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ELECTORAL POACHING AND PARTY IDENTIFICATION

by

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Electoral Poaching and Party Identification*

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Abstract

This paper studies electoral competition in a model of redistributive politics with deterministic voting and heterogeneous voter loyalties to political parties. We construct a natural measure of "party strength" based on the sizes and intensities of a party's loyal voter segments and demonstrate how party behavior varies with the two parties' strengths. In equilibrium, parties target or "poach" a strict subset of the opposition party's loyal voters: offering those voters a high expected transfer, while "freezing out" the remainder with a zero transfer. The size of the subset of opposition voters frozen out and, consequently, the level of inequality in utilities generated by a party's equilibrium redistribution schedule is increasing in the opposition party's strength. We also construct a measure of "political polarization" that is increasing in the sum and symmetry of the parties' strengths, and find that the expected ex-post inequality in utilities of the implemented policy is increasing in political polarization.

1 Introduction

In the model of redistributive politics, political parties compete for representation in a legislature by simultaneously announcing binding commitments as to how they will allocate a budget across voters. Each voter votes for the party offering the highest level of utility, and each party's payoff is its representation in the legislature, which under proportional representation is equal to the fraction of votes received by that party. Originally formulated by Myerson (1993), the model has served as a fundamental tool in the analysis of electoral competition. In recent years, the model has attracted renewed interest through its application to the study of the inequality created by political competition (Laslier (2002), Laslier and Picard (2002)), incentives for generating budget deficits (Lizzeri (2002)), inefficiency of public good provision (Lizzeri and Persico (2001,2002)), and campaign spending regulation (Sahuguet and Persico (2006)).

This paper extends the model of redistributive politics to allow for heterogeneous voter loyalties to political parties and shows that this has important implications for the nature of redistributive competition. Voters are distinguished by the party with which they identify, if any, and the intensity of their attachment, or "loyalty," to that party. We assume that parties are able to perfectly discriminate across voters by their party affiliation and the intensity of their attachment (including the set of "swing voters" who have no attachment to either party). Parties compete by simultaneously announcing offer distributions to each of the identified voter segments. When integrated over all segments, each party's offer distributions must satisfy a common aggregate budget constraint.[1] As in Myerson, each voter is assumed to vote (sincerely) for the party that offers the higher level of utility which, in our model, reflects both the transfer offered and the voter's loyalty.

We completely characterize the unique Nash equilibrium of this model and, explore its

[1]As in Myerson (1993) each offer distribution is a probability distribution over the nonnegative real numbers with the measure over each interval interpreted as the fraction of the particular loyal voter segment for whom the party's transfer has value in that interval. Since we assume a continuum of voters in each segment and offers that are independent across voters (each voter takes an independent draw from the offer distribution) we may appeal to Judd (1985) in assuming that the aggregate budget constraint holds with probability one and not just in expectation.

qualitative nature.[2] In equilibrium, within any given voter segment, the expected transfers from the two parties' offer distributions are identical. However, we find that voters pay a price for being loyal to a party. For a given distribution of voters' attachments to political parties, the expected transfer that voters receive is strictly decreasing in the voters' intensity of attachment (regardless of party affiliation). This monotonicity of transfers also translates into a monotonicity of utility. Although the expected utility provided by a party's redistribution schedule is identical for all of its loyal voter segments and equal to the expected utility that the swing voters receive from each party's redistribution schedule, the expected utility that a party's loyal voters receive from the opposition party's redistribution schedule is decreasing in the voters' level of attachment.

Moreover, we find that the parties have an incentive to target or "poach" a subset of the opposition party's loyal voters, in an effort to induce those voters to vote against their party. By "poaching" we mean a strategy of targeting each segment of the opposition party's loyal voters with a redistribution schedule that "freezes out" a portion of the segment with a zero transfer, but gives the remaining voters in the segment non-zero transfers which are higher in expectation than the opposition party's offers to the same segment. This captures the notion that a party may try to selectively induce a strict subset of the opposition's loyal voters to defect by offering them a higher transfer.

To facilitate our analysis we also construct a natural measure of "party strength" based on both the sizes and intensities of a party's loyal voter segments and show how party behavior varies with the two parties' strengths. We demonstrate that each party's vote share is increasing (decreasing) in its own (opponent's) party strength. We also find that as the opposition party's strength increases, a party's equilibrium redistribution schedule freezes out a larger set of the opposition's loyal voters and gives a higher expected transfer to those not frozen out. The party's own loyal voter segments also receive a higher expected transfer. Although it is not obvious from these effects, the level of inequality in utilities (as measured by the Gini-coefficient) from a party's equilibrium redistribution schedule is also increasing in the opposition party's strength.

As is common in models of electoral competition, the policy implemented by the legislature is assumed to be a probabilistic compromise of the parties' equilibrium redistribution schedules.[3]

[2] As formulated, our game is constant sum, so the parties' equilibrium strategies are maximin strategies.

[3] If, alternatively, the policy implemented is that of the party receiving a majority (measure) of votes, our

The probability that a party's schedule is adopted is proportional to the size of its legislative contingent.[4] From the characterization of equilibrium described above, it immediately follows that for a given distribution of voters' attachments to the political parties, the equilibrium expected transfers and resulting expected utilities from the implemented policy are highest for swing voters and strictly decreasing in the intensity of attachment.[5] Moreover, defining the "level of partisanship" as the sum of the parties' strengths, we find that partisanship preserving transformations of the electorate that increase the strength of party i at the expense of party $-i$ result in party i's loyal voters receiving higher expected utilities and party $-i$'s loyal voters receiving lower expected utilities from the implemented policy.

We also develop a measure of "political polarization" that is increasing in the sum and symmetry of the parties' strengths and show that the expected ex-post inequality in utilities (as measured by the expected Gini-coefficient) under the implemented policy is increasing in political polarization. In particular, partisanship preserving transformations of the electorate that decrease the difference in the parties' strengths increase the expected ex-post inequality in utilities of the implemented policy. Hence, for a given level of partisanship, the expected ex-post inequality in utilities is maximized when the parties are of equal strength. In addition, holding constant the difference in the parties' strengths, the expected ex-post inequality in utilities increases as the level of partisanship increases. That is, higher levels of partisanship and more symmetry in the parties' strengths generate inequality.

One related paper is Laslier (2002). Laslier (2002) examines the issue of tyranny of the majority[6] in a model of redistributive politics with a segmented homogeneous electorate and

results demonstrate that the party with the greater "strength" implements its redistribution schedule with certainty.

[4]This interpretation is due to Grossman and Helpman (1996). Probabilistic compromise can also be viewed as a system under which each party distributes a fraction of the budget, proportional to its representation in the legislature, according to its announced schedule. This approach is taken in Myerson (1993).

[5]For expected transfers this result holds regardless of party affiliation; for expected utilities it holds within each party.

[6]Tocqueville describes tyranny of the majority as follows,

> "For what is a majority taken collectively if not an individual with opinions and, more often than not, interests contrary to those of another individual known as the minority. Now, if you are willing to concede that a man to whom omnipotence has been granted can abuse it to the detriment of his adversaries, why will you not concede that the same may be true of a majority?" (pp. 288-289)

intra-segment homogeneity in a party's offers. That is, within each voter segment, a party's offer distribution is assumed to be degenerate with all mass on the fixed offer for that segment (although offers may vary across segments). In this context, Laslier finds that there is no tyranny of the majority as long as there does not exist a segment that contains over half of the voters. However, if any segment contains over half of the voters, each party uses its entire budget on that segment, thereby freezing out the remaining voters.

Our model extends the Laslier model in two ways. First, we allow for a heterogeneous electorate, partitioned into distinct segments of homogeneous voters. Second, we allow for intra-segment heterogeneity in a party's offers, as represented by the (general, non-decreasing) segment specific offer distributions. Since our model assumes that the implemented policy is a probabilistic compromise of the parties' redistribution schedules, a natural analogue of "tyranny of the majority" is the degree to which the implemented policy tyrannizes a minority by driving them down to their reservation utility level. In our model this arises when the implemented policy freezes out voters by giving them an ex-post transfer of zero.[7] Indeed, under our assumption that the probability that a party's schedule is adopted is equal to the size of its vote share, the expected measure of the set of voters receiving a transfer of zero under the implemented policy is proportional to our measure of polarization. That is, *polarization leads to tyranny*.[8]

A related literature follows from Cox and McCubbins (1986), Lindbeck and Weibull (1987), and Dixit and Londregan (1995, 1996). Dixit and Londregan (1996, henceforth D-L) is perhaps

[7] An alternative interpretation of tyranny of the majority refers to an outcome in which a majority receives a higher utility than some designated minority. In the equilibrium in our model, the expected utility, conditional on receiving a positive transfer in the implemented policy, is identical for all voters. Hence, among voters not frozen out there is a form of (conditional) equal treatment. However, within each party, the greater a voter's intensity of attachment, the lower his expected utility from the implemented policy. This arises because parties never freeze out their own loyal voters and the probability that a party's offer distribution freezes out an opposition voter is increasing in that voter's attachment.

[8] This formulation of tyranny would not apply to Myerson's interpretation of probabilistic compromise as a system under which each party distributes a fraction of the budget, proportional to its representation in the legislature, according to its announced schedule. Under this interpretation, no voters would be frozen out ex post in the implemented policy. However, under the implemented policy the unequal treatment (in utilities) of the more loyal voter segments within each party would continue to hold.

closest to our paper in focus. Both papers assume voters derive utility from redistribution and party identification.[9] Both assume a heterogeneous electorate, partitioned into distinct voter segments. While D-L assume a non-degenerate distribution of voter attachments within each segment (represented by a segment-specific density), in our model voters within a given segment are homogeneous, so that parties are able to directly target voters by party affiliation and intensity of attachment. Moreover, like Laslier, D-L assume intra-segment homogeneity in a party's offers. That is, within each voter segment, a party's offer distribution is assumed to be degenerate with all mass on the fixed offer for that segment. This, together with intra-segment heterogeneity of voters, precludes the ability to directly target voters by intensity of attachment or party affiliation. That is, the D-L model examines a type of third-degree transfer discrimination. In contrast, our model allows for perfect discrimination by party affiliation and intensity of attachment. In addition, parties in our model may (anonymously) offer discriminate across the identical voters within the given segment by employing a (general, non-decreasing) segment-specific offer distribution. Hence, our model features first-degree transfer discrimination.

In section 2 we present the model and characterize the unique Nash equilibrium of the game of redistributive politics with party identification. Section 3 explores the qualitative nature of the equilibrium and presents comparative statics results with respect to changes in measures of party strength, partisanship, and political polarization. Section 4 concludes.

[9] Formally, the D-L preference structure may be viewed as a reduced form for one in which voters' preferences over parties are represented by utilities that are additively separable in the two aggregate variables, party identification and the redistributive transfer. The former subutility, corresponding to party identification, is determined by the (fixed) ideological positions of the party. The difference in the subutilities generated by the two parties determines the intensity of attachment of the voter to one of the parties. Throughout much of the D-L analysis the subutility of the redistributive transfer is assumed to be isoelastic in the transfer. In contrast, our preference structure can be viewed as a reduced form for one in which voter utilities are multiplicatively separable in the same two aggregate variables, with the subutility of consumption also assumed to be isoelastic. Under these assumptions, all of our results on the properties of equilibrium transfers and vote shares, and in particular Theorem 1, hold. However, since the transformations of utility carried out to obtain our reduced form are generally not affine and vary across voter segments, some of our corollaries on the rankings of distributions of utility in the reduced form will generally fail to hold for other utilities representing the same ordinal voter preferences over the parties' offers.

2 The Model

Political Parties and the Legislature

Our model extends Myerson's (1993) two-party model of redistributive competition by including heterogeneous voter loyalties to political parties. Two parties, A and B, make simultaneous offers to each of a continuum of voters of unit measure. Each voter votes for the party offering the higher level of utility, and each party's payoff is its representation in the legislature, which under proportional representation is equal to the fraction of votes received by that party. All offers must be nonnegative and each party has a budget of 1, which corresponds to 1 unit of a homogeneous good per voter. Parties are assumed to have complete information regarding the party preferences of all voters. While this is a stylized assumption, this is not an unreasonable benchmark given the high level of organization of modern political parties.[10]

As is commonly assumed in the literature on electoral competition, the legislature implements a policy that is a probabilistic compromise of the parties' redistribution schedules. The policy that the legislature implements is a random variable which takes on party A's equilibrium redistribution schedule with probability equal to party A's equilibrium vote share and takes on party B's equilibrium redistribution schedule with probability equal to party B's equilibrium vote share.

> **Definition D.1:** The *implemented policy* is a random variable that takes on party A's equilibrium redistribution schedule with probability equal to party A's vote share and party B's equilibrium redistribution schedule with probability equal to party B's equilibrium vote share.

Voters

Voters are distinguished by the party with which they identify, if any, and the intensity of their attachment to that party. In this paper, we consider only distributions of voters' attachments to the political parties with support on a finite set of intensities of attachment. Let $\delta_i^j \in (0, 1)$ represent the number of units of the homogeneous good that party i must offer a loyal voter in

[10] See for example Dretzin (2004), an episode of the PBS television series Frontline, which discusses the high level of information that national political parties have access to and use to target voters.

its own loyal segment j in order to make that voter indifferent between the two parties when party $-i$ offers one unit of the homogeneous good.[11] Thus, the utility that each loyal voter in party i's segment j receives from an offer of x^A from party A is

$$u_i^j\left(x^A\right) = \begin{cases} x^A & if\ i = B \\ \frac{x^A}{\delta_A^j} & if\ i = A \end{cases}.$$

Define $\mathfrak{a}_i^j = 1 - \delta_i^j$ to be the intensity of attachment of party i's loyal voter segment j. Party A's loyal voters have a finite number, n_A, of different intensities of attachment. Let $\mathcal{A}$ be the set of all indices of intensity of attachment for voters loyal to party A. Each index of intensity $j \in \mathcal{A}$ corresponds to a segment of voters with intensity of attachment $\mathfrak{a}_A^j$ and measure $m_j > 0$. The *size of party* A is denoted by $M_A = \sum_{j \in \mathcal{A}} m_j$. Similarly, party B's loyal voters have a finite number, n_B, of different intensities of attachment. Let $\mathcal{B}$ be the set of all indices of intensity of attachment for voters loyal to party B, where $\mathcal{A}$ and $\mathcal{B}$ are disjoint sets. Each index of intensity $k \in \mathcal{B}$ corresponds to a segment of voters with intensity of attachment $\mathfrak{a}_B^k$ and measure $m_k > 0$. The *size of party* B is denoted by $M_B = \sum_{k \in \mathcal{B}} m_k$. There are also swing voters who do not identify with either party. Letting S be the index for no attachment to either party, the utility

[11] This type of effectiveness advantage originates, to the best of our knowledge, with Lein (1990) and is frequently used in the literature on unfair contests (see for instance: Clark and Riis (2000), Konrad (2002), and Sahuguet and Persico (2006)). As noted in footnote 9, the preferences over the two parties represented by this utility are identical to those arising in a model in which voters' preferences are represented by a multiplicatively separable utility function in two aggregate variables, party identification and the redistributive transfer, where the latter subutility is isoleastic in the transfer. In this case, suppose voter ν's utility when party A wins is $[K_\nu \frac{(x^A)^{1-\epsilon}}{1-\epsilon}]\Delta_\nu^A$ and when party B wins is $[K_\nu \frac{(x^B)^{1-\epsilon}}{1-\epsilon}]\Delta_\nu^B$, where $K_\nu > 0$, $0 < \epsilon < 1$, and $\Delta_\nu^A, \Delta_\nu^B > 0$. If $\Delta_\nu^A > \Delta_\nu^B$, then the voter is loyal to party A. Multiplying utility by $\frac{1-\epsilon}{K_\nu \Delta_\nu^B}$ and then raising it to the $\left(\frac{1}{1-\epsilon}\right) th$ power yields the utility that we employ. Note that, under this transformation, voter ν is in a segment j loyal to party A for which $\delta_A^j = \left(\frac{\Delta_\nu^B}{\Delta_\nu^A}\right)^{\frac{1}{1-\epsilon}}$. If for voter ν, $\Delta_\nu^B > \Delta_\nu^A$, an analogous transformation would show that voter ν is in a segment j loyal to party B for which $\delta_B^j = \left(\frac{\Delta_\nu^A}{\Delta_\nu^B}\right)^{\frac{1}{1-\epsilon}}$.

The fact that both of these utility functions represent the same preference over the two parties is sufficient to guarantee that all of our results on the nature of equilibrium transfers and vote shares, including Theorem 1, also hold for this multiplicatively separable utility function, as long as the set of Δ_ν^A and Δ_ν^B generate a finite number of equivalence classes corresponding to δ_A^j and δ_B^j. Of course, our corollaries on the rankings of distributions of utility will generally fail to hold, since the transformations of utility carried out above generally are both not affine and vary across voter segments (even if we took them to be identical within a specific voter segment).

that each swing voter receives from an offer of x^j from party j is

$$u_S\left(x^j\right) = \ x^j \quad for\ j = A, B\ .$$

The measure of swing voters is $m_S \equiv 1 - M_A - M_B \geq 0$. To summarize $\left\{\left\{m_j, \mathfrak{a}_A^j\right\}_{j\in\mathcal{A}}, \{m_k, \mathfrak{a}_B^k\}_{k\in\mathcal{B}}\right\}$ is a feasible distribution of voters' attachments to the political parties if n_A and n_B are finite, $M_A + M_B \leq 1$, and $m_j > 0$ for all $j \in \mathcal{A} \cup \mathcal{B}$. An example distribution of voters' attachments to the political parties is shown graphically in Figure 1.

[Insert Figure 1 here]

Each voter votes for the party that provides them the higher utility. Thus each swing voter votes for the party that makes them the higher transfer, while each loyal voter requires a proportionally higher transfer from the rival party in order to induce him to cross over. Representation in the legislature is allocated proportionally. Thus, we normalize each party's representation in the legislature to be equal to the fraction of the votes received by that party.

One simple yet important summary statistic of a party's distribution of loyal voters is the sum across segments of each segment's intensity of attachment weighted by the measure of the set of voters in that segment.

Definition D.2: The *strength of party A* is denoted by $\sigma_A \equiv \sum_{j\in\mathcal{A}} m_j \mathfrak{a}_A^j$. The *strength of party B* is denoted by $\sigma_B \equiv \sum_{k\in\mathcal{B}} m_k \mathfrak{a}_B^k$

Several properties of this summary statistic should be noted. First, holding constant the size of each of a party's loyal segments, the party's strength is strictly increasing in the intensity of the attachment of any of these segments. Second, holding constant the intensity of attachment of each of its loyal segments, the party's strength is strictly increasing in the size of each of these segments. Finally, holding constant a party's size, the party's strength is strictly increasing as loyal voters shift from weaker intensities of attachment to stronger intensities of attachment.

Given the parties' strengths, σ_A and σ_B, it is useful to derive two simple measures for the distribution of voters' attachments to the political parties.

Definition D.3: The *level of partisanship* is the sum of the parties' strengths, which is denoted by $\sigma \equiv \sigma_A + \sigma_B$.

Definition D.4: The *effective strength of party i* is denoted by $\hat{\sigma}_i \equiv \sigma_i - \sigma_{-i}$.

The level of partisanship is the sum across the entire electorate of each segment's intensity of attachment, to either party, weighted by the measure of the set of voters in that segment. The properties of the level of partisanship are similar to those of the parties' strengths. Holding constant the size of each loyal segment, the level of partisanship is strictly increasing in the intensity of attachment of each segment. In addition, the level of partisanship is strictly increasing as loyal voters shift from weaker intensities of attachment to stronger intensities of attachment or as swing voters become affiliated with a political party. The effective strength of party i measures the asymmetry between party i and party $-i$. If the parties have symmetric strengths then each party has an effective strength of 0.

Redistributive Competition

A strategy, which we label a *redistributive schedule* (or *offer distribution*), for party i is a set of cumulative distribution functions,[12] $\left\{F_i^j\right\}_{j\in\mathcal{A}\cup S\cup\mathcal{B}}$, one distribution function for each segment $j\in\mathcal{A}$ of voters loyal to party A, the segment of swing voters S, and each segment $k\in\mathcal{B}$ of voters loyal to party B. As in Myerson (1993) each $F_i^j(x)$ denotes the fraction of voters in segment j whom party i will offer a transfer less than or equal to x. The only restrictions that are placed on the set of feasible strategies is that each offer must be nonnegative and the set of cumulative distribution functions must satisfy the budget constraint:

$$\sum_{j\in\mathcal{A}\cup S\cup\mathcal{B}} m_j\int_0^\infty x dF_i^j\leq 1. \tag{1}$$

Redistributive competition is the one-shot game, which we label

$$G\left(\left\{\left\{m_j,\mathfrak{a}_A^j\right\}_{j\in\mathcal{A}},\left\{m_k,\mathfrak{a}_B^k\right\}_{k\in\mathcal{B}}\right\}\right),$$

in which parties attempt to maximize their representation in the legislature by simultaneously announcing redistributive schedules, subject to a budget constraint.

Optimal Strategies

The following theorem characterizes the equilibrium of the redistributive competition game.

[12] In this case the focus is on the distributions within each segment (marginal distributions) rather than an n-variate joint distribution. As discussed in the appendix, an n-variate joint distribution is trivial to obtain and adds nothing to the problem analyzed here.

Theorem 1: The unique Nash equilibrium of the redistributive competition game $G\left(\left\{\left\{m_j, \mathfrak{a}_A^j\right\}_{j\in\mathcal{A}}, \left\{m_k, \mathfrak{a}_B^k\right\}_{k\in\mathcal{B}}\right\}\right)$ is for each party i to choose offers according to the following distributions. For party A

$$\begin{array}{llr} \forall\, j \in \mathcal{A} & F_A^j(x) = \frac{x}{z\left(1-\mathfrak{a}_A^j\right)} & x \in \left[0, z\left(1-\mathfrak{a}_A^j\right)\right] \\ & F_A^S(x) = \frac{x}{z} & x \in [0, z] \\ \forall\, k \in \mathcal{B} & F_A^k(x) = \mathfrak{a}_B^k + \left(1-\mathfrak{a}_B^k\right)\frac{x}{z} & x \in [0, z]\,. \end{array}$$

Similarly for party B

$$\begin{array}{llr} \forall\, k \in \mathcal{B} & F_B^k(x) = \frac{x}{z\left(1-\mathfrak{a}_B^k\right)} & x \in \left[0, z\left(1-\mathfrak{a}_B^k\right)\right] \\ & F_B^S(x) = \frac{x}{z} & x \in [0, z] \\ \forall\, j \in \mathcal{A} & F_B^j(x) = \mathfrak{a}_A^j + \left(1-\mathfrak{a}_A^j\right)\frac{x}{z} & x \in [0, z]\,, \end{array}$$

where $z = \frac{2}{1-\sigma} = \frac{2}{1-\sigma_A-\sigma_B}$. In equilibrium, party A's share of the vote is $\frac{1+\hat{\sigma}_A}{2} = \frac{1+\sigma_A-\sigma_B}{2}$ and party B's share of the vote is $\frac{1+\hat{\sigma}_B}{2} = \frac{1+\sigma_B-\sigma_A}{2}$.

Proof: We begin by showing that this is an equilibrium. First, this is a feasible strategy since:

$$\sum_{j\in\mathcal{A}\cup S\cup\mathcal{B}} m_j \int_0^\infty x dF_i^j = 1.$$

Then given that party B is following the equilibrium strategy, the vote share $\pi_A(\cdot)$ for party A, when it chooses to provide transfers according to an arbitrary strategy $\left\{\bar{F}_A^j\right\}_{j\in\mathcal{A}\cup S\cup\mathcal{B}}$ is:

$$\begin{aligned} \pi_A\left(\left\{\bar{F}_A^j, F_B^j\right\}_{j\in\mathcal{A}\cup S\cup\mathcal{B}}\right) = & \textstyle\sum_{j\in\mathcal{A}} m_j \int_0^\infty F_B^j\left(\frac{x}{1-\mathfrak{a}_A^j}\right) d\bar{F}_A^j(x) \\ & + m_S \textstyle\int_0^\infty F_B^S(x)\, d\bar{F}_A^S(x) \\ & + \textstyle\sum_{k\in\mathcal{B}} m_k \int_0^\infty F_B^k\left(x\delta_B^k\right) d\bar{F}_A^k(x)\,. \end{aligned}$$

Since it is never a best response for party A to provide offers outside the support of party B's offers, we have:

$$\begin{aligned} \pi_A\left(\left\{\bar{F}_A^j, F_B^j\right\}_{j\in\mathcal{A}\cup S\cup\mathcal{B}}\right) = & \tfrac{1}{z}\textstyle\sum_{j\in\mathcal{A}} m_j \int_0^{z\left(1-\mathfrak{a}_A^j\right)} x d\bar{F}_A^j(x) \\ & + \textstyle\sum_{j\in\mathcal{A}} m_j \mathfrak{a}_A^j + \frac{m_S}{z}\int_0^z x d\bar{F}_A^S(x) \\ & + \tfrac{1}{z}\textstyle\sum_{k\in\mathcal{B}} m_k \int_0^z x d\bar{F}_A^k(x)\,. \end{aligned}$$

But from the budget constraint given in equation (1) it follows that

$$\pi_A\left(\left\{\bar{F}_A^j, F_B^j\right\}_{j\in\mathcal{A}\cup\mathcal{S}\cup\mathcal{B}}\right) \leq \frac{1}{z}+\sum_{j\in\mathcal{A}} m_j \mathfrak{a}_A^j = \frac{1+\sigma_A-\sigma_B}{2}$$

which holds with equality if $\left\{\bar{F}_A^j\right\}_{j\in\mathcal{A}\cup\mathcal{S}\cup\mathcal{B}}$ is the equilibrium strategy. Thus party A's vote share cannot be increased by deviating to another strategy. The argument for party B is symmetric.

In the appendix, the strategic equivalence between two-party games of redistributive politics with segmented voters and a unique set of appropriately chosen independent simultaneous two-bidder all-pay auctions is established. The proof of uniqueness then follows from the arguments appearing in Hillman and Riley (1989) and Baye, Kovenock and de Vries (1996). Q.E.D.

The following example illustrates the key features of Theorem 1.

Example: Assume that there are only two types of voters: voters loyal to party A and voters loyal to party B. Let $m_A = \frac{1}{3}$, $\mathfrak{a}_A = \frac{1}{2}$, $m_B = \frac{2}{3}$, and $\mathfrak{a}_B = \frac{3}{4}$. Party A's and party B's strengths are $\sigma_A = \frac{1}{3}\left(\frac{1}{2}\right) = \frac{1}{6}$ and $\sigma_B = \frac{2}{3}\left(\frac{3}{4}\right) = \frac{1}{2}$, respectively. Party A's and party B's equilibrium vote shares are $\frac{1+\hat{\sigma}_A}{2} = \frac{1+\sigma_A-\sigma_B}{2} = \frac{1}{3}$ and $\frac{1+\hat{\sigma}_B}{2} = \frac{1+\sigma_B-\sigma_A}{2} = \frac{2}{3}$, respectively. The transfers and resulting utilities from the unique equilibrium redistribution schedules given by Theorem 1 are shown in Figure 2 below. As party $-i$'s loyal voters' intensity of attachment, $\mathfrak{a}_{-i}$, increases, party i freezes out a larger proportion of $-i$'s loyal voters with a zero transfer. This is represented graphically in Figure 2(a) and 2(b) as shift up of $F_i^{-i}(0) = \mathfrak{a}_{-i}$.

[Insert Figure 2 here]

Note that each party's equilibrium vote share is increasing (decreasing) in its own (opponent's) party strength. In fact, since our game is constant sum, equilibrium strategies are maximin strategies and, from Theorem 1, party i can guarantee itself a vote share $\frac{1+\sigma_i-\sigma_{-i}}{2}$. The effectively stronger party (in the sense made precise in D.4) guarantees itself a majority (measure) of the votes. If $\sigma_i > \sigma_{-i}$, this then identifies the winning party in the analogous game in which the objective is not to maximize vote share but to obtain a majority of the votes. In this sense our results generalize the two-candidate results in Myerson (1993). Of course, there may be many strategies which assure a party a majority. The strategies specified in Theorem 1 are those that arise if each party attempts to maximize its vote share among all those strategies that guarantee its value in the analogous majority rule game.

Party identification also creates an incentive for parties to utilize a poaching strategy which freezes out a portion of the opposition's loyal voters with a zero transfer, but gives the remaining opposition voters non-zero transfers which are higher in expectation than the opposition party's offers. A similar poaching effect has been addressed in the industrial organization literature on brand loyalty and brand switching. For example, Fudenberg and Tirole (2000)[13] examine a duopoly model of brand loyalty and brand switching where firms try to poach the competitor's loyal consumers. The electoral poaching examined here differs from Fudenberg and Tirole (2000) in that the focus is on redistribution rather than short-term versus long-term contracts.

3 Transformations of the Electorate

We now apply Theorem 1 to explore the qualitative nature of the equilibrium and present comparative statics results with respect to changes in measures of party strength, partisanship, and political polarization. We begin with the nature of the equilibrium for a given distribution of voter attachments. In redistributive competition with heterogeneous voter loyalties, each party announces a distribution of offers for each segment of the electorate. In the discussion that follows we refer to the expectation of a party's equilibrium distribution of offers over a segment as that segment's equilibrium expected transfer from the party's redistribution schedule. A segment's equilibrium expected utility from the party's redistribution schedule is similarly defined, as are both the equilibrium expected transfer and utility from the implemented policy.

Despite the fact that from Theorem 1 the parties' equilibrium redistributive schedules differ in all segments of loyal voters, for each segment, the expected transfer from each party, and thus from the implemented policy, is the same. Furthermore, for a given distribution of voters' attachments to the political parties, the expected transfers are highest for the swing segment and are strictly decreasing in the intensity of attachment to a party. Thus, voters with the

[13] See also Lee (1997).

highest intensity of attachment receive the lowest expected transfers and swing voters receive the highest expected transfers. However, the poaching strategies utilized by the parties freeze out a portion of the opposition party's loyal voters with a zero transfer, and offer the remaining portion of the opposition party's loyal voters non-zero transfers which are higher in expectation than the opposition party's transfers. For each segment, conditional on receiving a positive transfer from the opposition party the expected transfer from the opposition party is equal to the expected transfer of the swing segment.

Corollary 1: Within any given voter segment, the expected transfers from the two parties are identical. For a given distribution of voters' attachments to the political parties, the expected transfers are strictly decreasing in the intensity of attachment (regardless of party affiliation). Conditional on receiving a positive transfer from the opposition party, within each loyal voter segment the expected transfer from the opposition party is equal to that of the swing voter segment.

Proof: From Theorem 1 the swing voters equilibrium expected transfer from each party and from the implemented policy $E^S(\cdot)$ is

$$E^S\left(\left\{\left\{m_j, \mathfrak{a}_A^j\right\}_{j\in\mathcal{A}}, \left\{m_k, \mathfrak{a}_B^k\right\}_{k\in\mathcal{B}}\right\}\right) = \tfrac{1}{1-\sigma} \ .$$

Similarly, for each segment $j \in \mathcal{A}$ of party A's loyal voters the equilibrium expected transfer from each party and from the implemented policy $E^j(\cdot)$ is

$$E^j\left(\left\{\left\{m_j, \mathfrak{a}_A^j\right\}_{j\in\mathcal{A}}, \left\{m_k, \mathfrak{a}_B^k\right\}_{k\in\mathcal{B}}\right\}\right) = \tfrac{1-\mathfrak{a}_A^j}{1-\sigma} \ .$$

Conditional on receiving a positive transfer from party B, for each segment $j \in \mathcal{A}$ of party A's loyal voters the equilibrium expected transfer from party B $E_+^j(\cdot)$ is

$$E_+^j\left(\left\{\left\{m_j, \mathfrak{a}_A^j\right\}_{j\in\mathcal{A}}, \left\{m_k, \mathfrak{a}_B^k\right\}_{k\in\mathcal{B}}\right\}\right) = \tfrac{1}{1-\sigma} \ .$$

The argument for voters loyal to party B is symmetric. Q.E.D.

The swing voter segment is the most contested segment since neither party has an advantage, and, thus, the equilibrium transfers are the highest in this segment. The presence of voter loyalties to the political parties creates an incentive for the parties to target or poach a subset of the opposition party's loyal voters. However, as a segment's intensity of attachment increases it becomes more difficult for the opposition party to induce a voter in that segment to vote against their party. Thus, the proportion of a segment's loyal voters that the opposition party targets with non-zero transfers is decreasing in the intensity of attachment. As the more attached segments are targeted less by the opposition, the affiliated party optimally diverts resources away from its most attached loyal voter segments to the other segments. This result is independent of the measures of the segments and the parties' strengths.

One difference between our results on expected transfers and the analysis of the resulting utilities is that for loyal voters the expected utilities from the affiliated party's redistribution schedule are higher than the unconditional expected utilities from the opposition party's redistribution schedule. In fact, the expected utility that each segment of loyal voters receives from the affiliated party's redistribution schedule is equal to the expected utility that the swing voters receive from either party's redistribution schedule. In addition, conditional on receiving a positive transfer, the expected utility that each subset of loyal voters receives from the opposition party's redistribution schedule is also equal to the expected utility that the swing voters receive. Thus, since the proportion of a segment's loyal voters that is targeted with non-zero transfers is decreasing in the intensity of attachment, the unconditional expected utility that each segment of loyal voters' receives from the opposition party's redistribution schedule is strictly decreasing in the intensity of attachment.

Corollary 2: For all loyal voter segments, the expected utility from the affiliated party's redistribution schedule and the expected utility conditional on receiving a positive transfer from the opposition party's redistribution schedule are identical and equal to the expected utility that the swing voters receive from either party's redistribution schedule. For a given distribution of voters' attachments to the political parties, loyal voters' unconditional expected utilities from the opposition party's redistribution schedule are strictly decreasing in the intensity of attachment.

Proof: We present the argument for party A's loyal segments. The argument for party B's segments is symmetric. From Theorem 1, for each segment $j \in \mathcal{A}$ of party A's loyal voters the equilibrium expected utility from party A, and the expected utility conditional on receiving a positive transfer from party B, $EU_A^j(\cdot)$

and $EU^j_{B+}(\cdot)$ respectively, are

$$EU^j_A\left(\left\{\left\{m_j,\mathfrak{a}^j_A\right\}_{j\in\mathcal{A}},\left\{m_k,\mathfrak{a}^k_B\right\}_{k\in\mathcal{B}}\right\}\right)=$$
$$EU^j_{B+}\left(\left\{\left\{m_j,\mathfrak{a}^j_A\right\}_{j\in\mathcal{A}},\left\{m_k,\mathfrak{a}^k_B\right\}_{k\in\mathcal{B}}\right\}\right)=\tfrac{1}{1-\sigma}.$$

From Corollary 1, this is equal to the expected utility for swing voters, $EU^S = E^S$.

The second part of the corollary follows from the fact that for each segment $j \in \mathcal{A}$ of party A's loyal voters the equilibrium unconditional expected utility from the opposition party's redistribution schedule, $EU^j_B(\cdot)$, is

$$EU^j_B\left(\left\{\left\{m_j,\mathfrak{a}^j_A\right\}_{j\in\mathcal{A}},\left\{m_k,\mathfrak{a}^k_B\right\}_{k\in\mathcal{B}}\right\}\right)=E^j=\tfrac{1-\mathfrak{a}^j_A}{1-\sigma}\ .$$

Q.E.D.

In fact, the equivalence between loyal voters' utilities from the affiliated party's redistribution schedule, the targeted loyal voters' utilities from the opposition party's redistribution schedule, and the swing voters utilities from both schedules is stronger than stated. The distribution of loyal voters' utilities from the affiliated party, the distribution of targeted loyal voters' utilities from the opposition party, and the distributions of swing voters utilities from both parties are identical.

Given these static properties of the equilibrium transfers and resulting utilities we now examine comparative statics with respect to transformations of the electorate. We will focus mainly on two simple transformations of the electorate. The first, *a partisanship preserving transformation of the electorate*, reflects a change in the symmetry of the parties' strengths while holding the level of partisanship constant. The second, *an effective party-strength preserving transformation of the electorate*, reflects a change in the level of partisanship while holding the absolute difference in the parties' strengths constant. These two types of transformations are represented graphically in Figure 3. In (σ_A, σ_B) space, for a given level of partisanship, the set of partisanship preserving transformations forms a line with slope of -1, and for fixed effective party strengths, the set of effective party-strength preserving transformations forms a line with slope of $+1$.

[Insert Figure 3 here]

Another transformation that we examine is one that holds constant or fixed the intensities of the attachment to parties, while shifting the electorate across the given set of intensities. The following corollary examines how, given fixed intensities of attachment, partisanship preserving and partisanship increasing transformations of the electorate change each voter segment's expected transfers and utilities. Proof of the corollary follows directly from Corollaries 1 and 2.

> **Corollary 3:** Given $\left\{\left\{m_j, \mathfrak{a}_A^j\right\}_{j\in\mathcal{A}}, \left\{m_k, \mathfrak{a}_B^k\right\}_{k\in\mathcal{B}}\right\}$, a partisanship preserving (resp., increasing) transformation of the electorate that leaves the intensities of attachment, $\left\{\mathfrak{a}_A^j\right\}_{j\in\mathcal{A}}$, $\left\{\mathfrak{a}_B^k\right\}_{k\in\mathcal{B}}$, fixed leaves invariant (resp., increases) the expected transfer and utility received from each party's redistribution schedule by voters within a given voter segment $j \in \mathcal{A}$, $k \in \mathcal{B}$, or S.

Given fixed intensities of attachment, partisanship preserving transformations of the electorate hold constant both the measure of the set of voters that receives a zero transfer from one of the two parties and the expected utility, conditional upon receiving a positive offer, received from either party's redistribution schedule. In addition note that in party i's equilibrium redistribution schedule the proportion of party $-i$'s loyal voter segment j that receives a zero transfer is equal to segment j's intensity of attachment, $\mathfrak{a}_{-i}^j$. Thus, the measure of party $-i$'s loyal voters who receive a zero transfer from party i is equal to party $-i$'s strength, σ_{-i}. Regardless of which party gains and which party loses, in a partisanship preserving transformation of the electorate the measure of the set of voters that receive a transfer of 0 from one of the two parties remains invariant. Similarly, partisanship increasing transformations of the electorate result in an increase in the measure of the set of voters who receive a transfer of 0 from one of the two parties.

These corollaries highlight several features of the nature of equilibrium poaching. These are summarized in the following corollary.

> **Corollary 4:** The proportion of loyal voter segment j of party i that receives a transfer of 0 from the redistribution schedule of party $-i$ is $\mathfrak{a}_i^j$. Conditional upon receiving a positive transfer from party $-i$, the expected transfer and utility received by a loyal voter in segment j of party i is $\frac{1}{1-\sigma}$. The unconditional expected transfer

to a voter in segment j of party i from the redistribution schedule of party $-i$ is $\frac{1-\alpha_i^j}{1-\sigma}$. The proportion of party i's loyal voters who receive a transfer of 0 from party $-i$ is $\frac{\sigma_i}{M_i}$.

This characterization of each party's equilibrium poaching raises the question of how swing voters fare with changes in voter loyalty to political parties. Since, by Theorem 1, swing voters receive an offer distribution which is uniform over $[0, z]$, $z = \frac{2}{1-\sigma}$, the answer is immediate. The salient effects are summarized in the following corollary.

Corollary 5: The offer distribution faced by each swing voter depends on the distribution of voters' attachments to parties only through the level of partisanship, σ. The expected transfer received by each swing voter, $\frac{1}{1-\sigma}$, is strictly increasing in σ. That is, partisanship preserving (increasing) transformations of the electorate leave invariant (increase) the expected transfer of swing voters.

The characterization of equilibrium poaching also raises the question of how changes in voter loyalty to political parties affect the inequality arising from equilibrium redistribution schedules. Remarkably, the comparative statics analysis of changes in inequality in the distribution of transfers is considerably more complex than the analysis of changes in the distribution of utilities. The Lorenz curves for the distributions of transfers arising from each party's redistribution schedule are piecewise quadratic functions that depend critically on each parameter in the distribution of voters' attachments to the parties. The kinks in these curves make it difficult to obtain unambiguous comparative statics results. It turns out that comparative statics results on the inequality in utility are more straightforward. Corollary 6 addresses inequality in the distribution of utilities arising from each party's equilibrium offer distribution as measured by the Gini-coefficient of inequality.

Corollary 6: For each party $i = A, B$, the inequality (as measured by the Gini-coefficient of inequality) arising from the party's equilibrium redistribution schedule is increasing in the opposition party's strength. More precisely, the Gini-coefficient of party i's equilibrium redistribution schedule is $C_i = \frac{1}{3} + \frac{2\sigma_{-i}}{3}$, $i = A, B$.

Proof: From Theorem 1, the measure of the set of voters who receive a utility level from party A's equilibrium redistribution schedule that is less than or equal to x is

$$\tilde{F}_A(x) = \sum_{k\in\mathcal{B}} m_k \mathfrak{a}_B^k + \frac{x}{z}\left(\sum_{k\in\mathcal{B}} m_k\left(1-\mathfrak{a}_B^k\right) + \sum_{j\in\mathcal{A}\cup S} m_j\right)$$

for $x \in [0, z]$. Simplifying, $\tilde{F}_A(x) = \sigma_B + \frac{x}{z}(1-\sigma_B)$ for $x \in [0, z]$.

By definition the Lorenz curve for $\tilde{F}_A$ is

$$L_A(y) = \frac{\int_0^y \tilde{F}_A^{-1}(x)dx}{\int_0^1 \tilde{F}_A^{-1}(x)dx}, \quad y \in [0,1],$$

which is equivalent to

$$L_A(y) = \begin{cases} 0 & if\ y \in [0, \sigma_B] \\ \frac{(y-\sigma_B)^2}{(1-\sigma_B)^2} & if\ y \in (\sigma_B, 1] \end{cases}.$$

By definition, the Gini-coefficient for $\tilde{F}_A$ is

$$C_A\left(\left\{\left\{m_j, \delta_A^j\right\}_{j\in\mathcal{A}}, \left\{m_k, \delta_B^k\right\}_{k\in\mathcal{B}}\right\}\right) = 1 - 2\int_{\sigma_B}^1 L_A(x)\,dx.$$

Simplifying we have $C_A = \frac{1}{3} + \frac{2\sigma_B}{3}$. It follows that $\frac{\partial C_A}{\partial \sigma_B} > 0$. A similar argument establishes the property for party B's equilibrium redistribution schedule. Q.E.D.

Party i has an incentive to target a different proportion of the voters from each of party $-i$'s loyal segments. As the intensity of attachment of a given segment of $-i$'s voters increases, the proportion of that segment that receives a transfer of 0 increases. As a result, the aggregate inequality in party i's equilibrium redistribution schedule increases.

More generally, as Corollary 6 states, any change in the distribution of voters' attachments to the political parties that leads to an increase in the strength of party $-i$, results in an increase in the aggregate inequality of party i's equilibrium redistribution schedule. Moreover, freezing out by party i increases in the sense that the measure of party $-i$'s loyal voters that receive a transfer of 0 from party i increases.

Given the assumption that the legislature implements a probabilistic compromise of the parties' equilibrium redistribution schedules, we can also examine the expected utilities and the expected ex-post inequality of utilities from the implemented policy. To measure changes in the expected utility from the implemented policy, we must take into account changes both in the level of partisanship and in the parties' effective strengths. In particular, for fixed intensities of attachment, partisanship preserving transformations of the electorate that increase the strength of party i increase party i's loyal voters' expected utilities from the implemented policy and decrease party $-i$'s loyal voters' expected utilities from the implemented policy. Conversely, effective party-strength preserving transformations of the electorate that increase the level of partisanship increase all voters' expected utilities.

Corollary 7: Given fixed intensities of attachment to the parties, partisanship preserving transformations of the electorate that increase the strength of party i increase party i's loyal voters' expected utilities and decrease party $-i$'s loyal voters' expected utilities from the implemented policy. In addition, effective party-strength preserving transformations of the electorate that increase the level of partisanship increase all voters' expected utilities from the implemented policy.

Proof: From Theorem 1, for each segment $j \in \mathcal{A}$ of party A's loyal voters the equilibrium expected transfer from the policy implemented by the legislature $EU^j(\cdot)$ is

$$EU^j\left(\left\{\left\{m_j, \mathfrak{a}_A^j\right\}_{j\in\mathcal{A}}, \left\{m_k, \mathfrak{a}_B^k\right\}_{k\in\mathcal{B}}\right\}\right) = \left(\tfrac{1+\hat{\sigma}_A}{2}\right)\left(\tfrac{1}{1-\sigma}\right) + \left(\tfrac{1-\hat{\sigma}_A}{2}\right)\left(\tfrac{1-\mathfrak{a}_A^j}{1-\sigma}\right)$$

which is increasing in $\hat{\sigma}_A$ and thus decreasing in $\hat{\sigma}_B = -\hat{\sigma}_A$. The argument for voters loyal to party B is symmetric.

The second part of the corollary follows from the fact that for each segment $j \in \mathcal{A}$

$$\frac{\partial EU^j}{\partial \sigma} > 0.$$

The argument for swing voters and voters loyal to party B follows directly. Q.E.D.

The implications of these results for the expected ex-post inequality of utilities from the implemented policy are examined in the following corollary. We use the expected Gini-coefficient to measure expected ex-post inequality and refer to the expected Gini-coefficient as the "*aggregate inequality.*"

Corollary 8: Partisanship preserving transformations of the electorate that increase the symmetry in the parties' strengths increase the aggregate inequality of the implemented policy. Moreover, for a given level of partisanship, σ, the aggregate inequality arising from the implemented policy is maximized when the parties are of equal strength, $\sigma_A = \sigma_B$. Conversely, effective party-strength preserving transformations of the electorate that increase the level of partisanship increase the aggregate inequality of the implemented policy.

Proof: From Corollary 5, the aggregate inequality arising from the implemented policy is

$$I(\sigma_A, \sigma_B) = \left(\tfrac{1+\hat{\sigma}_A}{2}\right)\left(\tfrac{1+2\sigma_B}{3}\right) + \left(\tfrac{1-\hat{\sigma}_A}{2}\right)\left(\tfrac{1+2\sigma_A}{3}\right) .$$

Simplifying we have $I(\sigma_A, \sigma_B) = \frac{1}{3} + \frac{\sigma - (\hat{\sigma}_A)^2}{3}$.

The first and third parts of the corollary follow directly. The second part follows from the fact that for a given level of partisanship, σ, $I(\sigma_A, \sigma_B)$ is maximized when $\hat{\sigma}_A = 0$, or $\sigma_A = \sigma_B = \frac{\sigma}{2}$. Q.E.D.

Hence, for a given level of partisanship symmetry in party strength generates inequality. Similarly, for given effective party-strengths, partisanship generates inequality.

Our results on party strength and inequality are closely related to issues arising in the literature on polarization.[14] Although much of this literature deals with the distribution of income, its tenets can be adapted to our context of redistributive politics. An interesting question that arises is whether there is a simple measure, of "political polarization," defined over the primitives of the model, with the property that the aggregate inequality from the implemented policy is increasing in the measure. It turns out that the answer is yes. Indeed, we base this measure solely on the party strengths. Setting

$$P(\sigma_A, \sigma_B) \equiv \sigma - (\hat{\sigma}_A)^2 = \sigma - (\hat{\sigma}_B)^2$$

it is easily demonstrated that the aggregate inequality in utilities arising from the implemented policy is increasing in $P(\cdot, \cdot)$.

Corollary 9: The aggregate inequality in utilities arising from the implemented policy is increasing in the measure of political polarization $P(\sigma_A, \sigma_B)$.

[14] See for example: Esteban and Ray (1994), Wolfson (1994), Wang and Tsui (2000), and Rodriguez and Salas (2003).

The level curves of the political polarization measure and the aggregate inequality of utilities from the implemented policy are shown in Figure 4 below.

[Insert Figure 4 here]

Several properties of these level curves should be mentioned. First a level of partisanship defines a 'budget' line over possible combinations of party strengths. Thus, the properties of aggregate inequality from the implemented policy addressed in Corollary 8 can be seen graphically in Figure 4. Second, given that the parties have symmetric strengths, an increase in either party's strength increases polarization and thus aggregate inequality. That is

$$\frac{\partial I}{\partial \sigma_i}|_{\hat{\sigma}_A=0} > 0.$$

Furthermore, for $\sigma_i < \frac{1}{2} + \sigma_{-i}$, a small increase in party i's strength increases the aggregate inequality arising from the implemented policy. That is

$$\frac{\partial I}{\partial \sigma_i}|_{\sigma_i<\frac{1}{2}+\sigma_{-i}} > 0.$$

For $\sigma_i > \frac{1}{2} + \sigma_{-i}$ the effect on aggregate inequality is reversed. That is, increasing the strength of a party that already had a sufficiently large effective strength decreases aggregate inequality.

Our results on inequality and political polarization are closely related to the incentive, created by party identification, to freeze out a portion of the opposition party's loyal voters with a zero transfer. Freezing out opposition voters is also closely related to the classic issue of the tyranny of the majority, in which a majority of voters expropriates from a minority. In Laslier (2002), a minority is frozen out only if a single voter segment contains a majority of voters. In contrast, in our model the expected measure of the set of voters frozen out by the implemented policy depends on the parties' strengths and is increasing in the level of political polarization. That is tyranny is increasing in polarization.

Corollary 10: The expected measure of the set of voters that receive a zero transfer from the implemented policy is $\frac{P(\cdot,\cdot)}{2}$. That is tyranny is increasing in the political polarization measure $P(\cdot,\cdot)$.

Proof: From Theorem 1, for each segment $j \in \mathcal{A}$ of party A's loyal voters the probability of receiving a zero transfer from the implemented policy is $\frac{1-\hat{\sigma}_A}{2}\left(\mathfrak{a}_A^j\right)$. Similarly for each segment $k \in \mathcal{B}$ of party B's loyal voters the probability of receiving a zero transfer from the implemented policy is $\frac{1+\hat{\sigma}_A}{2}\left(\mathfrak{a}_B^k\right)$. The result follows directly. Q.E.D.

4 Conclusion

This paper extends Myerson's (1993) model of redistributive politics to allow for heterogeneous voter loyalties to political parties. Parties segment voters by the party with which they identify, if any, and the intensity of their attachment, or "loyalty," to that party. We find that voters pay a price for party loyalty. For a given distribution of voters' attachments to the political parties, in the implemented policy, the segment of swing voters has the highest expected transfer and expected utility, and the expected transfers and utilities for loyal voter segments are strictly decreasing in the intensity of attachment. Using our measure of "party strength," based on both the sizes and intensities of attachment of a party's loyal voter segments, we demonstrate that each party's representation in the legislature is increasing (decreasing) in its own (opponent's) party strength. In addition, parties poach a subset of the opposition party's loyal voters, in an effort to induce those voters to vote against the opposition party. The level of inequality in and the size of the set of opposition party voters frozen out by a party's equilibrium redistribution schedule are increasing in the opposition party's strength.

We also develop a measure of "political polarization" that is increasing in the sum and symmetry of the party strengths, and find that aggregate inequality is increasing in political polarization. That is, higher levels of partisanship and more symmetry in the parties' strengths generate inequality. In addition, the expected measure of the set of voters that receive a zero transfer (and, hence, their secure utility level) from the implemented policy is increasing in the level of political polarization. In this sense polarization increases tyranny.

There are several potential directions for future research based on our model that appear to be particularly fruitful. The model can be applied to shed light on topics previously studied in the redistributive politics literature, such as candidate valence issues. In addition, this paper's focus on identifiable voter segments is immediately applicable to the study of transfers targeted by geographical region or other identifiable characteristics.

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Appendix

Sahuguet and Persico (2006) establish an equivalence between the two-party model of redistributive politics and an appropriately chosen two-bidder all-pay auction. We now extend this result to establish an equivalence between the two-party model of redistributive politics with segmented loyal voters and an appropriately chosen set of two-bidder independent simultaneous all-pay auctions.

We begin by reviewing the characterization of n simultaneous two-bidder all-pay auctions with complete information. Let F_i^j represent bidder i's distribution of bids for auction j, and v_i^j represent the value of auction j for bidder i. Each bidder i's problem is

$$\max_{\{F_i^j\}_{j=1}^n} \sum_{j=1}^{n} \int_0^\infty \left[v_i^j F_{-i}^j (x) - x \right] dF_i^j .$$

Since each auction is independent, the unique equilibrium is for each bidder to choose F_i^j as if auction j was the only auction. The case of a single all-pay auction with complete information is studied by Hillman and Riley (1989) and Baye, Kovenock, de Vries (1996). Thus, for each auction j and bidder i we have the following three cases:

$$\begin{array}{lll} if\ v_i^j > v_{-i}^j & F_i^j (x) = \frac{x}{v_{-i}^j} & x \in \left[0, v_{-i}^j\right], \\ if\ v_i^j = v_{-i}^j & F_i^j (x) = \frac{x}{v_i^j} & x \in \left[0, v_i^j\right], \\ if\ v_i^j < v_{-i}^j & F_i^j (x) = \left(\frac{v_{-i}^j - v_i^j}{v_{-i}^j}\right) + \frac{x}{v_{-i}^j} & x \in \left[0, v_i^j\right]. \end{array}$$

In addition, without a binding cap on bids, there is no reason to construct an n-variate distribution function from these marginal distributions.[15]

Now consider two-party redistributive competition with segmented loyal voters, and assume that the parties face the budget constraint

$$\sum_{j \in \mathcal{A} \cup \mathcal{S} \cup \mathcal{B}} m_j \int_0^\infty x dF_i^j \leq 1,$$

[15] Without a binding cap on bids, it is trivial to construct an n-variate distribution since any n-variate copula is sufficient. Given the Fréchet-Hoeffding bounds for n-variate copulas, the range of sufficient n-variate copulas is quite large. For this reason the n-variate joint distribution adds nothing to the problem analyzed here. See Nelson (1999) for an introduction to copulas.

where F_i^j represents party i's distribution of offers for voters in segment j and $m_j > 0$ is the measure of voters in segment j such that $\sum_{j\in\mathcal{A}\cup S\cup\mathcal{B}} m_j = 1$. In the discussion that follows the notation for the intensity of loyal voter attachment is modified in the following way: for each segment $j \in \mathcal{A}\cup S\cup\mathcal{B}$ if $j = S$, the swing segment, or $j \in \mathcal{B}$, one of party B's loyal voter segments, then $\mathfrak{a}_A^j = 0$, thus $\delta_A^j = 1$, and the same holds for $\mathfrak{a}_B^k$ if $k \in \mathcal{A}\cup S$. Each party i's problem is

$$\max_{\{F_i^j\}_{j\in\mathcal{A}\cup S\cup\mathcal{B}}} \sum_{j\in\mathcal{A}\cup S\cup\mathcal{B}} m_j \int_0^\infty F_{-i}^j\left(\frac{x\delta_{-i}^j}{\delta_i^j}\right) dF_i^j$$

subject to the budget constraint $\sum_{j\in\mathcal{A}\cup S\cup\mathcal{B}} m_j \int_0^\infty x dF_i^j(x) \le 1$. The associated Lagrangian is

$$\max_{\{F_i^j\}_{j\in\mathcal{A}\cup S\cup\mathcal{B}}} \sum_{j\in\mathcal{A}\cup S\cup\mathcal{B}} \left[m_j\lambda_i \int_0^\infty \left[\frac{1}{\lambda_i}F_{-i}^j\left(\frac{x\delta_{-i}^j}{\delta_i^j}\right) - x\right] dF_i^j(x)\right] + \lambda_i.$$

We can now proceed to the proof of the equivalence between the two-party model of redistributive politics with segmented loyal voters and an appropriately chosen set of two-bidder independent simultaneous all-pay auctions. In the discussion that follows, $\bar{s}_i^j$ and $\underline{s}_i^j$ are the upper and lower bounds of candidate i's distribution of offers in segment j.

Theorem 2: For each feasible distribution of voters' attachments to the political parties, there exists a one-to-one correspondence between the equilibria of the two-party model of redistributive politics with segmented loyal voters and the equilibria of a unique set of appropriately chosen two-bidder independent simultaneous all-pay auctions.

Proof: The proof, which is contained in the following lemmas, is instructive in that it establishes the uniqueness of the equilibrium given in Theorem 1.

The first three lemmas follow from lines drawn by Baye, Kovenock, and de Vries (1996).

Lemma 1: For each $j \in \mathcal{A}\cup S\cup\mathcal{B}$, $\frac{\bar{s}_{-i}^j\delta_i^j}{\delta_{-i}^j} = \bar{s}_i^j$.

Lemma 2: In any equilibrium $\left\{F_i^j, F_{-i}^j\right\}_{j\in\mathcal{A}\cup S\cup\mathcal{B}}$, no F_i^j can place an atom in the half open interval $\left(0, \bar{s}_i^j\right]$.

Lemma 3: For each $j \in \mathcal{A}\cup S\cup\mathcal{B}$ and for each $i \in \{A, B\}$, $\frac{1}{\lambda_i}F_{-i}^j\left(\frac{x\delta_{-i}^j}{\delta_i^j}\right) - x$ is constant $\forall\ x \in \left(0, \bar{s}_i^j\right]$.

The following lemma characterizes the relationship between λ_i and λ_{-i}.

Lemma 4: In equilibrium $\lambda_i = \lambda_{-i}$.

Proof: By way of contradiction suppose $\lambda_i \neq \lambda_{-i}$. In any equilibrium each party must use their entire budget, thus

$$\sum_{j \in \mathcal{A} \cup \mathcal{S} \cup \mathcal{B}} m_j \int_0^{\bar{s}_i^j} x dF_i^j(x) = \sum_{j \in \mathcal{A} \cup \mathcal{S} \cup \mathcal{B}} m_j \int_0^{\bar{s}_{-i}^j} x dF_{-i}^j(x). \tag{2}$$

But, from lemmas 2 and 3, it follows that

$$dF_i^j(x) = \lambda_{-i} \frac{\delta_{-i}^j}{\delta_i^j} dx \tag{3}$$

for all $x \in \left(0, \bar{s}_i^j\right]$, and

$$dF_{-i}^j(x) = \lambda_i \frac{\delta_i^j}{\delta_{-i}^j} dx \tag{4}$$

for all $x \in \left(0, \bar{s}_{-i}^j\right]$. Substituting equations 3 and 4 into equation 2, and applying lemma 1 we have

$$\lambda_{-i} \sum_{j \in \mathcal{A} \cup \mathcal{S} \cup \mathcal{B}} m_j \int_0^{\frac{\bar{s}_{-i}^j \delta_i^j}{\delta_{-i}^j}} x \frac{\delta_{-i}^j}{\delta_i^j} dx = \lambda_i \sum_{j \in \mathcal{A} \cup \mathcal{S} \cup \mathcal{B}} m_j \int_0^{\bar{s}_{-i}^j} x \frac{\delta_i^j}{\delta_{-i}^j} dx$$

which is a contradiction since

$$\sum_{j \in \mathcal{A} \cup \mathcal{S} \cup \mathcal{B}} m_j \int_0^{\frac{\bar{s}_{-i}^j \delta_i^j}{\delta_{-i}^j}} x \frac{\delta_{-i}^j}{\delta_i^j} dx = \sum_{j \in \mathcal{A} \cup \mathcal{S} \cup \mathcal{B}} m_j \int_0^{\bar{s}_{-i}^j} x \frac{\delta_i^j}{\delta_{-i}^j} dx$$

but $\lambda_i \neq \lambda_{-i}$. Q.E.D.

Let $\lambda \equiv \lambda_i = \lambda_{-i}$. The following lemma establishes the value of $\bar{s}_i^j$.

Lemma 5: $\bar{s}_i^j = \frac{\delta_i^j}{\lambda}$ $\forall$ i and j.

Proof: From lemmas 3 and 4, we know that for each party i and any segment j

$$\frac{1}{\lambda} F_{-i}^j \left(\frac{x \delta_{-i}^j}{\delta_i^j} \right) - x$$

is constant $\forall$ $x \in \left(0, \bar{s}_i^j\right]$. It then follows that party i would never use a strategy that provides offers in $\left(\frac{1}{\lambda}, \infty\right)$ since an offer of zero strictly dominates such a strategy. The result follows directly. Q.E.D.

The following lemma establishes that there exists a unique λ that satisfies the budget constraint.

Lemma 6: There exists a unique value for λ, and this value is $\frac{1-\sum_{j\in\mathcal{A}} m_j\left(1-\delta_A^j\right)-\sum_{k\in\mathcal{B}} m_k\left(1-\delta_B^k\right)}{2} = \frac{1-\sigma_A-\sigma_B}{2}$.

Proof: The budget constraint determines the unique value of λ. Thus, λ solves

$$\lambda \sum_{j\in\mathcal{A}\cup\mathcal{S}\cup\mathcal{B}} m_j \int_0^{\frac{\delta_i^j}{\lambda}} x \frac{\delta_{-i}^j}{\delta_i^j} dx = 1.$$

Solving for λ we have that

$$\lambda = \frac{1+\sum_{j\in\mathcal{A}} m_j\left(\delta_A^j-1\right)+\sum_{k\in\mathcal{B}} m_k\left(\delta_B^k-1\right)}{2} = \frac{1-\sigma_A-\sigma_B}{2}.$$

Q.E.D.

This completes the proof of Theorem 2.

To construct the unique Nash equilibrium of the redistributive politics game, note that the intensity of attachment parameters, $\mathfrak{a}_i^j = 1-\delta_i^j$, are isomorphic to differences in bidders' valuations in an all-pay auction. Thus, in each segment of voters loyal to party $-i$, party i places mass equal to $\frac{\frac{1}{\lambda}-\frac{1}{\lambda}\delta_{-i}}{\frac{1}{\lambda}} = 1-\delta_{-i}$ at 0. Then letting $z \equiv \frac{1}{\lambda}$, the unique equilibrium is for party A to offer redistribution according to

$$\begin{array}{lll} \forall\, j\in\mathcal{A} & F_A^j(x) = \frac{x}{z\delta_A^j} & x\in\left[0, z\delta_A^j\right] \\ & F_A^S(x) = \frac{x}{z} & x\in[0,z] \\ \forall\, k\in\mathcal{B} & F_A^k(x) = \left(1-\delta_B^k\right)+\frac{\delta_B^k x}{z} & x\in[0,z] \end{array}$$

and for party B to offer redistribution according to

$$\begin{array}{lll} \forall\, k\in\mathcal{B} & F_B^k(x) = \frac{x}{z\delta_B^k} & x\in\left[0, z\delta_B^k\right] \\ & F_B^S(x) = \frac{x}{z} & x\in[0,z] \\ \forall\, j\in\mathcal{A} & F_B^j(x) = \left(1-\delta_A^j\right)+\frac{\delta_A^j x}{z} & x\in[0,z] \end{array}$$

where $z = \frac{1}{\lambda} = \frac{2}{1-\sigma_A-\sigma_B}$.

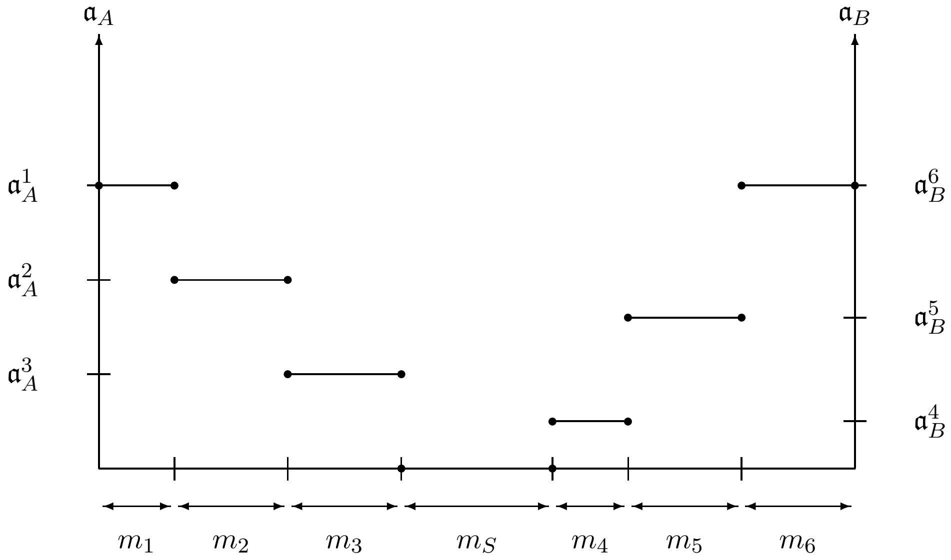
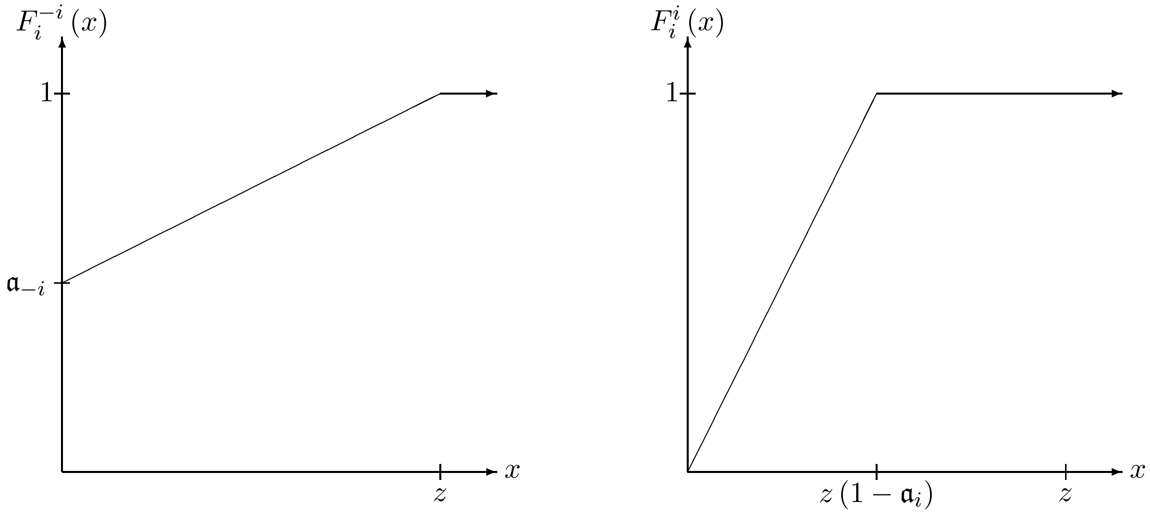


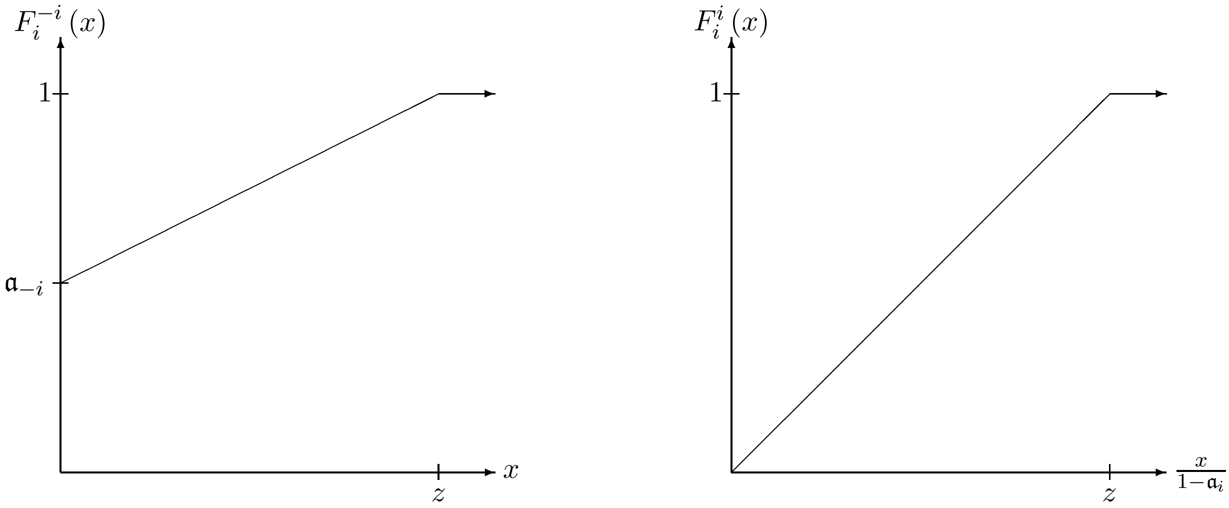
Figure 1: A distribution of voters' attachments to the political parties where segments 1, 2, and 3 are loyal to party A and segments 4, 5, and 6 are loyal to party B.



Party -i's Loyal Voters

Party i's Loyal Voters

Figure 2(a): Transfers from party i's equilibrium redistribution schedule



Party -i's Loyal Voters

Party i's Loyal Voters

Figure 2(b): Utilities from party i's equilibrium redistribution schedule

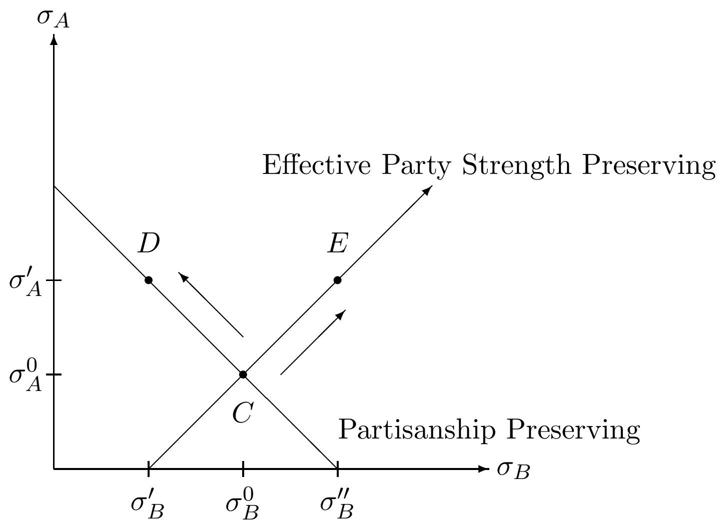


Figure 3: A transformation of the electorate that changes party strengths from $C = \left(\sigma^0_B, \sigma^0_A\right)$ to $D = (\sigma'_B, \sigma'_A)$ is a partisanship preserving transformation. A change from C to $E = (\sigma''_B, \sigma'_A)$ is an effective party-strength preserving transformation.

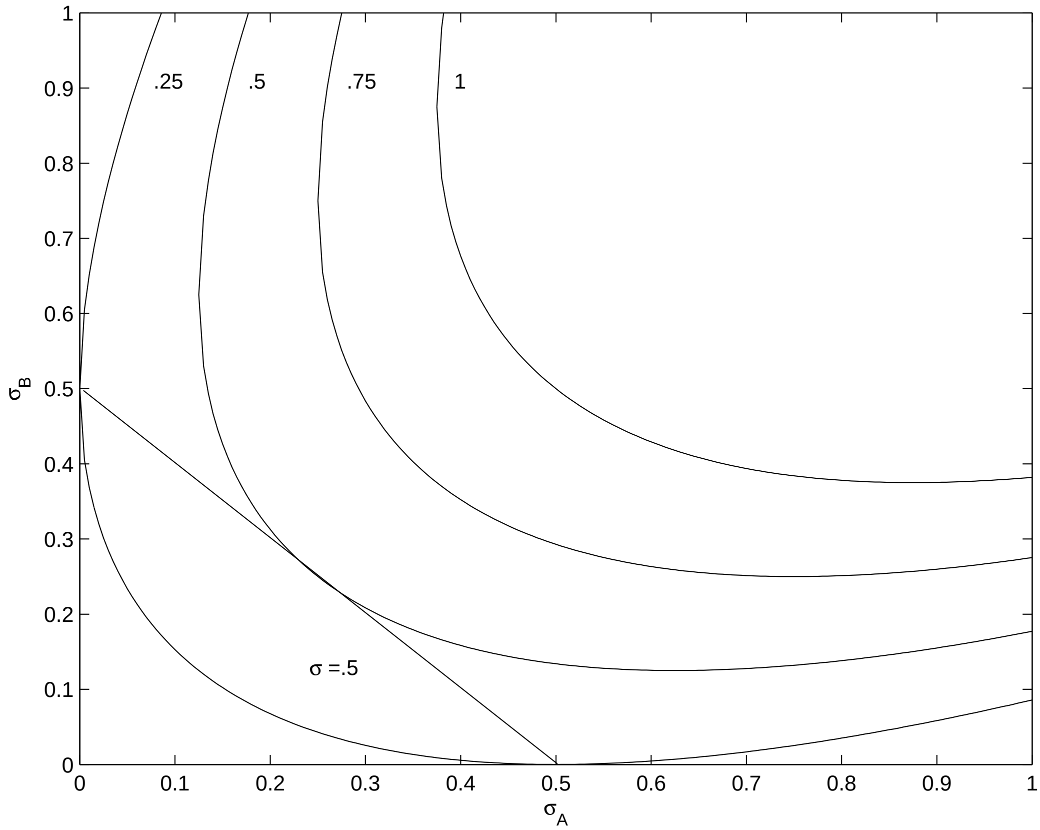


Figure 4: Level curves of political polarization