum subsidies are not available. This paper focuses exclusively on the constrained optimum, which for brevity I refer to as the social optimum.

II. Multiple-Technologies with Unrestricted Access

A. Market Equilibria

Now imagine that in the differentiated sector there exists a set of available technologies and all firms are free to choose any technology from this set. In particular, the firm i which has chosen to produce output x_i using technology t_j has the continuous cost function $C_j(x_i)$:

$$(9) \quad C_j(0) > 0 \quad C_j'(x_i) > 0 \quad C_j''(x_i) \geq 0$$

All other features of the DS (1977) model are preserved. Expanding the set of technologies is in accordance with the definition³ of a “large group” Chamberlinian monopolistic competition: there is a free entry condition, and any firm is ‘atomic’ relatively to the size of industry.

Obviously the profit maximizing firms always choose the technologies which guarantee them the highest profit. Let us define the set of such technologies, as a set of

² DS (1977), p.300

³ The concise definition is given among others by Paul R. Krugman (1980), Hart (1985), Jean Tirole (1988).

dominant technologies, T . Then the DS (1977) model can be viewed as a special case of this model with T being a singleton. The more general case, however, allows T to consist of any arbitrary number of technologies.

For each technology $t_j \in T$ the market price can be found from the firm's profit maximization problem, and the zero-profit condition allows us to find the firm's output:

$$(10) \quad p_{ej} = \frac{\sigma}{\sigma-1} C_j'(x_{ej}) \quad x_{ej} = \frac{C_j(x_{ej})}{C_j'(x_{ej})} \left(1 - \frac{1}{\sigma}\right)$$

Note, that the conditions imposed on the cost functions in (9) guarantee that

$$(11) \quad \partial \left(x_{ej} - \frac{C_j(x_{ej})}{C_j'(x_{ej})} \left(1 - \frac{1}{\sigma}\right) \right) / \partial x_{ej} > 0$$

Thus, the market price and zero-profit output are uniquely determined by (10) for all firms using the same technology. Combining (5) and (10) we can relate the cost functions to the corresponding outputs and prices of any two dominant technologies t_j

and t_k :

$$(12) \quad \frac{x_{ej}^{(\sigma-1)/\sigma}}{C_j(x_{ej})} = \frac{x_{ek}^{(\sigma-1)/\sigma}}{C_k(x_{ek})} \quad \frac{p_{ej}^{-(\sigma-1)}}{C_j(x_{ej})} = \frac{p_{ek}^{-(\sigma-1)}}{C_k(x_{ek})}$$

Since the firms using dominant technologies can at best earn zero profit, an attempt to use a non-dominant technology will ultimately result in earning negative profits. Thus if we compare any dominant technology, $t_j \in T$, to any non-dominant one,

$t_l \notin T$, we will get:

$$(13) \quad \frac{x_{ej}^{(\sigma-1)/\sigma}}{C_j(x_{ej})} > \frac{x_{el}^{(\sigma-1)/\sigma}}{C_l(x_{el})} \quad \frac{p_{ej}^{-(\sigma-1)}}{C_j(x_{ej})} > \frac{p_{el}^{-(\sigma-1)}}{C_l(x_{el})}$$

From (12) and (13) we can see that the firms' potential profits can be easily ranked by looking at their profit-maximizing output levels and average costs. In particular, the smaller is the product $x_{ej}^{1/\sigma} * (C_j(x_{ej})/x_{ej})$, the higher is the profit.⁴ This might be helpful for the graphical representation of the multiple-technology world modeled in this section.

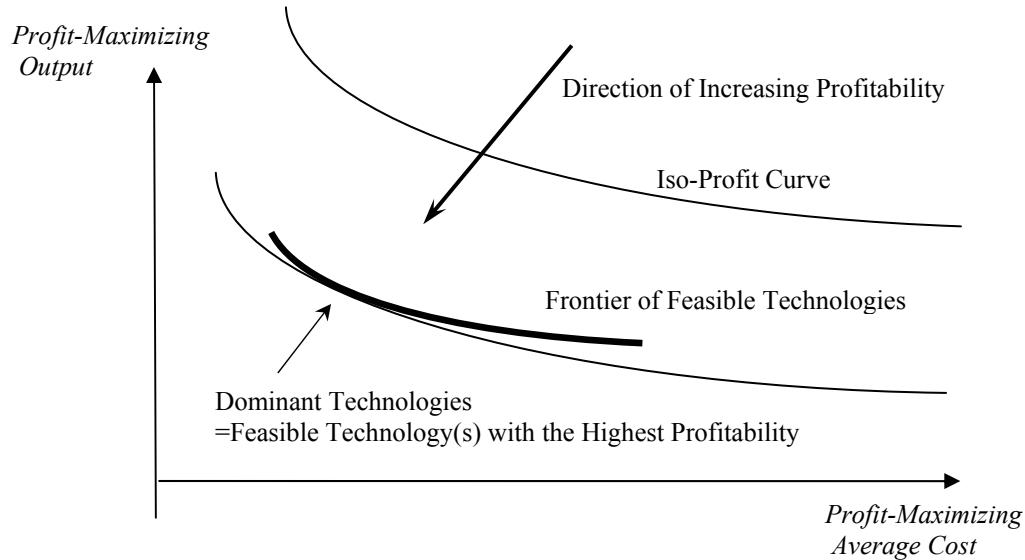


Figure 1. Multiple Technologies with Unrestricted Access.

From (11) we know that profit maximizing output is unique for each technology, and thus every technology has a corresponding unique point on this graph. However, the opposite statement is not true, since nothing restricts different technologies from having the same profit maximizing output and average cost. The closer to origin iso-profit curves represent technologies with higher potential profitability. The bold line represents the frontier of feasible technologies. I do not make any assumptions about its shape, and thus it may have an arbitrary number of common points with the best achievable iso-profit curve. These points represent the dominant technologies.

⁴ Note that "efficiency versus diversity" was put as a central issue of optimality of product diversification by Chamberlin (1950, p.89). At the same time when a profit-maximizing firm chooses a technology it faces a similar dilemma, since the average cost can be interpreted as a measure of efficiency and firm's size as an indirect measure of diversity.

The next question is whether the market equilibrium is unique. Since an active firm expects to earn zero profit by choosing any dominant technology, we cannot predict which one it will choose. Then if T includes more than one technology there exists a plethora of market equilibria, which I define as E . It is possible to show that all of these equilibria are welfare-equivalent.

Let us start the proof of this statement by choosing an arbitrary market equilibrium $e \in E$. We know that only dominant technologies are used in e , and thus there exists a technology $t_k \in T$ which is used in e by N_k firms, where $N_k > 0$. Imagine that we will close all these firms and reallocate the released resources to open the new firms using $t_j \in T$ such that each of the firms has price and quantity defined by (10). The number of new firms, N_j^{new} , can then be calculated as:

$$(14) \quad N_j^{new} = \frac{N_k C_k(x_{ek})}{C_j(x_{ej})}$$

Next let us calculate the updated price and quantity indices:

$$(15) \quad \begin{aligned} y' &= \left\{ \sum_{i \in T} N_i x_{ei}^{(\sigma-1)/\sigma} - N_k x_{ek}^{(\sigma-1)/\sigma} + N_j^{new} x_{ej}^{(\sigma-1)/\sigma} \right\}^{\frac{\sigma}{\sigma-1}} \\ q' &= \left\{ \sum_{i \in T} N_i p_{ei}^{-(\sigma-1)} - N_k p_{ek}^{-(\sigma-1)} + N_j^{new} p_{ej}^{-(\sigma-1)} \right\}^{-\frac{1}{\sigma-1}} \end{aligned}$$

By plugging (12) and (14) into (15) it is easy to show that the aggregate price and quantity indices (and thus the utility level) do not change due to such reallocation:

$$(16) \quad \begin{aligned} y' &= \left\{ \sum_{i=1} N_i (x_e)_i^{(\sigma-1)/\sigma} \right\}^{\frac{\sigma}{\sigma-1}} = y \\ q' &= \left\{ \sum_{i=1} N_i (p_e)_i^{-(\sigma-1)} \right\}^{-\frac{1}{\sigma-1}} = q \end{aligned}$$

By repeating this step with all other technologies used in e , we can replace all technologies used in e for t_j without affecting the utility level. Since all new firms which use $t_j \in T$ have market price and output levels, such replacement will result in the market equilibrium $e_j(t_j) \in E$ in which only technology t_j is used. This equilibrium will have the same utility level as e , and thus e and $e_j(t_j)$ are welfare-equivalent. Given that e was chosen arbitrarily, by the transitivity property, all equilibria in E are welfare-equivalent. Moreover, since t_j was chosen randomly, we can claim that if t_j is a dominant technology, there exists a market equilibrium $e_j(t_j) \in E$ in which only this technology is used.

B. Social Optima

Extending the definition of the constrained optimum given by DS (1977), I define a social optimum as a set of technologies, goods, individual prices and quantities which maximize utility satisfying the demand function and keeping the profit of each firm nonnegative. In the presence of multiple technologies we cannot guarantee the uniqueness of the social optimum, and thus I define O to be a set of social optima, where all elements of O provide the same utility level.

We will start the description of O by analyzing the properties of an arbitrarily chosen element of O , $o \in O$. First let us show that the individual price and output level (denoted as p_{oj} and x_{oj}) are uniquely defined for every technology t_j used in o . To do it, let us solve the following maximization problem:

$$(17) \quad \max_{x_{oj}} \frac{x_{oj}^{\frac{\sigma-1}{\sigma}}}{C_j(x_{oj})}$$

The first-order condition holds at:

$$(18) \quad x_{oj} = \frac{C_j(x_{oj})}{C'_j(x_{oj})} \left(1 - \frac{1}{\sigma}\right)$$

and from (11) we know that it is a maximum and that it is unique. From DS (1977) we know that all firms are earning zero profit in the social optimum⁵, which allows us to find the corresponding price:

$$(19) \quad p_{oj} = C'_j(x_{oj}) \frac{\sigma}{1-\sigma}$$

Now assume that N'_j firms using technology t_j chose their output levels and prices to satisfy the demand and to earn nonnegative profits, but to be different from those defined by (18) and (19). I claim that it is impossible in any $o \in O$. To see it we should close all these firms and reallocate the released resources to the new firms using t_j and operating according to (18) and (19). Given that (18) uniquely maximizes (17), it is possible to show that such reallocation will increase the quantity index and decrease the price index, both of which will increase the utility level. Consequently, since it is impossible to increase the utility level of the social optimum, all firms operating in o and using t_j have prices and output levels as defined by (18) and (19).

By comparing (18) and (19) to the corresponding values in (10) we can see that the firms using the same technology will have the same output levels and prices both in the market equilibrium and in the social optimum. This fact helps us to make the

⁵ The proof is given in DS (1975) working paper. It heavily relies on the results of Dixit (1975), and it is robust to the introduction of heterogeneity across firms.

following statements. First, using (13) it is easy to show that replacement of the non-dominant technology by the dominant one in the above described fashion will be always welfare improving, and thus only dominant technologies can be used in the social optimum. Second, by using the same set of arguments as in the '*Market Equilibria*' subsection, it is easy to show that if t_j is a dominant technology, then there exists a social optimum $o_j(t_j) \in O$ which is achieved by using only this technology.

From the previous subsection we are aware of existence of the corresponding market equilibrium $e_j(t_j) \in E$. And from the single-technology model analyzed by DS (1977, 1993) we know that the social optimum $o_j(t_j)$ will coincide with the market equilibrium $e_j(t_j)$. Moreover, since all market equilibria in E are welfare-equivalent, as well as are all optima in O , by transitivity all market equilibria are welfare equivalent to all social optima.

III. Ricardian Heterogeneity

Now imagine that firms have asymmetric access to the set of available technologies, so that in the market equilibrium there are Ricardian gaps in the profit levels across firms. The free entry condition guarantees that the marginal firm(s) will make exactly zero profit, but I assume that there will be at least one intramarginal firm which will earn strictly positive profit. A graphical representation of this model is shown in Figure 2.

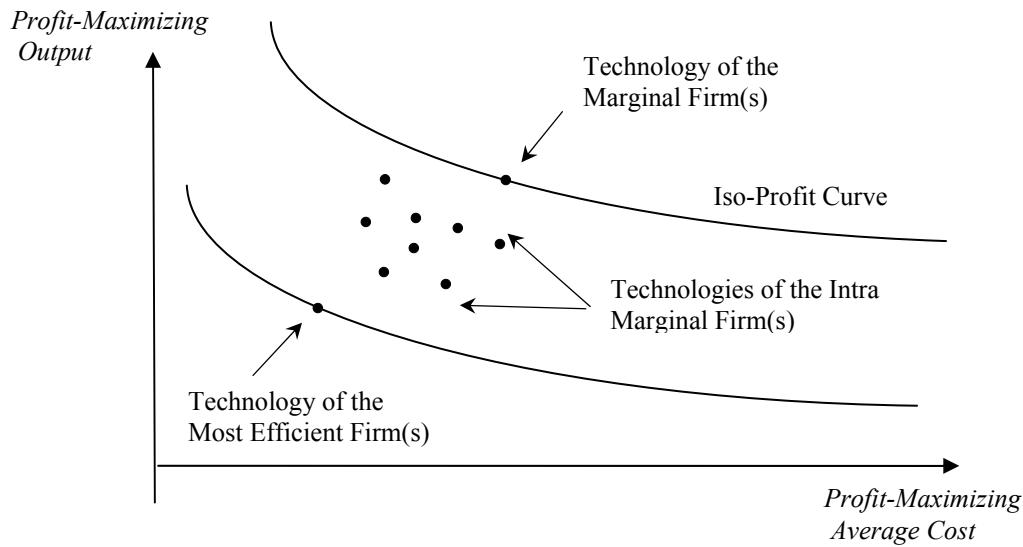


Figure 2. Ricardian Heterogeneity.

Comparison of Figure 1 and Figure 2 clearly demonstrates the main difference between the corresponding models. Under Ricardian heterogeneity each technology can be used only by a limited number of firms, and thus not every firm can choose the best technologies as in the ‘unrestricted access’ case. It turns out that this difference in the nature of firms’ heterogeneity is crucial for the optimality result of the model. In particular, under Ricardian heterogeneity the market equilibrium is not optimal.

To prove this statement I will use the fact that all models presented in this paper can be considered as special cases of the model used by Dixit (1975). Another common feature of these models is that in the market equilibrium all firms charge the same markup over the marginal cost, and all active firms earn non negative profits. What makes the ‘Ricardian Heterogeneity’ model different is that at least one firm earns strictly positive profit. This allows us to use the Theorem 1 of Dixit (1975) according to which we can improve the utility level achieved in the market equilibrium by the following regulation: i) slightly lower all prices towards the corresponding marginal costs in the

same proportion; ii) apply the lump sum tax to the remaining profits; iii) use the collected taxes to pay the lump sum subsidies to the firms, which earn negative profits due to the lowering of their prices.

In the original DS (1977) model and in the multiple-technology model presented in Section II such a policy was impossible, since all firms were making exactly zero profit in the market equilibrium. Thus lowering their prices would be inconsistent with the nonnegative profitability, since all firms would be incurring losses, and there would be no sources for subsidies. However in the model with Ricardian gaps in profit levels, the subsidies to the less efficient firms, which earn negative profits due to the price cut, can be financed using the profits of the more efficient firms, which still make positive profits even after the price cut. Certainly, the price cut should be small enough, so that the amount of total subsidies does not exceed the amount of total profits. However it is not a problem, since we can always choose the change in the price as small as we need to satisfy this condition.

Consequently, since the market equilibrium can be improved, it is no longer optimal under Ricardian technological heterogeneity.

IV. Conclusions and Possible Extensions

While most of the research associated with the Dixit-Stiglitz (1977) model of monopolistic competition assumes a single technology, empirical work demonstrates that technology differs significantly across the firms. In this paper I explore how allowing for multiple technologies affects the central result of the Dixit-Stiglitz framework – the

equivalence of the monopolistically competitive equilibrium to the social optimum. I show that the answer crucially depends on the nature of the technological heterogeneity.

In particular if all firms can choose from any of the existing technologies, the market equilibrium is still optimal even if multiple technologies are available. Moreover if the set of dominant technologies contains more than one technology, there exists a plethora of monopolistically competitive equilibria, all of which are equivalent among themselves and to the social optimum in terms of levels of utility, aggregate outputs and aggregate prices. In this case the optimal number of varieties is not necessarily unique, as it is in the original model, and different degrees of products differentiation might be optimal for the same economy.

The unrestricted access to all technologies is not only a sufficient, but also a necessary condition for the optimality of the market equilibrium if we allow for the multiple technologies in the DS (1977) model. Thus, the monopolistically competitive equilibrium is not optimal if the heterogeneity across firms is introduced in the Ricardian firm-specific fashion, such that some firms earn positive profits.

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