The Alliance Formation Puzzle and Capacity Constraints*

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Abstract

The formation of an alliance in conflict situations is known to suffer from a collective action problem and from the potential of internal conflict. We show that budget constraints of an intermediate size can overcome this strong disadvantage and explain the formation of alliances.

Keywords: alliance formation puzzle, military conflict, budget constraints

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1 Introduction

The formation of an alliance constitutes a puzzle. Alliances are very common in many applications that constitute contests or tournaments, including military conflict, R&D tournaments, lobbying and political campaigning. But, as shown by Esteban and Sákovics (2003), alliance formation involves severe strategic disadvantages that make them undesirable under a wide range of circumstances if the members of the alliance can join and coordinate their efforts when fighting against an external enemy, but behave non-cooperatively vis-a-vis each other.[1] First, the alliance as a group faces a collective action problem. Second, once an alliance wins the conflict against a joint enemy,

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[1] Other analyses of alliance formation focussing on different aspects are Skaperdas (1998), Garfinkel (2004) and Kovenock and Roberson (2007).

peace inside the alliance may end and the members of the alliance may turn against each other.[2] The anticipated cost of this internal conflict also makes alliance formation less attractive. We show that capacity constraints on effort can make such a non-cooperative alliance between equal alliance members profitable for its members, compared to the grand contest in which all players compete simultaneously in one single stage.

2 The alliance formation puzzle

Consider a set of three players $N = \{A, B, C\}$ who contest for a prize that is valued equally by all three players, and normalize this value to 1. The players B and C may form an alliance against player A.[3] Once the alliance has formed, two groups $\{A\}$ and $\{B, C\}$ contest with each other in an all-pay auction without noise as characterized by Hillman and Riley (1989) and Baye, Kovenock and deVries (1996). Let the effort choices of A, B and C in this inter-group contest (stage 1) be $x_A \in [0, m_A]$, $x_B \in [0, m_B]$ and $x_C \in [0, m_C]$, where m_A, m_B and m_C describe the maximum effort that can be mobilized at this stage, due to some capacity constraints. Assume further that the efforts of alliance members add up to the effort $x_B + x_C$ of the alliance. The probability that A wins is $p_{\{A\}} = 1$ if $x_A > x_B + x_C$, $p_{\{A\}} = 0$ if $x_A < x_B + x_C$, and $p_{\{A\}} = 1/2$ if $x_A = x_B + x_C$, and $p_{\{B,C\}} = 1 - p_{\{A\}}$.

The game ends if player A wins this contest and receives the prize. If the alliance $\{B, C\}$ wins, the two alliance members need to determine how

[2] Katz and Tokatlidu (1996) and Wärneryd (1998) address the problem of internal conflict that may emerge inside an alliance that is victorious in the contest with another group, and analyse this nested structure in the context of a Tullock contest. To illustrate the structure with an example, consider the alliance between the USA and Russia during World War II. The alliance was seemingly useful for the Allied Forces in their goal of defeating Nazi Germany and its coalition. However, when the end of the war came closer, a divergence of interests among the Allied Forces became visible. It was not difficult to anticipate that the alliance would break up after the defeat of Germany and that a conflict between the two large victorious countries would emerge. Rational players may therefore have anticipated what is now known as the Cold War. From this perspective, the conferences that took place prior to the termination of the war (e.g., the Moskow Conference, October 1944, and the Yalta Conference, February 1945) can be seen as attempts to reach an agreement regarding the division of the gains of winning the war. Part of the agreement, for instance, was the division of the defeated region into spheres of influence. As is well-known now, these attempts have not been very successful. The agreements were not time consistent. They did not solve the fundamental conflict and competition about the future social and economic world order and political dominance.

[3] Alliance formation is typically a voluntary process. Describing this endogeneity is a different and difficult problem.

the prize is shared between them. Due to the lack of enforceable contracts between B and C the two allies will themselves enter into a distributional conflict (stage 2). They simultaneously choose conflict effort $y_B \in [0, m_B]$ and $y_C \in [0, m_C]$, or random distributions described by cumulative distribution functions G_B and G_C on these intervals, and win probabilities are $p_B = 1$ if $y_B > y_C$, $p_B = 0$ if $y_B < y_C$, and $p_B = 1/2$ if $y_B = y_C$. Each player's payoff is equal to his probability of winning the prize, minus the player's own cost of contest effort(s), i.e.,

$$\begin{aligned} \pi_A &= p_{\{A\}} - x_A \\ \pi_B &= (1 - p_{\{A\}}) p_B - x_B - y_B \\ \pi_C &= (1 - p_{\{A\}}) p_C - x_C - y_C. \end{aligned} \tag{1}$$

We examine an extended game in which effort choices in stage 1 also occur simultaneously. $\{A\}$ chooses x_A or a mixed strategy described by a cumulative distribution function $F_{\{A\}}$. The members of the alliance may correlate their choices by observing the draw z from a random variable that has a cumulative distribution function[4]

$$F_Z(z) = \begin{cases} z & \text{for} \quad z \in [0, m_A) \\ 1 & \text{for} \quad z \geq m_A \end{cases} \tag{2}$$

They choose their efforts x_B and x_C independently from each other and may make their choices dependent on z. Instead of pure strategies $x_i(z) \in [0, m_i]$ players $i \in \{B, C\}$ may also choose cumulative distribution functions F_i as functions of z. When choosing their efforts, B and C individually maximize their individual payoffs, respectively.

Proposition 1 *Let B and C form an alliance. Let $m_B = m_C \equiv m \in (\frac{m_A}{2}, \frac{1}{2} - \frac{m_A}{2})$. Then a subgame perfect equilibrium of the extended game exists with $\pi_A = 0$, $\pi_B + \pi_C = 1 - m_A - 2m$.*

Proof. Consider stage 2. If $\{A\}$ was the winner of the inter-group contest, A receives the full prize, without further fighting. If $\{B, C\}$ wins the prize in stage 1, B and C fight over the distribution of the prize in an all-pay auction. Their effort choices are y_B and y_C, with $y_i \in [0, m]$ for $i \in \{B, C\}$. The solution of this contest depends on the size of m. Since $m \leq \frac{1}{2} - \frac{m_A}{2} < 1/2$, both B and C expend the maximum effort in the unique equilibrium of this

[4] See Harris, Reny, and Robson (1995) who introduce the concept of an extended game with public randomization to restore existence of equilibrium in continuous games with almost perfect information. In our game only the members of the alliance observe the realization of z.

subgame, each wins with a probability of 1/2, and each has a payoff from participating in this subgame of $v \equiv \frac{1}{2} - m > 0$.

Turning to stage 1, consider the following effort choices as a candidate equilibrium: Let $x_B(z) = x_C(z) = \frac{z}{2}$. Further, let player A choose a mixed strategy

$$F_{\{A\}}(x_A) = \begin{cases} \frac{v-m_A}{v} + \frac{x_A}{v} & \text{for} \quad x_A \in [0, m_A] \\ 1 & \text{for} \quad x_A > m_A. \end{cases} \tag{3}$$

We show that, given the equilibrium play in the subgame in stage 2, these effort choices constitute optimal replies vis-a-vis each other.

Consider first A. This player anticipates that the joint effort $x = x_B + x_C$ of B and C is distributed according to a cumulative distribution function

$$F_{\{B,C\}}(x) = \begin{cases} x & \text{for} \quad x \in (0, m_A) \\ 1 & \text{for} \quad x \geq m_A. \end{cases} \tag{4}$$

Hence, his payoff from choosing x_A is $x_A \cdot 1 - x_A = 0$ for $x_A \in [0, m_A)$, and $\frac{m_A}{1} \cdot \frac{1}{2} - m_A \leq 0$ for $x_A = m_A$. This makes A indifferent as regards the choice of $x_A \in [0, m_A)$, and makes $F_A(x_A)$ in (3) an optimal reply.

Consider now B. This player attributes the value $v = \frac{1}{2} - m$ to a victory of the alliance in stage 1. Observing z and taking the probability $F_A(x)$ that $x_A \leq x$ in (3) and $x_C = z/2$ as given, B maximizes

$$\pi_B(x_B; x_C = \frac{z}{2}) = \begin{cases} (\frac{v-m_A}{v} + \frac{x_B + z/2}{v})v - x_B & \text{for} \quad x_B \in [0, m_A - \frac{z}{2}] \\ v - x_B & \text{for} \quad x_B \in [m_A - \frac{z}{2}, m_B]. \end{cases} \tag{5}$$

As $(\frac{v-m_A}{v} + \frac{x_B+z/2}{v})v - x_B = v - m_A + \frac{z}{2}$, any $x_B \in [0, m_A - \frac{z}{2}]$ is maximizing this $\pi_B(x_B)$; this makes $x_B(z) = z/2$ an optimal reply. The proof for why $x_C(z) = z/2$ is an optimal reply for C is fully analogous. Moreover, if $x_B(z) = x_C(z) = z/2$, this together with F_Z generates (4). ■

The contest problem combines the all-pay auction problem with different budget limits as in Che and Gale (1997) with a collective action problem in stage 1 and a potential all-pay auction in stage 2.[5] In the equilibrium the members of the alliance are able to overcome the collective action problem in stage 1, using perfectly correlated strategies. The solution requires a randomization device that is observable for B and C, but not for A. For simplicity, we assumed that this device is exogenously given to the alliance.

[5] A similar equilibrium with correlated efforts, but without budget constraits is analysed in Konrad and Leininger (2007). However, they use multiplicity of subgames in later stages and the threat of deviating to an inferior equilibrium to enforce high collective efforts. In our context, the equilibrium efforts are self-enforced without such threats.

Our result constrasts with the findings in Esteban and Sákovics (2003). They consider a structurally very similar nested conflict, with a Tullock contest success function, instead of perfect discrimination without noise. Also, they allow for convex costs of effort for each player, and their players are not capacity constrained. They find that the payoffs of the members of the alliance are lower and the payoff of the single player is higher than if all three players compete in a single-stage Tullock contest. We find the opposite. If A, B and C contest in a single-stage all-pay auction without noise and have capacity constraints $m_A > m_B = m_C \equiv m$, an equilibrium has $\pi_A = m_A - m > 0$, and $\pi_B = \pi_C = 0$. This shows that alliance formation can benefit the players who form the alliance and harm the player who is not a member of the alliance.

The specific choice of budget limits in this example is not arbitrary. First, consider the role of $m \in (\frac{m_A}{2}, \frac{1}{2} - \frac{m_A}{2})$ for this result. If $m < \frac{m_A}{2}$, then, by a choice of $x_A = 2m + \epsilon$, player A can guarantee himself a payoff that is arbitrarily close to $1-2m$, and of $1-m$ in a game without alliance formation. Second, A is given a larger budget than B and C, so that alliance formation allows for higher joint efforts by B and C than A's maximum effort. To overbid A's maximum effort was infeasible if players B and C stayed alone. Given $m_A > m_B = m_C = m$, the equilibrium outcome of a three-player "playing the field" all-pay auction without noise led to equilibrium payoffs $\pi_A = 1 - m$, and $\pi_B = \pi_C = 0$.

Third, $m_B = m_C$ has been chosen in order to avoid giving B or C a strict advantage in the intra-group conflict in stage 2. If $m_B > m_C$, then B and C face a more difficult collective action problem, as B can outbid C in the intra-alliance contest in stage 2. In turn, this causes $v_B = 1 - m_C$ and $v_C = 0$ and this would prevent C from contributing in the inter-group contest. This problem regarding $m_B \neq m_C$ should not be taken too literally. First, if one assumes a contest success function with noise in stage 2, a small difference in budgets does not change the results qualitatively. Second, if B's and C's budgets are not certain, but uncertain and drawn from a distribution at the beginning of stage 2, then the strong hold-up that is generated by $m_B \neq m_C$ also disappears. Third, and more strictly within our framework with given and known m_B and m_C and an all-pay auction without noise in stage 2, if B and C correctly anticipate the incentive problems this implies, they might be able to make a transfer of capacity between them in stage 1 and have an option to influence the effort limits they have available in both the inter-group conflict and in the intra-alliance conflict, so that B and C can escape from the hold-up from $m_B \neq m_C$ in this way, again making the the outcome underlying Proposition 1 an equilibrium.

3 Conclusions

If players would like to expend more than their budget in an all-pay auction, the player who has a higher budget than his competitor can make a bid that is slightly higher than his competitor's budget, and guarantee a victory. An alliance may increase the range of possible effort, compared to players' options in a stand alone situation. The formation of an alliance may allow the group to outbid budget constrained competitors. This is an advantage from joining forces. On the other hand, in the literature alliances have been seen as being very detrimental to its members because of the later conflict between these members. With sufficiently low budget or capacity constraints, however, the strategic disadvantage is more than outweighed by the increase in competitive strength of the alliance. Capacity constraints also allow the alliance to overcome the collective action problem. Hence, the beneficial budget enlargement effect of forming an alliance may outweigh the potentially harmful strategic effects. For this to happen, the alliance needs to be sufficiently strong, but not too strong.

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